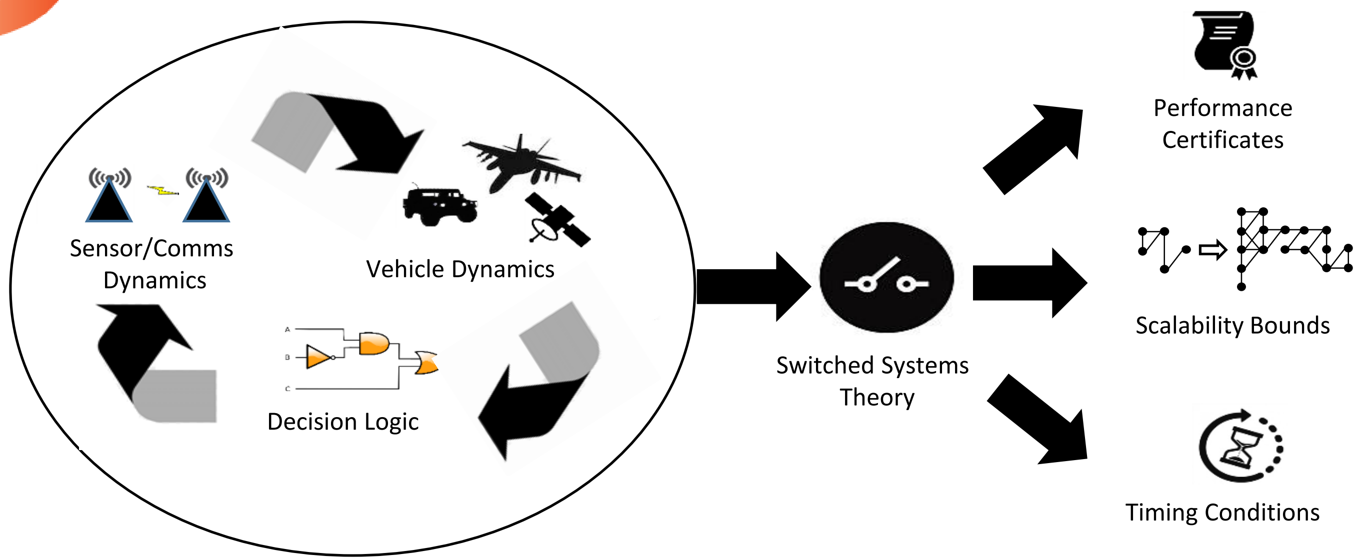
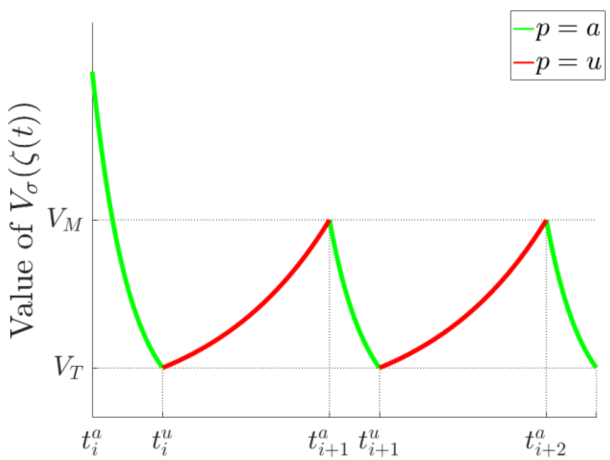
Background





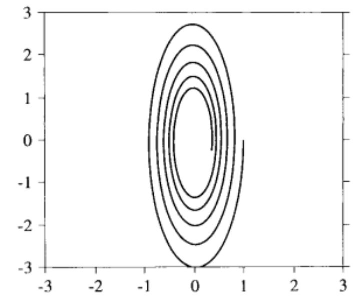
Representative illustration of a switched Lyapunov function

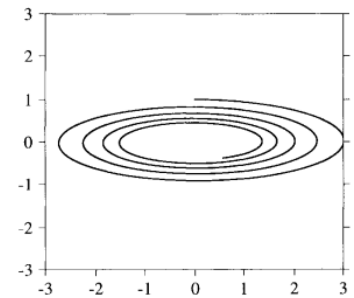
- Require stabilizing conditions
- User-defined thresholds for the error systems
- Dwell-time conditions
  - Ensure boundedness of the switched Lyapunov-like function
  - Prediction between feedback events

Background

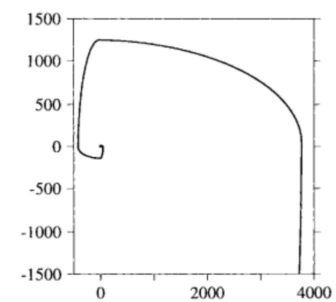
- Switched systems theory provides a framework for analyzing the stability and performance of systems of the form

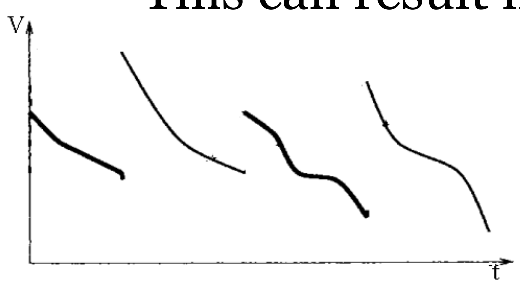
$$\dot{x} = f_p(x, t), \ \ \forall p \in \mathscr{P}$$





- Switched systems can become unstable even if every subsystem is stable
- Typical Lyapunov analysis is performed in two parts
  - Each subsystem is shown to be stable
  - Switching is shown to be non-destabilizing
  - This can result in dwell time conditions





Source: M.S. Branicky, “Multiple Lyapunov functions and other analysis tools for switched and hybrid systems”, IEEE Trans. Autom. Control, vol. 10, no. 4, pp.475-482, 1998

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Arbitrary Switching

The objective is to establish asymptotic properties of the generalized solutions of the switched system using asymptotic properties of the generalized solutions of the subsystems.

To facilitate the subsequent development, consider the dynamic system

$$\dot{x}(t) = f_{\rho(x(t),t)}(x(t), t) \qquad x(t) \triangleq \left[\ e^T(t), \quad \tilde{\theta}^T(t)\ \right]^T$$

where $\rho(x(t), t) : \mathbb{R}^n \times \mathbb{R}_{\geq t_0} \rightarrow \mathcal{N}$ denotes a piecewise continuous switching signal, the collection $\{f_\sigma : \mathbb{R}^n \times \mathbb{R}_{\geq t_0} \rightarrow \mathbb{R}^n\}_{\sigma \in \mathcal{N}}$ is assumed to be locally bounded, uniformly in σ and t, and the functions $f_\sigma(x(t), t)$ and $\rho(x(t), t)$ are assumed to be Lebesgue measurable $\forall \in x(t) \in \mathbb{R}^n$ and $\sigma \in \mathcal{N}$

Stable adaptive estimators (or asymptotic observers) can be designed individually to yield the following closed-loop subsystems

$$\dot{x}(t) = f_\sigma(x(t), t)$$

Arbitrary Switching

Generalized solutions via regularizations:

Filippov regularization $K_f[g](x,t) \triangleq \bigcap_{\delta>0} \bigcap_{\mu(N)=0} \overline{\mathrm{co}}\{g(y,t) \mid y \in B(x,\delta) \setminus N\}$

Krasovskii regularization $K_k[g](x,t) \triangleq \bigcap_{\delta>0} \overline{\mathrm{co}}\{g(y,t) \mid y \in B(x,\delta)\}$

Consider a differential inclusion of the form

$$\dot{x} \in F(x,t)$$

Set valued map

Focus on Lyapunov-based analysis of maximal solutions of set-valued maps that admit local solutions. Define generalized time derivative of a locally Lipschitz-continuous function

$$\dot{\bar{V}}_F(x,t) := \max_{p \in \partial V(x,t)} \max_{q \in F(x,t)} p^T [q; 1]$$

but the max-max definition is very conservative (Paden, Bacciotti)

- on-going work (AFOSR) to reduce conservativeness

Arbitrary Switching

Generalization of the LaSalle-Yoshizawa Thm. for switched systems (arbitrary switching).

