

Recent Advancements in Faster and Deeper Learning

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AFOSR Center of Excellence Review @ Zoom
November 9th , 2021





- Real-time **learning and control** of dynamical systems

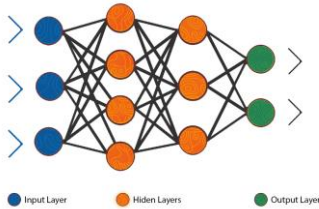
$$\dot{x} = f(x) + u$$

- Unstructured Uncertainty

- Deep Neural Networks to approximate model uncertainty
- Real-time learning of DNN weights with Lyapunov-based methods

$$f(x) = W^{*T} \sigma(\Phi(x))$$

$$\Phi = (V_k^{*T} \phi_k \circ \dots \circ V_1^{*T}) (V_0^{*T} x)$$



- Structured Uncertainty

- Dynamics modeled as linear regression model
- Connections between accelerated gradient methods in optimization and adaptive control

$$f(x) = Y(x) \theta^*$$

- R. Sun, M. Greene, D. Le, **Z. Bell**, G. Chowdhary, and W. Dixon, “Lyapunov-based Real-Time and Iterative Adjustment of Deep Neural Networks,” IEEE Control Sys. Lett. 2022, [Link](#)
- D. Le, M. Greene, W. Makumi, and W. Dixon, “Real-time Modular Deep Neural Network-Based Adaptive Control of Nonlinear Systems,” IEEE Control Sys. Lett. 2022, [Link](#)
- O. Patil, D. Le, M. Greene, and W. Dixon, “Lyapunov-Derived Control and Adaptive Update Laws for Inner and Outer Layer Weights of a Deep Neural Network,” (Submitted to LCSS/ACC)
- D. Le, O. Patil, P. Amy, and W. Dixon, “Integral Concurrent Learning-Based Accelerated Gradient Adaptive Control of Uncertain Euler-Lagrange Systems,” (Submitted to ACC 2022)

Integral Concurrent Learning- Based Accelerated Gradient Adaptive Control of Uncertain Euler-Lagrange Systems

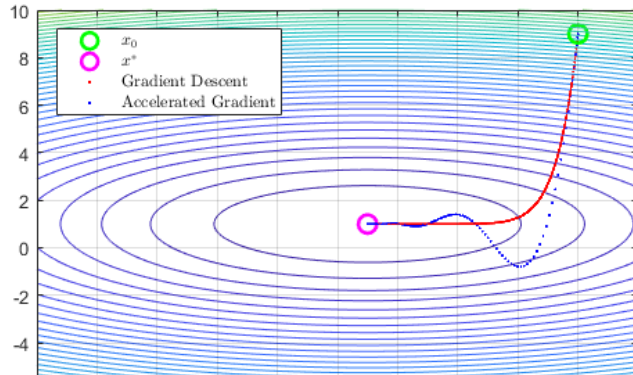
Daniel Le, Omkar Sudhir Patil, Patrick Amy, and Warren Dixon

Submitted to ACC 2022



- Nesterov's Accelerated Gradient

- Add “momentum” to the update law by adding the current step a weighted version of the previous step



- **GD** converges in 5574 iterations
- **NAG** converges in 447 iterations

- Connections to continuous time analogues*
 - Dynamical systems perspective and analysis
 - Insights on adaptation design

$$\ddot{\hat{\theta}} + c(t) \dot{\hat{\theta}} = -k \nabla f(\hat{\theta})$$

Technical Challenges

- Can not naively implement in closed-loop control
- Convergence of parameter estimations

*Su.Boyd.Candes, 2019
 Wibisono.Wilson.Jordan, 2016
 Wilson.Recht.Jordan, 2021

Problem Formulation

- General uncertain nonlinear Euler-Lagrange dynamics

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F\dot{q} = \tau$$

q, \dot{q}, \ddot{q} generalized position, velocity, and acceleration

M inertia

V_m centripetal coriolis

G generalized potential effects

F generalized dissipation effects

τ control input

Assumption. *The uncertain dynamics are linear in the parameters and can be expressed as*

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F\dot{q} = \underbrace{\Psi(q, \dot{q}, \ddot{q})}_{\text{Regressor Matrix}} \theta^* \quad \text{Unknown Parameters}$$



- Control objective
 - Trajectory tracking
 - Real-time parameter estimation
- Tracking and filtered tracking errors

$$e \triangleq q_d - q$$
$$r \triangleq \dot{e} + \alpha e$$

- Parameter estimation errors

$$\tilde{\theta} \triangleq \theta^* - \hat{\theta}$$



- Control Input $\tau \triangleq kr + Y\hat{\theta} + e - 2Y(\hat{\theta} - \nu)$
- Higher-order adaptation laws (implementable form)

$$\dot{\nu} \triangleq \Gamma \left(Y^T r + k_1 \sum_{i=1}^N \Psi_{f,i}^T (\tau_{f,i} - \Psi_{f,i}^T \nu) \right)$$

$$\dot{\hat{\theta}} \triangleq -\Gamma \left(k_2 (\hat{\theta} - \nu) - k_1 \sum_{i=1}^N \Psi_{f,i}^T (\tau_{f,i} - \Psi_{f,i}^T \nu) \right)$$

- Higher-order adaptation laws (analysis form)

$$\dot{\nu} = \Gamma \left(Y^T r + k_1 \sum_{i=1}^N \Psi_{f,i}^T \Psi_{f,i} (\theta^* - \nu) \right)$$

$$\dot{\hat{\theta}} = -\Gamma \left(k_2 (\hat{\theta} - \nu) - k_1 \sum_{i=1}^N \Psi_{f,i}^T \Psi_{f,i} (\theta^* - \nu) \right)$$

Assumption. *There exists a time $T \in \mathbb{R}_{>0}$ such that $\lambda_{\min} \left(\sum_{i=1}^N \Psi_{f,i}^T \Psi_{f,i} \right) \geq \gamma$, where $\gamma \in \mathbb{R}_{>0}$ is a user-defined parameter.*

