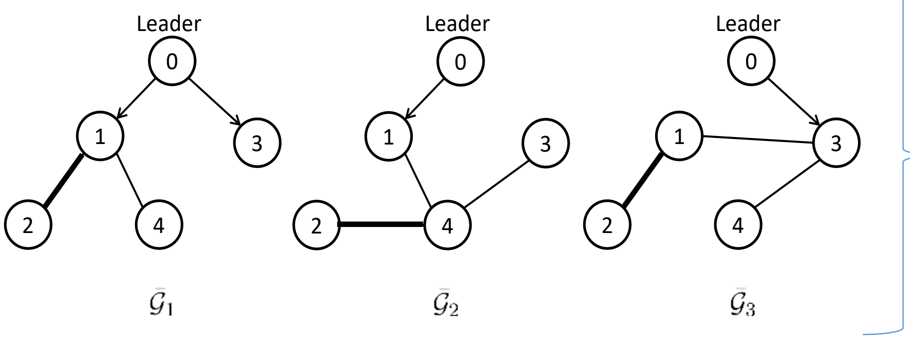
Event & Self-Triggered Approximate Leader-Follower Consensus with Resilience to Byzantine Adversaries

Intermittent Measurements

- Intermittency can result in time varying topologies



$$\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$$
$$H_p = L_p + D_p,\ p = 1,\ 2,\ 3$$

- Switched systems theory provides a framework for analyzing the stability and performance of the resulting switched/hybrid dynamic system

- Dynamics matter for these problems because of the need to develop predictors
  - Frameworks from Nonsmooth Analysis provide toolsets to allow switching with uncertainty
  - Network specific challenges: connectivity, fixed or time-varying topology, directed/undirected, signed/unsigned, resiliency

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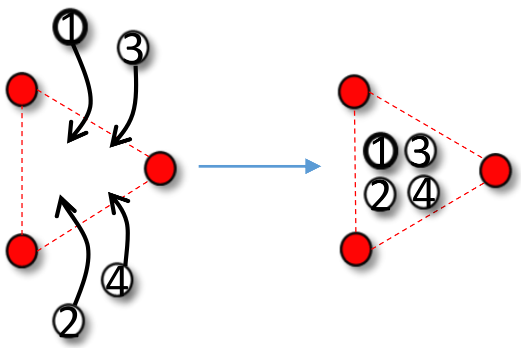
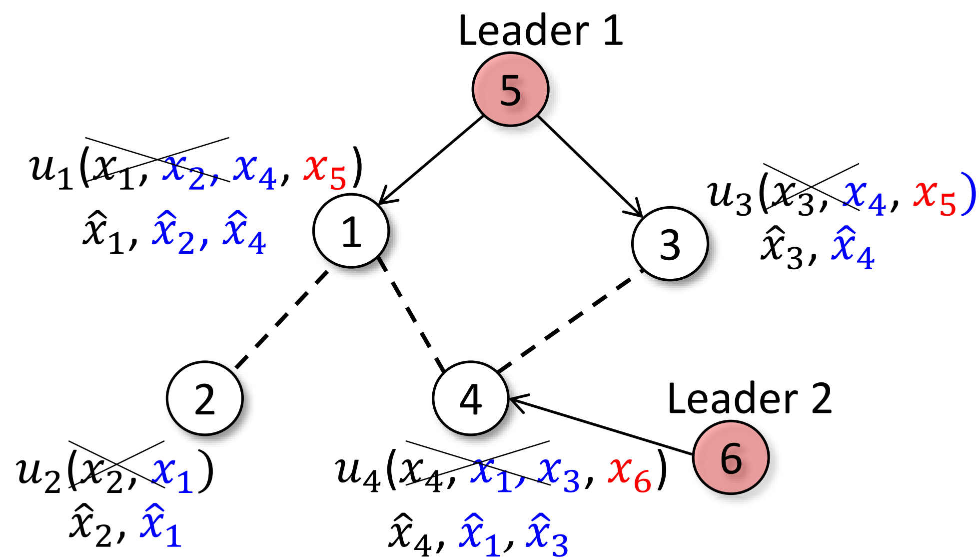
Example: Distributed Event-Trigger



Goal: Agents converge to the convex hull spanned by the leaders



Dynamics:

$$\dot{x}_i = Ax_i, \qquad i \in \mathcal{V}_{\mathcal{L}}$$
$$\dot{x}_i = Ax_i + Bu_i, \; i \in \mathcal{V}_{\mathcal{F}}$$

Estimate dynamics:

$$\dot{\hat{x}}_j(t) = A\hat{x}_j(t), \; j \in \{i\} \cup \mathcal{N}_{\mathcal{F}i}, \; t \in \left[t_k^j, t_{k+1}^j\right) \Rightarrow$$ **No Comm.**

$$\hat{x}_j\left(t_k^j\right) = x_j\left(t_k^j\right) \Rightarrow$$ **Comm.**

Controller Design

Controller:

$$u_i = K\hat{z}_i$$

$$\hat{z}_i = \sum_{j \in \mathcal{V}_\mathcal{F}} a_{ij}\left(\hat{x}_j - \hat{x}_i\right) + \sum_{j \in \mathcal{V}_\mathcal{L}} a_{ij}\left(x_j - \hat{x}_i\right), \; i \in \mathcal{V}_\mathcal{F}$$

Known

where $K = B^T P$

$$P : PA + A^T P - 2\delta_{\min} PBB^T P + \delta_{\min} I_n < 0$$

Estimate Error: $e_i(t) = \hat{x}_i(t) - x_i(t), \; i \in \mathcal{V}_\mathcal{F}$

Known

Closed-loop dynamics:

$$\varepsilon_i = \hat{z}_i - \sum_{j \in \mathcal{V}_\mathcal{F}} a_{ij}\left(e_i - e_j\right) - \sum_{j \in \mathcal{V}_\mathcal{F}} a_{ij} e_i, \quad i \in \mathcal{V}_\mathcal{F}$$

unknown unknown

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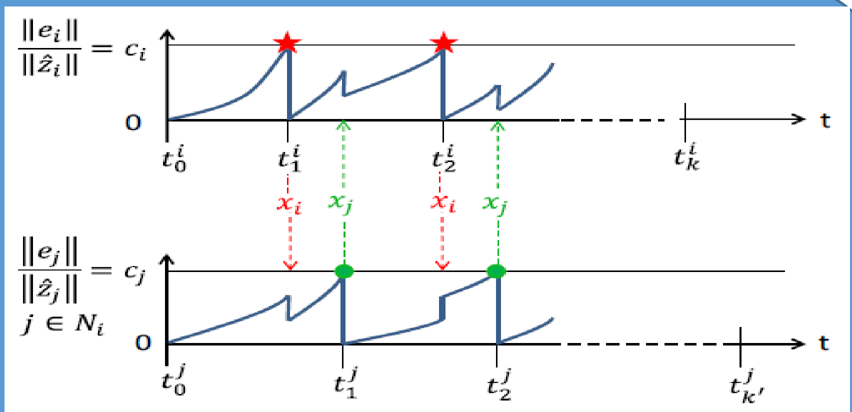
When to Communicate?



T. H. Cheng, Z. Kan, J. R. Klotz, J. M. Shea, W. E. Dixon, "Event-Triggered Control of Multi-Agent Systems for Fixed and Time-Varying Network Topologies," **IEEE Trans. Autom. Control**, Vol. 62(10), pp. 5365-5371, 2017.

Nonlinear Analysis

$$V = \varepsilon^T \left(I_F \otimes P\right) \varepsilon$$

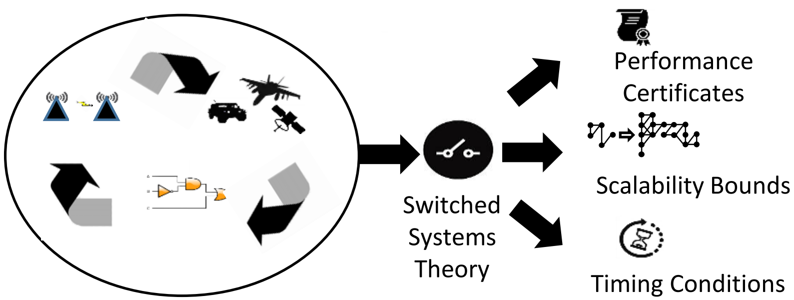
$$\dot{V} \leq -\sum_{i \in \mathcal{V}_\mathcal{F}} \left[\left(\delta_1 - \frac{k_2}{\beta} \right) \|\hat{z}_i\|^2 - (k_1 + k_2 \beta) \|e_i\|^2 \right] - \delta_2 \varepsilon^T \varepsilon$$

$$\dot{V} \leq -\delta_2 \varepsilon^T \varepsilon$$

$$\|\varepsilon(t)\| \leq \|\varepsilon(t_0)\| e^{-\gamma t}$$

$$x_\mathcal{F} \to -\left(\mathcal{L}_\mathcal{F}^{-1} \mathcal{L}_\mathcal{L} \otimes I_n\right) x_\mathcal{L} \quad \text{as} \quad t \to \infty$$

