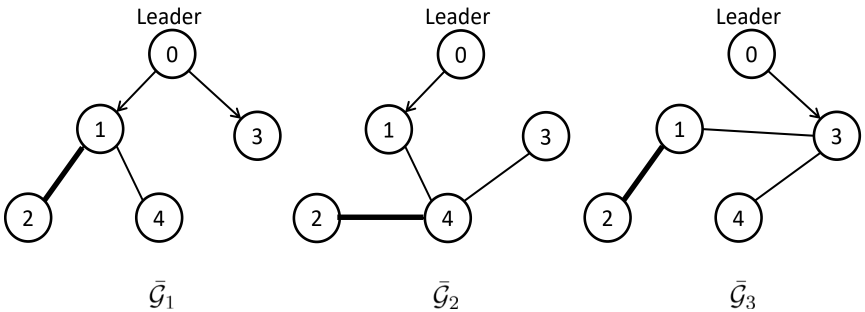
Event & Self-Triggered Approximate Leader-Follower Consensus with Resilience to Byzantine Adversaries

Intermittent Measurements

- Intermittency can result in time varying topologies

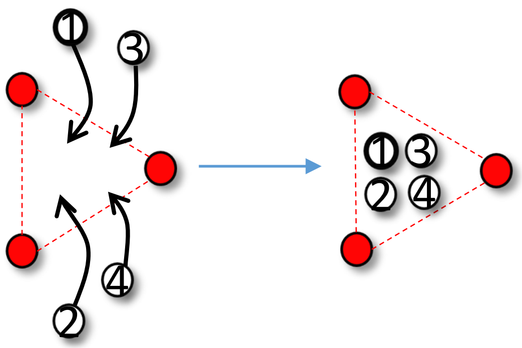


$$\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$$
$$H_p = L_p + D_p, \; p = 1,\, 2,\, 3$$

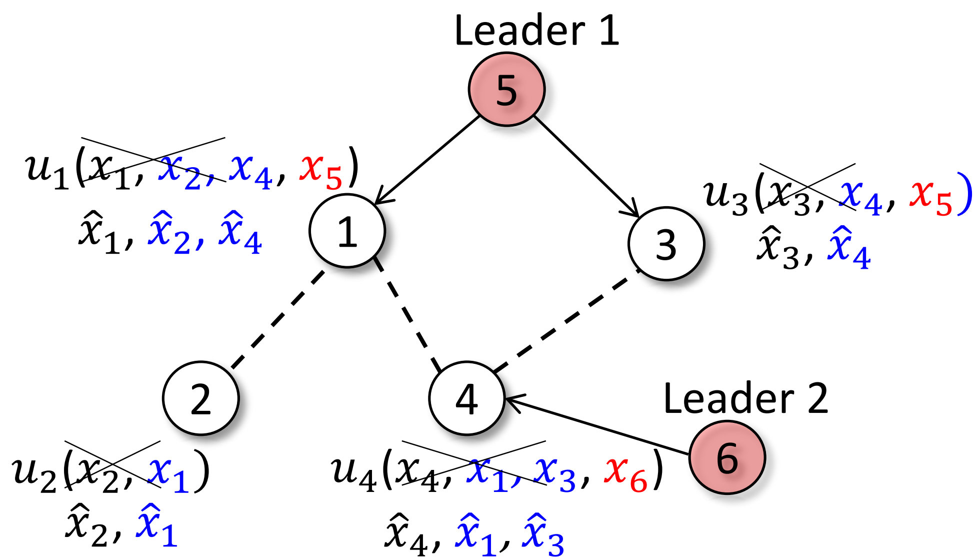
- Switched systems theory provides a framework for analyzing the stability and performance of the resulting switched/hybrid dynamic system

- Dynamics matter for these problems because of the need to develop predictors
  - Frameworks from Nonsmooth Analysis provide toolsets to allow switching with uncertainty
  - Network specific challenges: connectivity, fixed or time-varying topology, directed/undirected, signed/unsigned, resiliency

