October 29th / 30th 2020
AACE review

CubeSat Adaptive Attitude Control with Uncertain Drag Coefficient and Atmospheric Density

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Motivations & Contributions

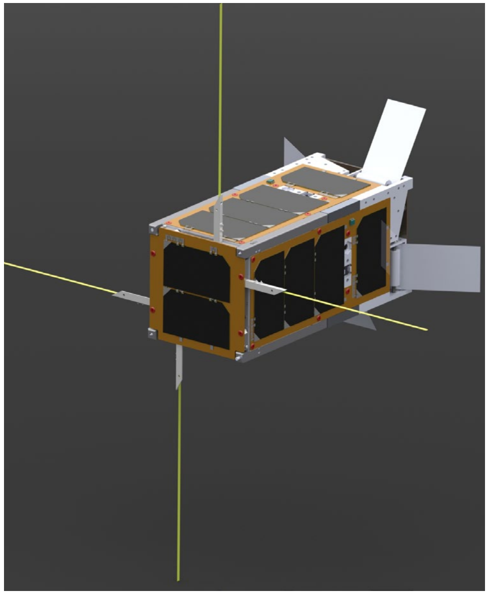
- Space is a contested warfighter operational scenario.

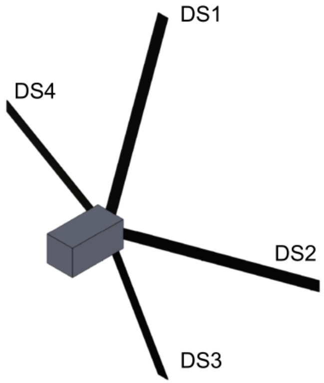
- Resident Space Object ID, on-orbit SSA, and multi-S/C coordination are key elements of the new scenario.

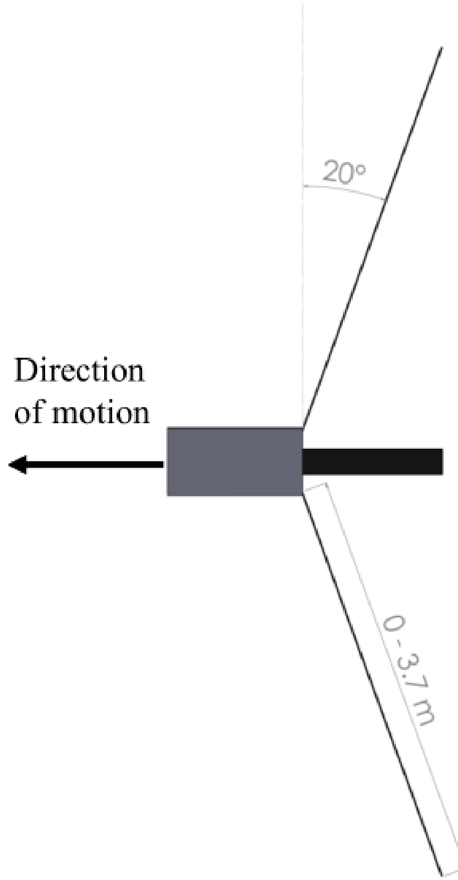
- **The ADAMUS efforts focus on solving the following problems (long term)**
  - Propellant-less relative maneuvering with respect to unknown space objects (cost effective & stealth).
  - Increase autonomy by eliminating dependence on atmospheric density forecasts for differential-drag based maneuvers.
  - Build the foundations to design complex multiple spacecraft missions. **RT4 (networked agent coordination)**

- **This talk shows**
  - Simultaneous attitude tracking and estimation of the S/C's drag parameters (drag coefficient and atmospheric density). **RT2 (Adaptation)**

Attitude Tracking & DMD







- ❑ Can be attached to standard CubeSats
- ❑ Enables the host CubeSat with four repeatedly extendable/retractable surfaces that allow changing the experienced environmental forces and torques.
- ❑ Changes in the matrix of inertia: Gravity Gradient Torque ($\boldsymbol{\tau}_{GG}$)
- ❑ Changes in the experienced aerodynamic torques ($\boldsymbol{\tau}_{AT}$).
- ❑ **GOAL**: use GGT and AT to control the S/C attitude.
- ❑ Dynamics governed by Euler's law:

$$\dot{J}\boldsymbol{\omega} + J\dot{\boldsymbol{\omega}} + \omega^{\times}J\boldsymbol{\omega} = \boldsymbol{\tau}_{AT} + \boldsymbol{\tau}_{GG}$$

$\boldsymbol{\omega}$: angular velocity w. r. t inertial frame
J: inertia matrix

- *D. Guglielmo, S. Omar, R. Bevilacqua, et al., "Drag De-Orbit Device - A New Standard Re-Entry Actuator for CubeSats", Journal of Spacecraft and Rockets, Vol. 56, No. 1 (2019), pp. 129-145.*