Nonsmooth Analysis



Trigger Condition

$$c_i = \sqrt{\frac{\eta_i \left(\delta_1 - \frac{k_2}{\beta} \right)}{(k_1 + k_2 \beta)}}$$

Minimum Interval Event Time

$$\tau \geq \frac{1}{\max\{\bar{c}_0, \bar{c}_1\}} \ln \left(\frac{1}{F} \sqrt{\frac{\eta_h \left(\delta_1 - \frac{k_2}{\beta} \right)}{(k_1 + k_2 \beta)}} + 1 \right)$$

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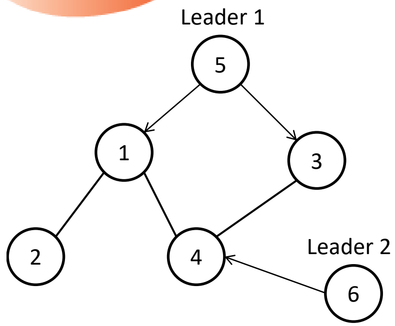
Simulation



$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

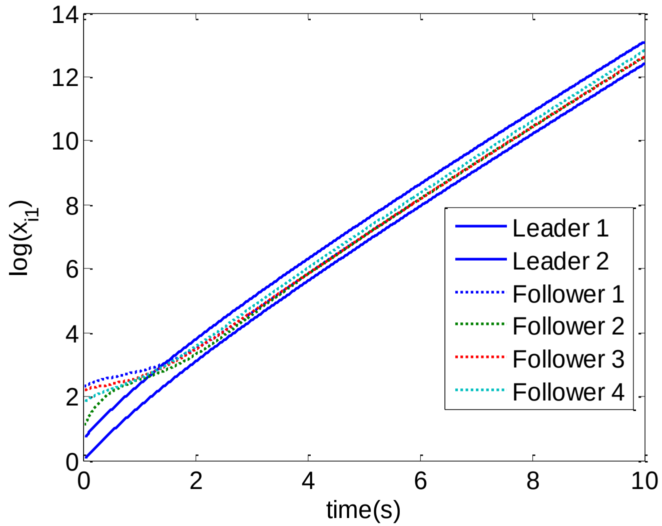
$$\mathcal{L}_{\mathcal{F}} = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}$$

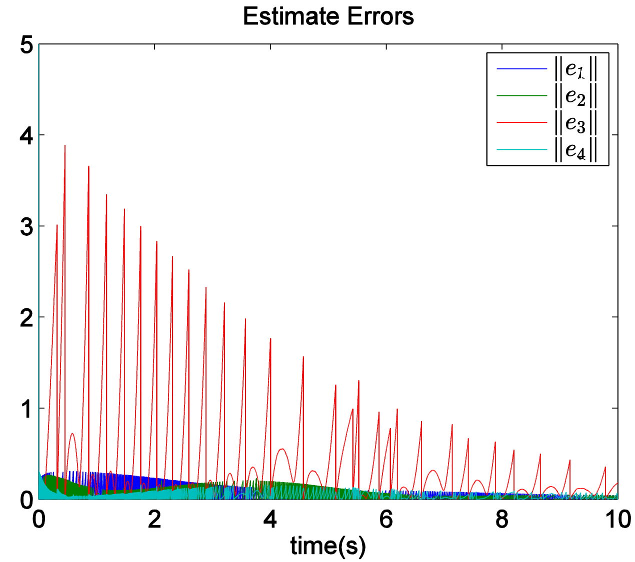
$$P = \begin{bmatrix} 15.897 & 5.969 \\ 5.969 & 5.266 \end{bmatrix}$$

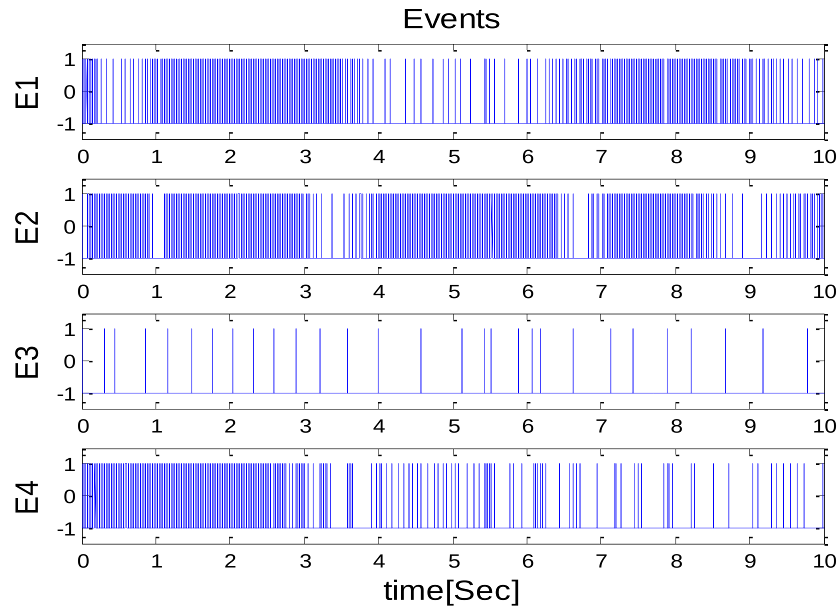
$$K = \begin{bmatrix} 5.969 & 5.266 \end{bmatrix}$$

1: Triggered

-1: Not triggered







Ave: 29 ms

Ave: 10 ms

Ave: 78 ms

Ave: 10 ms

On-going Efforts

Event-Triggered Control

- Opportunistically select when to communicate (dynamics-based trigger condition)
- Require continuous listening (expensive)

Self-Triggered Control

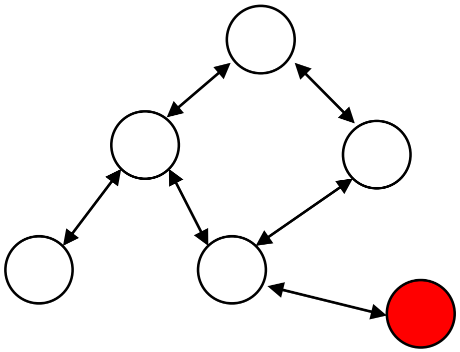
- Eliminates continuous listening (least expensive?)
- Predict (uncertainty?) when to send/listen (asynchrony?)

Byzantine adversary

- Categorize? False information (How to know/detect?)
- Impart undesirable influence on network
  - Partition, wrong objective, data exfiltration

Open Questions

- How to model?
  - Signed graphs? Adversary classification?
- Network characteristics?
  - Power boost? Connectivity? Asynchrony?
- Game Theory Methods?
- Resiliency? Protecting Information?

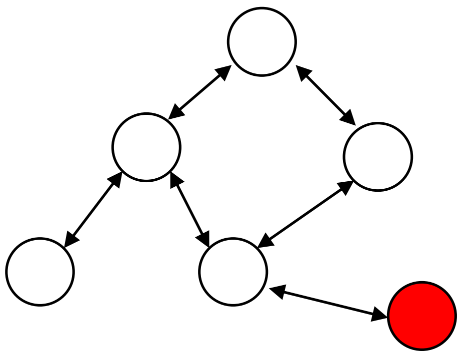


Example: Self-Trigger LF Consensus

- Undirected network of followers $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$
- Perform self-triggered approximate leader-follower consensus

$$\limsup_{t \to \infty} \|e_i(t)\| \leq \varepsilon \quad \forall i \in \mathcal{V}$$

$$e_{1,i}(t) \triangleq x_i(t) - x_0(t)$$



- Byzantine adversary detection error

$$e_{2,j}(t) = \hat{x}_j(t) - x_j(t)$$

- LTI dynamics of followers

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t)$$

- LTI dynamics of the leader

$$\dot{x}_0(t) = Ax_0(t) + Bu_0(t)$$

LTI known dynamics facilitate Byzantine agent detection.

How to extend to uncertain nonlinear dynamics?

