Lyapunov-based MTL Methods for Network Control

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AFOSR Center of Excellence Review

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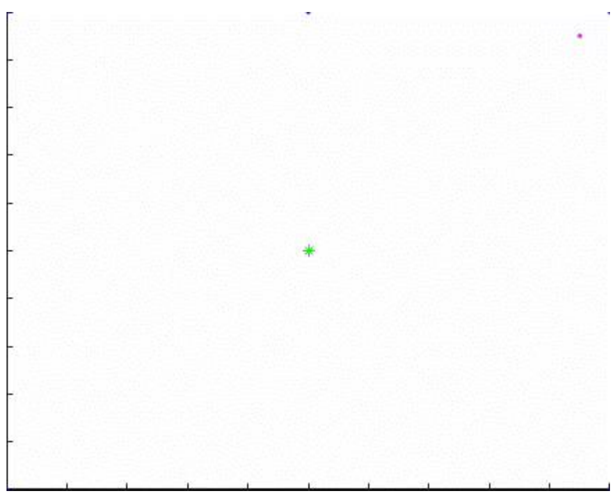
Collaborative efforts between UF + Duke + UTA

- Z. Xu, F. Zegers, B. Wu, A. Phillips, W. Dixon, U. Topcu, “Controller Synthesis for Multi-Agent Systems with Intermittent Communication and Metric Temporal Logic Specifications” (Submitted)
- D. Le, X. Luo, L. Bridgeman, M. Zavlanos, W. E. Dixon, "Single-Agent Indirect Herding of Multiple Targets using Metric Temporal Logic Switching," (Journal in preparation)
- D. Le, X. Luo, L. Bridgeman, M. Zavlanos, W. E. Dixon, "Single-Agent Indirect Herding of Multiple Targets using Metric Temporal Logic Switching," In Proc. IEEE Conf. Decis. Control, 2020.
- Z. Xu, F. M. Zegers, B. Wu, W. E. Dixon, U. Topcu, "Controller Synthesis For Multi-Agent Systems with Intermittent Communication: A Metric Temporal Logic Approach," In *Proc. Allerton Conf. on Commun., Control, and Compu.*, 2019.

Problem Formulation

- Consensus in multi-agent systems
  - In adversarial/contested environments
    - Intermittencies in communication
    - Noncooperative agents
- Switched System approach



Leader position
Goal location
Follower position
Predicted Follower position

F. Zegers, et. al, "A Switched Systems Approach to Consensus of a Distributed Multi-agent System with Intermittent Communication," American Control Conference
R. Licitra, et. al, "Single Agent Indirect Herding of Multiple Targets with Unknown Dynamics," *IEEE Transactions on Robotics.*

Dynamics

- Dynamic Model

Relay Explorer Agent Problem	Herding	
$\dot{x}_L = f_L\left(x_L, u_L\right)$	$\dot{x}_L = u$	Leader Agent
$\dot{x}_i = f_i\left(x_i, u_i\right) + d\left(t\right)$ $i = 1, 2, \ldots, N$	$\dot{x}_i = \alpha_i\left(\lVert x_i - x_L \rVert\right)\left(x_i - x_L\right) + f_i\left(x_i, t\right)$ $i = 1, 2, \ldots, N$	Follower Agents

REAP

- Leader agent
  - has state information
- Follower agents
  - No state sensing capabilities
  - Dead reckon (open-loop)

Herding

- Leader agent
  - Direct control
- Follower agents
  - Do not have direct control

