Event/Self-Triggered Approximate Leader-Follower Consensus with Resilience to Byzantine Adversaries

F. M. Zegers, P. Deptula, J. M. Shea, and W. E. Dixon, "Event/Self-Triggered Approximate Leader-Follower Consensus with Resilience to Byzantine Adversaries," in *IEEE Transactions on Automatic Control*, 2019 June. Under review.

F. M. Zegers, P. Deptula, J. M. Shea, and W. E. Dixon, "Event-Triggered Approximate Leader-Follower Consensus with Resilience to Byzantine Adversaries," in *IEEE Conference on Decision and Control*, Nice, FR, December 2019.

Consider a homogenous multi-agent system with *N* follower agents and a single leader. The model of agent *i* is given by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t).$$

The leader is indexed by 0 and the followers are indexed by $\mathcal{V} \triangleq \{1, 2, ..., N\}$.

The flow of information be between the followers is modeled with the (initially) undirected

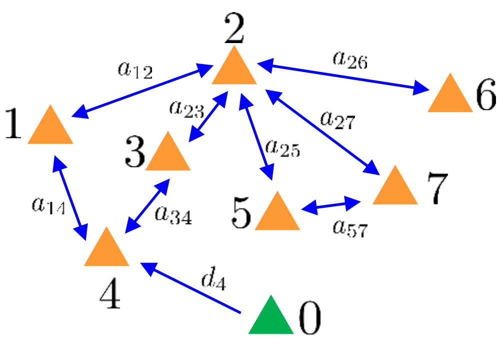
$$\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t), \mathcal{A}(t)).$$

Objective: Design a controller for the followers that

- synchronizes the cooperative follower trajectories with that of the leader,
- is distributed & event-triggered,
- is resilient to Byzantine adversaries.

Assumptions

- Each agent can measure its state for all time.
- The pair (A, B) is stabilizable.
- The control and state of the leader are bounded.
- The leader is a cooperative agent for all time.
- The graph of the CMAS is connected for all time.
- At least one cooperative follower is connected to the leader for all time.



Follower error signals:

$$e_{1,i}(t) = x_i(t) - x_0(t)$$
$$e_{2,i}(t) = \hat{x}_i(t) - x_i(t)$$

1, if agent *i* is connected to leader
0, otherwise

Follower controller:

$$u_i(t) = K(z_i(t) + e_{2,i}(t))$$
$$z_i(t) = \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(\hat{x}_j(t) - \hat{x}_i(t)) + d_i(t)(x_0(t) - \hat{x}_i(t))$$

$$K \triangleq B^T P$$ Solution to ARE

$$A^T P + PA - \lambda_{\min}(H_{\min}) 2PBB^T P + kI_m = 0_{m \times m}$$

Used to exclude Zeno behavior

The broadcast times of follower *j* are given by $\{t_k^j\}_{k=0}^{\infty}$

$$t_{k+1}^j = \inf \left\{ t > t_k^j : \phi_2 \left\| e_{2,j}(t) \right\|^2 \geq \phi_3 \left\| z_j(t) \right\|^2 + \frac{\theta}{N} \right\}.$$

Follower observer (open-loop):

$$\dot{\hat{x}}_j(t) = A\hat{x}_j(t), \; t \in \left[t_k^j, t_{k+1}^j\right) \quad i \in \mathcal{N}_j(t) \cup \{j\}$$

$$\hat{x}_j\left(t_k^j\right) = x_{j,i}\left(t_k^j\right)$$

State of agent *j* received by agent *i*.

The detector of follower *j* used to test the status of follower *i* at each broadcast time t_k^i is

$$\Xi_{i,k} \triangleq \left\| \hat{x}_i^-\left(t_k^i\right) - x_{i,j}\left(t_k^i\right) \right\| - \Psi_{i,k}.$$

Estimated state the instant before being reset

Potential state update

Model-based upper bound for the estimation error $e_{2,i}(t)$

Edge weight policy

$$a_{ij}(t) \triangleq \begin{cases} 1, & j \in \mathcal{C}_i(t) \\ 0, & j \in \mathcal{B}_i(t). \end{cases}$$