Byzantine Detection

Check if agent was cooperative during previous times

$$\hat{x}_j\left(t\right) = e^{A\left(t-t^i_{k-1}\right)}\hat{x}_j\left(t^i_{k-1}\right) \qquad x_j\left(t^i_s\right)\ \forall s \in \{0, 1, ..., k-1\}$$

Analyze the maximum growth rate for $e_{2,j}\left(t\right) = \hat{x}_j\left(t\right) - x_j\left(t\right)$

$$V_{2,i}\left(e_{2,i}\left(t\right)\right) \triangleq \frac{1}{2} e^T_{2,i}\left(t\right) e_{2,i}\left(t\right)$$

$$V_{2,i}\left(e_{2,i}\left(t\right)\right) \le \left(\frac{\sqrt{2}\xi_i\left(e^{\lambda_{max}(A)\left(t-t^i_k\right)}-1\right)}{2}\right)^2$$

$$\left\|e_{2,j}\left(t\right)\right\| \le \xi_j\left(e^{\lambda_{max}(A)\left(t-t^i_{k-1}\right)}-1\right) t \in \left[t^i_{k-1}, t^i_k\right)$$

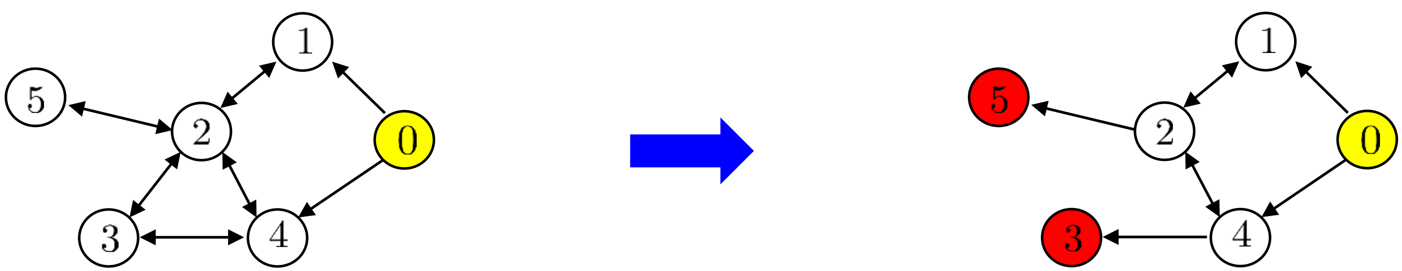
Detection Condition

$$\Xi_j\left(t^i_k\right) = \lim_{t\to t^i_k}\left(\left\|e_{2,j}\left(t\right)\right\| - \xi_j\left(e^{\lambda_{max}(A)\left(t-t^i_{k-1}\right)}-1\right)\right)$$

Agents alter the network topology due to the presence of the Byzantine agents

Fixed, Balanced, and Undirected Graph

Time-Varying, Unbalanced, and Directed Graph



Controller/Observer

Distributed controller

$$u_i(t) = K\tilde{z}_i(t) + K(\hat{x}_i(t) - x_i(t))$$

Connectivity parameter

$$\tilde{z}_i(t) = \sum_{j\in\mathcal{N}_i(\mathcal{G})} \mu_{ij} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + d_i(x_0(t) - \hat{x}_i(t))$$

$$\mu_{ij} = \begin{cases} 1, & j \in \mathcal{C}_i(t_k^i) \\ 0, & j \in \mathcal{B}_i(t_k^i) \end{cases}$$

Neighbor state estimator

$$\dot{\hat{x}}_j(t) = A\hat{x}_j(t), \quad t \in [t_k^i, t_{k+1}^i), \quad j \in \mathcal{N}_i(\mathcal{G}) \cup \{i\}$$

$$\hat{x}_j(t) = x_j(t_k^i)$$

Nonsmooth Stability Analysis

- Time-varying unbalanced directed graph
- Triggered communication

$$V_1(e_1(t)) \triangleq e_1^T(t)(I_N \otimes P)e_1(t)$$

$$\dot{V}_1(g(t)) \overset{a.e.}{\in} \dot{\tilde{V}}_1(g(t))$$

$$\dot{\tilde{V}}_1(g(t)) \subseteq \{e_1^T(t)(I_N \otimes (A^T P + PA))e_1(t)\} - \{e_1^T(t)(H_{\sigma(t)} \otimes 2PBB^T P)e_1(t)\}$$
$$+ \{e_2^T(t)((I_N - H_{\sigma(t)}) \otimes 2PBB^T P)e_1(t)\} - \{(1_N^T \otimes 2u_0^T(t)B^T P)e_1(t)\}$$

$$\dot{V}_1(e_1(t)) \overset{a.e.}{\leq} -\phi_1 \|e_1(t)\|^2 + \bar{\delta}$$

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Reputation-Based Event-Triggered Formation Control and Leader Tracking with Resilience to Byzantine Adversaries

Submitted ACC 2020

Motivation

Autonomous Escort: Leader-Follower Model

Goal: Perform formation control and leader tracking

……with controllers that are

- Distributed
- Event-Triggered
- Resilient to Byzantine adversaries

Byzantine Model

Common threats for a mobile network

- Denial-of-Service (DoS)
- Time-Delay Switch (TDS)
- False Data Injection (FDI)

Byzantine attack: a more general threat where communication can be delayed, corrupted, and/or interrupted arbitrarily

Current Assumptions:

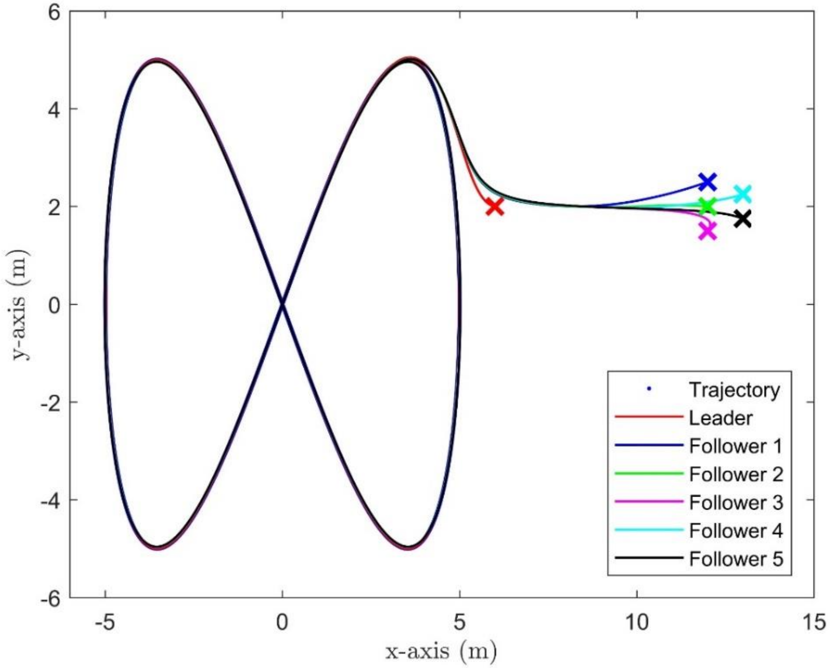
- Only followers can become Byzantine
- No teamwork between Byzantine agents

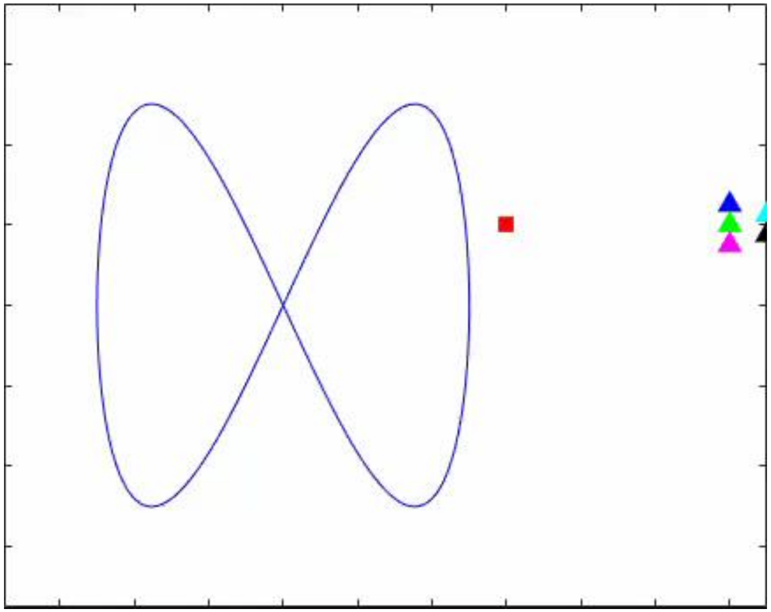
- **Type I** - Physically remains within network; FDI

- **Type II** - Abandons network

Preliminary Result (TAC 2017)