Example: Distributed Event-Trigger



Goal: Agents converge to the convex hull spanned by the leaders



Dynamics:

$$\dot{x}_i = Ax_i, \quad i \in \mathcal{V}_{\mathcal{L}}$$
$$\dot{x}_i = Ax_i + Bu_i, \; i \in \mathcal{V}_{\mathcal{F}}$$

Estimate dynamics:

$$\dot{\hat{x}}_j(t) = A\hat{x}_j(t), \; j \in \{i\} \cup \mathcal{N}_{\mathcal{F}i}, \; t \in \left[t_k^j, t_{k+1}^j\right) \Rightarrow \textbf{No Comm.}$$

$$\hat{x}_j\left(t_k^j\right) = x_j\left(t_k^j\right) \Rightarrow \textbf{Comm.}$$

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Controller Design

Controller:

$$u_i = K\hat{z}_i$$

$$\hat{z}_i = \sum_{j\in\mathcal{V}_\mathcal{F}} a_{ij}\left(\hat{x}_j - \hat{x}_i\right) + \sum_{j\in\mathcal{V}_\mathcal{L}} a_{ij}\left(x_j - \hat{x}_i\right),\ i \in \mathcal{V}_\mathcal{F}$$

Known

where $K = B^T P$

$$P : PA + A^T P - 2\delta_{\min} PBB^T P + \delta_{\min} I_n < 0$$

Estimate Error: $e_i(t) = \hat{x}_i(t) - x_i(t),\ i \in \mathcal{V}_\mathcal{F}$

Known

Closed-loop dynamics:

$$\varepsilon_i = \hat{z}_i - \sum_{j\in\mathcal{V}_\mathcal{F}} a_{ij}\left(e_i - e_j\right) - \sum_{j\in\mathcal{V}_\mathcal{F}} a_{ij} e_i, \quad i \in \mathcal{V}_\mathcal{F}$$

unknown

unknown

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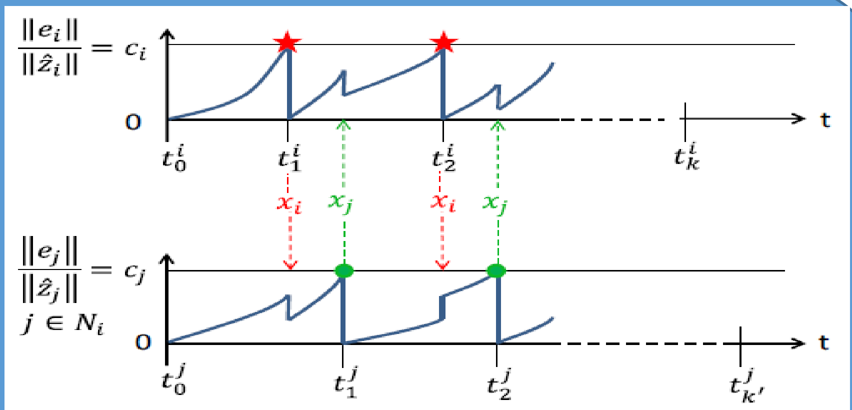
When to Communicate?



T. H. Cheng, Z. Kan, J. R. Klotz, J. M. Shea, W. E. Dixon, "Event-Triggered Control of Multi-Agent Systems for Fixed and Time-Varying Network Topologies," **IEEE Trans. Autom. Control**, Vol. 62(10), pp. 5365-5371, 2017.