Gravity Gradient Torque

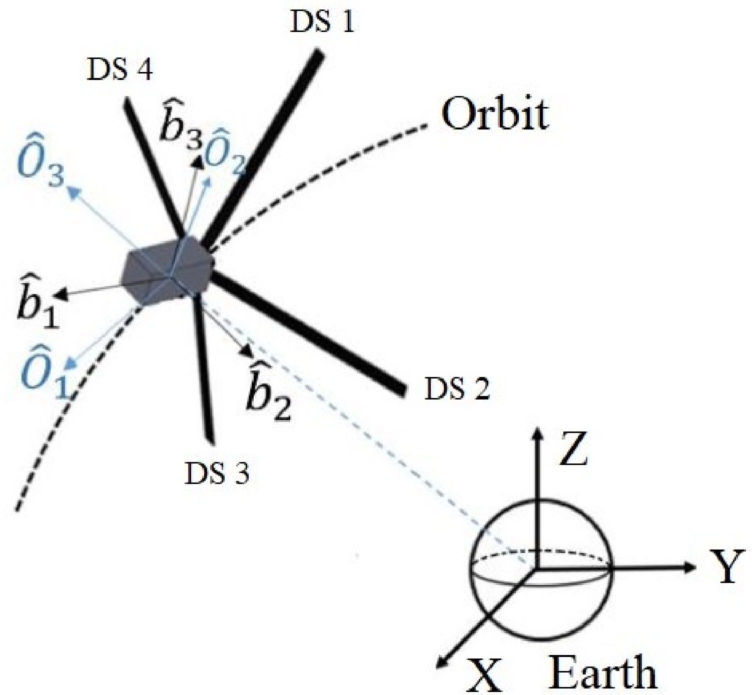
$$\tau_{GG} = \frac{3GM}{\|\boldsymbol{R_c}\|^5} R_c^{\times} J \boldsymbol{R_c}$$

G: Universal gravitational constant
$M_{\oplus}$: Mass of the Earth
$\boldsymbol{R_c}$: Vector from the center of the Earth to the S/C CoM

Aerodynamic Torque

$$\boldsymbol{\tau_{AT}} = \boldsymbol{r}^{\times}\left(-\frac{1}{2}\, C_D \rho L w_b \|\boldsymbol{v}_{\perp}\|^2 \widehat{\boldsymbol{v}_{\boldsymbol{r}}}\right)$$

$\boldsymbol{r}$: vector from CoM to center of pressure.
C_D: **Uncertain** drag coefficient.
ρ: **Uncertain** atmospheric density.
L, w_b: Length and width of drag surface.
$\boldsymbol{v_r}$: S/C-atmosphere velocity vector.
$\boldsymbol{v}_{\perp}$: component of $\boldsymbol{v_r}$ normal to the surface

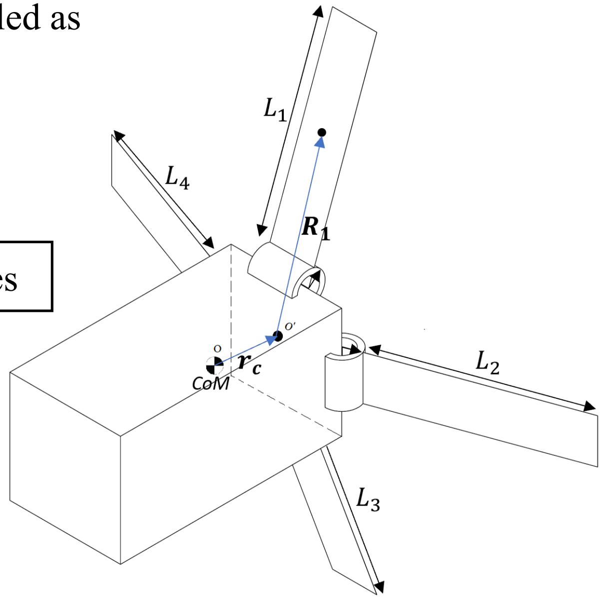


Spacecraft Attitude Dynamics

- The complete nonlinear S/C attitude dynamics in circular LEO can be modeled as

$$\dot{J}\boldsymbol{\omega} + J\dot{\boldsymbol{\omega}} + \omega^{\times}J\boldsymbol{\omega} = \boldsymbol{\tau}_{AT} + \boldsymbol{\tau}_{GG}$$

All altered by moving the drag surfaces



- Deployed portions: modeled as flat plates
- Rolled portions: modeled as thick walled cylinder
- CubeSat body: modeled as rectangular box

- GOAL: Vary “Ls” such that the spacecraft orientation follows a desired trajectory w.r.t the inertial frame.
- Use quaternions to express spacecraft orientation

S/C quaternion: $\boldsymbol{q} = [q_0 \;\; \boldsymbol{q}_v^T]^T$ **Desired quaternion:** $\boldsymbol{q_d} = [q_{0d} \; \boldsymbol{q_{vd}}^T]^T$	**Error quaternion:** $\boldsymbol{e} = [e_0 \;\; \boldsymbol{e_v}^T]^T$ $\dot{\boldsymbol{e}}_v = \frac{1}{2}(e_v^{\times} + e_0 I_3)\tilde{\boldsymbol{\omega}}, \qquad \dot{\boldsymbol{e}}_0 = -\frac{1}{2}\boldsymbol{e}_v^T\tilde{\boldsymbol{\omega}}$ $\lVert\boldsymbol{q}\rVert = \lVert\boldsymbol{q_d}\rVert = \lVert\boldsymbol{e}\rVert = 1$

Control Development

- The relative angular velocity

$$\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \tilde{R}\boldsymbol{\omega}_{\boldsymbol{d}}$$