Cooperative & Byzantine neighbor set of agent *i*

$$\mathcal{C}_i(t) \triangleq \left\{ j \in \mathcal{N}_i(t) : \forall t_k^j \leq t \; \Xi_{i,k} \leq 0 \right\},$$

$$\mathcal{B}_i(t) \triangleq \mathcal{N}_i(t) \setminus \mathcal{C}_i(t).$$

UF UNIVERSITY of FLORIDA

Duke UNIVERSITY

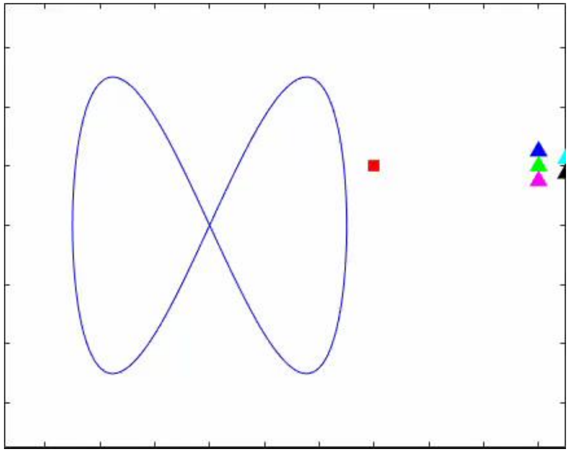
TEXAS The University of Texas at Austin

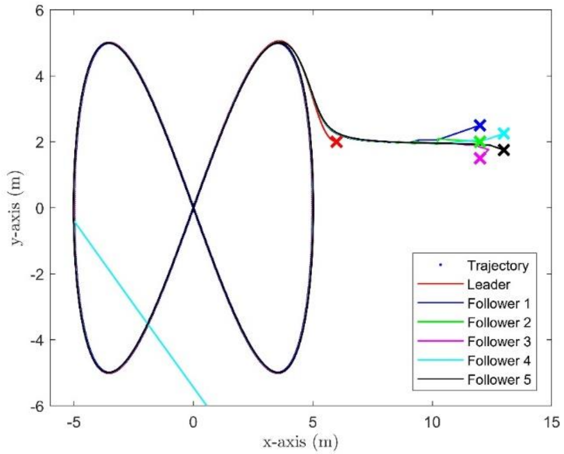
UC SANTA CRUZ

Theorem 1: The edge weight policy, controller, and state observer ensure the stacked leader-follower error $e_1(t)$ is globally uniformly ultimately bounded in the sense that

$$\|e_1(t)\| \leq \beta_1 + \beta_2 e^{-\beta_3 t}$$

where β_1, β_2, and β_3 known, positive bounding constants provided state feedback is available as dictated by the event trigger, Assumptions 1-6 are satisfied, and the sufficient user-defined parameter conditions (defined in the paper) are satisfied.





- Cooperative agents: L,1,2,3
- Byzantine agent: 4,5

Event-Triggered Formation Control and Leader Tracking with Resilience to Byzantine Adversaries: A Reputation-Based Approach

F. M. Zegers, M. T. Hale, J. M. Shea, and W. E. Dixon, "Event-Triggered Formation Control and Leader Tracking with Resilience to Byzantine Adversaries: A Reputation-Based Approach," in *IEEE Transactions on Control and Network Systems*. Under review.

F. M. Zegers, M. T. Hale, J. M. Shea, and W. E. Dixon, "Reputation-Based Event-Triggered Formation Control and Leader Tracking with Resilience to Byzantine Adversaries," in *American Control Conference*, Denver, CO, July 2020.

Consider a heterogeneous multi-agent system with *N* follower agents and a single leader. The model of agent *i* is given by

$$\dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))\, u_i(t) + d_i(t)\,.$$

The leader is indexed by 0 and the followers are indexed by $\mathcal{V} \triangleq \{1, 2, ..., N\}$.

The flow of information be between the followers is modeled with the (initially) undirected

$$\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t), \mathcal{A}(t))\,.$$

Objective: Design a controller for the followers that

- achieves formation control and leader tracking (FCLT),
- is distributed & event-triggered,
- is resilient to Byzantine adversaries.

Idea: Make edge weights a function of trust, use redundant state information.