Event-Triggered Consensus: Known Linear Dynamics





Controller

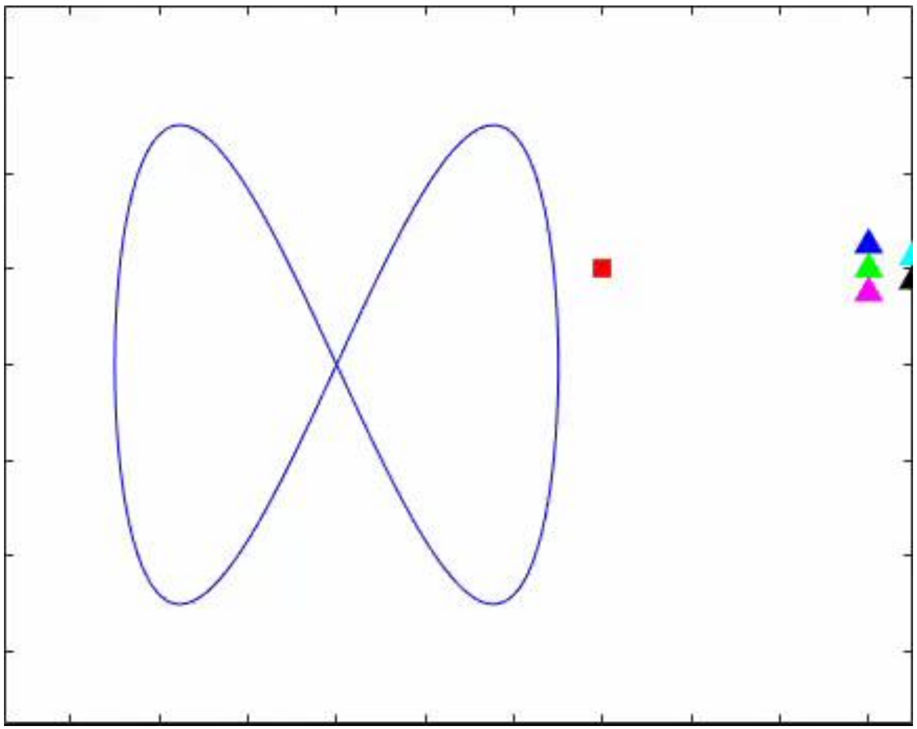
$$u_i(t) \triangleq K z_i(t) + K e_{2,i}(t) \quad e_{2,i}(t) \triangleq \hat{x}_i(t) - x_i(t)$$

$$z_i(t) \triangleq \sum_{j \in \mathcal{N}_i(t)} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + d_i\left(x_0(t) - \hat{x}_i(t)\right)$$

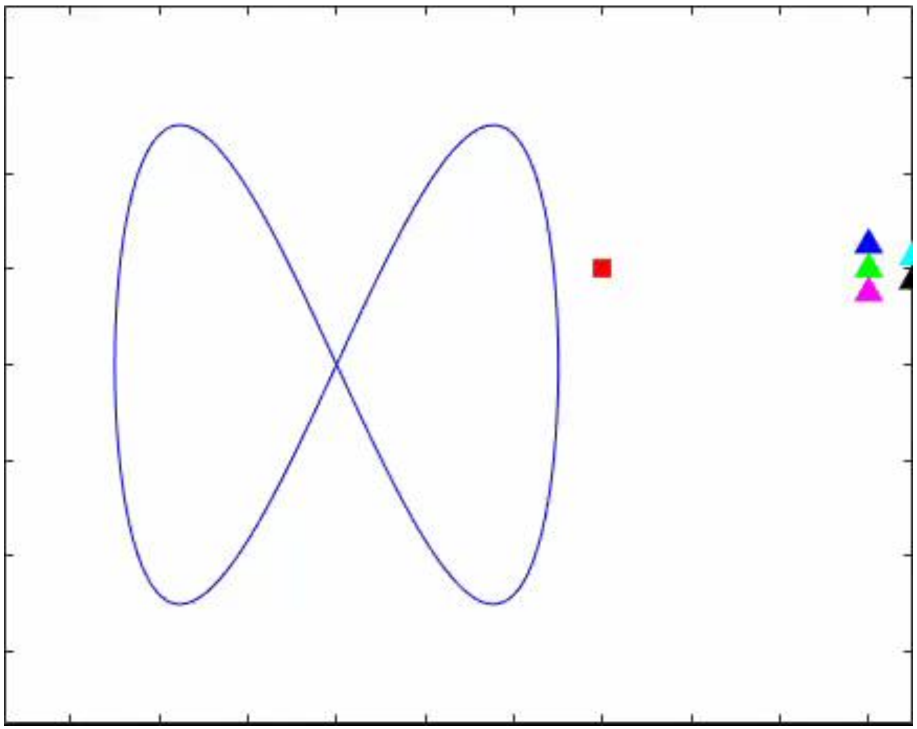
! Compute update based on what all neighbors are doin

Preliminary Result (TAC 2017)

Type I Byzantine Adversary

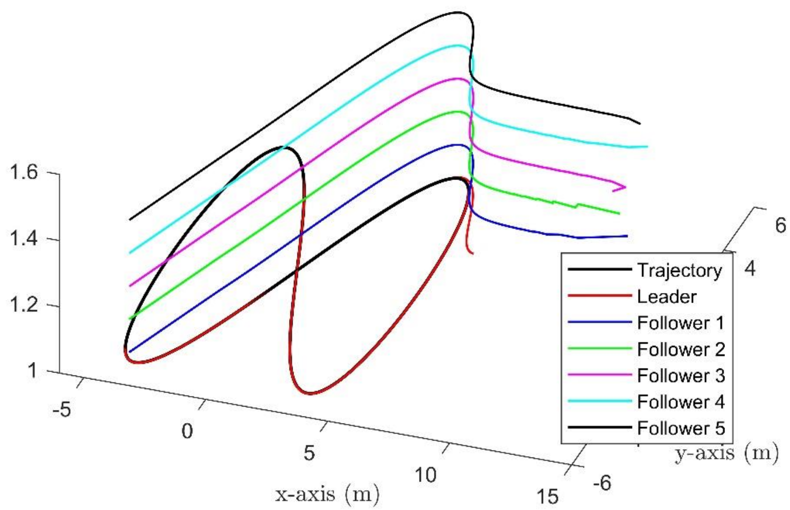


Type II Byzantine Adversary

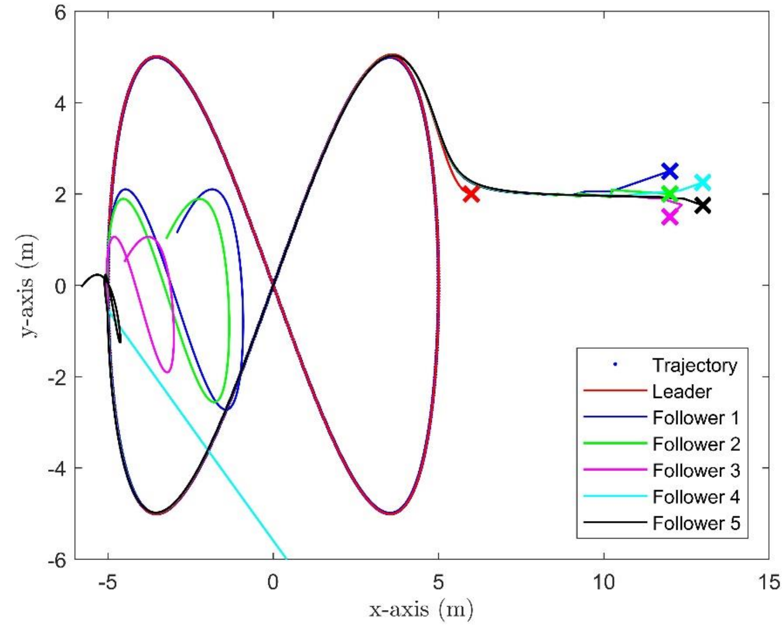


Preliminary Result (TAC 2017)

Type I Byzantine Adversary (Follower 5)



Type II Byzantine Adversary (Follower 4)