- Two-link robot manipulator



$$M = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix}$$

$$V_m = \begin{bmatrix} -p_3s_2\dot{q}_2 & -p_3s_2(\dot{q}_1 + \dot{q}_2) \\ p_3s_2\dot{q}_1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}$$

- Parameters: $p_1 = 3.473$, $p_2 = 0.196$, $p_3 = 0.242$, $f_1 = 5.3$, and $f_2 = 1.1$

- Desired trajectory $q_d \triangleq \begin{bmatrix} \cos(0.5t) \\ 2 \cos(t) \end{bmatrix}$



- **Simulation 1 – Standard Adaptive**

$$\tau \triangleq kr + Y\hat{\theta} + e$$

$$\dot{\hat{\theta}} \triangleq \Gamma Y^T r$$

- **Simulation 2 – ICL Adaptive**

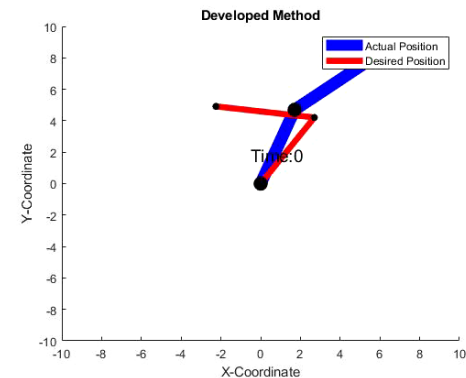
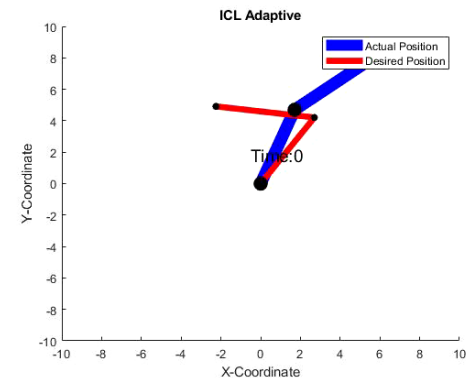
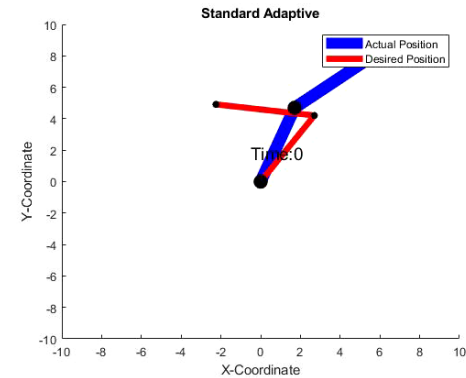
$$\dot{\hat{\theta}} \triangleq \Gamma \left(Y^T r + k_1 \sum_{i=1}^N \Psi_{f,i}^T (\tau_i - \Psi_{f,i} \hat{\theta}) \right)$$

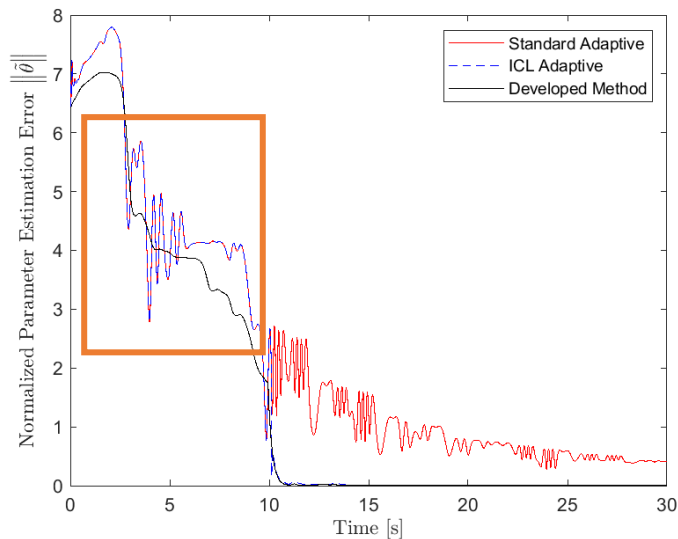
- **Simulation 3 – Developed Method**

$$\tau \triangleq kr + Y\hat{\theta} + e - 2Y(\hat{\theta} - \nu)$$

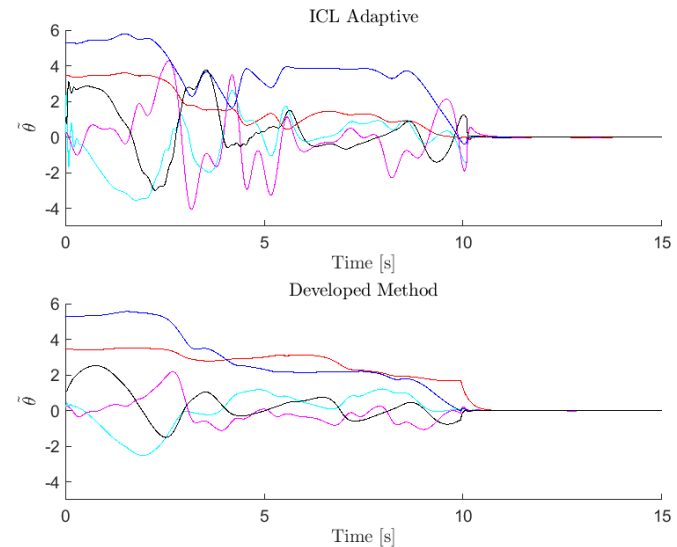
$$\dot{\nu} \triangleq \Gamma \left(Y^T r + k_1 \sum_{i=1}^N \Psi_{f,i}^T (\tau_{f,i} - \Psi_{f,i} \nu) \right)$$

$$\dot{\hat{\theta}} \triangleq -\Gamma \left(k_2 (\hat{\theta} - \nu) - k_1 \sum_{i=1}^N \Psi_{f,i}^T (\tau_{f,i} - \Psi_{f,i} \nu) \right)$$





Evolution of the normalized parameter estimation error trajectories for each simulation. The red line represents the simulation using the standard adaptive method. The blue line represents the simulation using the ICL adaptive method. The black line represents the simulation using the developed method.