Nonlinear Analysis

$$V = \varepsilon^T \left(I_F \otimes P\right) \varepsilon$$

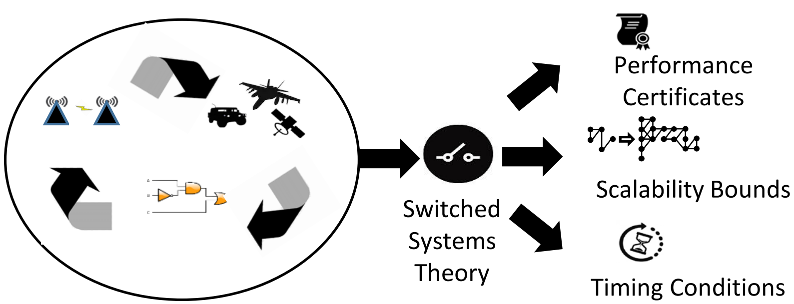
$$\dot{V} \leq -\sum_{i \in \mathcal{V}_{\mathcal{F}}} \left[\left(\delta_1 - \frac{k_2}{\beta}\right) \|\hat{z}_i\|^2 - (k_1 + k_2\beta) \|e_i\|^2 \right] - \delta_2 \varepsilon^T \varepsilon$$

$$\dot{V} \leq -\delta_2 \varepsilon^T \varepsilon$$

$$\|\varepsilon(t)\| \leq \|\varepsilon(t_0)\| e^{-\gamma t}$$

$$x_{\mathcal{F}} \to -\left(\mathcal{L}_{\mathcal{F}}^{-1} \mathcal{L}_{\mathcal{L}} \otimes I_n\right) x_{\mathcal{L}} \quad \text{as} \quad t \to \infty$$

Nonsmooth Analysis



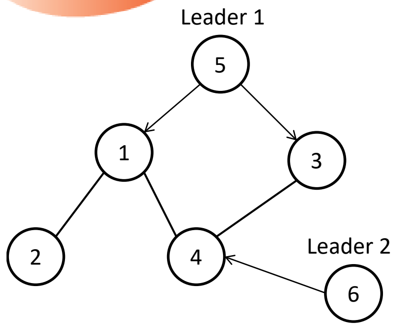
Trigger Condition

$$c_i = \sqrt{\frac{\eta_i \left(\delta_1 - \frac{k_2}{\beta}\right)}{(k_1 + k_2\beta)}}$$

Minimum Interval Event Time

$$\tau \geq \frac{1}{\max\{\bar{c}_0, \bar{c}_1\}} \ln \left(\frac{1}{F} \sqrt{\frac{\eta_h \left(\delta_1 - \frac{k_2}{\beta}\right)}{(k_1 + k_2\beta)}} + 1 \right)$$