Let $\{t_k^j\}_{k=0}^{\infty}$ denote the sequence of broadcast times of agent *j*, where

$$x_{j,1}\left(t_k^j\right) = \text{communicated state},$$

$$x_{j,2}\left(t_k^j\right) = \text{sensed state}.$$

The trust follower *i* has in follower $j \in \mathcal{N}_i\left(t_k^j\right)$ is defined as

$$\tau_{ij}\left(t\right) \triangleq \begin{cases} 1, & \left|S_j\right| = 0 \\ \frac{1}{\left|S_j\right|} \sum_{t_k^j \in S_j} e^{-s_1 \Psi_{ij}\left(t_k^j\right)}, & \left|S_j\right| \neq 0, \end{cases}$$

$$\Psi_{ij}\left(t_k^j\right) \triangleq \left\| x_{j,1}\left(t_k^j\right) - x_{j,2}\left(t_k^j\right) \right\|.$$

$$\mathcal{N}_{ij}(t) \triangleq \mathcal{N}_i(t) \cap \mathcal{N}_j(t)$$

$$\dot{\zeta}_{ij}(t) = \text{proj}\left(\underbrace{\eta_\tau\left(\tau_{ij}(t) - \zeta_{ij}(t)\right)}_{\text{Accounts for what } i \text{ thinks of } j} + \sum_{n \in \mathcal{N}_{ij}(t)} \underbrace{\eta_\zeta \zeta_{in}(t)\left(\zeta_{nj}\left(t_k^n\right) - \zeta_{ij}(t)\right)}_{\text{Accounts for what } k \text{ thinks of } j \text{ weighted by what } i \text{ thinks } k}\right)$$

Accounts for what *i* thinks of *j*

Accounts for what *k* thinks of *j* weighted by what *i* thinks *k*

$$\zeta_{ij}(t) \in [0, 1]\; \forall t \geq 0$$

Edge weight policy

$$a_{ij}(t) = \begin{cases} \zeta_{ij}(t), & \zeta_{ij}(t) \geq \zeta_{\min} \text{ and } j \in \mathcal{N}_i(t) \\ 0, & \zeta_{ij}(t) < \zeta_{\min} \text{ or } j \notin \mathcal{N}_i(t), \end{cases}$$

$$\zeta_{\min} \in [0,1].$$

Cooperative & Byzantine neighbor set of agent *i*

$$\mathcal{C}_i(t) \triangleq \{j \in \mathcal{N}_i(t) : a_{ij}(t) \neq 0\},$$

$$\mathcal{B}_i(t) \triangleq \mathcal{N}_i(t) \setminus \mathcal{C}_i(t).$$

Benefits

- No exact model knowledge needed for detection,
- No bounds on neighbor quantities needed,
- Enables re-integration of rehabilitated agents.

Follower error signals:

Desired relative position vector, fixed

$$e_{1,i}(t) = x_i(t) - x_0(t) - v_i$$
$$e_{2,i}(t) = \hat{x}_i(t) - x_i(t)$$

Follower controller:

Right pseudo inverse of $g_i(x_i(t))$

$$u_i(t) = g_i^{+}(x_i(t))\left(k_1 z_i(t) + k_2 e_{2,i}(t)\right)$$
$$z_i(t) = \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)\left(\hat{x}_j(t) - \hat{x}_i(t) - v_j + v_i\right) + b_i(t)\left(v_i + \hat{x}_0(t) - \hat{x}_i(t)\right)$$

Follower *i* knows the formation

Positive only if connected to leader

The event trigger of follower *i* and the leader are given by

$$t_{k+1}^{i} = \inf\left\{t > t_k^i : \phi_2 \left\|e_{2,i}(t)\right\|^2 \geq \phi_3 \left\|z_i(t)\right\|^2 + \frac{\varepsilon}{N}\right\},$$

$$t_{k+1}^{0} = \inf\left\{t > t_k^0 : N b_{\max}^2 \phi_3 \left\|e_{2,0}(t)\right\|^2 \geq c_0\right\}.$$