(top): Parameter estimation error using the ICL adaptive method. (bottom): Parameter estimation error using the developed method.

Iterative Deep Neural Network System Identification

Max L. Greene

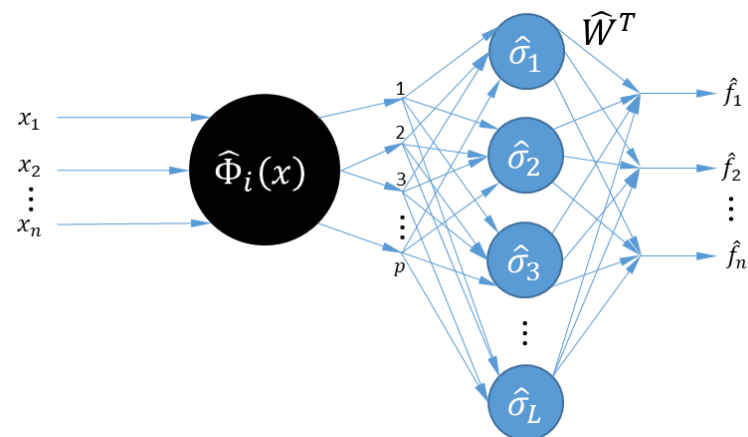
R. Sun, M. Greene, D. Le, Z. Bell, G. Chowdhary, and W. E. Dixon, "Lyapunov-Based Real-Time and Iterative Adjustment of Deep Neural Networks," IEEE Control Systems Letters, Vol. 6, pp. 193-198 (2021).





Iterative DNN Approximation

- Multi-Timescale DNN
 - Online system ID
 - Improvement over single hidden-layer
 - Inner features updated iteratively (switched)
 - Output-layer updated in real-time
 - Feedforward term in robust nonlinear controller



Inner Features (Trained Iteratively) Fully Connected Layer (Updated in real-time)



Real-Time DNN Approximation

Dynamical System

Given a control affine nonlinear dynamical system:

$$\dot{x} = f(x) + g(x)u$$

DNN Approximation

Approximate the drift dynamics with a DNN:

$$\text{Dynamics: } f(x) = \theta^T \phi(\Phi^*(x)) + \epsilon(x)$$

$$i^{\text{th}} \text{ DNN Approximation: } \hat{f}_i(x, \hat{\theta}) = \hat{\theta}^T \phi(\hat{\Phi}_i(x))$$

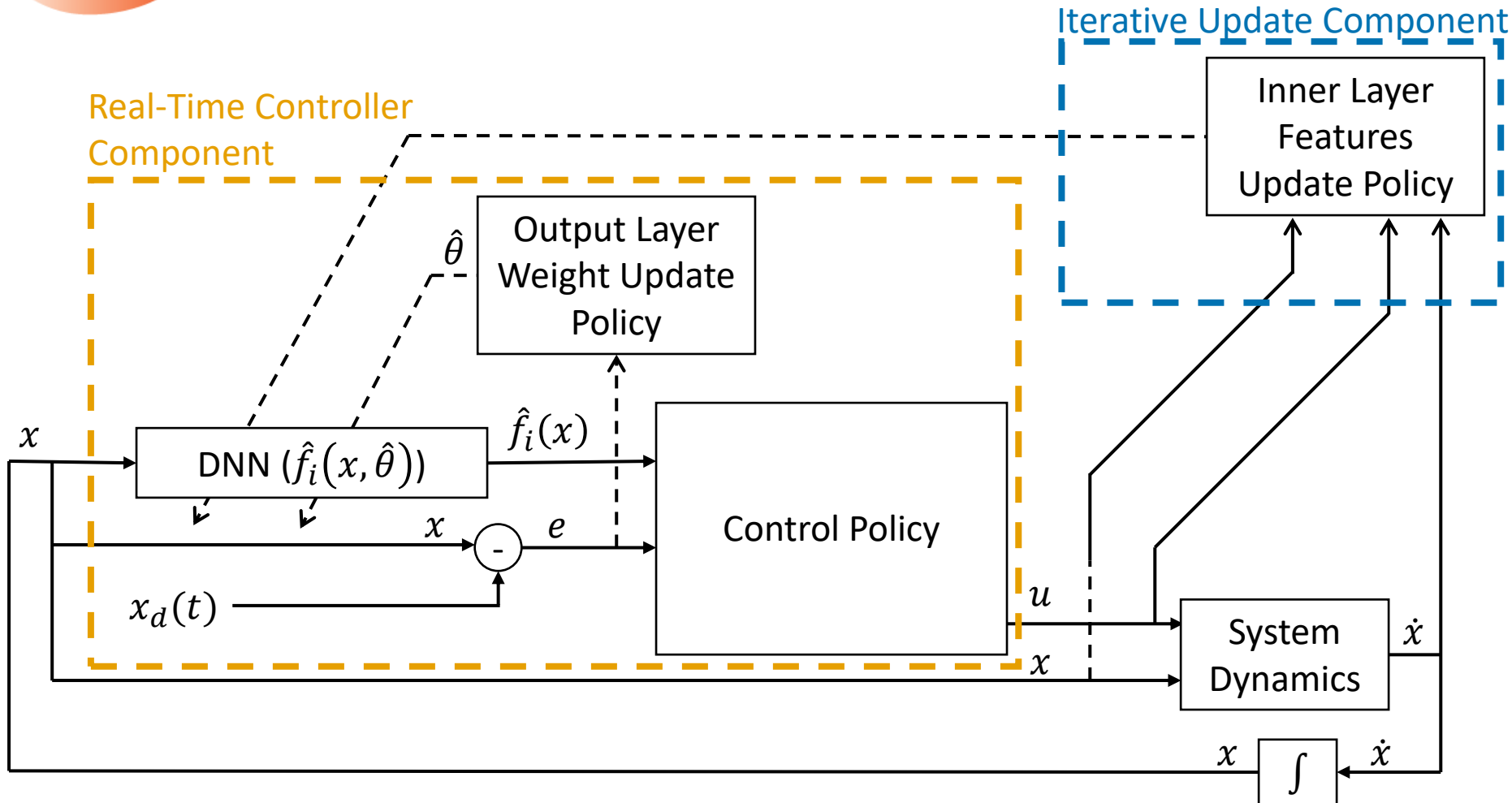
Looks Like:

Reminds us of linearly parameterizable dynamics:

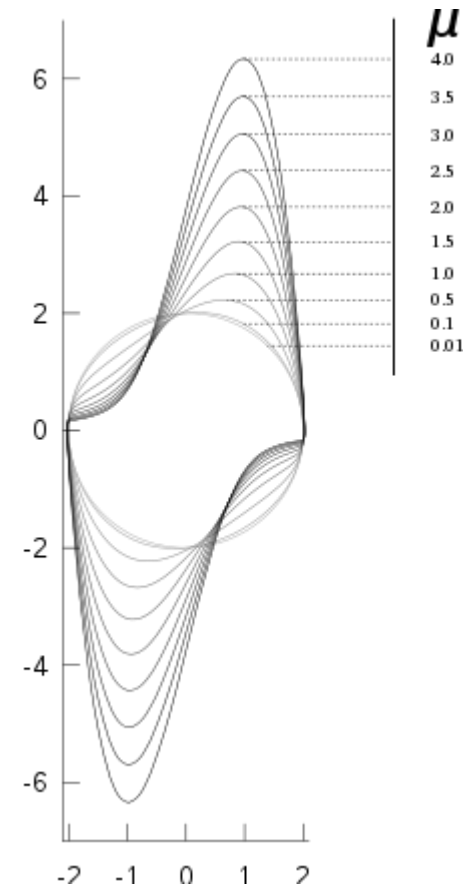
$$f(x) = \theta^T \phi(\Phi^*(x)) + \epsilon(x) \approx \theta^T Y(x) + \epsilon(x)$$



Real-Time DNN Approximation

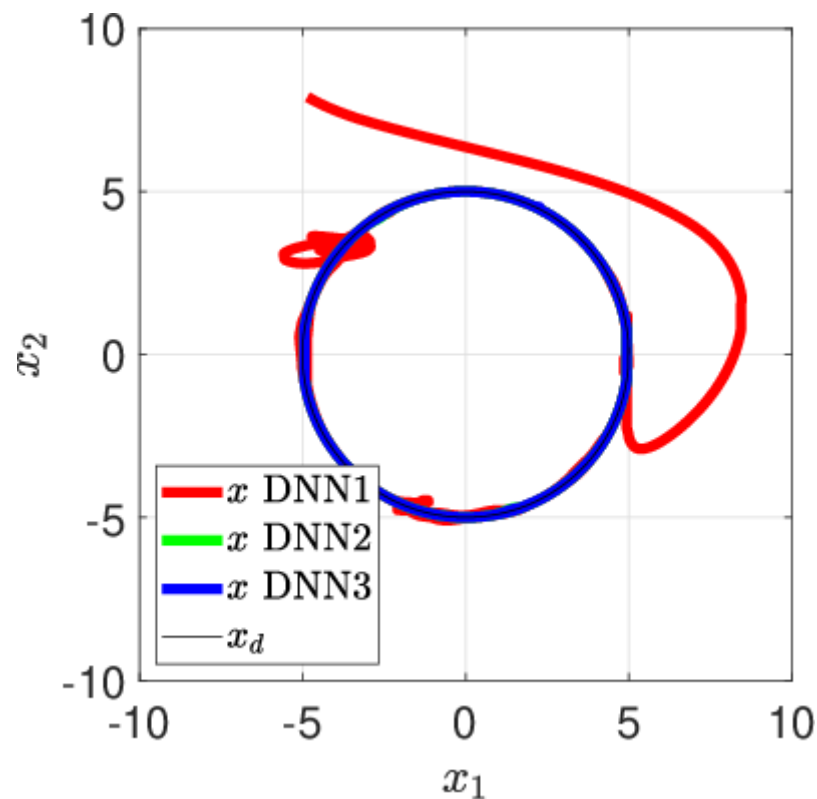
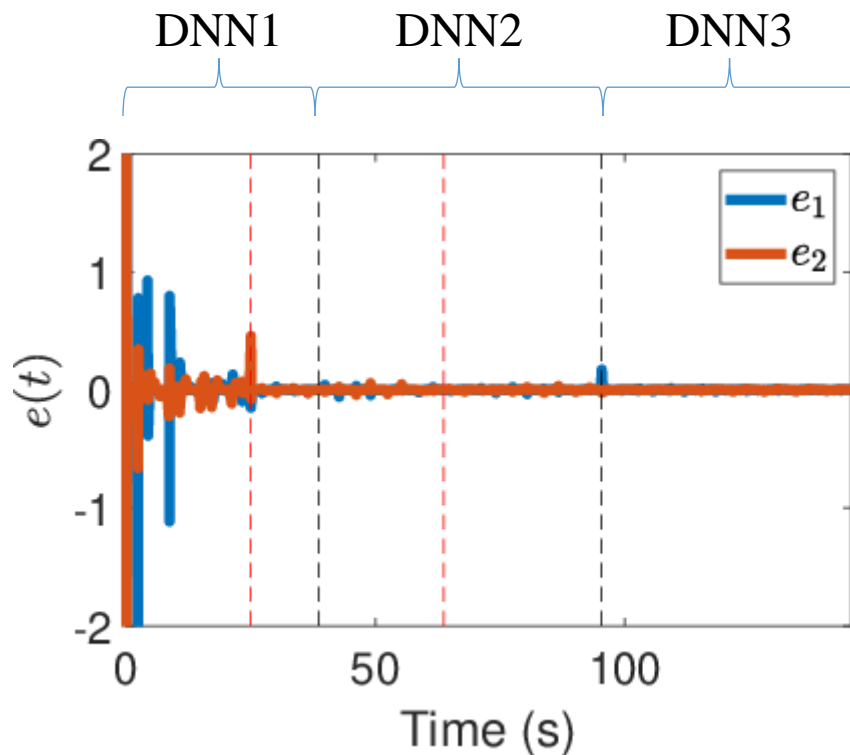