- The auxiliary error signal

$$\boldsymbol{r} = \dot{\boldsymbol{e}}_{\boldsymbol{v}} + \alpha \boldsymbol{e}_{\boldsymbol{v}}$$

- Open-loop error system

$$\dot{\boldsymbol{r}} = Y\boldsymbol{\Theta} + \alpha \dot{\boldsymbol{e}}_{\boldsymbol{v}},$$

$$Y\boldsymbol{\Theta} = \frac{1}{2}(\dot{e}_v^{\times} + \dot{\mathbf{e}}_{\mathbf{0}} I_3)\tilde{\boldsymbol{\omega}} + \frac{1}{2}(e_v^{\times} + \mathbf{e}_{\mathbf{0}} I_3)\left(J^{-1}\,\boldsymbol{\tau}_{\mathbf{AT}} + J^{-1}\,\boldsymbol{\tau}_{\mathbf{GG}} - J^{-1}\,\dot{J}\boldsymbol{\omega} - J^{-1}\,\omega^{\times}J\boldsymbol{\omega} + \tilde{\omega}^{\times}\tilde{R}\boldsymbol{\omega}_{\mathbf{d}} - \tilde{R}\dot{\boldsymbol{\omega}}_{\mathbf{d}}\right)$$

$$Y \in \mathbb{R}^{3\times 2},\ \ \boldsymbol{\Theta} = [C_D\rho \ \ 1]^T \in \mathbb{R}^2$$

- Auxiliary controller and desired auxiliary controller

$$\bar{\boldsymbol{u}} = Y\widehat{\boldsymbol{\Theta}}, \quad \widehat{\boldsymbol{\Theta}}\text{: estimate of } \boldsymbol{\Theta}$$

$$\bar{\boldsymbol{u}}_{\boldsymbol{d}} = -k\boldsymbol{r} - \alpha\dot{\boldsymbol{e}}_{\boldsymbol{v}} - \beta\boldsymbol{e}_{\boldsymbol{v}}$$

- Closed-loop error system

$$\dot{\boldsymbol{r}} = Y\widetilde{\boldsymbol{\Theta}} - k\boldsymbol{r} - \beta\boldsymbol{e}_{\boldsymbol{v}} + \chi$$

$$\chi = \bar{\boldsymbol{u}} - \bar{\boldsymbol{u}}_{\boldsymbol{d}}$$

$$\widetilde{\boldsymbol{\Theta}} = \boldsymbol{\Theta} - \widehat{\boldsymbol{\Theta}}$$

Control Development

- Assumption 1. The auxiliary term χ can be upper bounded by a positive constant, i.e., $\|\chi\| \leq \epsilon$ for $\epsilon \in \mathbb{R}_{>0}$

- The parameter adaptation law is

$$\dot{\hat{\Theta}} = \text{proj}\left(\Gamma_{ICL} Y^T \boldsymbol{r} + \Gamma_{\text{ICL}} k_{ICL} \sum_{i=1}^{N} \mathcal{Y}_i^T \left(\mathcal{U}_i - \mathcal{Y}_i \hat{\Theta}\right)\right)$$

$$\sum_{i=1}^{N} \mathcal{Y}_i^T \mathcal{Y}_i \tilde{\boldsymbol{\Theta}}$$

$$\boldsymbol{\mathcal{U}}(\Delta t, t) = \int_{t-\Delta t}^{t} (\dot{\boldsymbol{r}}(\sigma) - \alpha \dot{\boldsymbol{e}}_v(\sigma)) d\sigma$$

Collect input – output data

$$\mathcal{Y}(\Delta t, t) = \int_{t-\Delta t}^{t} Y(\sigma) d\sigma$$

- *Assumption 2*. The system is sufficiently excited over a finite duration of time (Finite Excitation "FE" condition).

$$\lambda_{min}\left\{\sum_{i=1}^{N} \mathcal{Y}_i^T \mathcal{Y}_i\right\} \geq \lambda$$

$\lambda \in \mathbb{R}_{>0}$: user defined threshold

Integral Concurrent Learning (ICL): Relax the traditional persistent excitation (PE) condition (cannot be guaranteed a priori for nonlinear systems and cannot be verified online) to identify constant unknown system parameters.

A. Parikh, R. Kamalapurkar, and W. E. Dixon, "Integral concurrent learning: Adaptive control with parameter convergence using finite excitation," Int. J. Adapt. Control and Signal Process, vol. 33, pp. 1775-1787, Dec. 2019.

Theorem

- *Theorem 1*. Consider the spacecraft attitude dynamics and satisfying Assumption 1. The auxiliary controller and the adaptive update law ensure the attitude tracking errors $\boldsymbol{r}(t)$ and $\boldsymbol{e}_v(t)$ remain bounded, provided the gain condition $\lambda_{min}\{k\} > \frac{1}{2}$ is satisfied in the sense that

$$\|\boldsymbol{y}(\boldsymbol{t})\|^2 \leq b_1 \exp(-b_2 t) + b_3$$

for all $t \in [0, \bar{T})$, where $\boldsymbol{y} = [\boldsymbol{r}^T \quad \boldsymbol{e}_v^T]^T$, $b_1 = \frac{B_{\bar{V}}}{B_V}\|\boldsymbol{y}(0)\|^2$, $b_2 = \frac{\lambda_1}{B_{\bar{V}}}$, $b_3 = \frac{B_{\bar{V}}}{2\lambda_1 B_V}\epsilon^2 + \frac{\bar{b}-b}{B_V}$,
$\lambda_1 = \min\left\{\lambda_{min}\{k\} - \frac{1}{2}, \beta\lambda_{min}\{\alpha\}\right\}$, $B_{\bar{V}} = \frac{1}{2}\max\{1,\beta\}$, $B_V = \frac{1}{2}\min\{1,\beta\}$, and $b, \bar{b} \in \mathbb{R}_{>0}$.