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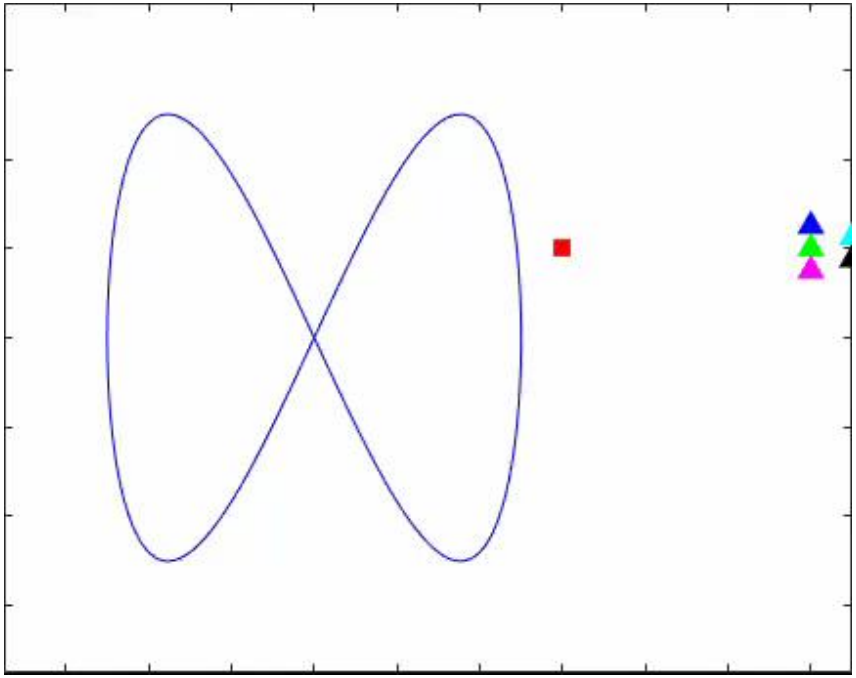
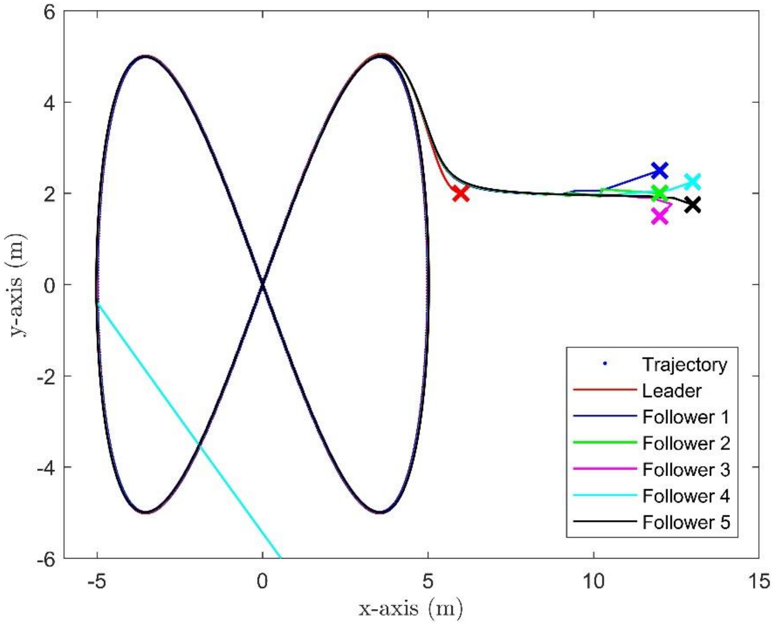
Preliminary Result (CDC2019)

Resilient Method with Type I & Type II Byzantine Agents





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Kickoff Result (CDC 2019)

Controller

$$u_i(t) \triangleq K\tilde{z}_i(t) + Ke_{2,i}(t)$$

$$\tilde{z}_i(t) \triangleq \sum_{j\in\mathcal{N}_i(t)} \mu_{ij}(t)\, a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + d_i\left(x_0(t) - \hat{x}_i(t)\right)$$

$$\mu_{ij}(t) \triangleq \begin{cases} 1, & j \in \mathcal{C}_i\left(t_k^i\right) \\ 0, & j \in \mathcal{B}_i\left(t_k^i\right) \end{cases}$$

Detector

$$\Xi_j\left(t_k^i\right) \triangleq \lim_{t\to t_k^i}\left(\left\|e_{2,j}(t)\right\| - \Psi_j\left(t - t_{k-1}^i\right)\right) \qquad j \in \mathcal{N}_i\left(t_k^i\right) \quad t \in \left[t_k^i, t_{k+1}^i\right)$$

$$\left\|e_{2,j}(t)\right\| = \left\|e^{A\left(t-t_{k-1}^i\right)} x_j\left(t_{k-1}^i\right) - x_j(t)\right\|$$

$$\Psi_j\left(t - t_{k-1}^i\right) \triangleq \frac{S_{max}(B) M_j}{S_{max}(A)}\left(e^{S_{max}(A)\left(t-t_{k-1}^j\right)} - 1\right) \qquad \left\|u_j(t)\right\| \le M_j \in \mathbb{R}_{>0}$$

Limitations of Detector

- Exact model knowledge
- Bound on neighbor’s control
- No re-integration

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Problem Formulation

Problem Formulation

- Consider a heterogeneous multi-agent system of *N* follower agents and a single leader
- Influence between followers: Weight Undirected Network Topology

$$\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$$
$$\mathcal{V} \triangleq \{1, 2, ..., N\}$$
$$\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$$
$$\mathcal{A} \triangleq [a_{ij}] \in \mathbb{R}_{\geq 0}^{N \times N}$$

- Dynamics of agent *i* (control affine)

$$\dot{x}_i(t) \triangleq f_i(x_i(t)) + g_i(x_i(t))\, u_i(t) + d_i(t)$$

$x_i =$ Position of agent *i*

$x_i : [0, \infty) \to \mathbb{R}^m$

$u_i =$ Control of agent *i*

$u_i : [0, \infty) \to \mathbb{R}^n$

Uncertain drift dynamics of agent *i*: $f_i : \mathbb{R}^m \to \mathbb{R}^m$

Known control effectiveness of agent *i*: $g_i : \mathbb{R}^m \to \mathbb{R}^{m \times n}$

Disturbance acting on agent *i*: $d_i : [0, \infty) \to \mathbb{R}^m$

Problem Formulation

Objective: Design a controller for the followers

- Formation control and leader tracking (FCLT)
- Distributed & Event-Triggered
- Resilient to Byzantine adversaries

Assumptions

- The uncertain drift dynamics are continuously differentiable and bounded given a bounded argument
- The control effectiveness is full-row rank and bounded given a bounded argument
- The disturbance is bounded
- All followers are initially cooperative
- The leader is cooperative for all time
- All agents can measure their state
- The control and state of the leader are bounded
- The uncertain drift dynamics are linear in the uncertain parameters
- At least one state measurement is accurate (used in trust model)
- The graph $\mathcal{G}_C(t)$ is connected for all time

Trust Model

Idea: Make edge weights a function of trust

Idea: Multi-point authentication

Given r state measurements from neighbor $j \in \mathcal{N}_i\left(t_k^i\right)$

$$\Psi_{ij}\left(t_k^i\right) \triangleq \sum_{p=1}^{r-1} \sum_{q>p}^{r} \left\| x_{j,p}\left(t_k^i\right) - x_{j,q}\left(t_k^i\right) \right\|$$

$x_{j,1}\left(t_k^i\right)$ = communicated state

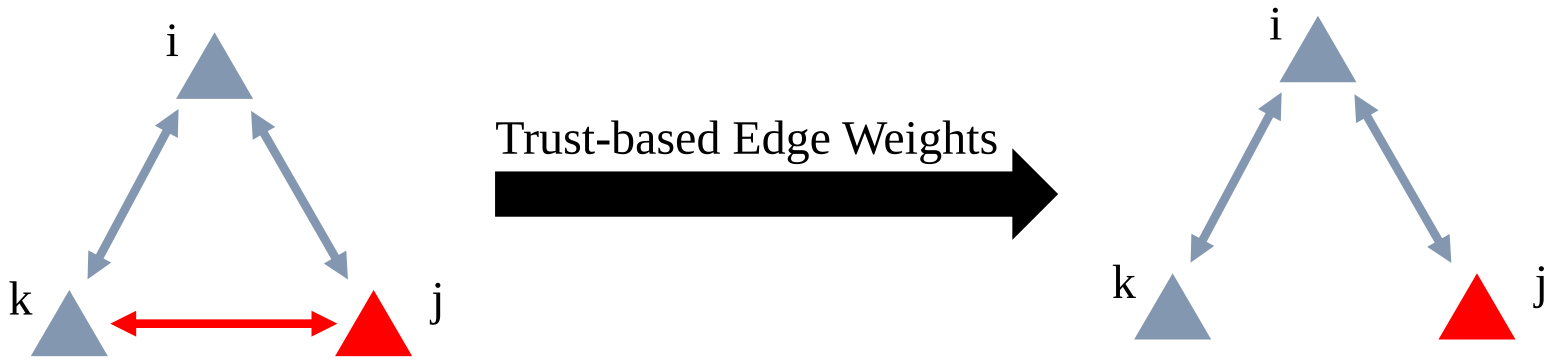
$x_{j,2}\left(t_k^i\right)$ = sensed state

Let $S_i \triangleq \left\{ t_k^i \in \mathbb{R}_{\geq 0} : t - t_{\text{reset}}^i \leq t_k^i < t \right\}$

$$\tau_{ij}(t) \triangleq \begin{cases} 1, & |S_i| = 0 \\ \frac{1}{|S_i|} \sum_{t_k^i \in S_i} e^{-s_1 \Psi_{ij}\left(t_k^i\right)}, & |S_i| \neq 0 \end{cases}$$

Controls rate of change of trust

Reputation Model



Cannot isolate Byzantine agent from MAS

$$\mathcal{N}_{ij}(t) \triangleq \mathcal{N}_i(t) \cap \mathcal{N}_j(t)$$

$$\dot{\zeta}_{ij}\left(t_{k+1}^i\right) \triangleq \operatorname{proj}\left(\underbrace{\eta_{\tau i}\left(\tau_{ij}(t) - \zeta_{ij}\left(t_k^i\right)\right)}_{\text{Accounts for what i thinks of j}} + \sum_{n \in \mathcal{N}_{ij}\left(t_k^i\right)} \eta_{\zeta i} \underbrace{\zeta_{in}\left(t_k^i\right)\left(\zeta_{nj}\left(t_k^i\right) - \zeta_{ij}\left(t_k^i\right)\right)}_{\text{Accounts for what k thinks of j weighted by what i thinks k}}\right)$$