- Van der Pol Oscillator
- Trained with 600s of simulation data
- Transient response is fast relative to the overall timescale





Trained on Identical Dynamics





- Adversarial Attacks

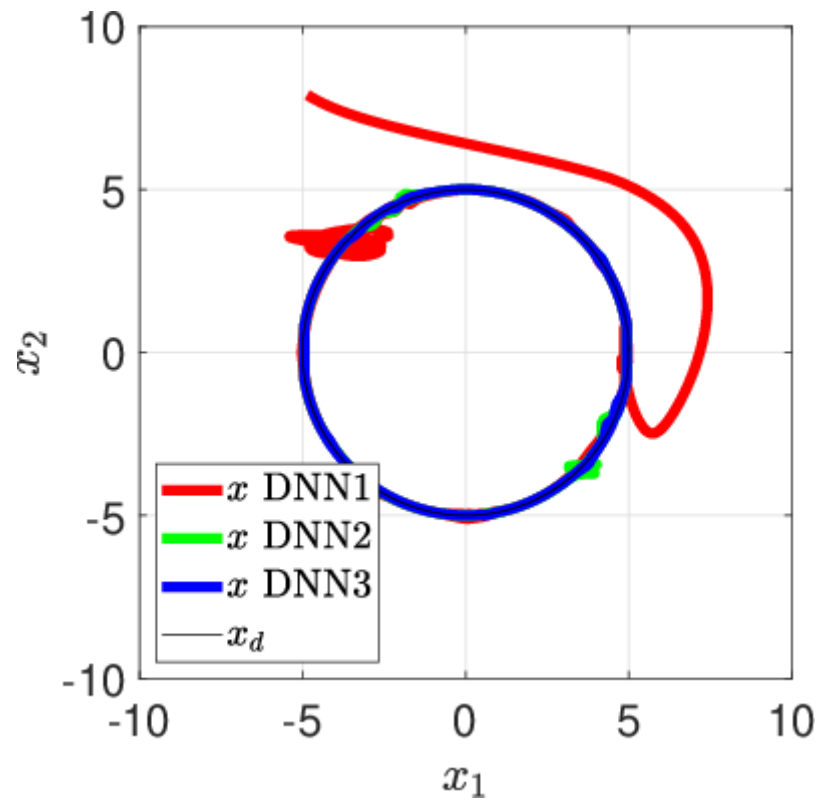
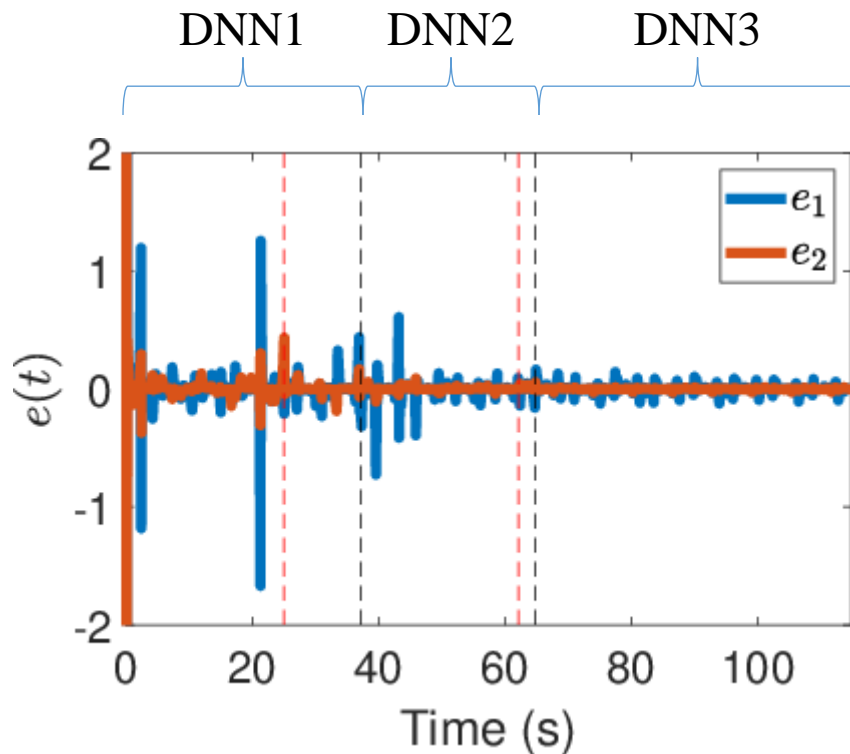
- Recall, $\hat{f}_i(x, \hat{\theta}) = \hat{\theta}^T \phi(\hat{\Phi}_i(x))$
- What if an adversary alters $\hat{\Phi}_i$?
 - Initially Poor Approximation
 - Output-layer Adjustment
 - Robustness to Adversarial Attacks
- Adaptation to unknown/uncertain environments