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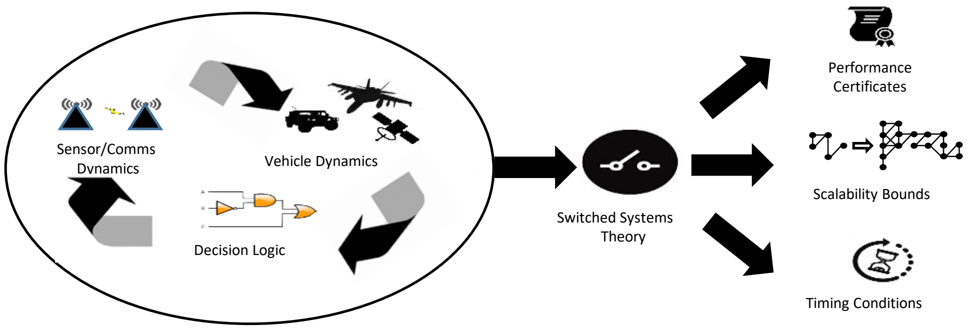
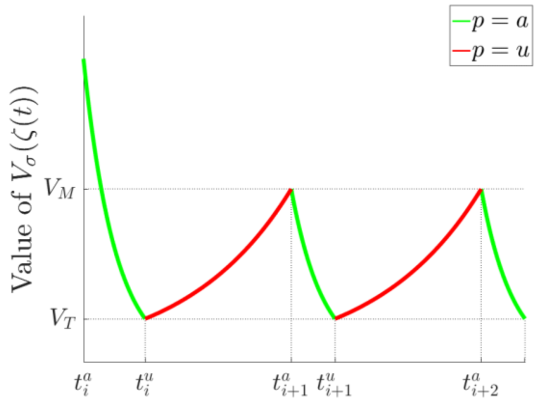
Switched Systems

- Switched Systems
  - Operational modes
  - “closed-loop/open-loop”
  - “chased/unchased”