Accounts for what i thinks of j

Accounts for what k thinks of j weighted by what i thinks k

$$\operatorname{proj}(\dot{x}(t)) \triangleq \begin{cases} \dot{x}(t), & x_{\min} < x(t) \wedge x(t) < x_{\max} \\ \dot{x}(t), & x_{\min} = x(t) \wedge \dot{x}(t) > 0 \\ \dot{x}(t), & x_{\max} = x(t) \wedge \dot{x}(t) < 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\zeta_{ij}(t) \in [0, \zeta_{\max}]$$

Edge Weight

Edge weight

$$a_{ij}(t) \triangleq \begin{cases} \frac{\zeta_{ij}(t_k^i)}{\zeta_{\max}}, & \frac{\zeta_{ij}(t_k^i)}{\zeta_{\max}} \geq \zeta_{\min} \wedge j \in \mathcal{N}_i(t) \\ 0, & \frac{\zeta_{ij}(t_k^i)}{\zeta_{\max}} < \zeta_{\min} \vee j \notin \mathcal{N}_i(t) \end{cases} \qquad \zeta_{\min} \in [0,1]$$

Cooperative & Byzantine neighbor set

$$\mathcal{C}_i(t) \triangleq \{j \in \mathcal{N}_i(t) : a_{ij}(t) \neq 0\}$$

$$\mathcal{B}_i(t) \triangleq \mathcal{N}_i(t) \setminus \mathcal{C}_i(t)$$

Benefits

- No exact model knowledge needed
- No bounds on neighbor quantities needed
- Enables re-integration of rehabilitated agents

Dynamics

Assumption: The uncertain drift dynamics are linear in the uncertain parameters, i.e.,

$$f_i\left(x_i\left(t\right)\right) = y_i\left(x_i\left(t\right)\right)\theta_i$$

where $y_i : \mathbb{R}^m \to \mathbb{R}^{m\times q_i}$ is a measurable regressor matrix and $\theta_i \in \mathbb{R}^{q_i}$ is a column vector of bounded but unknown coefficients.

Follower *i* dynamics:

$$\dot{x}_i\left(t\right) = y_i\left(x_i\left(t\right)\right)\theta_i + g_i\left(x_i\left(t\right)\right)u_i\left(t\right) + d_i\left(t\right)$$

Estimate of uncertain parameters: $\hat{\theta}_i : [0,\infty) \to \mathbb{R}^{q_i}$

where $\tilde{\theta}_i : [0,\infty) \to \mathbb{R}^{q_i}$

$$\tilde{\theta}_i\left(t\right) \triangleq \theta_i - \hat{\theta}_i\left(t\right)$$

Controller, Observer, and Event-Trigger of Follower *i*:

$$u_i(t) \triangleq g_i^+(x_i(t))\left(k_1 z_i(t) + k_2 e_{2,i}(t)\right)$$

$$z_i(t) \triangleq \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\left(\hat{x}_j(t) - \hat{x}_i(t) - v_j + v_i\right) + b_i(t)\left(v_i + x_0(t) - \hat{x}_i(t)\right)$$

Follower *i* knows the formation

Positive only if connected to leader

$$\dot{\hat{x}}_j(t) \triangleq y_j(\hat{x}_j(t))\hat{\theta}_j(t), \; t\in\left[t_k^i, t_{k+1}^i\right) \qquad j\in\mathcal{N}_i(t)\cup\{i\}$$

$$\hat{x}_j\left(t_k^i\right) \triangleq x_{j,1}\left(t_k^i\right)$$

$$\dot{\hat{\theta}}_j(t) \triangleq -\mathrm{proj}\left(\Gamma_j^{-1} y_j^T(\hat{x}_j(t))\, e_{2,i}(t)\right)$$

User-defined positive definite matrix

Parameter used to exclude Zeno behavior

$$t_{k+1}^i \triangleq \inf\left\{t > t_k^i : \phi_3\left\|e_{2,i}(t)\right\|^2 \geq \phi_4\left\|z_i(t)\right\|^2 + \frac{\varepsilon}{N}\right\}$$

Positive parameters

Agent-level FCLT error $e_{1,i} : [0, \infty) \to \mathbb{R}^m$

$$e_{1,i}(t) \triangleq x_i(t) - x_0(t) - v_i$$

Estimation error $e_{2,i} : [0, \infty) \to \mathbb{R}^m$

Desired relative orientation

$$e_{2,i}(t) \triangleq \hat{x}_i(t) - x_i(t)$$

$\hat{x}_i : [0, \infty) \to \mathbb{R}^m$ is the estimate of $x_i(t)$

Agent-level closed-loop error systems

$$\dot{e}_{1,i}(t) = f_i(x_i(t)) + k_1 \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(e_{2,j}(t) - e_{2,i}(t)) - k_1 b_i(t) e_{2,i} + k_1 \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(e_{1,j}(t) - e_{1,i}(t))$$
$$-k_1 b_i(t) e_{1,i}(t) + k_2 e_{2,i}(t) + d_i(t) - \dot{x}_0(t)$$

OK

$$\dot{e}_{2,i}(t) = -y_i(\hat{x}_i(t))\tilde{\theta}_i(t) + (y_i(\hat{x}_i(t)) - y_i(x_i(t)))\theta_i - k_1 \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(e_{2,j}(t) - e_{2,i}(t))$$
$$-k_1 \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(e_{1,j}(t) - e_{1,i}(t)) + k_1 b_i(t) e_{2,i}(t) + k_1 b_i(t) e_{1,i}(t) - k_2 e_{2,i}(t) - d_i(t)$$

Result

Ensemble-level closed-loop error systems

$$\dot{E}_1 = \widetilde{N} + N_d - k_1 \left(H\left(t\right) \otimes I_m\right) E_2 - k_1 \left(H\left(t\right) \otimes I_m\right) E_1 + k_2 E_2$$

$$\dot{E}_2 = -Y\left(\hat{X}\right)\widetilde{\Theta} + \left(Y\left(\hat{X}\right) - Y\left(X\right)\right)\Theta - k_2 E_2 - D + k_1 \left(H\left(t\right) \otimes I_m\right) E_2 + k_1 \left(H\left(t\right) \otimes I_m\right) E_1$$

Theorem 1:

The trust model, reputation model, edge weight policy, state observer, and controller ensure E_1 is globally uniformly ultimately bounded in the sense that

$$\left\|E_1\right\| \leq \beta_1 + \beta_2 e^{-\beta_3 t}$$

where $\beta_1,\ \beta_2,\ \beta_3 \in \mathbb{R}_{\geq 0}$ are known constants provided state feedback is available as dictated by the event-trigger and all assumptions are satisfied, including some sufficient gain conditions are satisfied

Sketch of Proof

$$V_1\left(W\left(t\right)\right) \triangleq \frac{1}{2}E_1^T E_1 + \frac{1}{2}E_2^T E_2 + \frac{1}{2}\widetilde{\Theta}^T \Gamma \widetilde{\Theta}$$

$$\frac{1}{2}\left\|E\right\|^2 \leq V_1\left(W\left(t\right)\right) \leq \frac{1}{2}\left\|E\right\|^2 + \frac{\lambda_{\max}(\Gamma)c_6}{2}$$

$$\dot{V}_1\left(W\left(t\right)\right) \overset{a.e.}{\leq} -\phi_1\left\|E_1\right\|^2 - \phi_2\left\|E_2\right\|^2 + \bar{\delta} + \sum_{i\in\mathcal{V}}\left(\phi_3\left\|e_{2,i}\left(t\right)\right\|^2 - \phi_4\left\|z_i\left(t\right)\right\|^2 - \frac{\varepsilon}{N}\right)$$

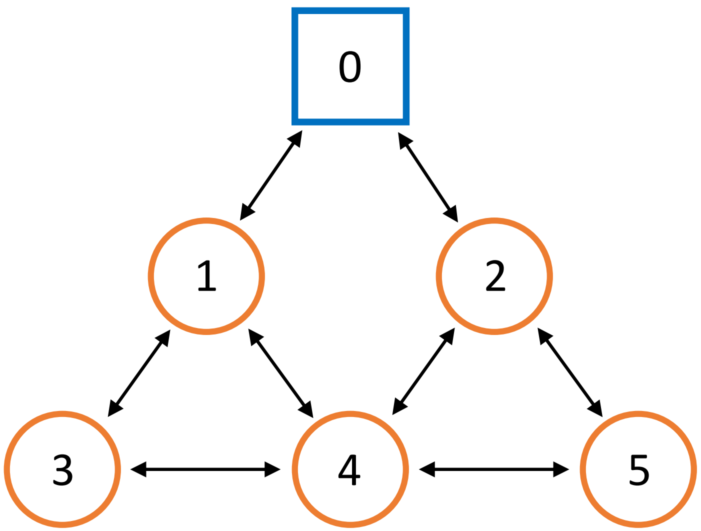
$$\dot{V}_1\left(W\left(t\right)\right) \overset{a.e.}{\leq} -\phi\left(\left\|E_1\right\|^2 + \left\|E_2\right\|^2\right) + \bar{\delta}$$

$$\dot{V}_1\left(W\left(t\right)\right) \overset{a.e.}{\leq} -\phi 2 V_1\left(W\left(t\right)\right) + \phi\lambda_{\max}\left(\Gamma\right)c_6 + \bar{\delta}$$