Simulation

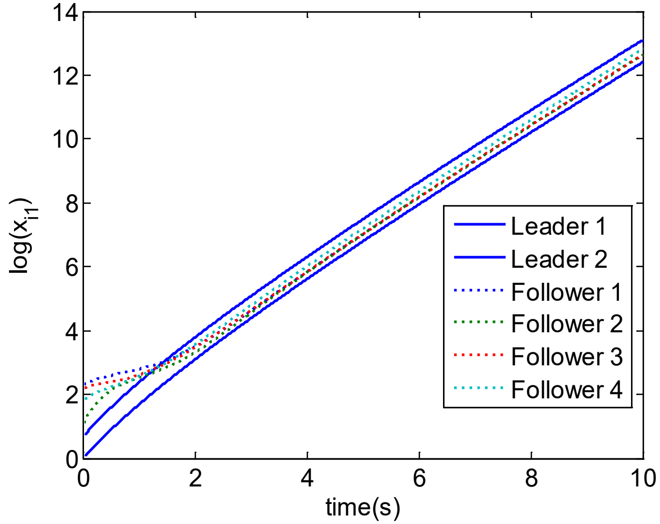


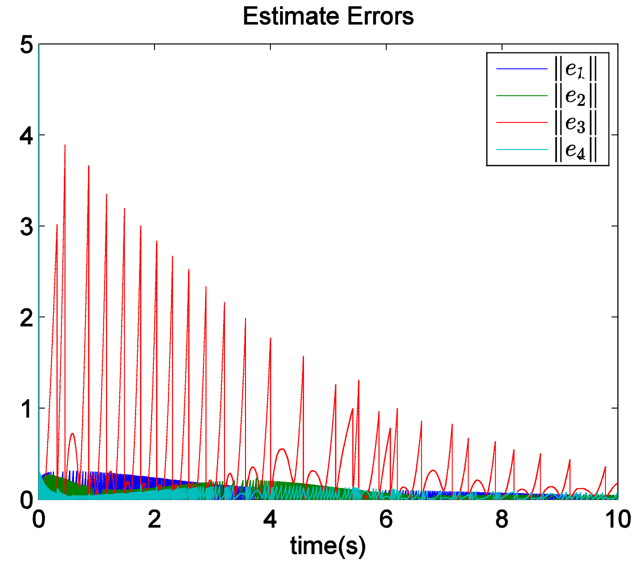
$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

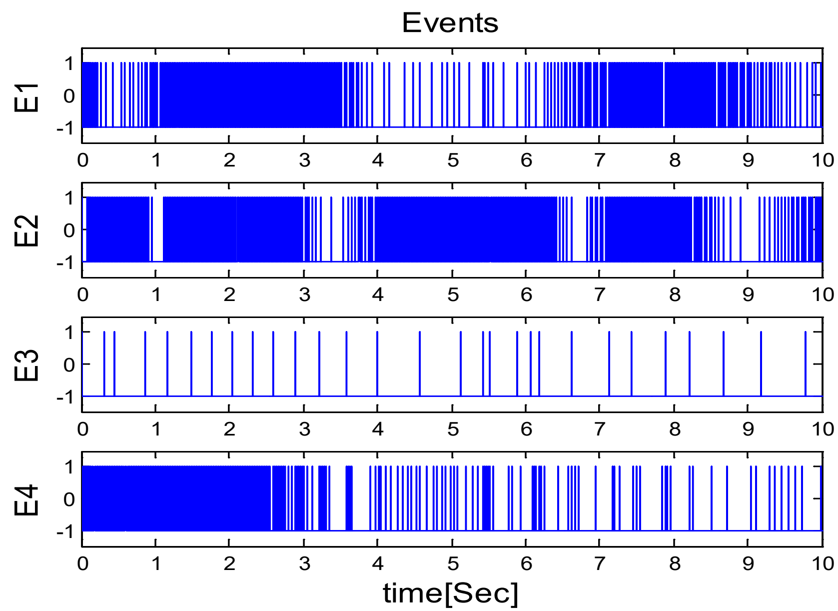
$$\mathcal{L}_{\mathcal{F}} = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}$$

$$P = \begin{bmatrix} 15.897 & 5.969 \\ 5.969 & 5.266 \end{bmatrix}$$

$$K = \begin{bmatrix} 5.969 & 5.266 \end{bmatrix}$$







Ave: 29 ms

Ave: 10 ms

Ave: 78 ms

Ave: 10 ms

1: Triggered

-1: Not triggered

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On-going Efforts

Event-Triggered Control

- Opportunistically select when to communicate (dynamics-based trigger condition)
- Require continuous listening (expensive)

Self-Triggered Control

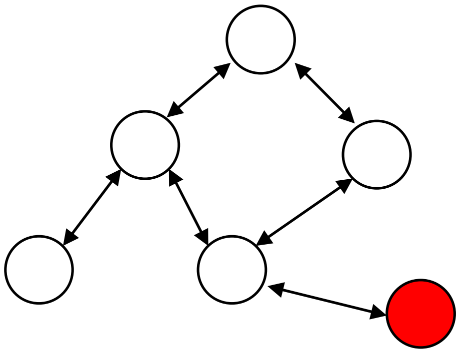
- Eliminates continuous listening (least expensive?)
- Predict (uncertainty?) when to send/listen (asynchrony?)