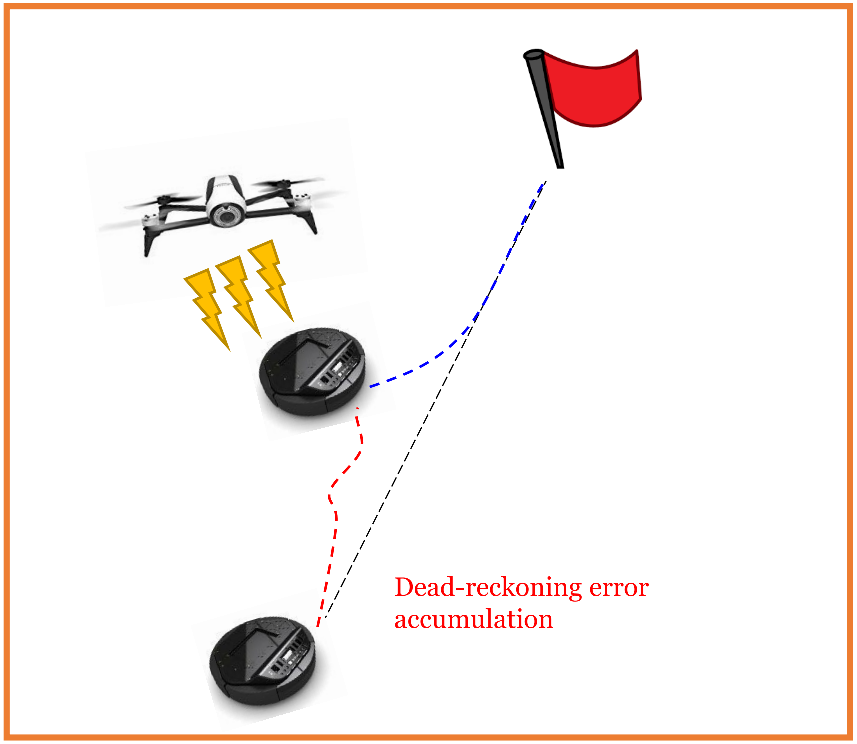


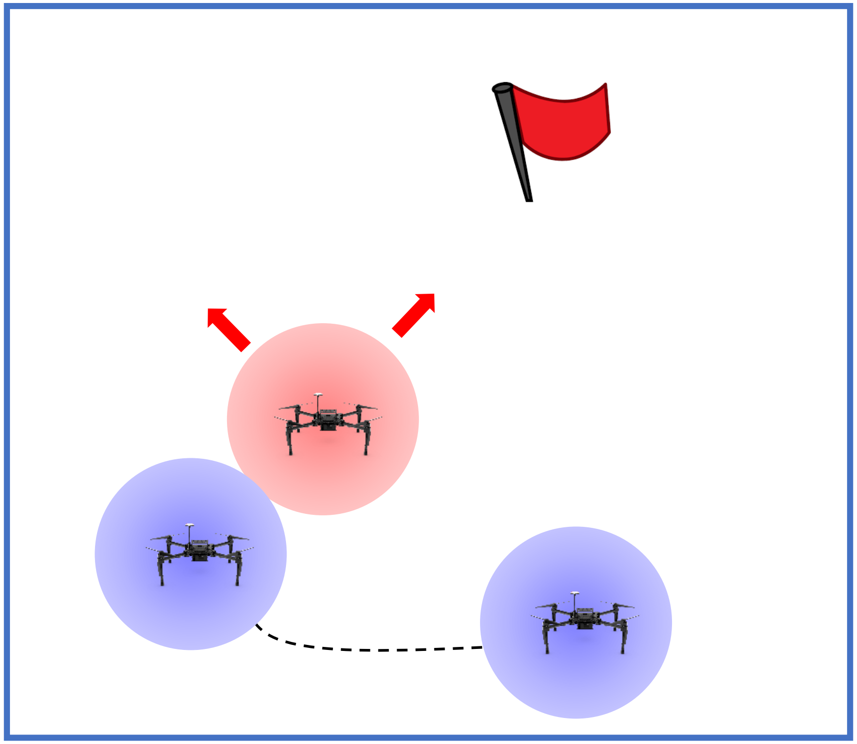


- Dwell-time analysis
  - Maximum dwell-time
  - Minimum dwell-time
  - Average dwell-time
  - Only provide sufficient Conditions!
- How/When to switch between modes?

Low-Level Control Design

- Trajectory tracking
  - Analyze trajectory of each agent
  - Switched system: sufficient conditions for stability guarantees in solutions of the switched system
- Challenges
  - Which agent to service/chase? When to switch modes?
  - Other specified tasks?

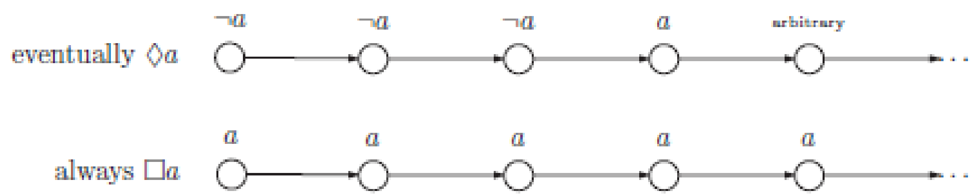




Temporal Logic

- Goal: specify a behavior
- Linear Temporal Logic

$\Diamond$ eventually (satisfied at some point)
$\Box$ always (satisfied now and forever)



- Metric Temporal Logic

$\Diamond_{\mathcal{I}}$ eventually in the time interval $\mathcal{I}$
$\Box_{\mathcal{I}}$ always in the time interval $\mathcal{I}$

$\Box\Diamond_{\mathcal{I}} p$ always eventually p in the interval $\mathcal{I}$

$\Box_{\mathcal{I}} p \implies \Box_{\mathcal{I}} q$ always p implies always q

C. Baier and J. Katoen, "Principles of Model Checking," *MIT Press*

Metric Temporal Logic

- Encoding maximum dwell-time
  - The leader agent should be in the communication radius of each follower agents estimated position at least once in a specified time period

$$\phi_1 = \bigwedge_{1 \leq i \leq N} \left(\Box \Diamond_{[0,n_i]} \|x_L - \hat{x}_i\| \leq r_{com} \right)$$

- Encoding minimum dwell-time
  - The leader agent should service each follower agent for a specified time period

$$\phi_2 = \bigwedge_{1 \leq i \leq N} \left(\Box \left(\|x_L - \hat{x}_i\| \leq r_{com} \implies \Box_{[1,m_i]} \|x_L - \hat{x}_i\| > r_{com} \right) \right)$$

- Encoding complex constraints
  - Reach the charging station G1 or G2 every 6 time units and always stay in the specified region D

$$\phi_3 = \Box \Diamond_{[0,6]} \left((x_L \in G_1) \vee (x_L \in G_2) \right) \wedge \Box \left(x_L \in D \right)$$

Control Synthesis

- Optimal Control Formulation (REAP)

$$\underset{u_L^l}{\operatorname{argmin}} \quad J$$

Leader control input

subject to:

$$x_L^{(k+1)} = \overline{f}_L\left(x_L^k, u_L^k\right)$$
$$\forall k = l, \ldots, l+N-1$$
$$\hat{x}_i^{(k+1)} = \overline{f}_i\left(x_i^k, u_i^k\right)$$
$$\forall k = l, \ldots, l+N-1, \forall i = 1, \ldots n$$