Theorem (Thm 2). Let r > 0 be selected such that $\overline{\mathbf{B}}(0,r) \subset \mathcal{D}$ and let $\Omega \coloneqq \mathcal{D} \times \mathbb{R}_{\geq t_0}$. If the (Filippov) Krasovskii regularizations of the subsystems $f_\sigma\,(x,t)$ admit a common non-strict Lyapunov function $V\colon \Omega \to \mathbb{R}$, then every solution of the (Filippov) Krasovskii regularization of the switched system $f\,(x,t)$ such that $x(t_0) \in \{x \in \mathbf{B}(0,r) | \overline{W}(x) \leq c\}$, where $c \in (0, \min_{\|x\|_2=r} \underline{W})$, is complete, bounded, and satisfies $\lim_{t\to\infty} W\,(x\,(t)) = 0$

In addition, if $\mathcal{D} \in \mathbb{R}^n$ and if the sets $\{\, x \in \mathbb{R}^n | \underline{W}(x) \leq\ c\}$ are compact, $\forall\, c \in \mathbb{R}_{>0}$, then the result is global.

- Also generalized the result for switched differential inclusions (Thm 3).

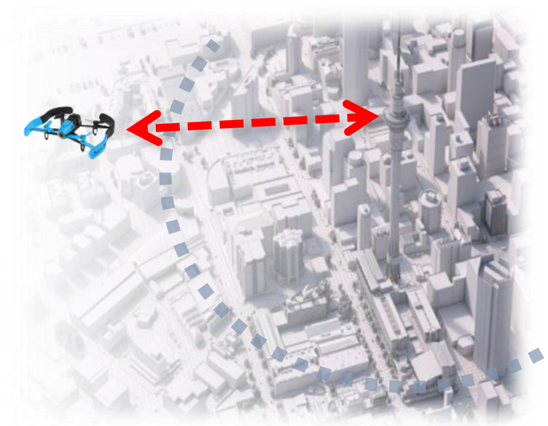
R. Kamalapurkar, J. A. Rosenfeld, A. Parikh, A. R. Teel, W. E. Dixon, "**Invariance-like results for Nonautonomous Switched Systems,**" ***IEEE TAC*, to appear.**

Open Challenges

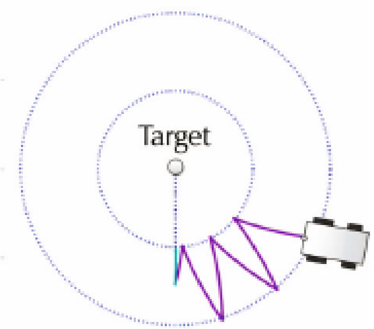
- Need for additional analysis tools
  - Example: ISS common method for robustness to disturbances
  - Some generalizations have been developed for hybrid systems, for strict Lyapunov functions (not the case for adaptive systems), but we seek arbitrary switching ISS conditions for nonstrict systems
  - Dwell-time concept for finite time convergent systems or asymptotically convergent systems? Is there a more general concept than a dwell time condition that accounts for spatial (only) and temporal changes in the Lyapunov-function
  - Ways to abstract time such as average dwell time condition for switching with unstable subsystems?
  - Dwell-time conditions with dynamics (e.g., stochastic learning for less conservative results)?

Intermittent Feedback

- Causes of temporary feedback loss
  - Task definition
    - Communication restricted operations
  - Operating environment
    - Intermittent occlusions of sensor signals
    - GPS denied regions/A2D2
  - Sensor modality
    - Limited camera field-of-view
  - Cyber effects/Adversarial Conditions



- Drawbacks of ensuring uninterrupted feedback
  - Limited operational range
  - Inefficient trajectories
  - Not feasible in confined environments



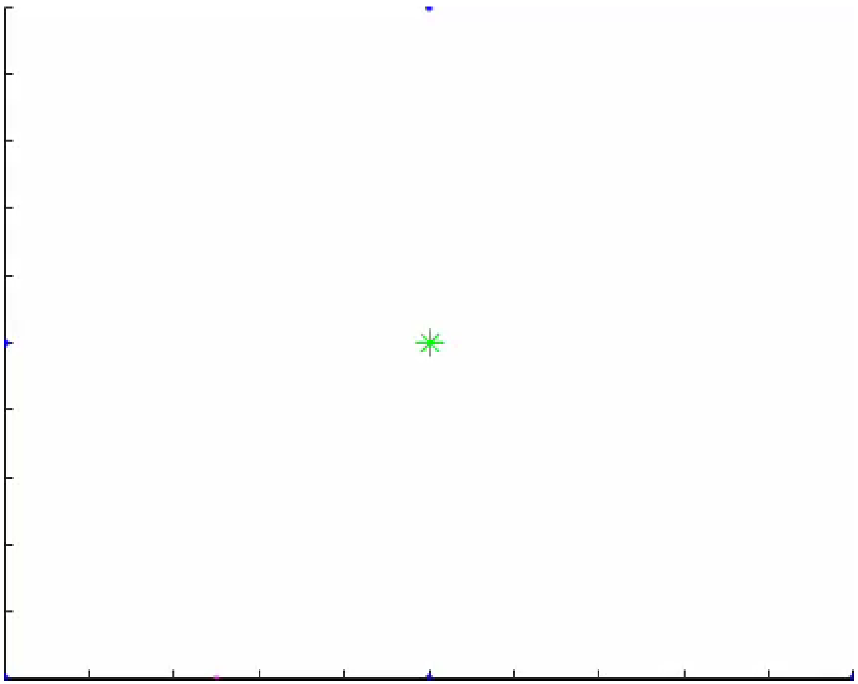
- **Motivated to account for intermittent feedback losses.**

Sensorless Example

Objective:

Regulate a set of followers to a desired location using a single leader

- Follower = mobile agent, no position sensors
- Leader = mobile agent, position sensors



Solution:

- Compute dwell-time
    - Find follower with shortest dwell-time
    - Update state, reset predictor error, recompute dwell-time
- Repeat

Sensorless Example

- Leader Dynamics

$$\dot{p}_L(t) = u_L(t)$$
$$p_L : [t_0, \infty) \to \mathbb{R}^3$$
$$u_L : [t_0, \infty) \to \mathbb{R}^3$$

- Follower Dynamics

$$\dot{p}_i(t) = f_i(p_i(t), t) + u_i(t) + d_i(t)$$
$$p_i : [t_0, \infty) \to \mathbb{R}^3$$
$$u_i : [t_0, \infty) \to \mathbb{R}^3$$

- Predictor Dynamics

$$\dot{\hat{p}}_i(t) = f_i(\hat{p}_i(t), t) + u_i(t)$$

p_L : Leader position (state)
u_L : Leader control
p_i : Follower *i* position (state)
u_i : Follower *i* control
f_i : Follower *i* drift dynamics
d_i : Follower *i* disturbance
F : Set of followers
$\hat{p}_i$: Follower *i* predicted position

$$f_i : \mathbb{R}^3 \times [t_0, \infty) \to \mathbb{R}^3$$
$$d_i : [t_0, \infty) \to \mathbb{R}^3$$
$$F \triangleq \{1, 2, \ldots, N\}$$