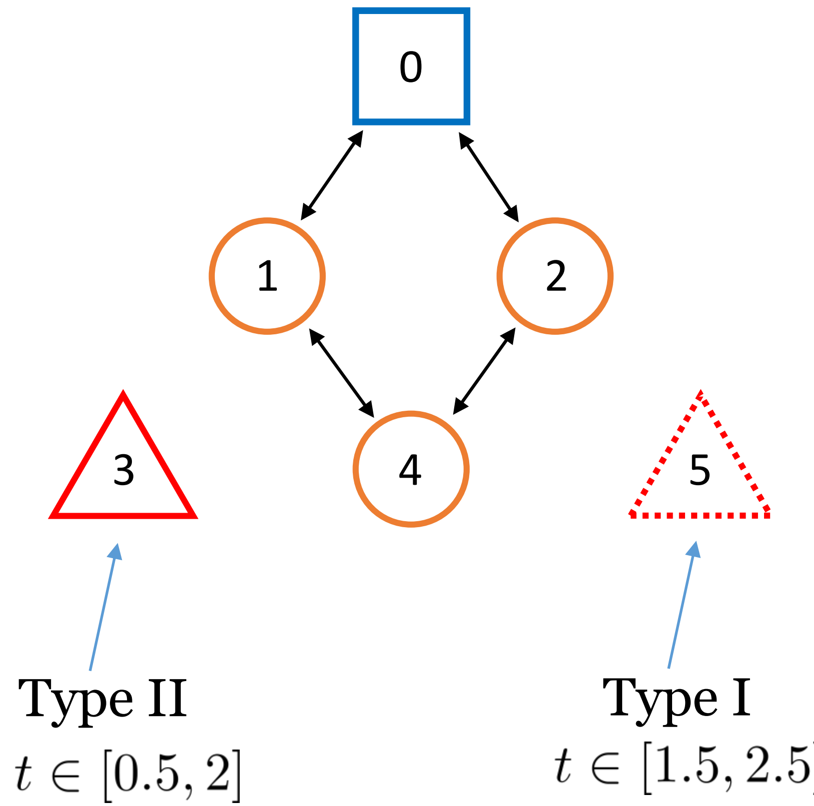
$$V_1\left(W\left(t\right)\right) \leq \frac{\beta_1^2}{2} + \frac{\beta_2^2}{2}e^{-2\beta_3 t}$$