Dynamic Constraints

$$u_L^k \in \mathcal{U}, \forall k = l, \ldots, l+N-1$$

Input Constraints

$$\left(\tilde{x}^{l:(l+N-1)}, 0\right) \models_w [\phi]_0^l$$

MTL Constraints

Control Synthesis

- Optimal Control Formulation (Herding)

$$\underset{\mathbf{b}}{\operatorname{argmin}} \quad J$$

Switching signal: which agent to chase

subject to:

$$\dot{x}_N(t) = \sum_{i=1}^{N-1} \sigma_i [k_2 \eta + \alpha_i e_i + (k_3 \|x_i - x_N\| + k_4) \operatorname{sgn}(\eta)],$$

$$\dot{x}_i(t) = (x_i - x_N)\,\alpha_i + f_i, \quad \forall i \in T,$$

Dynamic Constraints

$$\sum_{i=1}^{N-1} b_i(t_j) = 1, \quad \forall t_j \in \{l, \dots, l+n-1\},$$

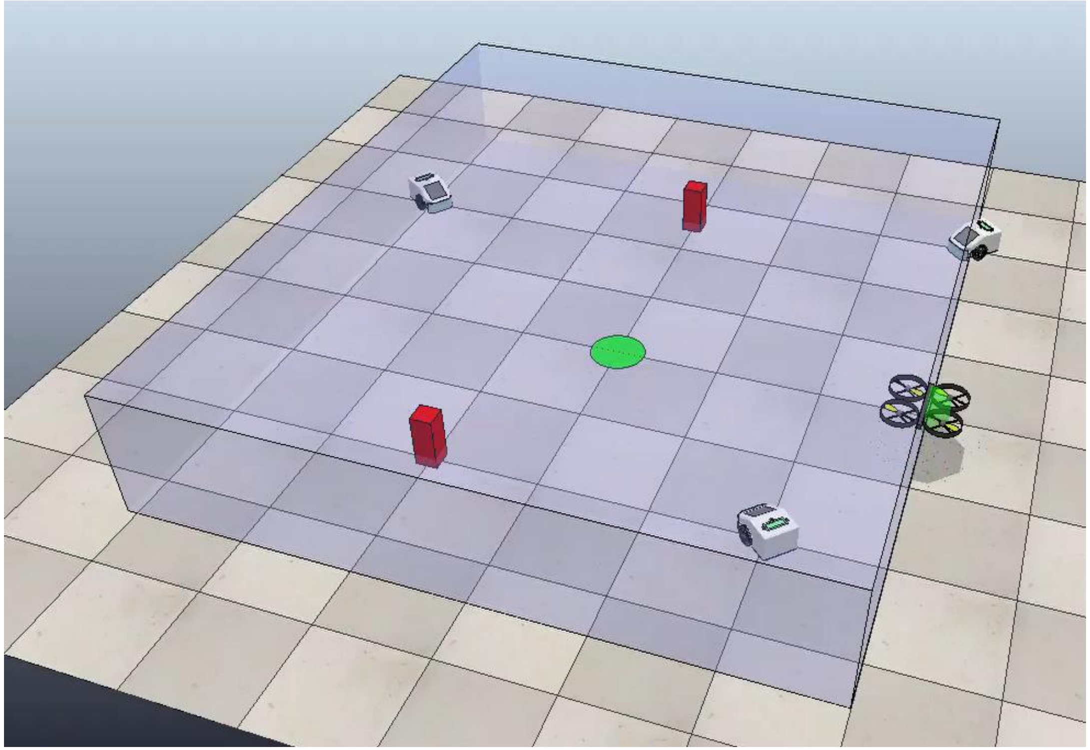
Input Constraints

$$[\mathbf{b}^l, \mathbf{b}], 0 \models_W [\phi]_0^l,$$

MTL Constraints

Simulation

- Specifications
  - **Maximum dwell-time constraints**: “do not leave an agent unattended for too long”
  - **Minimum dwell-time constraints**: “service the agent for a specified amount of time”
  - **Practical constraint**: “return to charging station every so often”



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NCR Links

Website: https://ncr.mae.ufl.edu/

YouTube: https://www.youtube.com/user/NCRatUF/videos

Thank you!