The observer of follower *i* is given by

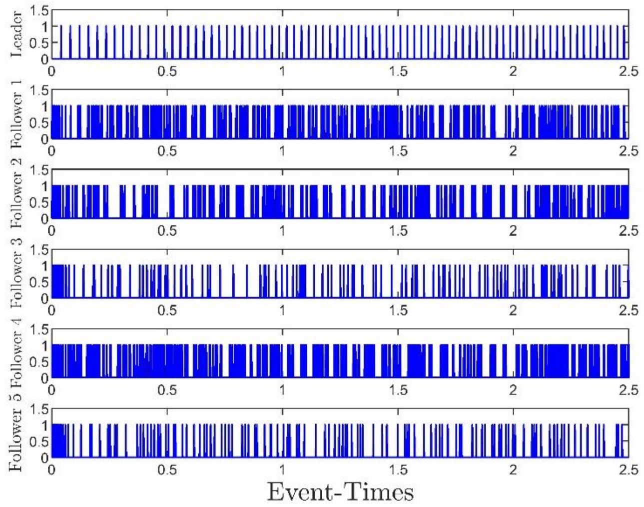
$$\hat{x}_j(t) = x_{j,1}\left(t_k^j\right),\ t \in \left[t_k^j, t_{k+1}^j\right)$$ ← Synchronized among all $i \in \mathcal{N}_j(t) \cup \{j\}$

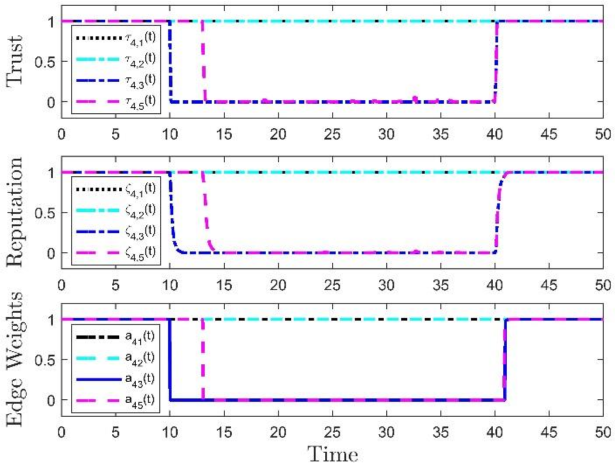
Theorem 1:
The trust model, reputation model, edge weight policy, state observer, and controller ensure E_1 is uniformly ultimately bounded in the sense that

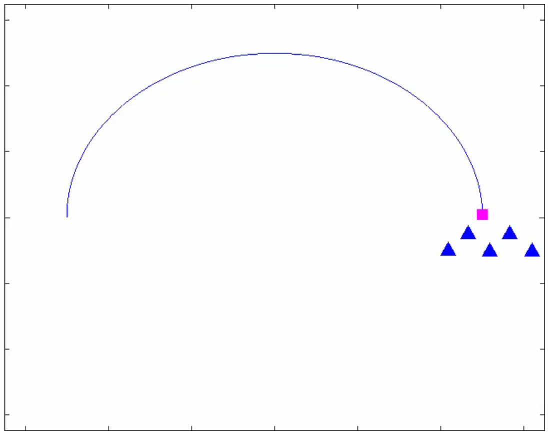
$$\limsup_{t\to\infty} \|E\| \leq 2\sqrt{\frac{4c_5^2+2\delta^*}{\phi_6}}$$ ← Can be made small

where $\beta_1,\ \beta_2,\ \beta_3 \in \mathbb{R}_{\geq 0}$ are known constants provided state feedback is available as dictated by the event-trigger, all assumptions are satisfied, and the sufficient user-defined parameter conditions (defined in the paper) are satisfied.

Simulation Results







- Purple = leader
- Blue = cooperative follower
- Red = Byzantine follower

Consensus over Clustered Networks with Asynchronous Inter-Cluster Communication

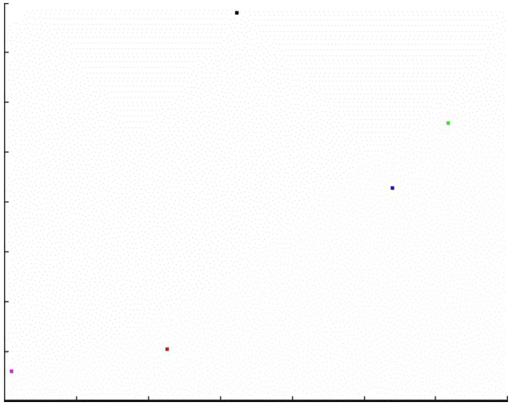
F. M. Zegers, S. Phillips, and W. E. Dixon, "Consensus over Clustered Networks with Asynchronous Inter-Cluster Communication" in *American Control Conference*. Submitted