Assumptions

- Initial follower positions are known
- Leader has full state knowledge
- Drift dynamics are known, locally Lipschitz
- Exogenous disturbance is bounded

Sensorless Example

- Error Systems

$e_{1,i}(t) = \hat{p}_i(t) - p_i(t)$ Only measureable when leader and follower are together

$e_{2,i}(t) = p_d - \hat{p}_i(t)$ Always measurable by agents and leader

$e_{3,i}(t) = \hat{p}_i(t) - p_L(t)$ Prediction error, always available to leader

- Controllers

$$u_i(t) = -f_i(\hat{p}_i(t), t) + k_{2,i} e_{2,i}(t)$$

$$u_L(t) = k_{2,i} e_{2,i}(t) + k_{3,n} e_{3,1}(t)$$

- Reset Map

$$\psi : \hat{p}_i\left(t_{n,i}^S\right) \mapsto p_i\left(t_{n,i}^S\right)$$

- Closed-loop Error Systems

$\dot{e}_{1,i}(t) = f_i(\hat{p}_i(t), t) - f_i(p_i(t), t) - d_i(t)$ Potentially unstable

$\dot{e}_{2,i}(t) = -k_{2,i} e_{2,i}(t)$ Exponentially stable

$\dot{e}_{3,i}(t) = -k_{3,n} e_{3,1}(t)$ Exponentially stable

Sensorless Example

The controllers given by $u_i(t) = -f_i(\hat{p}_i(t), t) + k_{2,i} e_{2,i}(t),$

and state estimate update laws given $\dot{\hat{p}}_i(t) = f_i(\hat{p}_i(t), t) + u_i(t),$

and a piece-wise continuous signal $\sigma: [t_0, \infty) \to \{S, U\}$ ensures the regulation of the follower agents to p_d is GUUB to a ball of radius $\sqrt{2V_T} \in \mathbb{R}_+$ provided the switching signal satisfies the maximum unregulated dwell-time condition

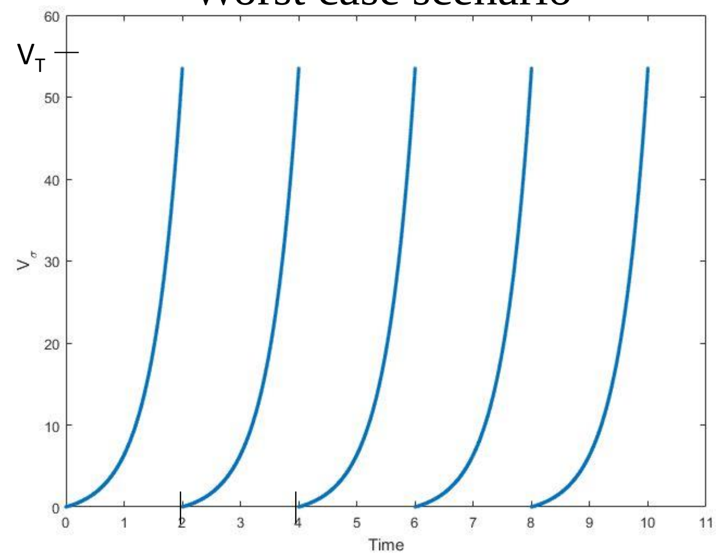
$$\Delta t_{n,i}^U < \frac{1}{\widetilde{\lambda}_i^U} \ln\left(\frac{V_T - \widetilde{\beta}_i}{\widetilde{\beta}_i}\right),$$

where

$$\widetilde{\beta}_i = \frac{1}{2}\left(\frac{\bar{d}_i}{L_i}\right)^2, \quad \widetilde{\lambda}_i^U = 2L_i$$

and $V_T \in \mathbb{R}_+$ is a user defined constant, $\bar{d}_i$ is the positive constant bound on the disturbance, and L_i is the Lipschitz constant for f_i.

Worst case scenario



$t_{n,i}^S \quad t_{n+1,i}^S$

Sensorless Example

Results: Planar Simulation

$$f_i(p_i(t),t) \triangleq p_i(t),\ f_i(\hat{p}_i(t),t) \triangleq \hat{p}_i(t)$$

$$d_1(t) \triangleq \left[sin(t)\ cos(t) \right]^T$$

$$d_2(t) \triangleq \left[cos(t)\ sin(t) \right]^T$$

$$d_3(t) \triangleq \left[-cos(t)\ sin(t) \right]^T$$

$$d_4(t) \triangleq \left[cos(t)\ -sin(t) \right]^T$$

$$d_5(t) \triangleq \left[-cos(t)\ -sin(t) \right]^T$$

TABLE 1: SIMULATION PARAMETERS.

$k_{2,i}$	M	L	C_{com}	V_T	ε
0.5	0.5	1	5	5	1

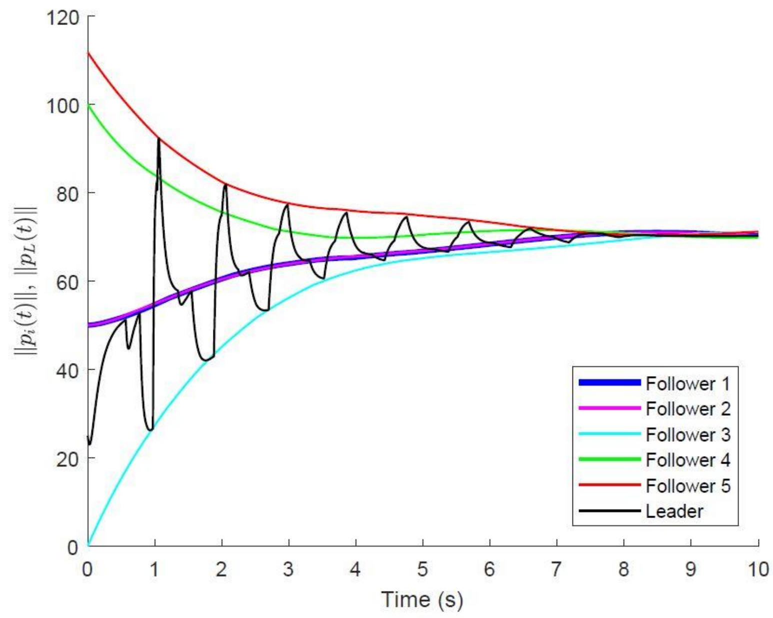


FIGURE 1: LEADER AND FOLLOWER POSITIONS WITH A CONTINUOUS DISTURBANCE WHERE $p_d = (50,50)$ METERS, AND HENCE $\|p_d\| = 50\sqrt{2}$ METERS.

Sensorless Example

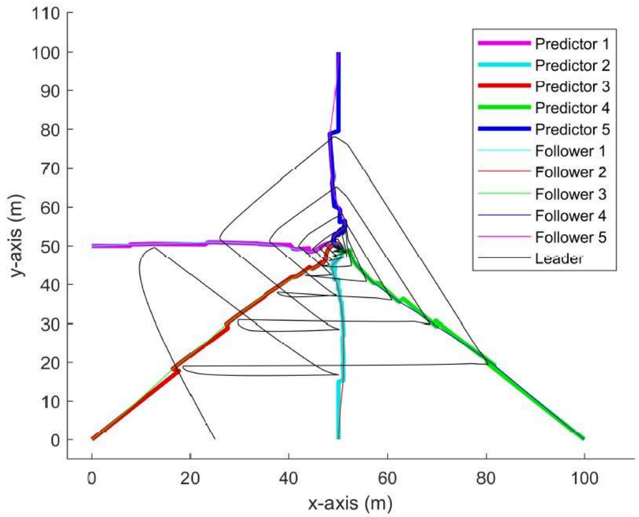


FIGURE 2: PLANAR REGULATION OF FIVE FOLLOWER AGENTS BY A SINGLE LEADER AGENT SUBJECT TO A CONTINUOUS DISTURBANCE.