$$\hat{f}_i(x, \hat{\theta}) = \hat{\theta}^T \phi(\hat{\Phi}_i(x))$$

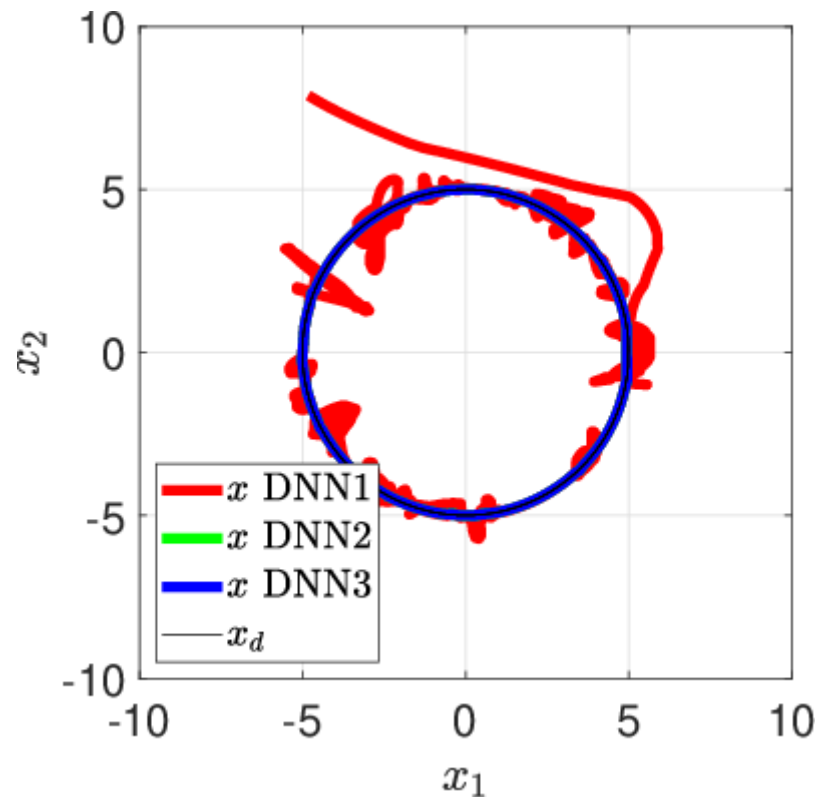
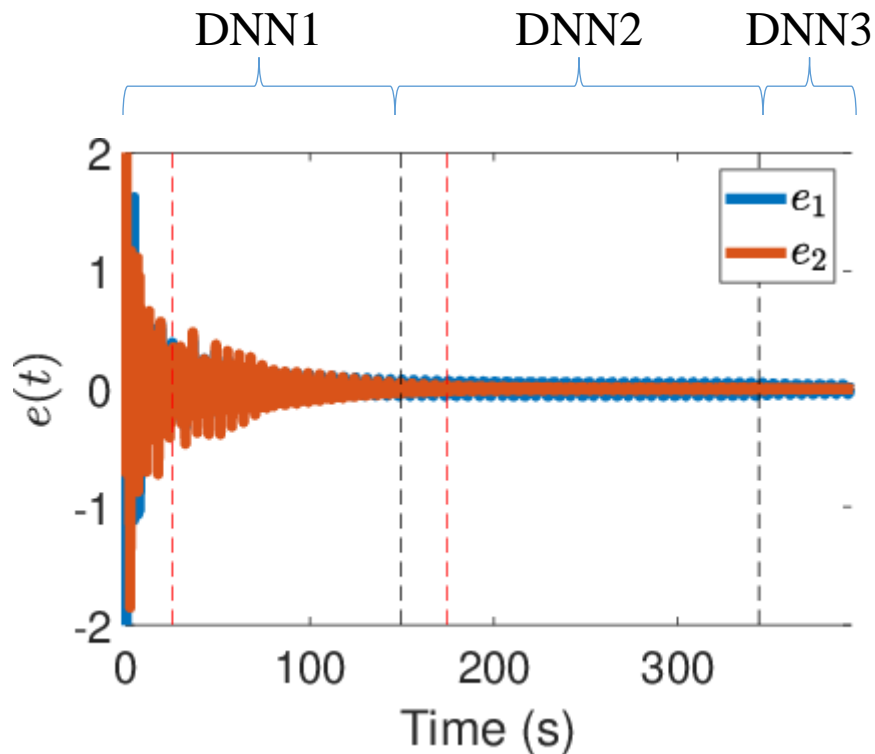
Accommodates for poor approximation due to $\hat{\Phi}_i(x)$



Inner-Layer Features Altered By Adversary



No offline training. Inner-layer DNN features are randomly initialized.



- Raised Questions
 - Training the output-layer is good
 - Can we update every layer online?
 - Class of update policies to update every layer?
 - Can every layer be updated simultaneously?

Questions?

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Real-time Modular Deep Neural Network Control

D. Le, M. Greene, W. Makumi, and W. Dixon, “Real-time Modular Deep Neural Network-Based Adaptive Control of Nonlinear Systems,” IEEE Control Sys. Lett. 2022

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- Nonlinear dynamic system

$$\dot{x} = f(x) + g(x)u$$

- DNN approximation
 - Using DNN to approximate the uncertain drift dynamics
 - Real-time learning of DNN weights with Lyapunov-based methods

$$f(x) = W^{*T} \sigma(\Phi^*(x)) + \varepsilon(x)$$

$$\Phi^*(x) = (V_k^{*T} \phi_k \circ \dots \circ V_1^{*T} \phi_1) (V_0^{*T} x)$$

- Control Objective

Track a user-defined time-varying trajectory x_d

$$e = x - x_d$$



- Control input

$$u = g^+(x) \left(\dot{x}_d - k_1 e - k_s \text{sgn}(e) - \hat{f}(x) \right)$$

- Output-layer weight adaptation

$$\dot{W} = \Gamma_W \sigma \left(\hat{\Phi}(x) \right) e^T$$

- Inner-layer weight adaptation

$$\dot{\hat{V}}_j = p_j(t) \nu_j(e, t) \mathbf{1}_{\left\{ \underline{\hat{V}}_j \leq \|\hat{V}_j\|_F \leq \overline{\hat{V}}_j \right\}}$$