Theorem

Candidate Lyapunov Function

$$V(t) = \frac{1}{2}\mathbf{r}^T\mathbf{r} + \frac{\beta}{2}\mathbf{e}_\mathbf{v}^\mathbf{T}\mathbf{e}_\mathbf{v} + \frac{1}{2}\widetilde{\boldsymbol{\Theta}}^T\Gamma_{ICL}^{-1}\widetilde{\boldsymbol{\Theta}}$$

→ **Composite Error Vector**

$$\boldsymbol{y}(t) = [\boldsymbol{r}^T \;\; \boldsymbol{e}_v^T]^T$$

→ **Time-derivative of Lyapunov Function**

$$\dot{V}(t)$$

↓ **Substituting in Closed-loop Error and Adaptation Law**

$$\dot{V} = -\mathbf{r}^T k\mathbf{r} + \mathbf{r}^T\boldsymbol{\chi} - \beta\mathbf{e}_\mathbf{v}^\mathbf{T}\alpha\mathbf{e}_\mathbf{v} + -\widetilde{\boldsymbol{\Theta}}^\mathbf{T}k_{ICL}\sum_{i=1}^{N}\mathcal{Y}_i^T\mathcal{Y}_i\widetilde{\boldsymbol{\Theta}}$$

$$\dot{V} \le -\boldsymbol{r}^T k\boldsymbol{r} - \beta\,\boldsymbol{e}_v^T\alpha\boldsymbol{e}_v + \|\boldsymbol{r}\|\epsilon$$

← **Invoking the Comparison Lemma**

$$V(t) \implies \|\boldsymbol{y}(t)\|^2$$

← **Upper Bound the Composite Vector**

$$\boldsymbol{y}(t) \in \mathcal{L}_\infty$$

↓ **Tracking Errors Remain Bounded**

$$\boldsymbol{r}(t), \boldsymbol{e}_v(t) \in \mathcal{L}_\infty$$

→ **Further Upper Bound Each Signal**

$$e_o(t), \dot{e}_0(t), \dot{\boldsymbol{e}}_v(t), \widetilde{\boldsymbol{\omega}}(t) \in \mathcal{L}_\infty$$

$$\overline{\boldsymbol{u}}_d(t), \overline{\boldsymbol{u}}(t), Y(t), \widehat{\boldsymbol{\Theta}}(t) \in \mathcal{L}_\infty$$

Theorem

- *Theorem 2*. Consider the spacecraft attitude dynamics and satisfying Assumptions 1 and 2. The auxiliary controller and the adaptive update law ensure the attitude tracking errors and the parameter estimation errors are uniformly ultimately bounded, provided the gain condition $\lambda_{min}\{k\} > \frac{1}{2}$ is satisfied in the sense that

$$\|\boldsymbol{z}(t)\|^2 \leq c_1 \exp(-c_2 t) + c_3$$

for all $t \in [0, \infty)$, where $\boldsymbol{z} = \left[\boldsymbol{r}^T \quad \boldsymbol{e}_v^T \quad \tilde{\boldsymbol{\Theta}}^T\right]^T$, $c_1 = \frac{C_{\overline{V}}}{C_V}\|\boldsymbol{z}(0)\|^2 \exp\left(\frac{\lambda_2}{C_{\overline{V}}}\overline{T}\right)$, $c_2 = \frac{\lambda_2}{C_{\overline{V}}}$,

$c_3 = \left(\frac{B_{\overline{V}}}{2\lambda_1 C_V}\epsilon^2 + \frac{\bar{b}}{C_V}\right)\exp\left(\frac{\lambda_2}{C_{\overline{V}}}\overline{T}\right) + \frac{C_{\overline{V}}}{2\lambda_2 C_V}\epsilon^2$, $\lambda_2 = \min\left\{\lambda_{min}\{k\} - \frac{1}{2}, \beta\lambda_{min}\{\alpha\}, \lambda\lambda_{min}\{k_{ICL}\}\right\}$,

$C_V = \frac{1}{2}\min\left\{1, \beta, \lambda_{min}\{\Gamma_{ICL}^{-1}\}\right\}$, and $C_{\overline{V}} = \frac{1}{2}\max\left\{1, \beta, \lambda_{max}\{\Gamma_{ICL}^{-1}\}\right\}$.