Byzantine adversary

- Categorize? False information (How to know/detect?)
- Impart undesirable influence on network
  - Partition, wrong objective, data exfiltration

Open Questions

- How to model?
  - Signed graphs? Adversary classification?
- Network characteristics?
  - Power boost? Connectivity? Asynchrony?
- Game Theory Methods?
- Resiliency? Protecting Information?



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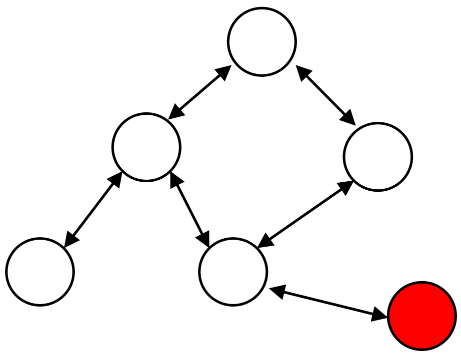
Example: Self-Trigger LF Consensus

- Undirected network of followers $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$
- Perform self-triggered approximate leader-follower consensus

$$\lim_{t \to \infty} \sup \|e_i(t)\| \leq \varepsilon \quad \forall i \in \mathcal{V}$$

$$e_{1,i}(t) \triangleq x_i(t) - x_0(t)$$



- Byzantine adversary detection error

$$e_{2,j}(t) = \hat{x}_j(t) - x_j(t)$$

- LTI dynamics of followers

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t)$$

- LTI dynamics of the leader

$$\dot{x}_0(t) = Ax_0(t) + Bu_0(t)$$

LTI known dynamics facilitate Byzantine agent detection.

How to extend to uncertain nonlinear dynamics?

Byzantine Detection

Check if agent was cooperative during previous times

$$\hat{x}_j(t) = e^{A\left(t-t_{k-1}^i\right)}\hat{x}_j\left(t_{k-1}^i\right) \qquad x_j\left(t_s^i\right)\ \forall s \in \{0,1,\ldots,k-1\}$$

Analyze the maximum growth rate for $e_{2,j}(t) = \hat{x}_j(t) - x_j(t)$

$$V_{2,i}\left(e_{2,i}(t)\right) \triangleq \frac{1}{2}e_{2,i}^T(t)\,e_{2,i}(t)$$

$$V_{2,i}\left(e_{2,i}(t)\right) \leq \left(\frac{\sqrt{2}\xi_i\left(e^{\lambda_{max}(A)\left(t-t_k^i\right)}-1\right)}{2}\right)^2$$

$$\left\|e_{2,j}(t)\right\| \leq \xi_j\left(e^{\lambda_{max}(A)\left(t-t_{k-1}^i\right)}-1\right) t \in \left[t_{k-1}^i, t_k^i\right)$$

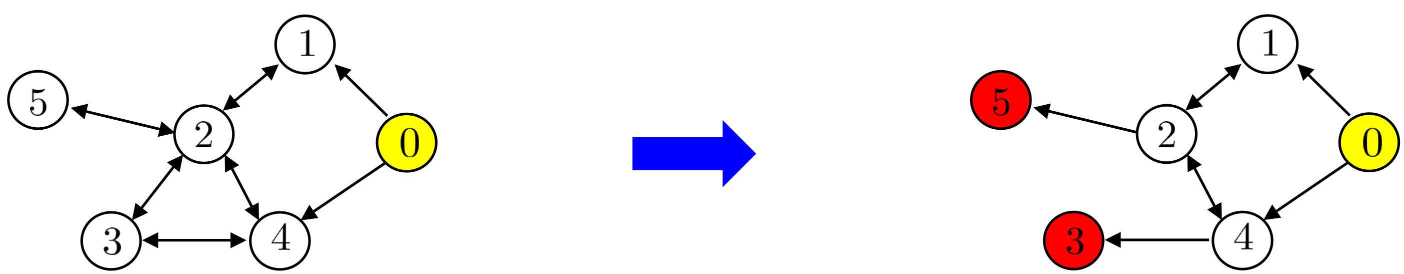
Detection Condition

$$\Xi_j\left(t_k^i\right) = \lim_{t\to t_k^i}\left(\left\|e_{2,j}(t)\right\| - \xi_j\left(e^{\lambda_{max}(A)\left(t-t_{k-1}^i\right)}-1\right)\right)$$