- Modular adaptive constraints

$$\|\nu_j\|_F \leq \rho(\|e\|) \|e\|$$

- Closed-loop error system

$$\dot{e} = W^{*T} \sigma(\Phi^*(x)) + \varepsilon(x) - k_1 e - k_s \text{sgn}(e) - \hat{W}^T \sigma(\hat{\Phi}(x))$$

- **Theorem.** The designed control input, output-layer weight adaptation law, and the family of potential inner-layer weight adaptation laws that satisfy the modular adaptive constraints ensure the closed-loop error system yields semi-global asymptotic tracking, provided the following sufficient gain condition is satisfied

$$k_s > 2\bar{\sigma}\bar{W}^* + \bar{\varepsilon} + 2\bar{V}^* (k+1) \rho \left(\sqrt{\frac{\bar{\alpha}}{\underline{\alpha}}} \|z(0)\| \right),$$

where $\underline{\alpha}, \bar{\alpha} \in \mathbb{R}_{\geq 0}$ are known constants.



- System Dynamics

$$f(x) = \left[-x_1 + x_2, -\frac{1}{2}x_1 + \frac{1}{2}x_2 \left(1 - \left(\cos(2x_1 + 2) \right)^2 \right) \right]^T$$

$$g(x) = \text{diag}[5, 3]$$

- DNN structure

- 6 layers: 12, 10, 15, 15, 20 neurons
- tanh activation function

- Inner-layer weight adaptation

- Switching signal $p_j(t) = \begin{cases} 1, & t \in [10j, 10(j+1)], \\ 0, & \text{else,} \end{cases}$
- Update law

$$\nu_j = \Gamma_{V_j} e \cdot \text{re} \left(\left(\tanh \circ \hat{\phi}_j^{-1} \circ \hat{V}_j^{+T} \hat{\phi}_{j+1}^{-1} \circ \hat{V}_5^{+T} \hat{\sigma}^{-1} \right) \left(\hat{W}^{+T} \dot{\hat{x}} \right) \right) \hat{V}_j$$

- Parameters

$$x_d = [3 \cos(t), 5 \sin(t)]^T$$

$$k_s = 0.5, k = 7$$

$$\Gamma_W = 10 \cdot \mathbf{1}_{L \times L}, \Gamma_{V_j} = 1000 \cdot \mathbf{1}_{L_j \times 2}$$

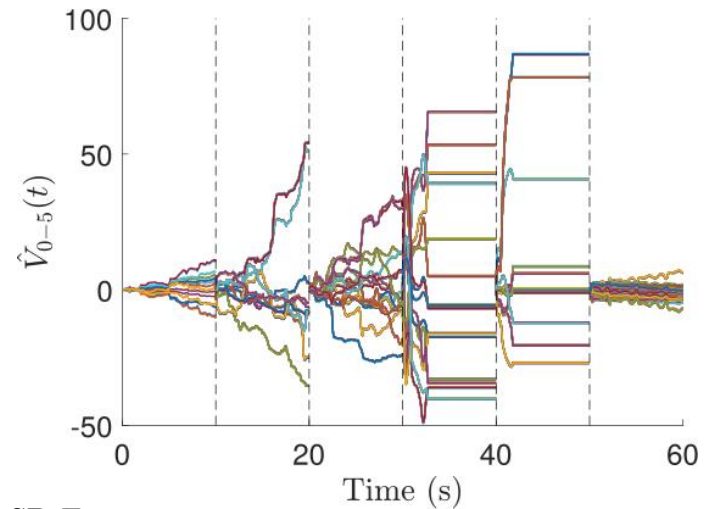
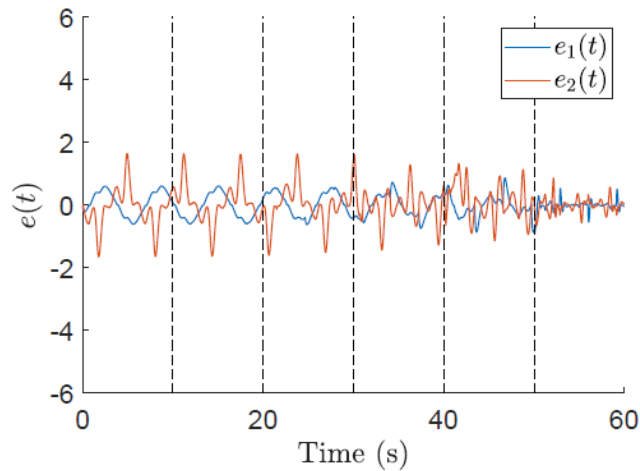


Table 1: RMS and SD Error

Active \hat{V}_j	RMS Error	SD Error
\hat{V}_0	0.735	0.145
\hat{V}_1	0.727	0.142
\hat{V}_2	0.710	0.136
\hat{V}_3	0.640	0.116
\hat{V}_4	0.658	0.087
\hat{V}_5	0.289	0.039

Lyapunov-Derived Control and Adaptive Update Laws for Inner and Outer Layer Weights of a Deep Neural Network

O. Patil, D. Le, M. Greene, and W. Dixon, “Lyapunov-Derived Control and Adaptive Update Laws for Inner and Outer Layer Weights of a Deep Neural Network,” submitted to IEEE Control Sys. Lett.

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- Dynamic Model

$$\dot{x} = f(x) + u$$

- Tracking Error $e \triangleq x - x_d$

where $x_d(t) \in \Omega$, given a compact set Ω .

