Adaptive Safety With Multiple Barrier Functions Using Integral Concurrent Learning

Axton Isaly, Omkar Sudhir Patil, Ricardo G. Sanfelice, Warren E. Dixon

Submitted to American Control Conference 2021

Control Objective

$$\dot{x} = Y(x,t)\,\theta + g(x)\,u$$

- $\theta \in \mathbb{R}^p$ is unknown
- Design a controller so that

$$\mathcal{S} \triangleq \{x \in \mathbb{R}^n : B(x) \leq 0\}$$

is forward invariant, where

$$B(x) \triangleq [B_1(x), B_2(x), \ldots, B_d(x)]^T$$

- Safe set described by multiple continuously differentiable functions

Motivation

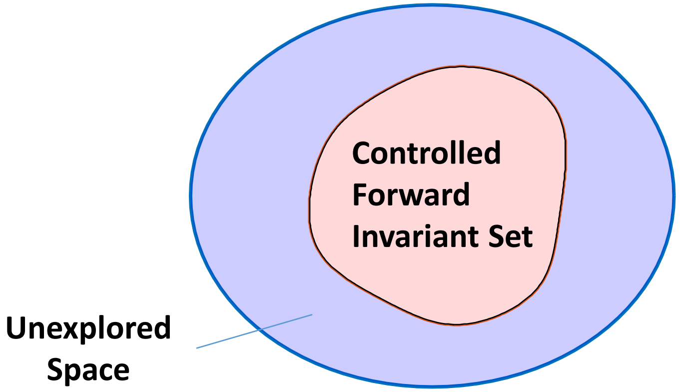
- Control objective can be accomplished by enforcing for each $i \in \{1,2,\dots,d\}$:

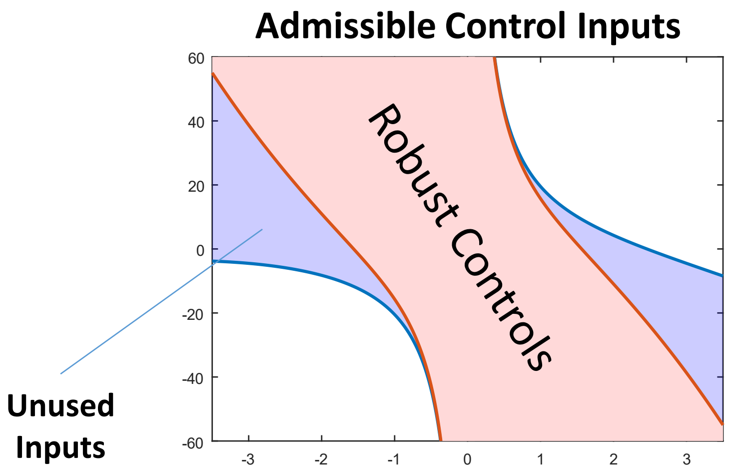
$$\nabla B_i^T(x)\left(Y(x,t)\theta + g(x)u\right) \le -\gamma_i(x)$$

for all x in an open set $\mathcal{D}$, where $\mathcal{S} \subset \mathcal{D}$

- $\gamma : \mathbb{R}^n \to \mathbb{R}^d$ is continuous
- $\gamma_i(x) \ge 0$ for all $x \in U(M_i)\setminus\mathcal{S}_i$, where $M_i \triangleq \{x \in \partial\mathcal{S} : B_i(x) = 0\}$
- **Example:** $\gamma_i(x) \triangleq B_i(x)$

- Using robust control techniques to compensate for uncertainty leads to conservative control action





Estimator Design

$$\tilde{\theta} \triangleq \theta - \hat{\theta}$$

$$\nabla B_i^T(x)\, Y(x,t)\, \theta = \nabla B_i^T(x) \left(Y(x,t)\, \hat{\theta} + Y(x,t)\, \tilde{\theta} \right)$$

Integral Concurrent Learning

Using the update law,

$$\dot{\hat{\theta}} \triangleq k_{CL} \sum_{i=1}^{N(t)} \mathcal{Y}_i^T \left(\phi(t_i) - \phi(t_i - \Delta t) - \mathcal{K}_i - \mathcal{Y}_i \hat{\theta} \right)$$

$$\mathcal{Y}(t) \triangleq \int_{t-\Delta t}^{t} Y(\phi(\tau), \tau)\, \mathrm{d}\tau$$

Leads to,

$$\dot{\tilde{\theta}} = -k_{CL} \sum_{i=1}^{N(t)} \mathcal{Y}_i^T \mathcal{Y}_i \tilde{\theta}$$

Compensation for Estimation Error

We show that,

$$\left\| \tilde{\theta}(t) \right\| \leq \tilde{\theta}_{UB}(t)$$

for all $t \in \operatorname{dom} \phi$, where

$$\tilde{\theta}_{UB}(t) \triangleq \left\| \tilde{\theta}(0) \right\| \exp\left(- \int_0^t k_{CL} \lambda_{min}(\tau)\, \mathrm{d}\tau \right)$$

$$\lambda_{min} \left\{ \sum_{i=1}^{N(t)} \mathcal{Y}_i^T \mathcal{Y}_i \right\}$$

