Optimizing Synchronization Times for Distributed Tracking of a Mobile Asset in GPS-denied Environments

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- Clock synchronization requires communication among sensors and localization may not be possible during the synchronization times
- Need to optimize between localization and synchronization to maximize performance

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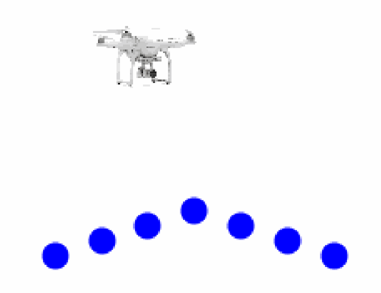
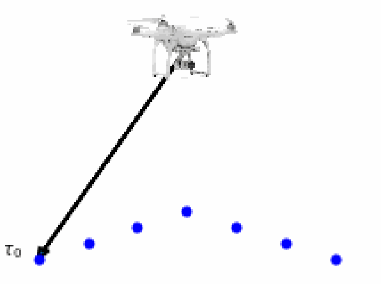


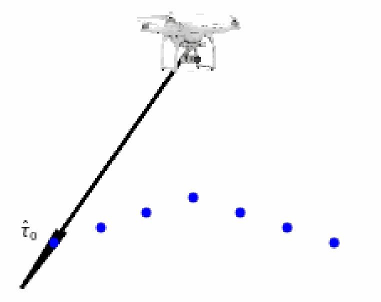
System Model 2

- Sensors measure time-of-flights (ToFs) of beacon signal and localizes (LOC) asset by fusing these measurements



System Model 3

- Each agent's clock drifts independently and variance of clock signals increase with time



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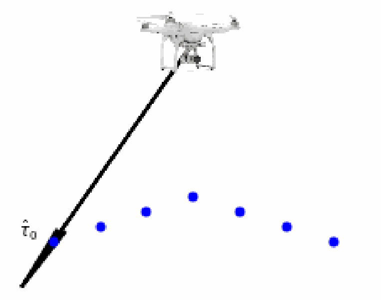

- Agents can synchronize (SYNCH) clocks at expense of not being able to measure ToFs during that time

Model-Free Localization

- Let coordinates of asset and sensor i in interval k be $(x_{k,a}, y_{k,a}, z_{k,a})$ and (x_i, y_i, z_i)

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- Let coordinates of asset and sensor i in interval k be $(x_{k,a}, y_{k,a}, z_{k,a})$ and (x_i, y_i, z_i)
- Using sensor $m-1$ as a reference, form linear equations $\mathbf{A} \cdot \mathbf{v}_k = \boldsymbol{\beta}_k$
- Here $\mathbf{v}_k = \left[x_{k,a}, y_{k,a}, z_{k,a}\right]^T$, $\mathbf{A}$ is a matrix with row i given by

$$\mathbf{A}_i = \left[2(x_i - x_{m-1}),\ 2(y_i - y_{m-1}),\ 2(z_i - z_{m-1})\right], \quad i \in \{0, 1, \ldots, m-2\},$$

and $\boldsymbol{\beta}_k$ is a column vector with component

$$\beta_i = c^2\left(\widehat{\tau}_{k,i}^2 - \widehat{\tau}_{k,m-1}^2\right) - \left(x_i^2 - x_{m-1}^2\right) - \left(y_i^2 - y_{m-1}^2\right) - \left(z_i^2 - z_{m-1}^2\right), \qquad i \in \{0, 1, \ldots, m-2\}$$

Localization Solution

- The least squares solution is given by $[\widehat{x}_{k,a}, \widehat{y}_{k,a}, \widehat{z}_{k,a}]^T = \mathbf{A}^\dagger \boldsymbol{\beta}_k$ where $\mathbf{A}^\dagger = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$ is the Moore-Penrose pseduo-inverse of $\mathbf{A}$

Improving Localization and Coordinating Synchronization

- Pure localization generally not good enough because of noisy clocks
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- Resolve both problems by treating tracking problem as HMM and treating choice of SYNC/LOC as control problem

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- Resolve both problems by treating tracking problem as HMM and treating choice of SYNC/LOC as control problem
- Since true state of asset never known, result is Partially Observable Markov Decision Process (POMDP)

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6. A cost function
   $c(x, u, z)$

Control Set

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  - u_s: synchronize (**synch**)

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  and
- Note that at time k, $T_k^{(s)}$ is known (deterministic) given the previous controls $u_0, u_1, \ldots, u_{k-1}$