Theorem

Candidate Lyapunov Function

$$V(t) = \frac{1}{2}\mathbf{r}^T\mathbf{r} + \frac{\beta}{2}\mathbf{e}_\mathbf{v}^\mathbf{T}\mathbf{e}_\mathbf{v} + \frac{1}{2}\widetilde{\mathbf{\Theta}}^T\Gamma_{ICL}^{-1}\widetilde{\mathbf{\Theta}}$$

→ **Composite Error Vector**

$$\boldsymbol{z} = \left[\boldsymbol{r}^T \;\; \boldsymbol{e}_v^T \;\; \widetilde{\mathbf{\Theta}}^T\right]^T$$

→ **Time-derivative of Lyapunov Function**

$$\dot{V}(t)$$

↓ **Substituting in Closed-loop Error and Adaptation Law**

$$\dot{V} = -\mathbf{r}^T k\mathbf{r} + \mathbf{r}^T\boldsymbol{\chi} - \beta\mathbf{e}_\mathbf{v}^\mathbf{T}\alpha\mathbf{e}_\mathbf{v} + -\widetilde{\mathbf{\Theta}}^\mathbf{T}k_{ICL}\sum_{i=1}^{N}\mathcal{Y}_i^T\mathcal{Y}_i\widetilde{\mathbf{\Theta}}$$

$$\dot{V} \leq -\left(\lambda_{min}\{k\} - \frac{1}{2}\right)\|\boldsymbol{r}\|^2 + -\beta\lambda_{min}\{\alpha\}\|\boldsymbol{e}_v\|^2 + -\lambda\lambda_{min}\{k_{ICL}\}\left\|\widetilde{\mathbf{\Theta}}\right\|^2 + \frac{1}{2}\epsilon^2$$

← **Invoking the Comparison Lemma**

$$V(t) \implies \|\boldsymbol{z}(t)\|^2$$

← **Upper Bound the Composite Vector**

$$\boldsymbol{z}(t) \in \mathcal{L}_\infty$$

↓ **Tracking Errors Remain Bounded**

$$\boldsymbol{r}(t), \boldsymbol{e}_v(t), \widetilde{\mathbf{\Theta}} \in \mathcal{L}_\infty$$

→ **Further Upper Bound Each Signal**

$$e_o(t), \dot{e}_0(t), \dot{\boldsymbol{e}}_v(t), \widetilde{\boldsymbol{\omega}}(t) \in \mathcal{L}_\infty$$

$$\overline{\boldsymbol{u}}_{\boldsymbol{d}}(t), \overline{\boldsymbol{u}}(t), Y(t), \widehat{\mathbf{\Theta}}(t) \in \mathcal{L}_\infty$$

Simulation Results

- Numerical simulation with the physical properties of a 2U DMD-equipped CubeSat for the nonlinear attitude dynamics.

Table 1 Initial conditions of the orbit for the spacecraft

Semi-major axis	Eccentricity	True anomaly	RAAN	Argument of perigee	Orbit inclination
6778 km	0	108.08 deg	206.36 deg	101.07 deg	51 deg

Table 2 Physical characteristics of the spacecraft

Mass of the body	Mass of the boom	Maximum boom length	Boom width	Nominal value of C_D
3.0 kg	9.0×10^{-2} kg	3.7 m	3.8×10^{-2} m	2.2

The controller gains were selected as

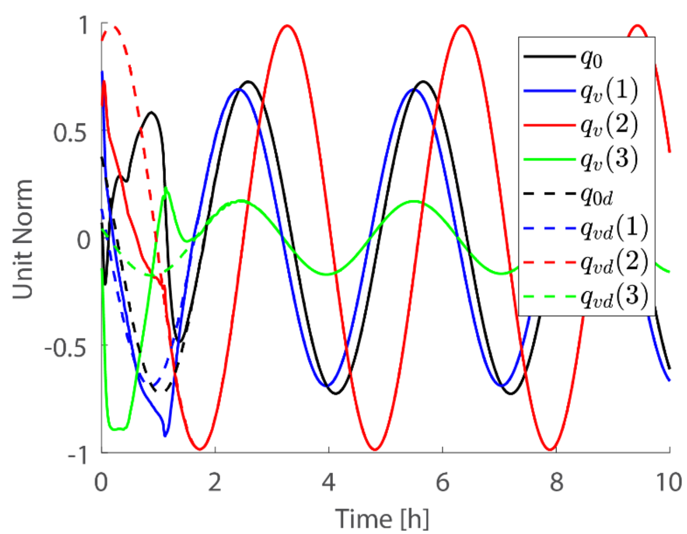
$$\alpha = 10^{-2} \times \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 2.2 \end{bmatrix}, \beta = 5.0 \times 10^{-7}, k = 10^{-2} \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 1.2 \end{bmatrix},$$

$$k_{\mathrm{ICL}} = 10^{-3} \begin{bmatrix} 5.0 & 0 \\ 0 & 1.0 \end{bmatrix}, \Gamma_{\mathrm{ICL}} = 10^{-17} \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix}.$$