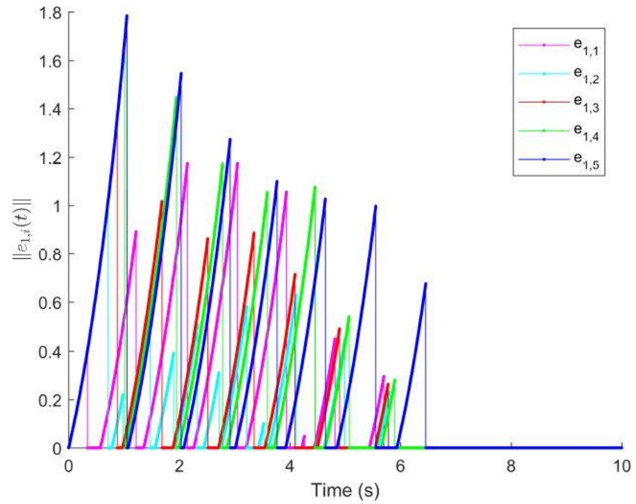


FIGURE 3: FOLLOWER PREDICTION ERROR WHERE THE FOLLOWER AGENTS ARE SUBJECT TO A CONTINUOUS DISTURBANCE. THE PREDICTION ERROR STOPS INCREASING BEYOND 6.5 SECONDS BECAUSE THE FOLLOWER AGENTS ARE CONTINUOUSLY WITHIN C_{com} WHERE THE RESET MAP FOR EACH FOLLOWER IS CONTINUOUSLY INVOKED.

Sensorless Example

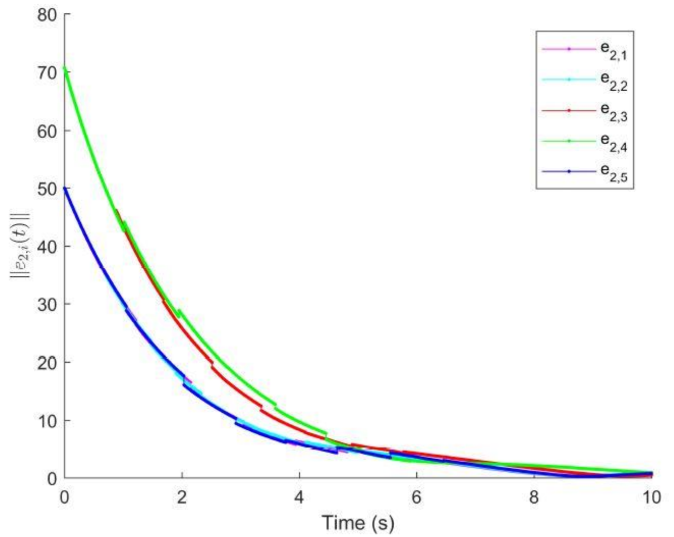


FIGURE 4: DIFFERENCE BETWEEN THE GOAL LOCATION AND THE FOLLOWER AGENT'S PREDICTED POSITION WHERE THE FOLLOWER AGENTS ARE SUBJECT TO A CONTINUOUS DISTURBANCE. JUMP DISCONTINUITIES ARE DUE TO THE ACTIVATION OF THE RESET MAP.

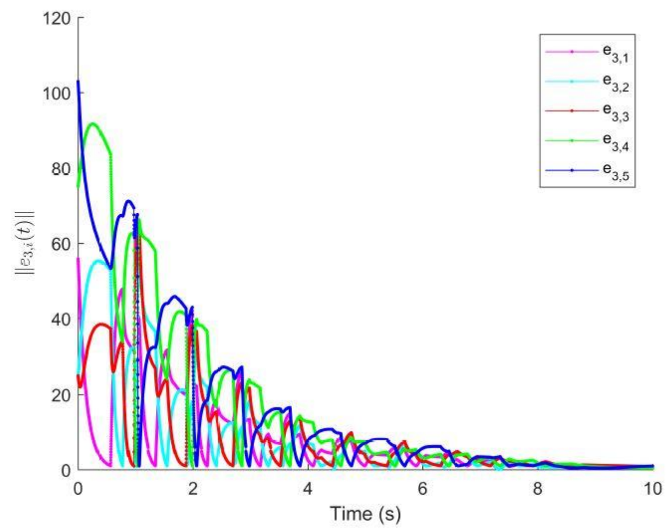
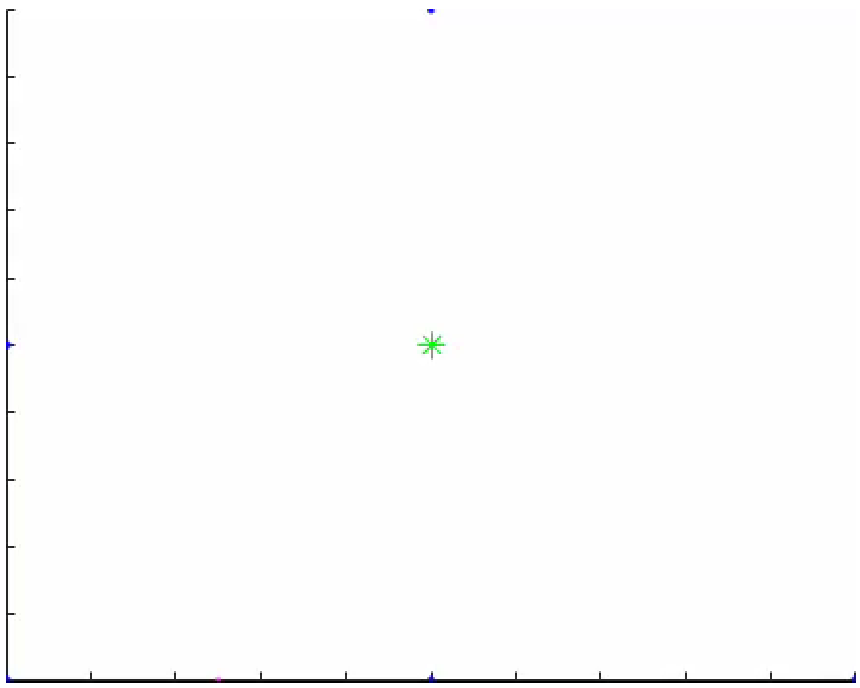
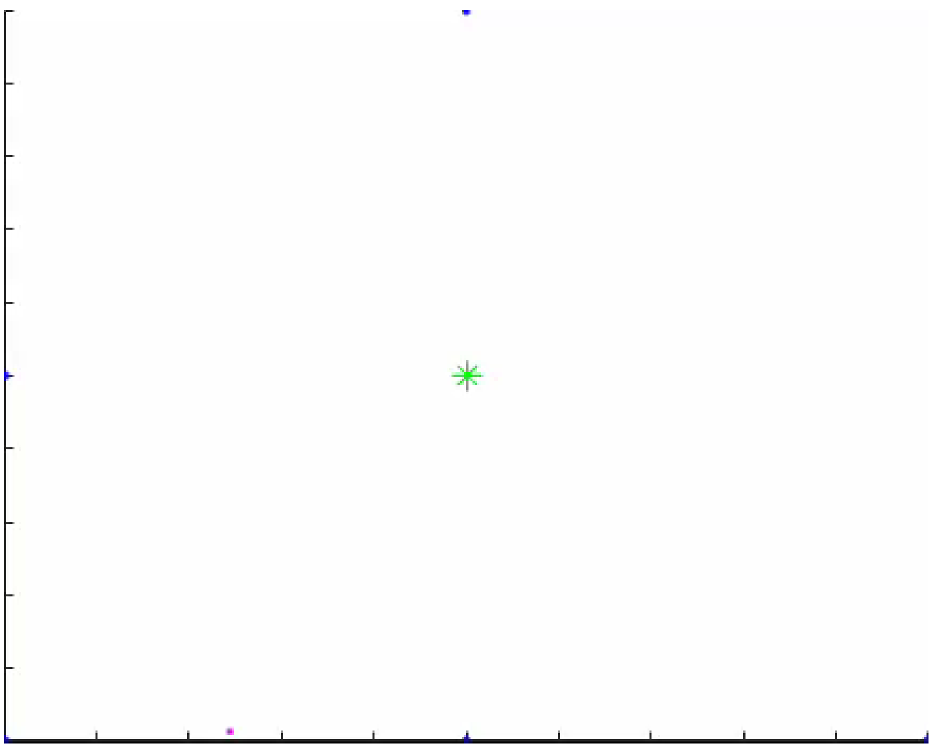