Belief States, Observation Sequences and Control Sequences

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  - $\mathbf{z}_k$: vector of observations up to interval k
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- Given:
  - $\mathbf{z}_k$: vector of observations up to interval k
  - $\mathbf{u}_{k-1}$: vector of controls leading up to interval $k-1$
- Belief state at interval k is $\mathbf{b}_k$:

$$b_k(x) = \Pr\left(X_k = x \middle| \mathbf{z}_k, \mathbf{u}_{k-1}\right)$$

Belief Update

- Continuous observation space (localization results) – most papers consider finite observation space

$$b_{k+1}(x_{k+1}) = \frac{f(\mathbf{z}_{k+1}, x_{k+1} \,|\mathbf{u}_k\,)}{f(\mathbf{z}_{k+1}\,|\mathbf{u}_k\,)}, \text{ where}$$

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$$\begin{aligned} f(\mathbf{z}_{k+1}, x_{k+1} \,|\mathbf{u}_k) &= \sum_{x_k \in \mathcal{X}} f(\mathbf{z}_{k+1}, x_{k+1} \,|\mathbf{z}_k, x_k, \mathbf{u}_k)\, f(\mathbf{z}_k, x_k \,|\mathbf{u}_k) \\ &= \sum_{x_k \in \mathcal{X}} f(z_{k+1}, x_{k+1} \,|x_k, u_k)\, f(\mathbf{z}_k, x_k \,|\mathbf{u}_{k-1}) \\ &= f(z_{k+1} \,|x_{k+1}) \sum_{x_k \in \mathcal{X}} \Pr(x_{k+1} \,|x_k, u_k)\, f(\mathbf{z}_k, x_k \,|\mathbf{u}_{k-1}) \end{aligned}$$

More on the Belief Update

- The conditional distribution of z_k given x_k is modeled as Gaussian: with mean determined by the ML state of $\mathbf{b}_k$ and variance $\left(T_k^{(s)}\right)^2$
- If the control is **sync**, then no measurement z_{k+1} is available; then, update the belief by applying the Markov model transitions probabilities

$$\mathbf{b}_{k+1} = \mathbf{P} \cdot \mathbf{b}_k$$

Cost Function

- Distance between asset's true location and the ML estimate from the belief state

$$c_k = |L(x_k) - L(\widehat{x}_k)|$$

where

  - x_k is true state
  - $\widehat{x}_k = \arg\max_{x \in \mathcal{X}} b_k(x)$

Movement Models

- Evaluate performance using simple location-only, one-dimensional Markov chains:

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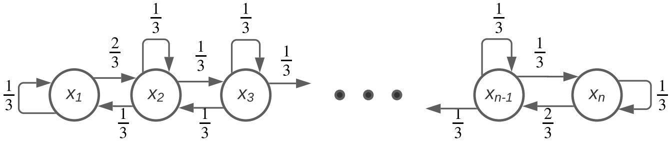
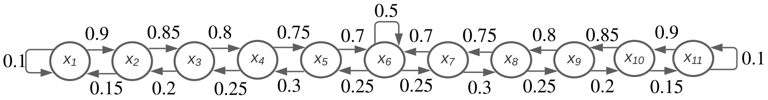
- Chain 1: uniform probability of staying or moving to adjacent states:



Movement Model 2

- Chain 2: model an asset that primarily loiters near middle of region, rarely transitions to the outer edges



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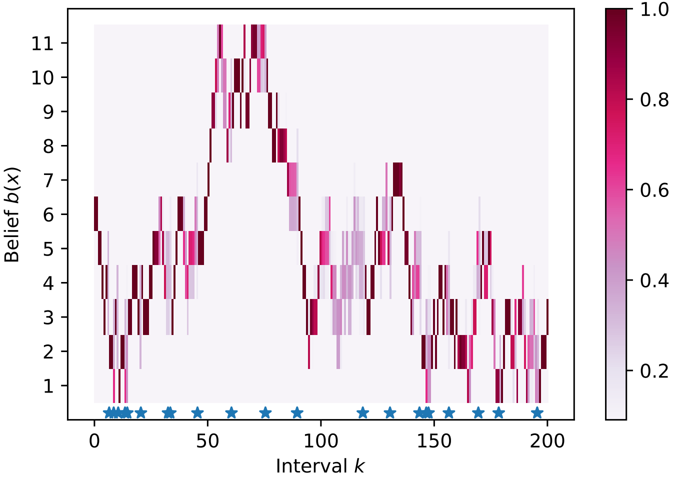