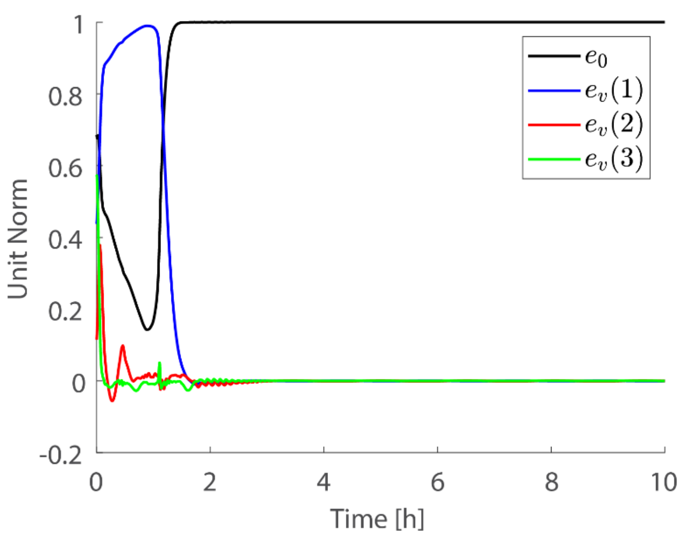
Simulation - Regulation

Table 3 Initial conditions for the regulation objective

$\phi_0 = 80$ deg	$\theta_0 = -60$ deg	$\psi_0 = 50$ deg
$\dot{\phi}_0 = 0.02 \frac{\text{deg}}{s}$	$\dot{\theta}_0 = -0.03 \frac{\text{deg}}{s}$	$\dot{\psi}_0 = 0.025 \frac{\text{deg}}{s}$
$\phi_d = 45$ deg	$\theta_d = 0$ deg	$\psi_d = 0$ deg
$\widehat{\Theta}_0 = \left[\begin{array}{cc} 1.4 \times 10^{-11} & 1 \end{array} \right]^T \frac{\text{kg}}{\text{m}^3}$		



- Spacecraft configuration tracking in quaternion representation.



- Spacecraft configuration error.

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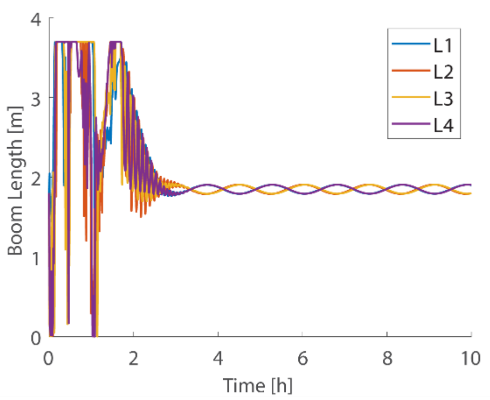
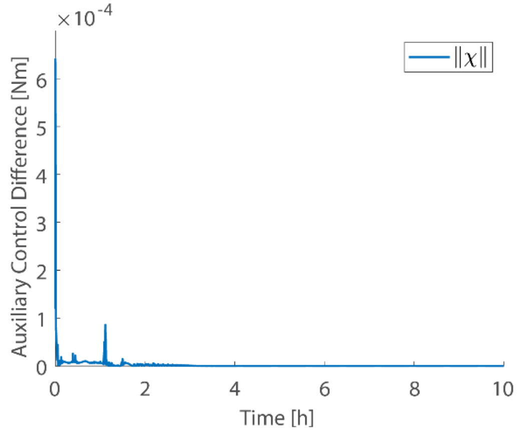
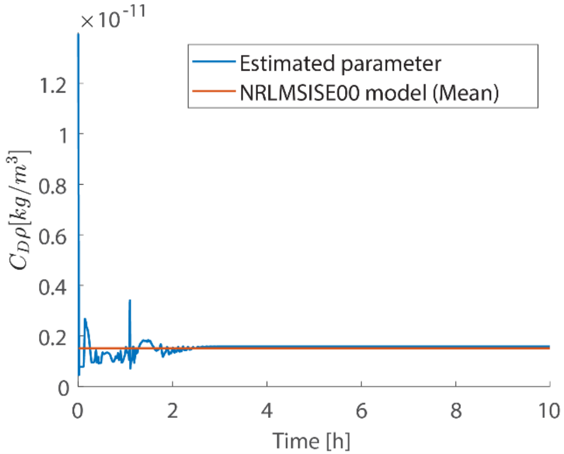
Simulation Regulation





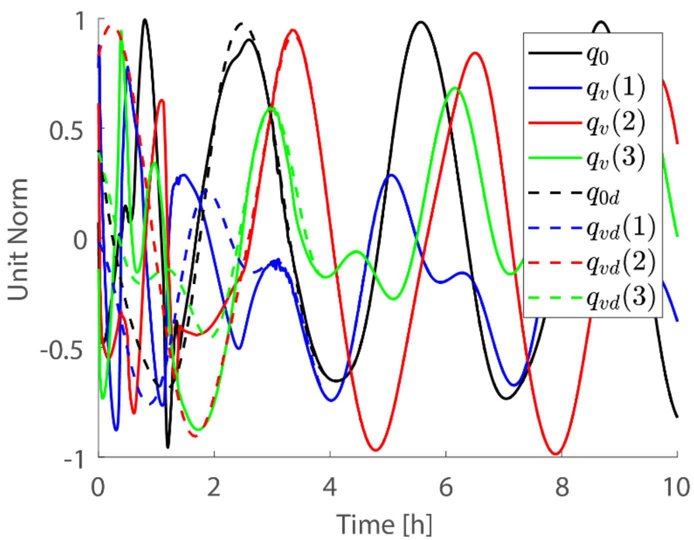


- Boom lengths of the CubeSat DMD in real-time.
- Auxiliary control difference, i.e., $\chi \triangleq \bar{u} - \bar{u}_d$.
- Estimated parameter value and true parameter value (NRLMSISE-00).

