Model Checking Meets Robot Planning: A Sampling-Based Framework for Large-Scale Optimal Temporal Logic Synthesis

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### **Robot Motion Planning**



#### **Point-to-point navigation tasks**

- "Starting from point A, reach point B while avoiding obstacles"
- L. Kavraki et al (TRA 1996), S. LaValle et al (IJJR 2001),
- S. Karaman et al (IJJR 2011), L. Janson (IJRR 2015)

#### <u>High-level complex tasks</u>

- "Pick up the mail by visiting houses in a given order"
- "Next visit a delivery site"
- "Never leave the delivery site until a ground robot is present to pick up the mail"
- "Repeat this process every day"

How to express complex tasks in a formal way? How to synthesize optimal and correct-byconstruction controllers?

M. Kloetzer et al (TRO 2010), S. Smith et al (IJRR 2011) A. Ulusoy et al (IJRR 2013), M. Guo et al (IJRR 2015)

















LTL is a formal type of logic that consists of Boolean and temporal operators defined over a set of atomic propositions/predicates.

Syntax:  $\phi ::= \text{true} \mid \pi \mid \phi_1 \land \phi_2 \mid \neg \phi \mid \bigcirc \phi \mid \phi_1 \mathcal{U} \phi_2$ 

Set  $\mathcal{AP}$  of Atomic Propositions (Boolean variables).

Other useful temporal operators:

- Always
- •
- Eventually
- Infinitely often  $\Box \diamondsuit$













## Expressing Complex Tasks using LTL



Reachability task $\Diamond \pi_i^{\text{room1}}$ Reachability with<br/>avoidance $\neg (\pi_i^{\text{room1}} \lor \pi_i^{\text{room2}}) \mathcal{U} \pi_i^{\text{room3}}$ Coverage task $\Diamond \pi_i^{\text{room1}} \land \Diamond \pi_i^{\text{room2}} \land \Diamond \pi_i^{\text{room3}}$ Sequencing $\Diamond (\pi_i^{\text{room1}} \land (\Diamond (\pi_i^{\text{room2}} \land \Diamond \pi_i^{\text{room3}})))$ Recurrent<br/>sequencing $\Box \Diamond (\pi_i^{\text{room1}} \land (\Diamond (\pi_i^{\text{room2}} \land \Diamond \pi_i^{\text{room3}})))$ 

Compositional tasks: 
$$\phi = \Box \Diamond (\pi_1^{\text{room1}}) \land (\neg \pi_1^{\text{room1}} \mathcal{U} \pi_2^{\text{room2}}) \land (\Diamond \Box (\pi_2^{\text{room3}}))$$
  
Robot 1: visit  
room1 infinitely often
Robot 1: never visit room1  
until robot 2 visits room 2
Robot 2: eventually  
always visit room 3
Robot 2: eventually

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Given N robots, an abstraction of the environment and robot dynamics

$$\dot{\mathbf{x}}_i(t) = f_i(\mathbf{x}_i(t), \mathbf{u}_i(t)), \ \forall i \in \{1, 2, \dots, N\}$$

and a collaborative task captured by a global LTL specification  $\phi$ , synthesize a discrete motion plan  $\tau$  such that  $\tau \models \phi$  and a user-specified metric  $J(\tau)$ , such as total traveled distance, is minimized.



$$\phi = \Diamond (\pi_i^{\text{room2}} \land (\Diamond \pi_i^{\text{room4}} \land (\Diamond \pi_i^{\text{room5}} \land (\Diamond \pi_i^{\text{room6}})))) \land (\Diamond \Box \pi_i^{\text{room6}}) \land (\Box \neg \pi_i^{\text{room3}})$$

 $\tau = \text{room1,room2,corridor,room4,}$ room5,corridor,room6, [room6]<sup>\u03c6</sup>













Challenges

#### **Optimal Control Synthesis**



### **Model Checking / Verification**

NuSMV 2, nUxmv, SPIN, SPOT

More scalable (~10<sup>30</sup> states) but no optimality guarantees. Return a feasible, and not the optimal, solution.