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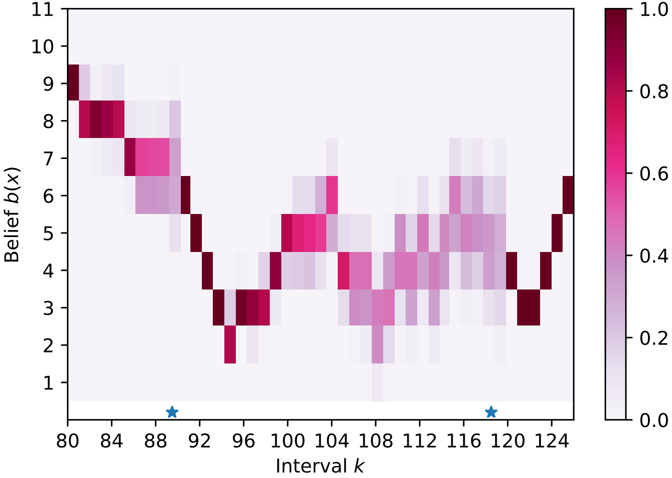

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  - $T_k^{(s)}$: is the number of time since last **sync**
  - $\hat{x}_k$: ML estimate for movement state
  - $\sigma_{k,x}^2$: Quantized variance of movement state

Belief State Compression 2

- Whereas spreading of beliefs is an implicit factor in original belief state, it becomes an explicit component of the compressed state through the variance measure
- Called: *Triple Q-Learning* (TQ-Learning)

TQ-Learning Update

- Use tabular Q-learning with usual update rule:

$$Q(\underline{x}, u) = Q(\underline{x}, u) + \alpha \left[c + \gamma \min_{u'} Q(g(\underline{x}, u), u') - Q(\underline{x}, u) \right]$$

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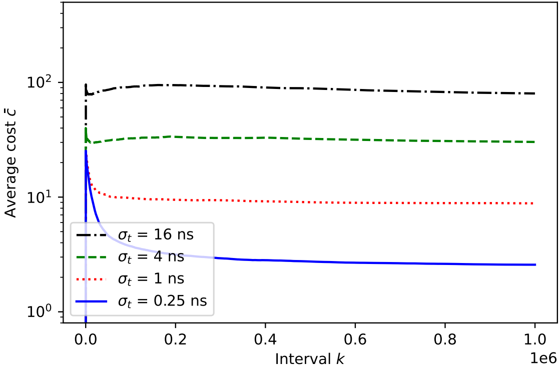
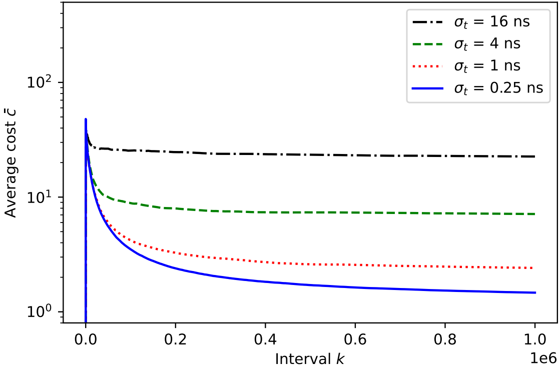
- Model-free (MF) localization: based on raw localization results from triangulation

Training Curves: P_1, $m = 3$



Training Curves, P_2, $m = 15$



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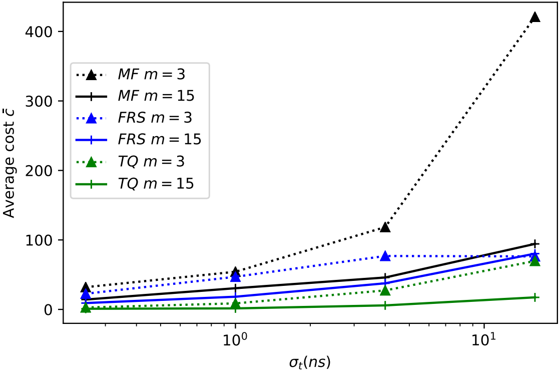
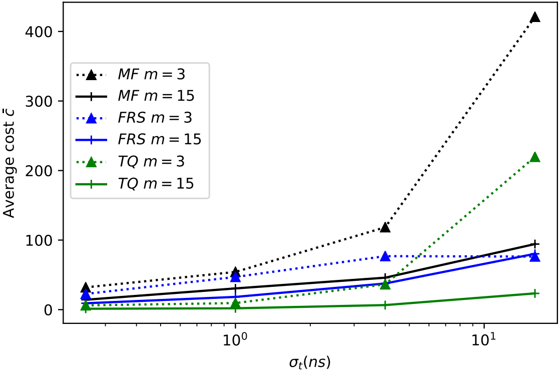


Testing Results: Model P2



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- Very early work: good candidate for deep Q-learning, want to consider RADAR problem, moving sensors, ...