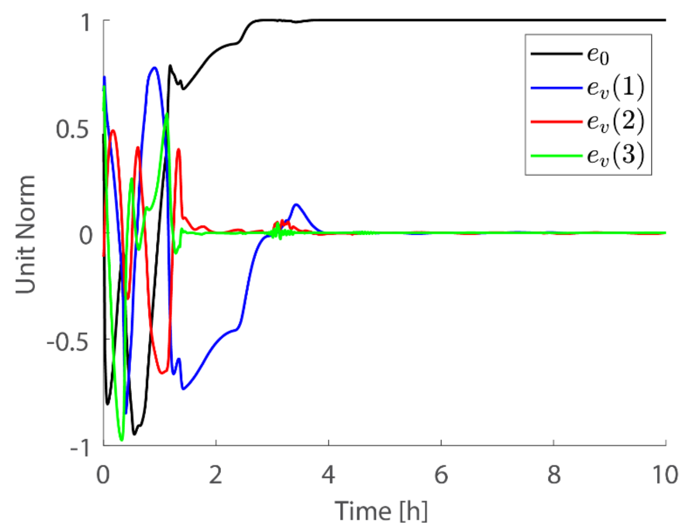
Simulation - Tracking

Table 4 Initial conditions for the tracking objective

$\phi_0 = 80$ deg	$\theta_0 = -60$ deg	$\psi_0 = 50$ deg
$\dot{\phi}_0 = 0.5 \frac{\text{deg}}{s}$	$\dot{\theta}_0 = -1 \frac{\text{deg}}{s}$	$\dot{\psi}_0 = 1 \frac{\text{deg}}{s}$
$\phi_d = \sin\left(\frac{\pi}{6000} t\right)$ deg	$\theta_d = 0$ deg	$\psi_d = 0$ deg
$\widehat{\Theta}_0 = \begin{bmatrix} 1.4 \times 10^{-11} & 1 \end{bmatrix}^T \frac{\text{kg}}{\text{m}^3}$		

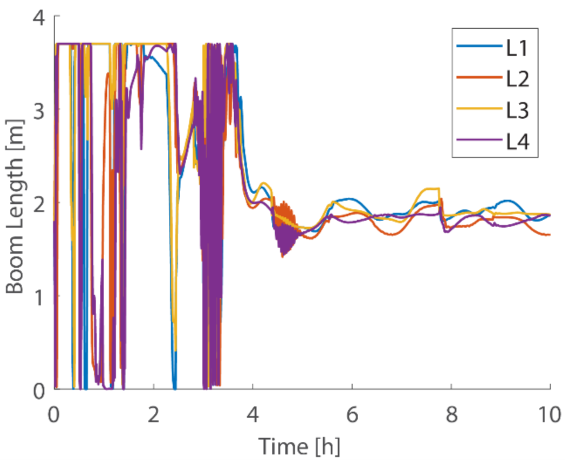


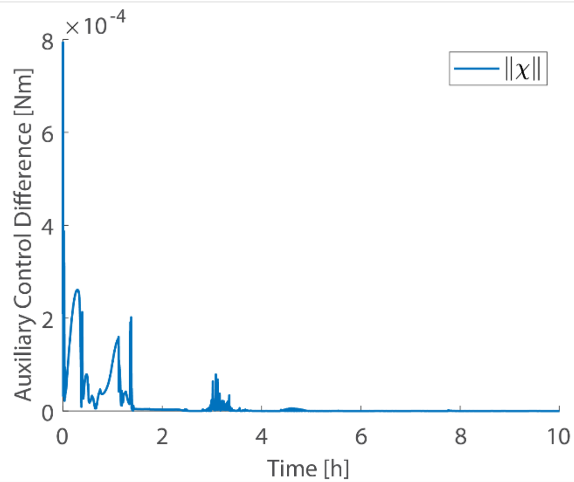
- Spacecraft configuration tracking in quaternion representation.

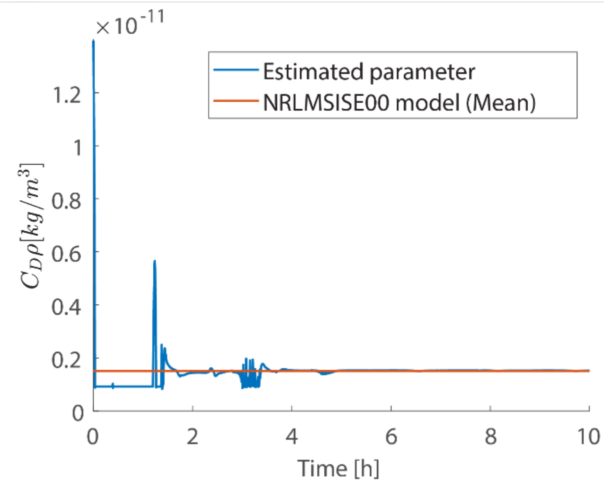


- Spacecraft configuration error.

Simulation Regulation







- Boom lengths of the CubeSat DMD in real-time.
- Auxiliary control difference, i.e., $\chi \triangleq \bar{u} - \bar{u}_d$.
- Estimated parameter value and true parameter value (NRLMSISE-00).