Estimator Design

$$\tilde{\theta} \triangleq \theta - \hat{\theta}$$

$$\nabla B_i^T(x) Y(x,t)\theta = \nabla B_i^T(x)\left(Y(x,t)\hat{\theta} + Y(x,t)\tilde{\theta}\right)$$

Assumption 1

When θ is bounded,
$\tilde{\theta}_{UB}(t)$ is computable

Assumption 2

When $\lambda_{min} \geq \underline{\lambda} > 0$ for all $t \geq T$,
then $\tilde{\theta}_{UB}(t)$ is exponentially regulated

Compensation for Estimation Error

We show that,

$$\left\|\tilde{\theta}(t)\right\| \leq \tilde{\theta}_{UB}(t)$$

for all $t \in \text{dom}\ \phi$, where

$$\tilde{\theta}_{UB}(t) \triangleq \left\|\tilde{\theta}(0)\right\| \exp\left(-\int_0^t k_{CL}\lambda_{min}(\tau)\, \mathrm{d}\tau\right)$$

$$\lambda_{min}\left\{\sum_{i=1}^{N(t)} \mathcal{Y}_i^T \mathcal{Y}_i\right\}$$

UF UNIVERSITY of FLORIDA · Duke UNIVERSITY · TEXAS The University of Texas at Austin · UC SANTA CRUZ

$$\nabla B_i^T(x)\,Y(x,t)\,\theta = \nabla B_i^T(x)\left(Y(x,t)\,\hat{\theta} + Y(x,t)\,\tilde{\theta}\right)$$

$$\left\|\tilde{\theta}(t)\right\| \le \tilde{\theta}_{UB}(t)$$

Compensate for estimation error using:

$$\nabla B_i^T(\phi(t))\,Y(\phi(t),t)\,\tilde{\theta}(t) \le \left\|\nabla B_i^T(\phi(t))\,Y(\phi(t),t)\right\|\tilde{\theta}_{UB}(t)$$

$$\theta_{con,i}(x,t) \triangleq \min\Big(\underbrace{\left\|\nabla B_i^T(x)\,Y(x,t)\right\|\bar{\theta}}_{\text{Robust Term}},\ \underbrace{\nabla B_i^T(x)\,Y(x,t)\,\hat{\theta}(t) + \left\|\nabla B_i^T(x)\,Y(x,t)\right\|\tilde{\theta}_{UB}(t)}_{\text{Adaptive Term}}\Big)$$

$$\nabla B_i^T(\phi(t))\,Y(\phi(t),t)\,\theta \le \theta_{con,i}(\phi(t),t)$$

Control Design

$$\nabla B_i^T(\phi(t))\, Y(\phi(t), t)\,\theta \le \theta_{con,i}(\phi(t), t)$$

Theorem 1. *Consider a continuously differentiable BF candidate* $B : \mathbb{R}^n \to \mathbb{R}^d$*, suppose that the set* $\mathcal{S}$ *is compact, and suppose that Assumption 1 holds. Let the function* γ *and the set* $\mathcal{D}$ *satisfy (C1) and (C2). Along any solution to the dynamic system, let* $\kappa^*(x, t)$ *be a control law generated by the following QP:*

$$\kappa^*(x,t) \triangleq \underset{u\in\mathbb{R}^m}{\arg\min} \; \|u - \kappa_{nom}(x,t)\|^2$$
$$s.t.\ \nabla B_i^T(x)\, g(x) u \le -\gamma_i(x) - \theta_{con,i}(x,t),\ \ \forall i \in [d],$$

where $\kappa_{nom} : \mathbb{R}^n \times \mathbb{R}_{\geq 0}$ *is a nominal controller. If* $\kappa^*(x,t)$ *is continuous in* x *and* t*, then the set* $\mathcal{S}$ *is forward invariant for the closed-loop dynamics. Furthermore, if Assumption 2 holds, then* $\tilde{\theta}$ *is exponentially regulated.*

Feasibility Condition (C2)

For each $x \in \mathcal{D}$ and $t \in \mathbb{R}_{\geq 0}$, there exists $u \in \mathbb{R}^m$ such that, for every $i \in [d]$,

$$\nabla B_i^T(x)\, g(x)\, u < -\gamma_i(x) - \left\|\nabla B_i^T(x)\, Y(x,t)\right\| \bar{\theta}$$

Recall, we wish to enforce:

$$\nabla B_i^T(x)\, Y(x,t)\,\theta + \nabla B_i^T(x)\, g(x)\, u \le -\gamma_i(x)$$

Simulation

$$\dot{x} = \begin{bmatrix} x_1^2 & \sin(x_2) & 0 & 0 \\ 0 & x_2 \sin(t) & x_1 & x_1 x_2 \end{bmatrix} \theta + u \qquad B(x) = \begin{bmatrix} x_1 + x_2 - c \\ -x_1 + x_2 - c \\ \frac{1}{c} x_1^2 - x_2 - c \end{bmatrix}$$