We propose an algorithm that can solve optimally hundreds of orders of magnitude larger planning problems than state-of-the-art methods (~10<sup>800</sup> states and beyond).















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### Weighted Transition Systems (wTS)

$$\dot{\mathbf{x}}_i(t) = f_i(\mathbf{x}_i(t), \dot{\mathbf{u}}_i(t))$$
  
 $\mathbf{x}_i(t) \in \mathcal{W}$ 

$$\mathrm{wTS}_i = (\mathcal{Q}_i, q_i^0, \rightarrow_i, w_i, \mathcal{AP}_i, L_i)$$

$\mathcal{Q}_i = \{q_i^{r_e}\}_{\forall  ext{regions } r_e}$	: set of states
$q_i^0$	: initial state
$ ightarrow_i \subseteq \mathcal{Q}_i  imes \mathcal{Q}_i$	: transition rule
$\mathcal{AP}_i = \cup_{r_e} \{\pi_i^{r_e}\}$	: set of APs
$w_i: \mathcal{Q}_i \times \mathcal{Q}_i \to R_+$	: cost function
$L_i: \mathcal{Q}_i \to 2^{\mathcal{AP}_i}$	: observation relation







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### Non-Deterministic Buchi Automaton (NBA)

Translate the LTL formula to a NBA:  $B = (\mathcal{Q}_B, \mathcal{Q}_B^0, \Sigma, \rightarrow_B, \mathcal{Q}_B^F)$ 





### Product Buchi Automaton (PBA)

Given N transition systems and a NBA, the PBA is:

$$P = \text{wTS}_1 \times \ldots \times \text{wTS}_N \times B = (\mathcal{Q}_P, \mathcal{Q}_P^0, \rightarrow_P, \mathcal{Q}_P^F)$$

$$\begin{aligned} \mathcal{Q}_{P} = & \mathcal{Q}_{1} \times \cdots \times \mathcal{Q}_{N} \times \mathcal{Q}_{B} & \text{Set of states} \\ \mathcal{Q}_{P}^{0} = & q_{1}^{0} \times \ldots q_{N}^{0} \times \mathcal{Q}_{B}^{0} & \text{Set of initial states} & \text{Feasible wTS} \\ & \rightarrow_{P} : & \mathcal{Q}_{P} \times \mathcal{Q}_{P} & \text{Transition rule} \\ & q_{PTS} & q_{PTS}' & q_{PTS}' & q_{HTS}' \\ & (q_{1}, q_{2}, \dots, q_{N}, q_{B}) \rightarrow_{P} & (q_{1}', q_{2}', \dots, q_{N}', q_{B}') & q_{B} \xrightarrow{L_{1}(q_{1}), \dots, L_{N}(q_{N})} \\ & q_{B} \xrightarrow{L_{1}(q_{1}), \dots, L_{N}(q_{N})} & p & q_{B}' \\ & w_{P}(q_{P}, q_{P}') = & \sum_{i=1}^{N} w_{i}(q_{i}, q_{i}') & \text{Cost function} \\ & \mathcal{Q}_{P}^{F} = & \mathcal{Q}_{1} \times \cdots \times \mathcal{Q}_{N} \times \mathcal{Q}_{B}^{F} & \text{Set of final states} \end{aligned}$$

The LTL formula is satisfied if the set of final states of the PBA is visited infinitely often.











### **Optimal Control Synthesis**





### Limitations of Existing Methods

#### **Optimal Control Synthesis**



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- Completely avoid taking the product among wTSs and NBA.
- Approximate representation of PBA by a tree  $\mathcal{G}_T = \{\mathcal{V}_T, \mathcal{E}_T\}$



#### **Approximate?**

 $\mathcal{V}_T \subseteq \mathcal{Q}_P$  $\mathcal{E}_T \subseteq \to_P$ 

#### Why trees?

Resource efficient (memory complexity):

 $O(|\mathcal{E}_T|) << O(|\mathcal{V}_P| + |\mathcal{E}_P|)$ 

Computationally inexpensive graph search methods.

#### How to build trees?

- Incrementally through sampling on the PBA state-space.
- Cycle-detection method.