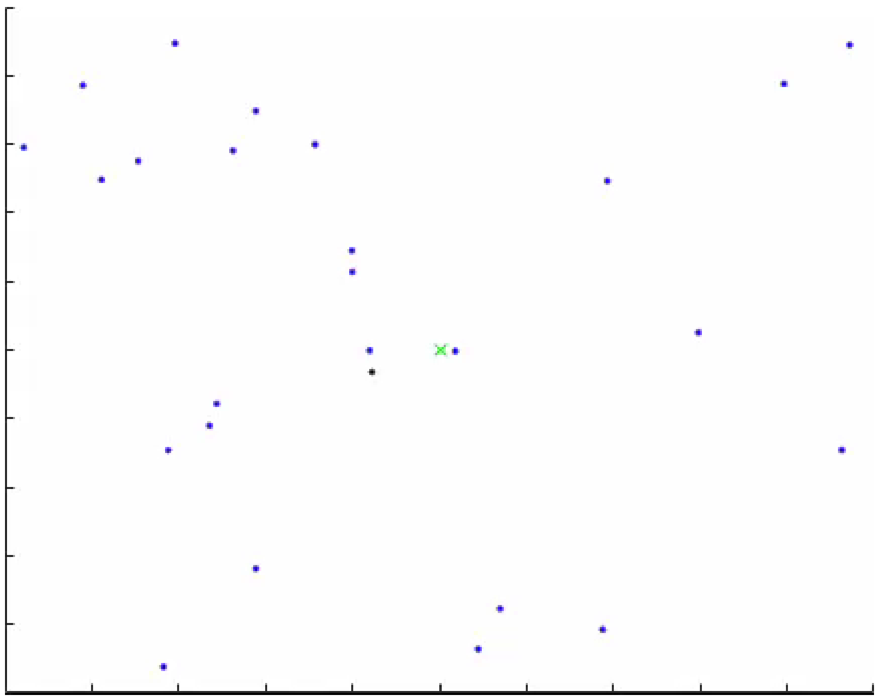
FIGURE 5: ERROR BETWEEN THE LEADER AND THE PREDICTED FOLLOWER POSITION, WHERE THE FOLLOWER AGENTS ARE SUBJECT TO A CONTINUOUS DISTURBANCE. AS THE LEADER SWITCHES BETWEEN FOLLOWERS, THE MULTI-AGENT SYSTEM CONVERGES TO p_d.