Consensus/Agreement Problem

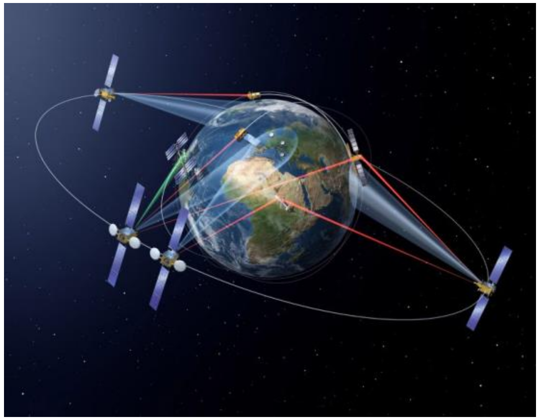
$$\{x_i(t)\}_{i\in I} \qquad \dot{x}_i = f_i(x_i(t), u_i(t))$$

$$\lim_{t\to\infty} \|x_i(t) - x_k(t)\| = 0 \quad \forall i, k \in I$$

Rendezvous



Attitude Control

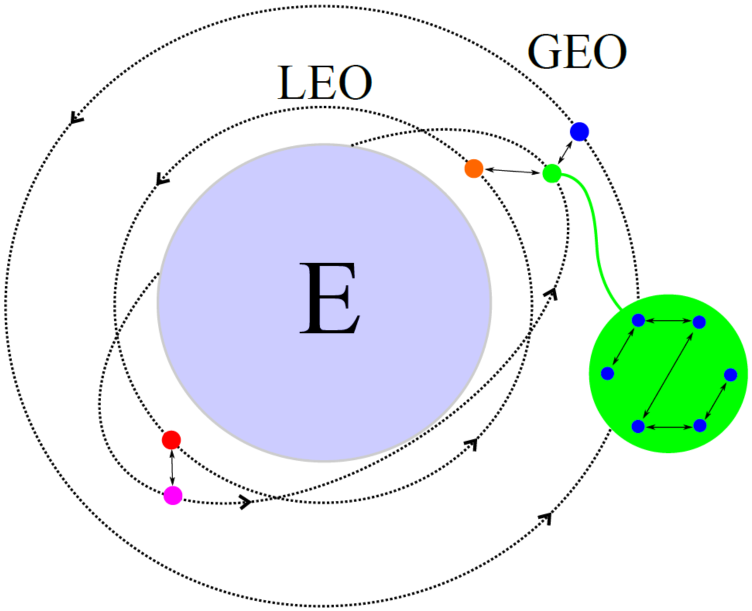


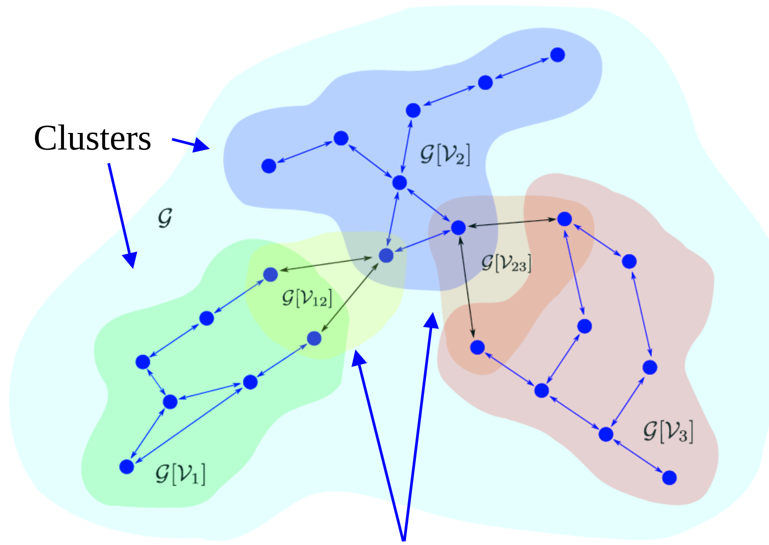
https://www.spatialsource.com

Blockchain Security

https://www.paymentsjournal.com

Clustered Networks





$$\mathcal{C} \triangleq \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_M\} \quad \mathcal{V}_p \cap \mathcal{V}_q = \varnothing \quad \bigcup_{p\in[M]} \mathcal{V}_p = \mathcal{V}$$

$$\mathcal{V}_p \subset \mathcal{V} \quad \forall p \in [M] \quad \forall p, q \in [M]$$