Simulation Results

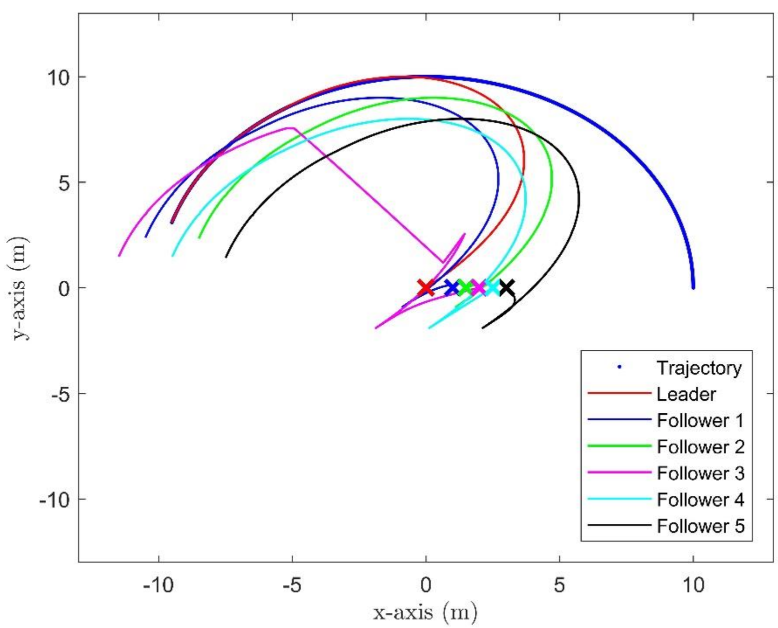
Desired formation

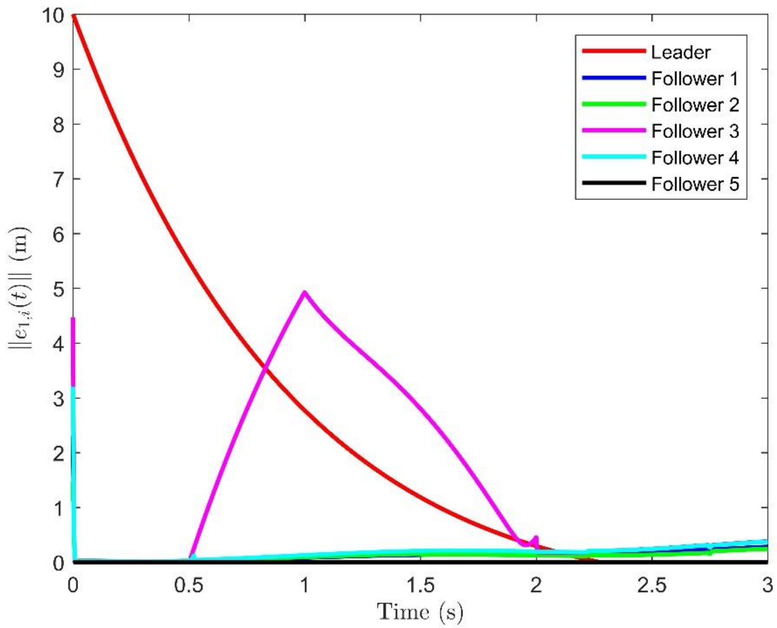


Compromised formation

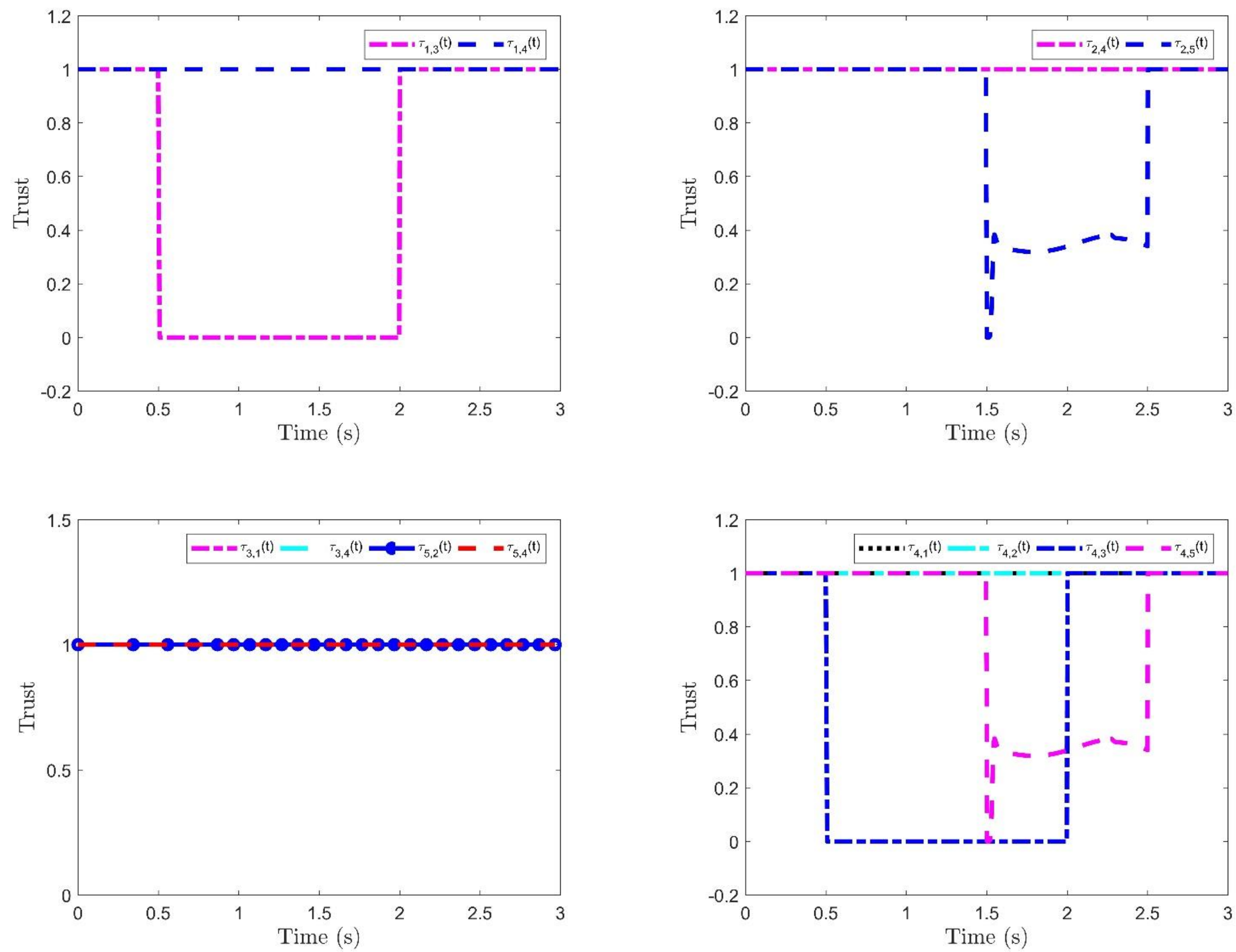


Simulation Results





Simulation Results



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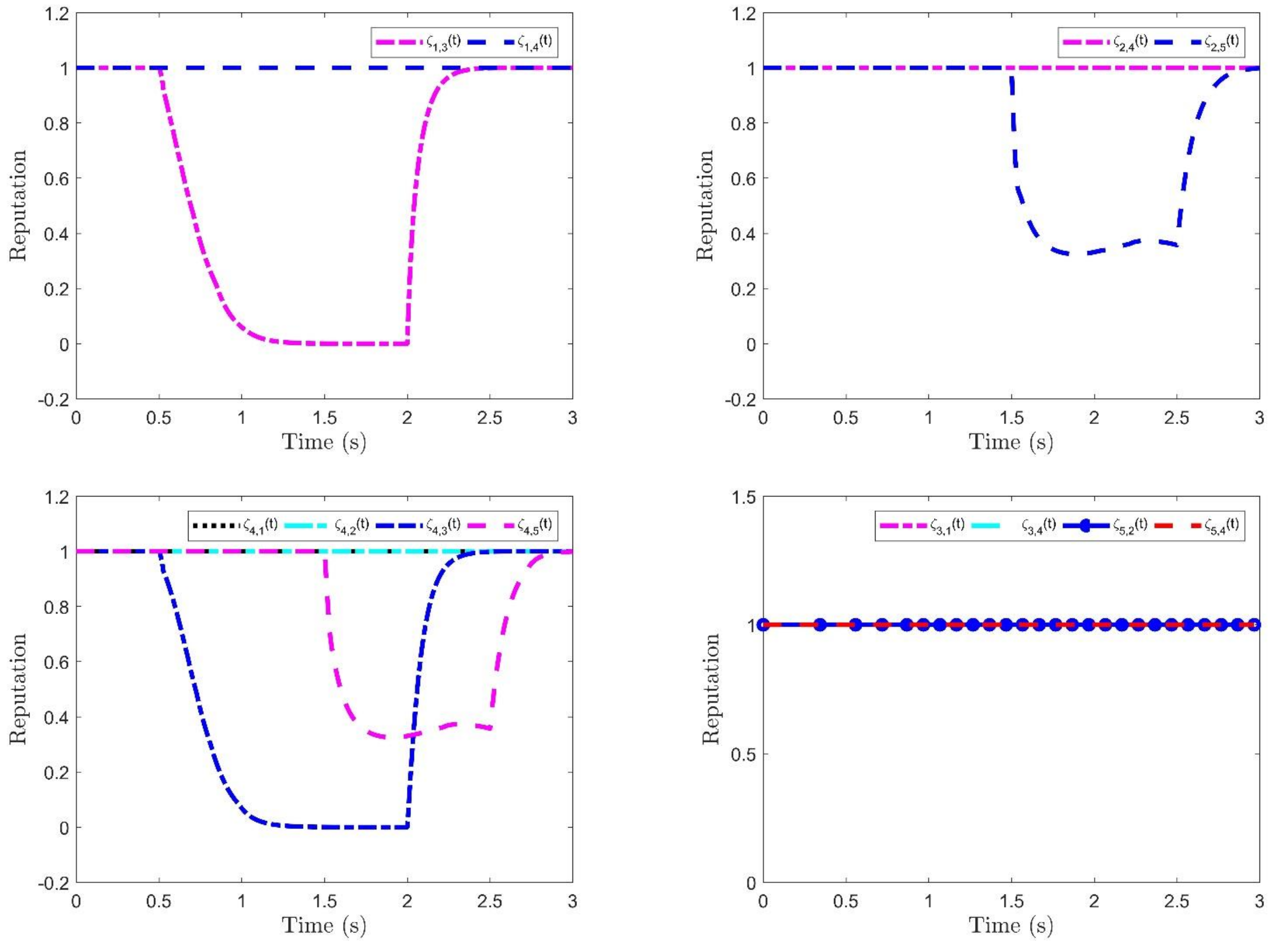
Simulation Results



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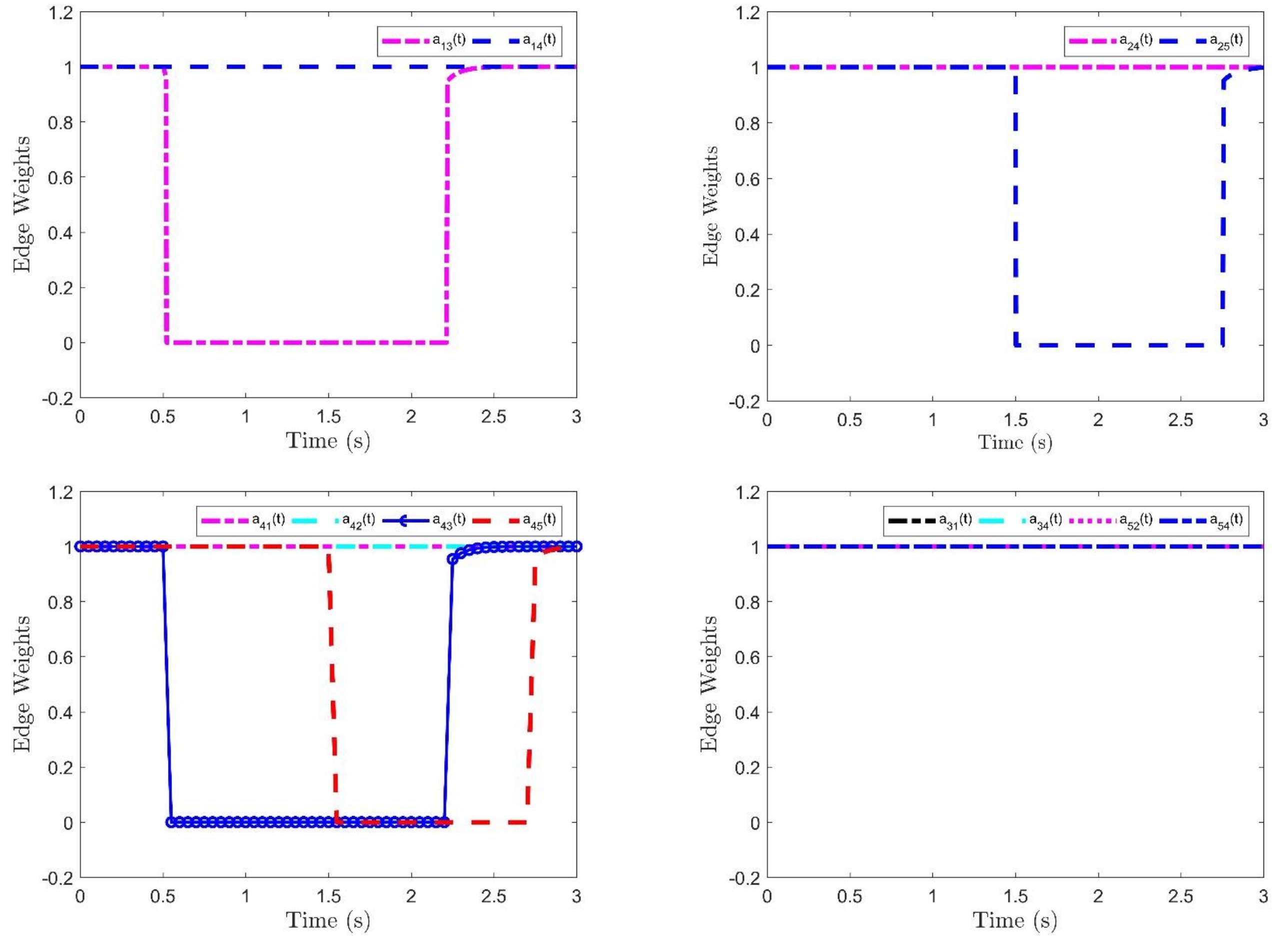

Simulation Results