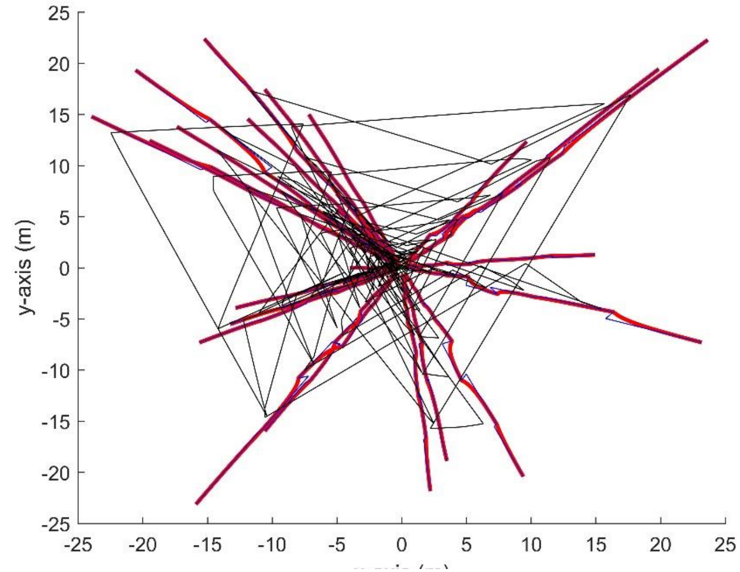
Sensorless Example

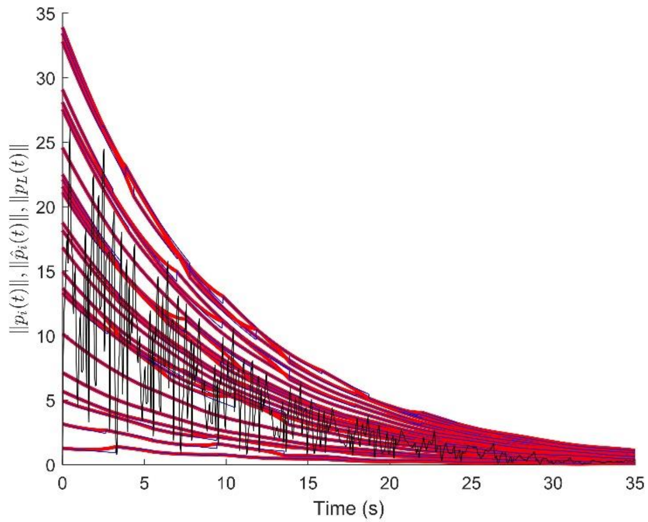




Scaled Sensorless Example







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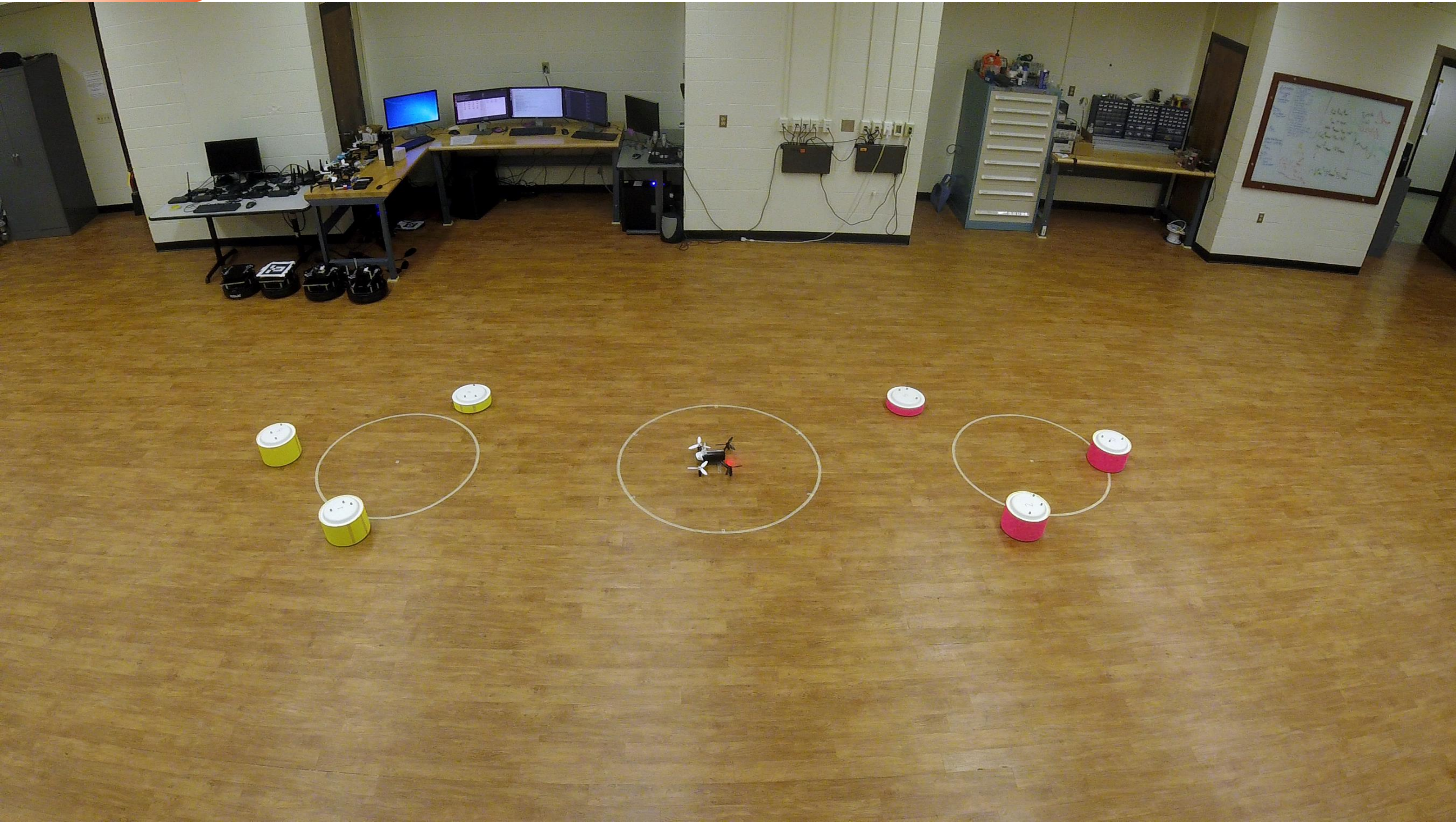
Scalability Example