Agents alter the network topology due to the presence of the Byzantine agents

Fixed, Balanced, and Undirected Graph

Time-Varying, Unbalanced, and Directed Graph



Controller/Observer

Distributed controller

$$u_i(t) = K\tilde{z}_i(t) + K\left(\hat{x}_i(t) - x_i(t)\right)$$

Connectivity parameter

$$\tilde{z}_i(t) = \sum_{j \in \mathcal{N}_i(\mathcal{G})} \mu_{ij} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + d_i\left(x_0(t) - \hat{x}_i(t)\right)$$

$$\mu_{ij} = \begin{cases} 1, & j \in \mathcal{C}_i\left(t_k^i\right) \\ 0, & j \in \mathcal{B}_i\left(t_k^i\right) \end{cases}$$

Neighbor state estimator

$$\dot{\hat{x}}_j(t) = A\hat{x}_j(t), \; t \in \left[t_k^i, t_{k+1}^i\right), \; j \in \mathcal{N}_i(\mathcal{G}) \cup \{i\}$$

$$\hat{x}_j(t) = x_j\left(t_k^i\right)$$

Nonsmooth Stability Analysis

- Time-varying unbalanced directed graph
- Triggered communication

$$V_1(e_1(t)) \triangleq e_1^T(t)\left(I_N \otimes P\right)e_1(t)$$

$$\dot{V}_1(g(t)) \overset{a.e.}{\in} \dot{\tilde{V}}_1(g(t))$$

$$\dot{\tilde{V}}_1(g(t)) \subseteq \left\{e_1^T(t)\left(I_N \otimes \left(A^TP + PA\right)\right)e_1(t)\right\} - \left\{e_1^T(t)\left(H_{\sigma(t)} \otimes 2PBB^TP\right)e_1(t)\right\}$$
$$+ \left\{e_2^T(t)\left(\left(I_N - H_{\sigma(t)}\right) \otimes 2PBB^TP\right)e_1(t)\right\} - \left\{\left(1_N^T \otimes 2u_0^T(t)B^TP\right)e_1(t)\right\}$$

$$\dot{V}_1(e_1(t)) \overset{a.e.}{\leq} -\phi_1\left\|e_1(t)\right\|^2 + \bar{\delta}$$

Event vs. Self-Triggering

Event-Trigger Condition

$$t_{k+1}^{i} = inf\left\{t > t_k^i : \left\|e_{2,i}(t)\right\| \geq \sqrt{\frac{\phi_1}{\phi_2}}\left\|\hat{z}_i(t)\right\|\right\}$$

Each $j \in \mathcal{N}_i(\mathcal{G})$ continuously senses

$$e_{2,i}(t) = \hat{x}_i(t) - x_i(t)$$

$$\hat{z}_i(t) = \sum_{j \in \mathcal{N}_i(\mathcal{G})} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + d_i\left(x_0(t) - \hat{x}_i(t)\right)$$

Self-Trigger Condition

$$\hat{t}_{k+1}^{i} = inf\left\{t > t_k^i : \breve{e}_{2,i}(t) \geq \sqrt{\frac{\phi_1}{\phi_2}}\breve{z}_i(t)\right\}.$$

$$\hat{t}_{k+1}^{i} \leq t_{k+1}^{i}$$

$$\breve{e}_{2,i}(t) = \xi_i\left(e^{\lambda_{max}(A)\left(t - t_k^i\right)} - 1\right)$$

$$\breve{z}_i(t) = min\left\{1, \Phi_i(t)\right\}\left\|e^{A\left(t - t_k^i\right)}\hat{z}_i\left(t_k^i\right)\right\|$$