Problem Formulation

Consider a MAS with

- N agents
- Communication graph (undirected)

  $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$

  $\mathcal{V} \triangleq \{1, 2, ..., N\}$

  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$
- Cluster set

  $\mathcal{C} \triangleq \{\mathcal{V}_1, \mathcal{V}_2, ..., \mathcal{V}_M\}$
- Process model

  $\dot{x}_i(t) = Ax_i(t) + Bu_i(t)$

  $x_i : [0, \infty) \to \mathbb{R}^n$

  $u_i : [0, \infty) \to \mathbb{R}^d$

  $A \in \mathbb{R}^{n \times n} \quad B \in \mathbb{R}^{n \times d}$

Design a distributed controller that enables

$$\lim_{t \to \infty} \|e_i(t)\| = 0 \quad \forall i \in \mathcal{V}$$

$$e_i \triangleq x_i - \frac{1}{N} \sum_{l \in \mathcal{V}} x_l$$

through asynchronous communication

Assumptions

- Each agent can measure their state for all time
- The union graph (static) is connected
- At most one inter-cluster between each cluster pair

Hybrid systems framework:

$$\mathcal{H} \triangleq \begin{cases} \dot{z} = f(z), & z \in C, \\ z^+ \in G(z), & z \in D, \end{cases}$$

system state

flow map

flow set

jump map

jump set

UF UNIVERSITY of FLORIDA · Duke UNIVERSITY · TEXAS The University of Texas at Austin · UC SANTA CRUZ

Timer dynamics: $r \in [M^*]$

$$\tau_r : [0,\infty) \to [0, T_2^r]$$

$$\dot{\tau}_r = -1, \qquad \tau_r \in [0, T_2^r]$$

$$\tau_r^+ \in [T_1^r, T_2^r], \quad \tau_r = 0.$$

Same cluster component:

$$\eta_i \triangleq \sum_{k \in \mathcal{N}_i^0} a_{ik}(x_k - x_i)$$

$$a_{ik} \triangleq \begin{cases} 1, & (i,k) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases}$$

Graph vs. timer:

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ models the potential flow of information.

τ_r determines inter-cluster communication.

Agent controller:

$$u_i \triangleq K\eta_i + K \sum_{r \in [M^*]} \eta_{i,r}$$

continuous comms

Intermittent/asynchronous comms

Inter-cluster r component:

$$\dot{\eta}_{i,r} = 0_n, \qquad \tau_r \in [0, T_2^r]$$

$$\eta_{i,r}^+ = \sum_{k \in \mathcal{N}_i^r} a_{ik}(x_k - x_i), \quad \tau_r = 0.$$

$$e \triangleq \left[e_1^\top, e_2^\top, ..., e_N^\top\right]^\top \in \mathbb{R}^{nN}$$
$$\eta \triangleq \left[\eta_1^\top, \eta_2^\top, ..., \eta_N^\top\right]^\top \in \mathbb{R}^{nN}$$
$$\theta_r \triangleq \left[\eta_{1,r}^\top, \eta_{2,r}^\top, ..., \eta_{N,r}^\top\right]^\top \in \mathbb{R}^{nN}$$
$$\Theta \triangleq \left[\theta_1^\top, \theta_2^\top, ..., \theta_{M^*}^\top\right]^\top \in \mathbb{R}^{nNM^*}$$
$$\tau \triangleq \left[\tau_1, \tau_2, ..., \tau_{M^*}\right]^\top \in \mathbb{R}^{M^*}$$

$$\tilde{\theta}_r \triangleq -\left(L_r \otimes I_n\right) e - \theta_r$$
$$\widetilde{\Theta} \triangleq \left[\tilde{\theta}_1^\top, \tilde{\theta}_2^\top, ..., \tilde{\theta}_{M^*}^\top\right]^\top \in \mathbb{R}^{nNM^*}$$
$$\mathcal{T} \triangleq [0, T_2^1] \times ... \times [0, T_2^{M^*}]$$