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Experiment #1: Target norms

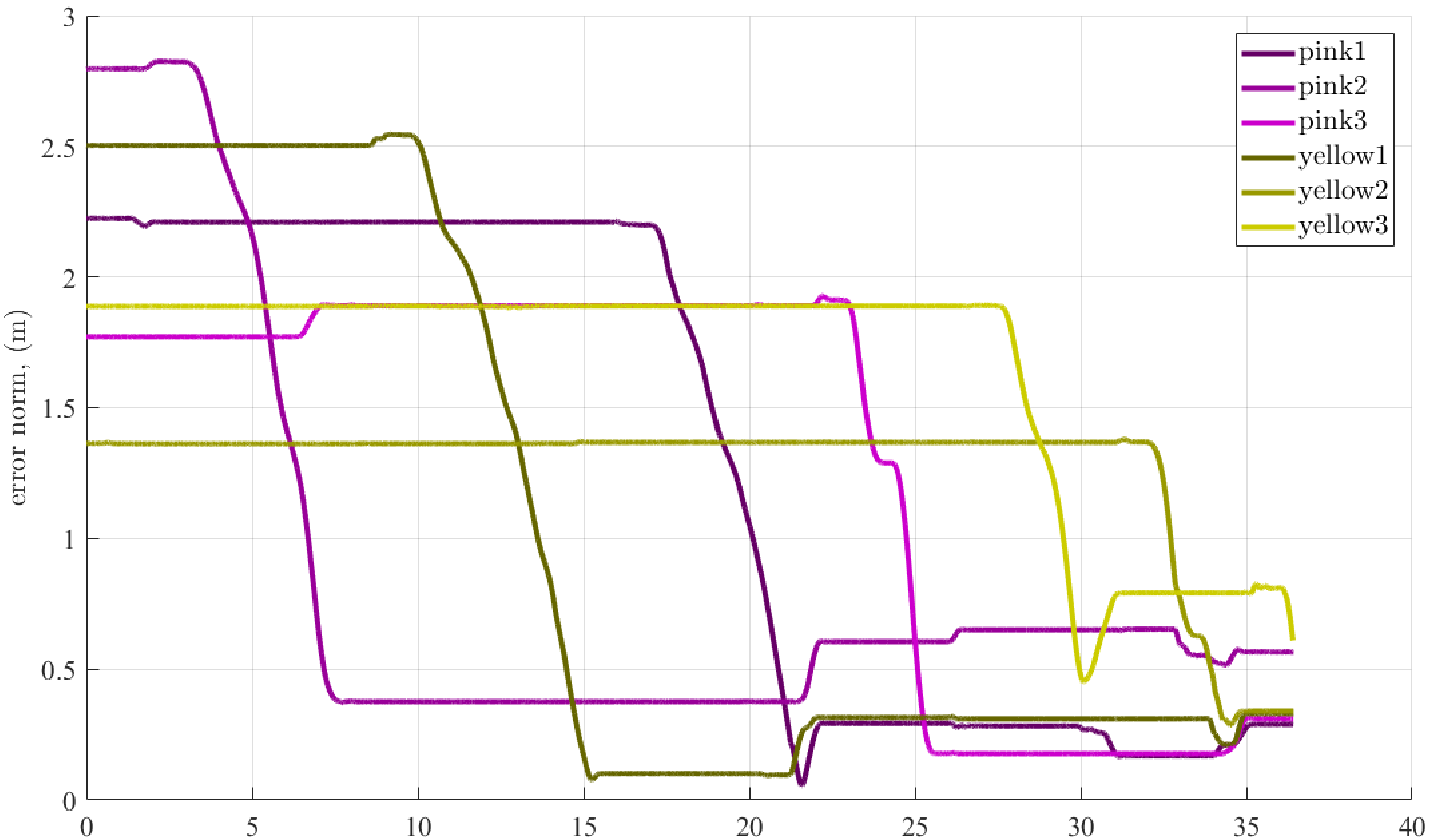


Image Intermittency Example

Moving overhead camera observing a moving target on the ground

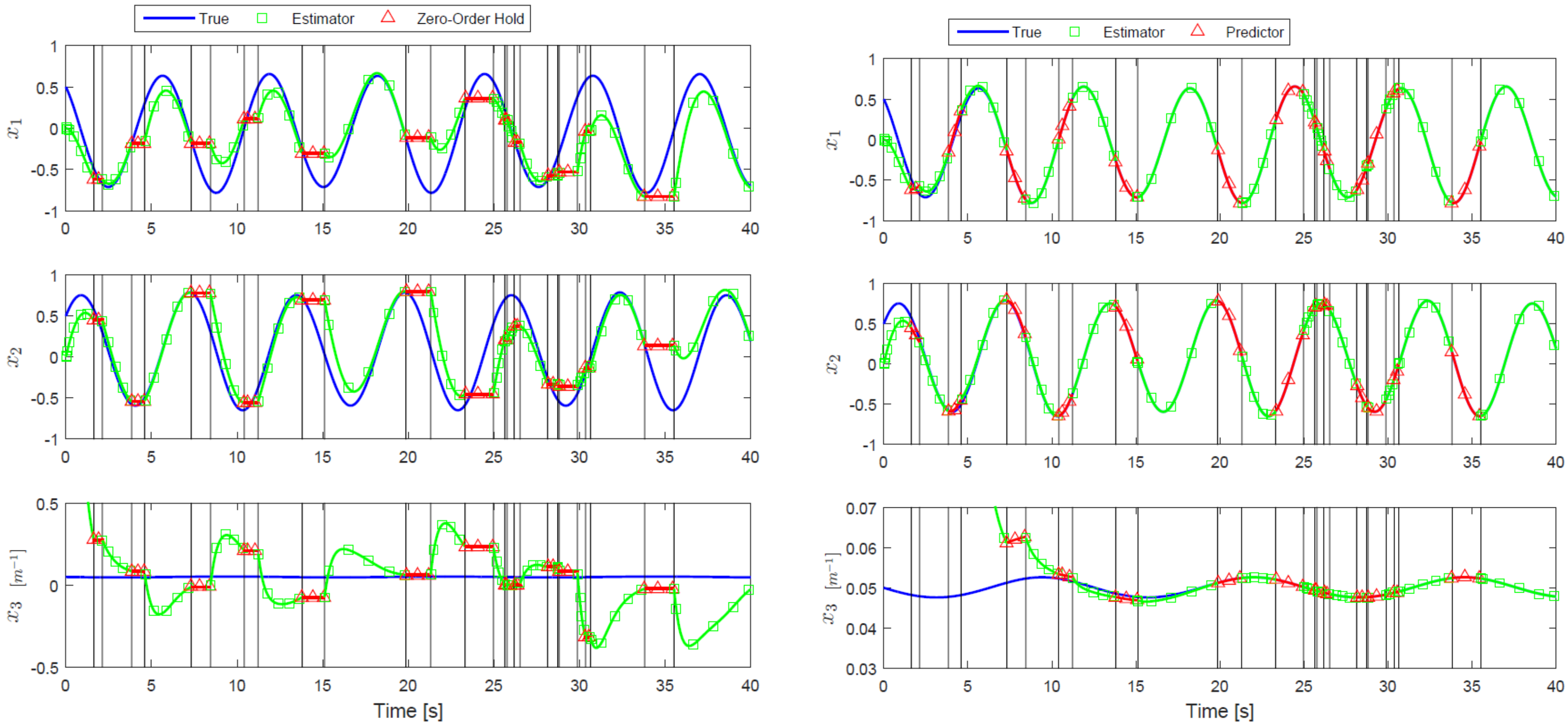


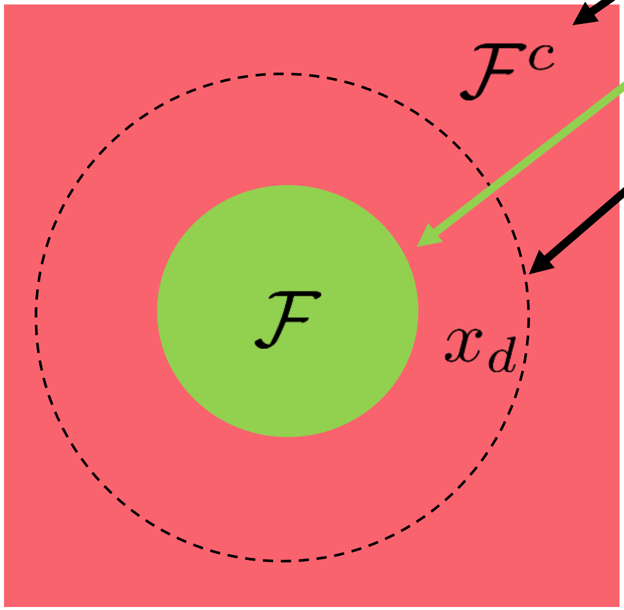
Image Intermittency Example

Image Intermittency Example

Regional Intermittency Example

Dynamic system with an exogenous disturbance

$$\dot{x}(t) = f(x(t), t) + v(x(t), t) + d(t)$$



Feedback-unavailable region $x(t) \in \mathcal{F}^c, \ p = u.$

Feedback-available region $x(t) \in \mathcal{F}, \ p = a.$

Desired path is outside the feedback region $x_d \subset \mathcal{F}^c$

A switched trajectory is designed to travel between the regions $x_\sigma(t) \in \mathbb{R}^n$

Trajectory tracking error $e(t) \triangleq x(t) - x_\sigma(t)$

Estimate tracking error $\hat{e}(t) \triangleq \hat{x}(t) - x_\sigma(t)$

Estimation error $\tilde{e}(t) \triangleq x(t) - \hat{x}(t)$

Regional Intermittency Example

- Generalized controller and update law designs

Generalized controller: $g(t, e(t))$; Generalized observer: $k(t, \hat{x}(t), u(t), \tilde{e}(t))$; System model: $f(t, \hat{x}(t), u(t))$

$$u(t) = \begin{cases} \underbrace{g(t, e(t))}_{\text{Generalized controller}}, & p = a \\ g(t, \hat{e}(t)), & p = u \end{cases} \qquad \dot{\hat{x}}(t) = \begin{cases} \overbrace{k(t, \hat{x}(t), u(t), \tilde{e}(t))}^{\text{Generalized observer}}, & p = a \\ \underbrace{f(t, \hat{x}(t), u(t))}_{\text{System model}}, & p = u \end{cases}$$

- Generalized error dynamics
  - Actual tracking error dynamics

$$\dot{e}(t) = \begin{cases} g_{e,p}\left(v(x,t), t\right), & p = a \quad \text{Stabilizable} \\ g_{e,p}\left(v(\hat{x},t), t\right), & p = u \quad \text{Potentially Unstable} \end{cases}$$

  - Estimate tracking error dynamics

$$\dot{\hat{e}}(t) = \begin{cases} g_{\hat{e},p}\left(v(x,t), t\right), & p = a \quad \text{Stabilizable} \\ g_{\hat{e},p}\left(v(\hat{x},t), t\right), & p = u \quad \text{Stabilizable} \end{cases}$$

  - Estimation error dynamics

$$\dot{\tilde{e}}(t) = g_{\tilde{e},p}\left(x, \hat{x}, t\right), \ \forall p \in \{a, u\} \quad \text{Potentially Unstable}$$