### Algorithm Outline









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### **Sampling Function**





The root of the tree is an initial state of the PBA.

Sample a state  $q_P^{\text{new}} = (q_{\text{PTS}}^{\text{new}}, q_B) \in \mathcal{Q}_P$  from the density function  $f_{\text{sample}} : \mathcal{Q}_P \to [0, 1]$ 

If  $q_P^{\text{new}} = (q_{\text{PTS}}^{\text{new}}, q_B) \in \mathcal{V}_T$ , then "Rewire" otherwise "Extend"

The choice of the sampling function affects the performance of the algorithm













### **Extend Function**





1) Collect all states in the tree that can directly reach  $q_P^{\text{new}}$  in a set of candidate parents:

$$\mathcal{R}_{\mathcal{V}_T}^{\to}(q_P^{\text{new}}) = \{ q_P \in \mathcal{V}_T \mid q_P \to_P q_P^{\text{new}} \}$$

2) Select as a parent for  $q_P^{\text{new}}$  the node  $q_P^{\text{prev}} \in \mathcal{R}_{\mathcal{V}_T}^{\rightarrow}(q_P^{\text{new}})$ 

that incurs the minimum cost from the root.



### **Rewire Function**





1) Collect all states in the tree that can be directly reached by  $q_P^{\text{new}}$  in a set of possible children:

$$\mathcal{R}_{\mathcal{V}_T}^{\leftarrow}(q_P^{\text{new}}) = \{q_P \in \mathcal{V}_T \mid q_P^{\text{new}} \to_P q_P\}$$

2) Rewire nodes  $q_P \in \mathcal{R}_{\mathcal{V}_T}^{\leftarrow}(q_P^{\text{new}})$  to  $q_P^{\text{new}}$ 

if their cost from the root can be further minimized.

**Complexity of rewiring:**  $O(|\mathcal{V}_T|(N+1))$ 















### **Construction of Prefix Plans**





The construction of the tree is **terminated** after a user-specified maximum number of iterations.

Compute paths (prefix parts) by tracking the sequence of parent nodes from final states to the **root**.

**Complexity of finding paths:**  $O(|\mathcal{V}_T|)$ 













### **Construction of Suffix Plans**





A tree graph rooted at a final state is built.

Every time a new node is added, check if a direct transition to the final state (root) is feasible, forming a suffix loop.













### Optimal Discrete Plan Synthesis

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#### Tree for suffix parts















#### **<u>Theorem</u>**: The proposed sampling-based algorithm is **probabilistically complete**.

#### **Theorem:** The proposed sampling-based algorithm is **asymptotically optimal**, i.e.,

$$\mathbb{P}\left(\left\{\lim_{\substack{n_{\max}^{\text{pre}}\to\infty, n_{\max}^{\text{suf}}\to\infty}}J(\tau_{n_{\max}^{\text{pre}}}^{n_{\max}^{\text{suf}}})=J^*\right\}\right)=1$$







**Theorem:** Let p denote a feasible prefix or suffix path.

$$\mathbf{p} = q_P^1, q_P^2, ..., q_P^{K-1}, q_P^K$$

Then there exist parameters  $\alpha_n(\mathbf{p}) \in (0, 1]$  such that the probability  $\prod_{\mathtt{suc}}(q_P^K)$  of finding the feasible prefix/suffix path p within  $n_{\mathtt{max}}$  iterations satisfies

$$1 \ge \Pi_{\texttt{suc}}(q_P^K) \ge 1 - e^{-\frac{\sum_{n=1}^{n_{\max}} \alpha_n(\mathbf{p})}{2} n_{\max} + K}, \quad \text{if } n_{\max} > K$$

Depend on the selected sampling functions

**<u>Theorem</u>**: Let p\* denote the optimal prefix or suffix path.

$$\mathbf{p}^* = q_P^1, q_P^2, ..., q_P^{K-1}, q_P^K$$

Then there exist parameters  $\alpha_n(\mathbf{p}^*) \in (0, 1]$  and  $\gamma_n(q_P^k) \in (0, 1]$  and iterations  $n_k$  for every state  $q_P^k$  in the optimal path such that the probability of finding the optimal path within  $n_{\max} > 2K$  iterations satisfies

$$\Pi_{\mathsf{opt}}(\mathbf{p}^*) \ge \left(1 - e^{-\frac{\sum_{n=1}^{\bar{n}} \alpha_n(\mathbf{p}^*)}{2} + K}\right) \prod_{k=1}^{K-1} \left(1 - e^{-\frac{\sum_{n=n_{k-1}}^{n_{\max}} \gamma_n(q_P^k)}{2} + 1}\right)$$