$$\xi \triangleq \left[e^\top, \widetilde{\Theta}^\top, \tau^\top\right]^\top \in \mathcal{X}$$

$$\mathcal{X} \triangleq \mathbb{R}^{nM} \times \mathbb{R}^{nNM^*} \times \mathcal{T}$$

$$C \triangleq \mathcal{X}$$
$$f(\xi) \triangleq \begin{bmatrix} \mathbf{A}z \\ -1_{M^*} \end{bmatrix} \qquad z \triangleq \left[e^\top, \widetilde{\Theta}^\top\right]^\top$$
$$\mathbf{A} \triangleq \begin{bmatrix} \bar{A} - \bar{B}\bar{L} & -\mathbf{A}^* \\ -\mathbf{L}\left(1_{M^*} \otimes \left(\bar{A} - \bar{B}\bar{L}\right)\right) & -\mathbf{L}\left(1_{M^*} \otimes \mathbf{A}^*\right) \end{bmatrix}$$
$$\mathbf{A}^* \triangleq \bar{B}\left(1_{M^*}^\top \otimes I_{nN}\right)$$
$$\mathbf{L} \triangleq \operatorname{diag}\left(L_1 \otimes I_n, ..., L_{M^*} \otimes I_n\right)$$

$$D \triangleq \cup_{r\in[M^*]} \{\xi \in \mathcal{X} : \tau_r = 0\}$$
$$G(\xi) \triangleq \begin{bmatrix} e \\ \left[\tilde{\theta}_1^\top, \tilde{\theta}_2^\top, ..., 0_{nN}^\top, ...\tilde{\theta}_{M^*}^\top\right]^\top \\ \left[\tau_1, \tau_2, ..., [T_1^r, T_2^r, ...\tau_{M^*}\right]^\top \end{bmatrix}$$

$$\mathcal{A} \triangleq \left\{ \xi \in C : \xi = \left[0_{nN}^{\top}, 0_{nNM^*}^{\top}, \nu^{\top} \right]^{\top}, \nu \in \mathcal{T} \right\}$$

Theorem:

Suppose Assumptions 1-3 are satisfied. Given $0 < T_2^r$ for all $r \in [M^*]$, the set $\mathcal{A}$ is globally exponentially stable for the hybrid system $\mathcal{H}$ with data (C, f, D, G) if there exists $\sigma > 0$, gain matrix $K \in \mathbb{R}^{d\times n}$, and symmetric positive definite matrices $P \in \mathbb{R}^{n(N-1)\times n(N-1)}$ and $Q_r \in \mathbb{R}^{n(N-1)\times n(N-1)}$ for each $r \in [M^*]$ such that

$$\widetilde{\mathbf{A}}^{\top} R(\tau) + R(\tau)\widetilde{\mathbf{A}} - \widetilde{R}(\tau) \leq -I_{nN(1+M^*)} \quad \forall \tau \in \mathcal{T},$$

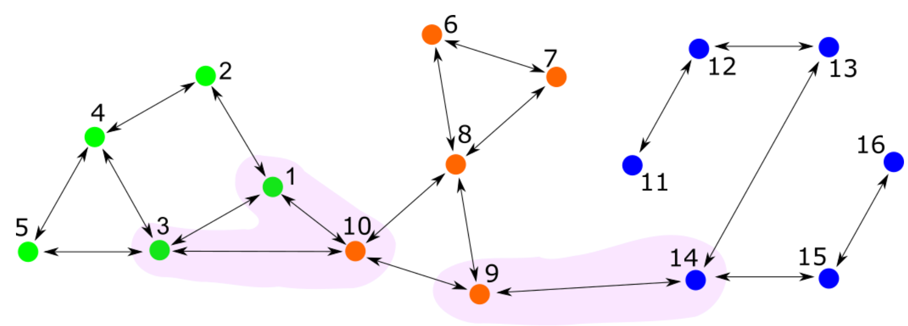
where $\widetilde{\Psi} \triangleq I_{(1+M^*)} \otimes \Psi \otimes I_n$, $R(\tau) \triangleq \mathrm{diag}\left(P, Q_1 e^{\sigma\tau_1}, \ldots, Q_{M^*} e^{\sigma\tau_{M^*}}\right)$, $\widetilde{R}(\tau) \triangleq \mathrm{diag}\left(0_{nN\times nN}, \sigma Q_1 e^{\sigma\tau_1}, \ldots, \sigma Q_{M^*} e^{\sigma\tau_{M^*}}\right)$, and $\widetilde{\mathbf{A}} \triangleq \widetilde{\Psi}^{\top}\mathbf{A}\widetilde{\Psi}$.