Regional Intermittency Example

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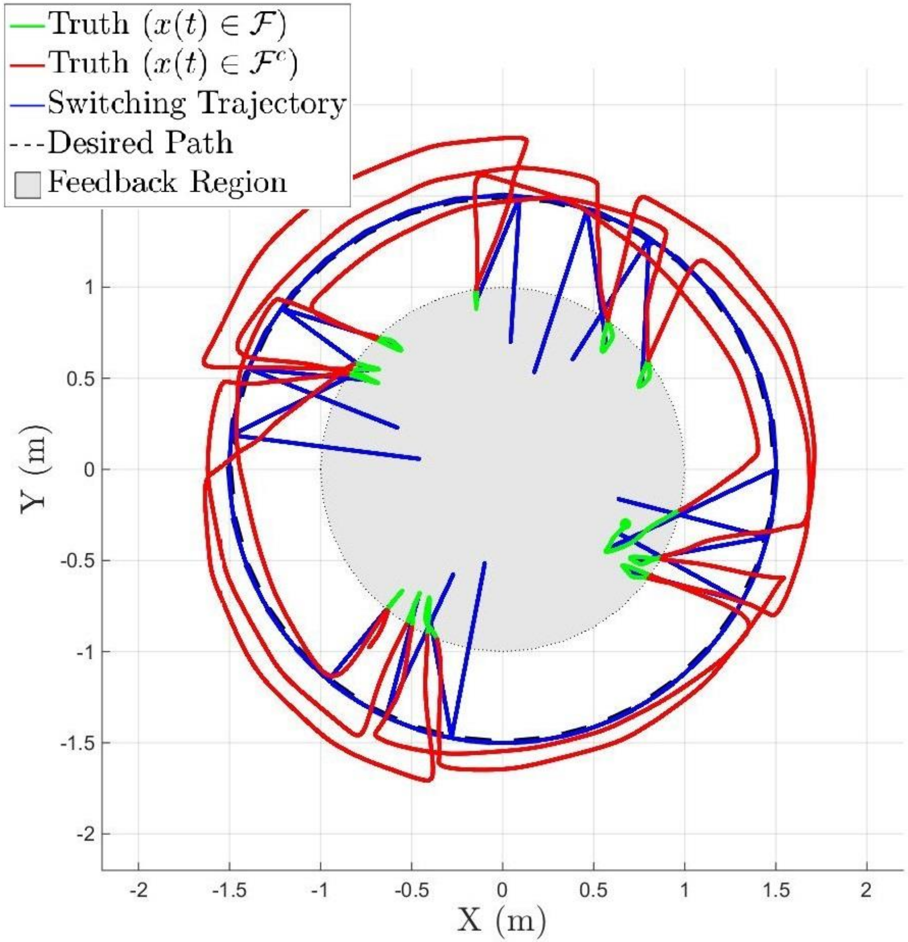
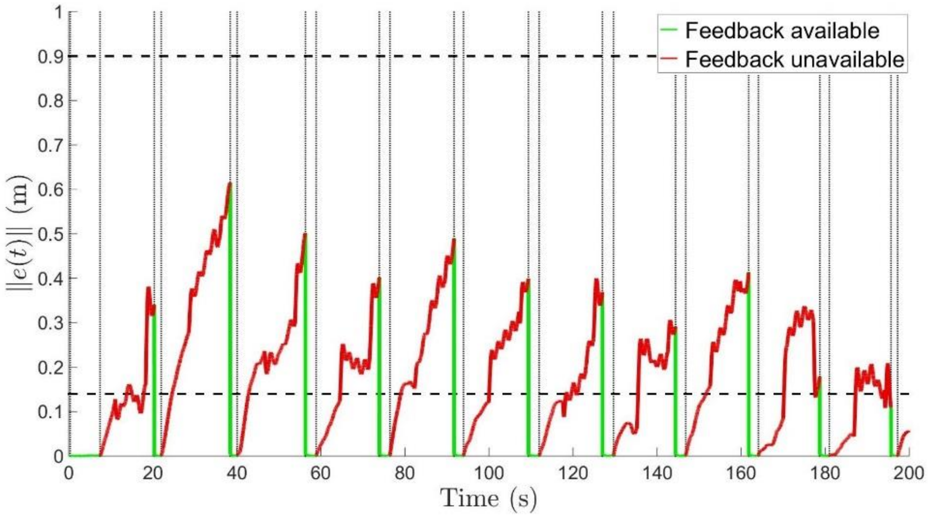
Regional Intermittency Example

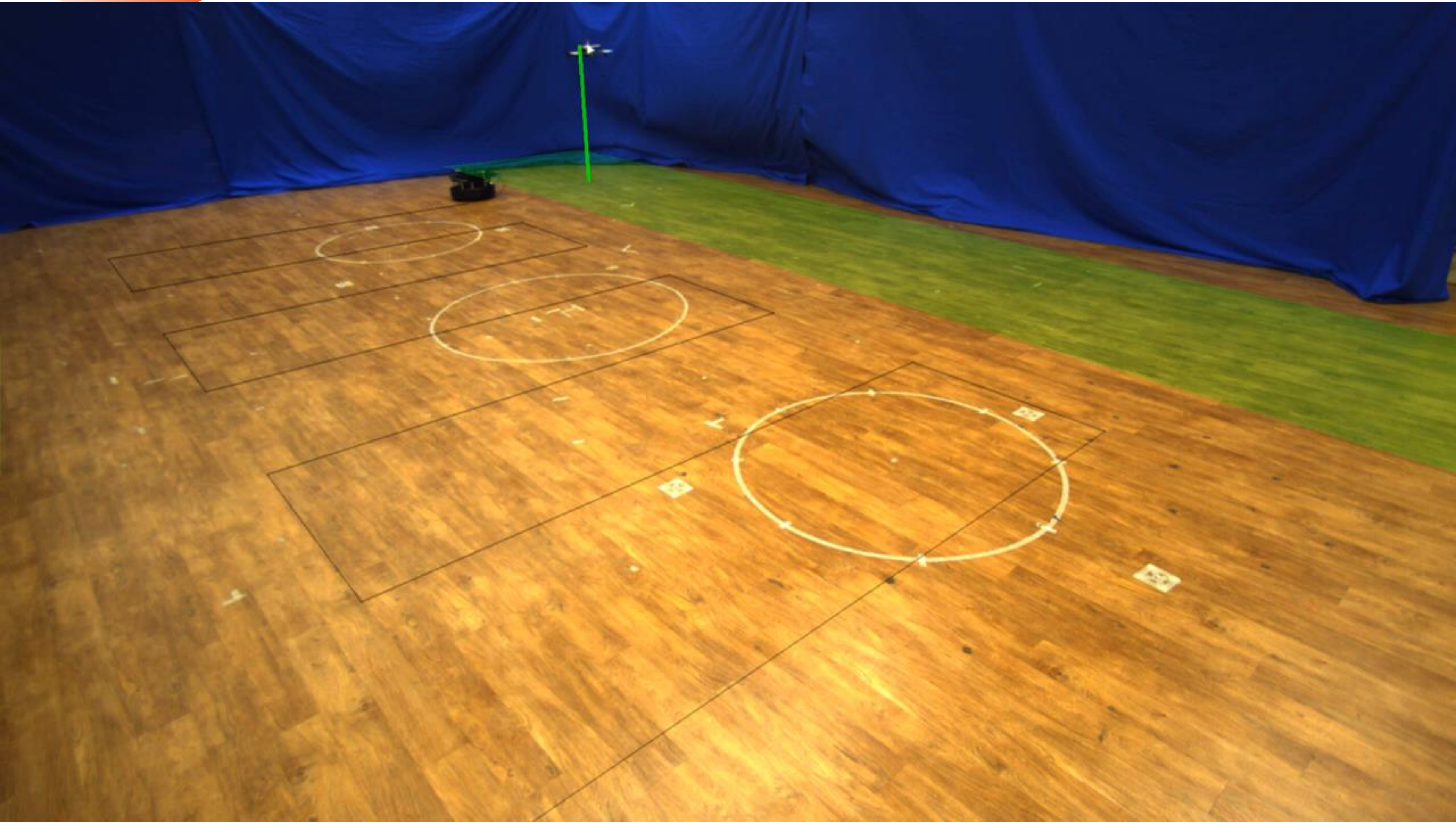




Relay Explorer (RE) Example

Multi-Agent RE Example

Path Dependent RE Example



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