Using **uniform sampling functions**, the **proposed method** detected the first feasible path in 1.6 hours with cost 568.1857 meters.

The existing optimal control synthesis method (e.g., Dijkstra, implicit graphs) failed to solve this problem (can solve problems with  $\sim 10^7$  states/edges).

NuSMV generated a feasible plan in 2 seconds with cost 672.2431 meters.

















 $\phi = \Box \Diamond (\pi_1^{\ell_6} \land \Diamond (\pi_2^{\ell_{14}})) \land \Box (\neg \pi_1^{\ell_9}) \land \Box (\pi_2^{\ell_{14}} \to \bigcirc (\neg \pi_2^{\ell_{14}} \mathcal{U} \pi_1^{\ell_4})) \land (\Diamond \pi_2^{\ell_{12}}) \land (\Box \Diamond \pi_2^{\ell_{10}})$ 



$$|Q_i| = 16, \ \forall i \in \{1, 2\}$$
  
 $|Q_B| = 24$   
 $|Q_P| = 6, 144$ 















 $J_{\rm NuSMV} = 30.883$  meters

### **Comparative Results: Small NBA**

MATLAB runtimes to detect



					the first f	the first feasible plan		
	TABLE I							
		FE	EASIBILITY AND	SCALABILITY ANAL	YSIS: $ \mathcal{Q}_B  = 21$			
N	$ \mathcal{Q}_i $	$ \mathcal{Q}_{\mathrm{P}} $	$n_{\rm Pre1} + n_{\rm Suf1}$	$ \mathcal{V}_T^{\operatorname{Pre1}}  +  \mathcal{V}_T^{\operatorname{Suf1}} $	Pre1+Suf1	NuSMV/nuXmv		
1	100	$10^{3}$	28 + 28	180 + 54	0.5+0.2 (secs)	< 1 sec		
1	1000	$10^{3}$	42 +31	338 + 119	0.9+0.7 (secs)	< 1 sec		
1	10000	$10^{4}$	71 + 43	512 + 131	19.2+11.2 (secs)	M/M		
9	9	$10^{10}$	36 + 37	373 + 83	0.78+0.29 (secs)	< 1 sec		
10	100	$10^{21}$	31+ 31	289 + 101	0.7+0.4 (secs)	$\approx 1.5$ secs		
10	1000	$10^{31}$	34 + 27	309 + 82	2.1+1.7 (secs)	≈50/40 secs		
10	2500	$10^{35}$	41 + 32	367 + 142	5.94+6.4 (secs)	$M \approx 31 \text{ mins}$		
10	10000	$10^{41}$	40 + 23	357+123	48.44+38.1 (secs)	M/M		
100	100	$10^{200}$	49 + 39	421 + 81	2.5+0.8 (secs)	F/F		
100	1000	$10^{300}$	30 + 38	254 + 110	12.6+24.1 (secs)	M/M		
100	10000	$10^{400}$	24 + 49	241 + 55	4.2+5.2 (mins)	M/M		
150	10000	$10^{600}$	29 + 87	382+530	6.1 + 27.71 (mins)	M/M		
200	10000	$10^{800}$	42 + 49	453 + 276	11.3+23.5 (mins)	M/M		







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### Comparative Results: Large NBA