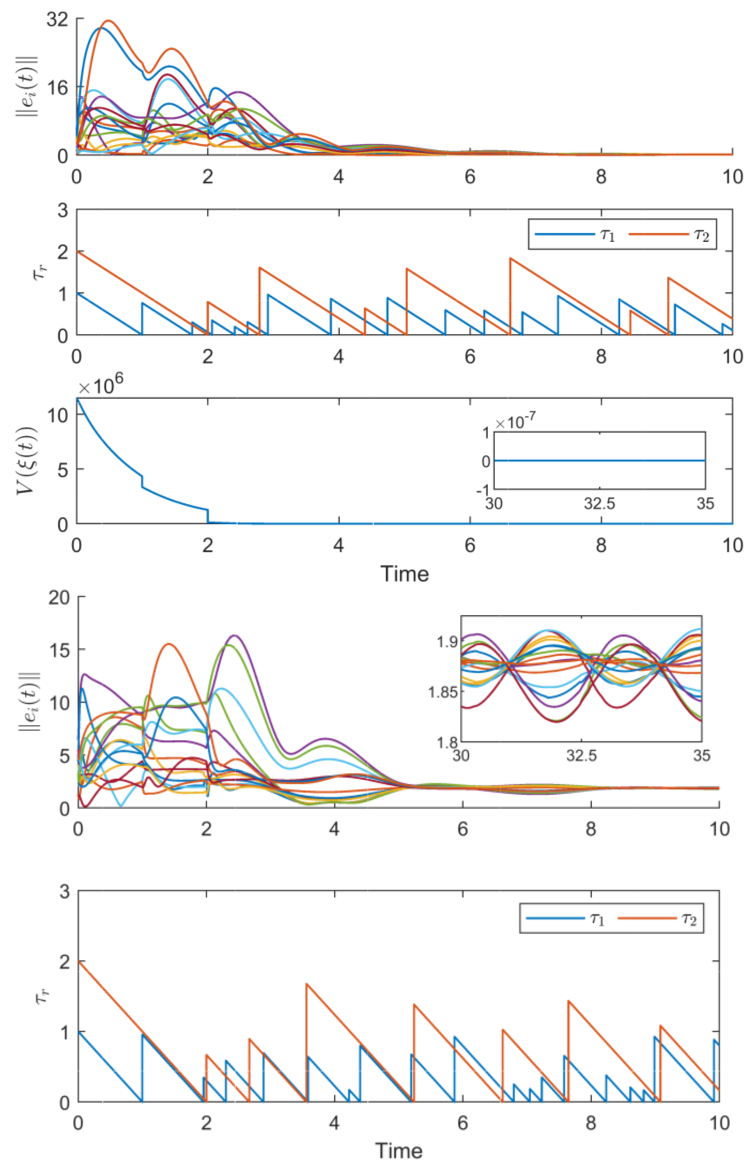
$$\dot{x}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$K = \begin{bmatrix} 26.5 & 10.5 \end{bmatrix}$$

$$\left.\begin{aligned} T_1^1 &= 0.1 \\ T_1^2 &= 0.1 \\ T_2^1 &= 1 \\ T_2^2 &= 2 \end{aligned}\right\} \quad T_1^r \le t_{w+1}^r - t_w^r \le T_2^r$$

Network topology: $N = 16$





A Switched Systems Approach to Multi-Agent System Consensus with Intermittent Communication

F. M. Zegers, P. Deptula, H.-Y. Chen, A. Isaly, and Warren E. Dixon, "A Switched Systems Approach to Multi-Agent System Consensus with Intermittent Communication," in *IEEE Transactions on Robotics*. In preparation.

Preliminary Experiment