Conclusion & Future Work

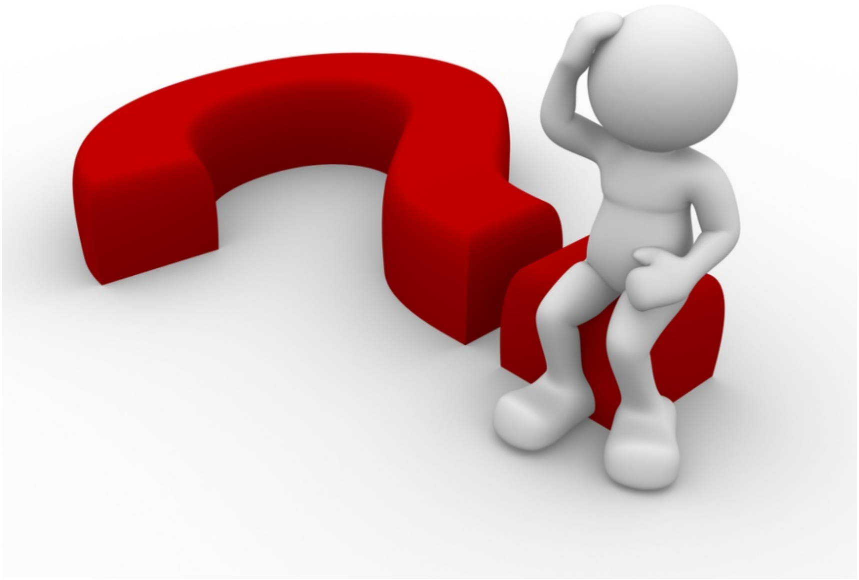
- Adaptive controller and data-based update law (ICL) have been proposed for autonomous spacecraft attitude tracking while simultaneously identify the uncertain drag coefficient and the average of the atmospheric density.

- Future work includes accounting for uncertainties in CoM location and inertia matrix.

- Potentially apply this controller for simultaneous roto-translational control.

Related Publications:

- C. Riano-Rios, R. Bevilacqua, W. E. Dixon, "Relative maneuvering for multiple spacecraft via differential drag using LQR and constrained least squares", 495 in: AAS Space Fight Mechanics Meeting, Maui, Hawaii, 2019, Paper No. AAS-19-346.
- C. Riano-Rios, S. Omar, R. Bevilacqua, W. Dixon, "Spacecraft attitude regulation in low earth orbit using natural torques", in: 2019 IEEE 4th Colombian Conference on Automatic Control (CCAC), Medellin, Colombia, 2019.
- C. Riano-Rios, R. Bevilacqua, W. E. Dixon, "Adaptive control for differential drag-based rendezvous maneuvers with an unknown target", Acta Astronautica, 2020.
- C. Riano-Rios, R. Bevilacqua, W. E. Dixon, "Differential Drag-Based Multiple Spacecraft Maneuvering and On-Line Parameter Estimation Using Integral Concurrent Learning", Acta Astronautica. 2020.
- R. Sun, C. Riano-Rios, R. Bevilacqua, N. G. Fitz-Coy, W. E. Dixon, "**CubeSat Adaptive Attitude Control with Uncertain Drag Coefficient and Atmospheric Density**," submitted to the Journal of Guidance, Control and Dynamics, 2020.



Backup Slide

- Quaternion representations.

$$q_v^T q_v + q_0^2 = 1$$

$$\dot{q}_v = \frac{1}{2}\left(q_v^\times + q_0 I_3\right)\omega$$

$$\dot{q}_0 = -\frac{1}{2} q_v^T \omega$$

$$\omega = 2\left(q_0 \dot{q}_v - q_v \dot{q}_0\right) - 2 q_v^\times \dot{q}_v$$

$$\omega_d = 2\left(q_{0d} \dot{q}_{vd} - q_{vd} \dot{q}_{0d}\right) - 2 q_{vd}^\times \dot{q}_{vd}$$

$$e_v^T e_v + e_0^2 = 1$$

$$e_v \triangleq q_{0d} q_v - q_0 q_{vd} + q_v^\times q_{vd}$$

$$e_0 \triangleq q_0 q_{0d} + q_v^T q_{vd}$$

$$\dot{e}_v = \frac{1}{2}\left(e_v^\times + e_0 I_3\right)\tilde{\omega}$$

$$\dot{e}_0 = -\frac{1}{2} e_v^T \tilde{\omega}$$

$$\widetilde{R} \triangleq R R_d^T = \left(e_0^2 - e_v^T e_v\right) I_3 + 2 e_v e_v^T - 2 e_0 e_v^\times$$

$$R \triangleq \left(q_0^2 - q_v^T q_v\right) I_3 + 2 q_v q_v^T - 2 q_0 q_v^\times$$

$$R_d \triangleq \left(q_{0d}^2 - q_{vd}^T q_{vd}\right) I_3 + 2 q_{vd} q_{vd}^T - 2 q_{0d} q_{vd}^\times$$

The objective is to regulate $\widetilde{R} \to I_3$ as $t \to \infty$,
can be achieved by regulating $\|e_v\| \to 0 \Rightarrow |e_0| \to 1.$