					MATLAB run	MATLAB runtimes to detect			
					the first fe	the first feasible plan			
	TABLE II								
	FEASIBILITY AND SCALABILITY ANALYSIS: $ \mathcal{Q}_B =59$								
N	$ \mathcal{Q}_i $	$ \mathcal{Q}_{\mathrm{P}} $	$n_{\rm Pre1} + n_{\rm Suf1}$	$ \mathcal{V}_T^{ ext{Prel}}  +  \mathcal{V}_T^{ ext{Sufl}} $	Pre1+Suf1	NuSMV/nuXmv			
1	100	$10^{3}$	54 + 92	533 + 274	2.18+1.55 (secs)	< 1 sec			
1	1000	$10^{3}$	78 +51	326 + 252	1.84+1.37 (secs)	< 1 sec			
1	10000	$10^{4}$	150 + 107	769 + 364	19.2+11.2 (secs)	M/M			
9	9	$10^{10}$	93 + 27	400 + 168	20.7+18.9 (secs)	< 1 sec			
10	100	$10^{21}$	51+ 39	650 + 239	2.1+0.74 (secs)	$\approx$ 3/2 secs			
10	1000	$10^{31}$	36 + 154	450 + 404	3.9+6.1 (secs)	$\approx$ 80/65 secs			
10	2500	$10^{35}$	61 + 98	710 + 516	10.4+11.9 (secs)	$M \approx 32 \text{ mins}$			
10	10000	$10^{41}$	47 + 164	722+604	56.6 + 98.1(secs)	M/M			
100	100	$10^{200}$	21 + 117	154 + 1431	1.6+18.5 (secs)	F/F			
100	1000	$10^{300}$	52 + 74	401 + 856	19.8+53.32 (secs)	M/M			
100	10000	$10^{400}$	39 + 89	398 + 1621	5.1+28.3 (mins)	M/M			
$\overline{150}$	10000	$10^{600}$	39 + 112	526+1864	8.3 + 60.11 (mins)	M/M			
200	10000	$10^{800}$	48 + 103	588 + 1926	11.7+65.9 (mins)	M/M			





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wTS with self loops or tasks that do not require robots to wait

**NuSMV** is more scalable if the words/guards on the NBA transitions cannot be classified as feasible/infeasible.

**NuSMV** does not provide any optimality guarantees. **STyLuS\*** is asymptotically optimal and can find "good" plans fast enough.















Summary

#### **Optimal Control Synthesis**



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### **Open Problems**



#### Distributed Control & Optimization for Networked Robots

- Reactive controllers for dynamic and uncertain environments
- Robust controllers (robot failures?)
- Realistic communication models (e.g., acoustic channels for underwater applications)
- Joint optimal task planning and communication control

#### Formal Methods/Control Synthesis for Cyber-Physical Systems

- Reactivity and learning
- Probabilistic control synthesis for large-scale networks
- Secure optimal control synthesis
- Trade-offs between security and optimality
- Model-free optimal control synthesis
- Human-in-the-loop control synthesis (e.g., human-robot collaborative tasks)



















### Thank You



### Sampling-Based Optimal Control Synthesis

- Y. Kantaros and M. M. Zavlanos, "Sampling-Based Optimal Control Synthesis for Multi-Robot Systems under Global Temporal Tasks," IEEE Transactions on Automatic Control, 2018.
- Y. Kantaros, B. Johnson, S. Chowdhury, D. J. Cappelleri, and M. M. Zavlanos, "Control of Magnetic Microrobot Teams for Temporal Micromanipulation Tasks," IEEE Transactions on Robotics, 2018.

#### STyLuS\*: large-<u>S</u>cale <u>T</u>emporal <u>L</u>ogic optimal <u>S</u>ynthesis

- Y. Kantaros and M. M. Zavlanos, "STyLuS\*: A Temporal Logic Optimal Control Synthesis Algorithm for Large-Scale Multi-Robot Systems," International Journal of Robotics Research, under review.
- Y. Kantaros and M. M. Zavlanos, "Temporal Logic Optimal Control for Large-Scale Multi-Robot Systems: 10<sup>400</sup> States and Beyond," 57th IEEE Conference on Decision and Control, 2018.

#### Distributed Sampling-Based Optimal Control Synthesis

• Y. Kantaros and M. M. Zavlanos, "Distributed Optimal Control Synthesis for Multi-Robot Systems under Global Temporal Tasks," 9th ACM/IEEE International Conference on Cyber- Physical Systems, 2018.

