- Control Objective : $e(t) \rightarrow 0$ as $t \rightarrow \infty$



- Fully-connected feedforward DNN

$$\Phi(x_d, V_0, V_1, \dots, V_k) \triangleq (V_k^T \phi_k \circ \dots \circ V_1^T \phi_1) (V_0^T x_{da})$$

- Recursive representation

$$\Phi_j \triangleq \begin{cases} V_j^T \phi_j (\Phi_{j-1}), & j \in \{1, \dots, k\}, \\ V_0^T x_{da}, & j = 0, \end{cases}$$

- Universal Approximation Property

$$\sup_{x_d \in \Omega} \|f(x_d) - \Phi(x_d, V_0^*, V_1^*, \dots, V_k^*)\| \leq \bar{\varepsilon}$$

- Then

$$f(x_d) = \Phi(x_d, V_0^*, V_1^*, \dots, V_k^*) + \varepsilon(x_d)$$

where $\sup_{x_d \in \Omega} \|\varepsilon(x_d)\| \leq \bar{\varepsilon}$.



- Adaptive Feedforward DNN term

$$\hat{\Phi} \triangleq \Phi(x_d, \hat{V}_0, \dots, \hat{V}_k)$$

where $\hat{V}_0, \dots, \hat{V}_k$ denote weight estimates.

- Control Law

$$u \triangleq \dot{x}_d - \underbrace{\rho(\|e\|)e}_{\text{Robust terms}} - k_1 e - \underbrace{k_s \text{sgn}(e)}_{\text{Robust terms}} - \underbrace{\hat{\Phi}}_{\text{DNN}}$$

where $\|f(x) - f(x_d)\| \leq \rho(\|e\|) \|e\|$

- $\hat{\Phi}_j \triangleq \Phi_j(x_d, \hat{V}_0, \dots, \hat{V}_j)$ $\Phi_j^* \triangleq \Phi_j(x_d, V_0^*, \dots, V_j^*)$
- $\hat{\phi}_j \triangleq \phi_j(\hat{\Phi}_{j-1})$ $\phi_j^* \triangleq \phi_j(\Phi_{j-1}^*)$
- $\hat{\phi}'_j \triangleq \phi'_j(\hat{\Phi}_{j-1})$ $\tilde{\Phi}_j \triangleq \Phi_j^* - \hat{\Phi}_j$

- Adaptation law for the j^{th} layer

$$\text{vec}(\dot{\hat{V}}_j) \triangleq \text{proj}(\Gamma_j \Lambda_j^T e)$$

$$\Lambda_0 \triangleq \Xi_0 (I_{L_1} \otimes x_{da}^T) \quad \Lambda_j \triangleq \Xi_j (I_{L_{j+1}} \otimes \hat{\phi}_j^T)$$

$$\Xi_0 \triangleq \overset{\curvearrowright k}{\prod}_{l=1} \hat{V}_l^T \hat{\phi}'_l \quad \Xi_j \triangleq \overset{\curvearrowright k}{\prod}_{l=1} \hat{V}_l^T \hat{\phi}'_l$$

- Projection operator guarantees weight estimates stay in the set

$$\mathcal{B}_j \triangleq \{\theta \in \mathbb{R}^{L_j L_{j+1}} : \|\theta\|_F \leq \bar{V}\}$$

- First-order Taylor series approximation

$$\phi_j^* = \hat{\phi}_j + \hat{\phi}'_j \tilde{\Phi}_{j-1} + \mathcal{O}^2 \left(\tilde{\Phi}_{j-1} \right)$$

- Output mismatch at the j^{th} layer

$$\tilde{\Phi}_j = \underbrace{\left(I_{L_{j+1}} \otimes \hat{\phi}_j^T \right) \text{vec} \left(\tilde{V}_j \right)}_{\text{Linearly-parameterized}} + \underbrace{\hat{V}_j^T \hat{\phi}'_j \tilde{\Phi}_{j-1}}_{\text{Recursion}} + \underbrace{\Delta_j}_{\text{Higher-Order Terms}}$$

where $\Delta_j \triangleq \tilde{V}_j^T \hat{\phi}'_j \tilde{\Phi}_{j-1} + V_j^{*T} \mathcal{O}^2 \left(\tilde{\Phi}_{j-1} \right)$

- DNNs with nonsmooth units are modeled via a switching mechanism involving smooth DNNs.

$$f(x_d) = \Phi_{k,\sigma}(x_d, V_0^*, V_1^*, \dots, V_k^*) + \varepsilon_\sigma(x_d)$$

- Using recursion

$$\tilde{\Phi} = \tilde{\Phi}_{k,\sigma} = \sum_{j=0}^k \Lambda_{j,\sigma} \text{vec}(\tilde{V}_j) + \sum_{j=1}^k \Xi_{j,\sigma} \Delta_{j,\sigma}$$

- Tracking Error Dynamics

$$\begin{aligned} \dot{e} = & f(x) - f(x_d) + \sum_{j=0}^k \Lambda_{j,\sigma} \text{vec}(\tilde{V}_j) + \sum_{j=1}^k \Xi_{j,\sigma} \Delta_{j,\sigma} \\ & + \varepsilon_\sigma(x_d) - \rho(\|e\|)e - k_1 e - k_s \text{sgn}(e) \end{aligned}$$

- Weight Estimation Error Dynamics

$$\text{vec}(\dot{\tilde{V}}_j) = -\text{proj}(\Gamma_j \Lambda_{j,\sigma}^T e)$$

- Theorem 1. The designed controller and adaptation laws ensure global asymptotic tracking error convergence in the sense that

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0$$

provided the following gain condition is satisfied

$$k_s > \bar{\varepsilon} + c$$

- Candidate Common Lyapunov Function

$$\mathcal{V}_L(z) \triangleq \frac{1}{2} e^T e + \frac{1}{2} \sum_{j=0}^k \text{vec}(\tilde{V}_j)^T \Gamma_j^{-1} \text{vec}(\tilde{V}_j),$$

- Generalized time-derivative

$$\dot{\mathcal{V}}_\sigma(z, t) \triangleq \max_{p \in \partial \mathcal{V}_L(z)} \max_{q \in F_\sigma(z, t)} p^T q$$

$$\begin{aligned} \dot{\mathcal{V}}_\sigma(z, t) &\stackrel{a.e.}{\leq} e^T (f_e + \sum_{j=1}^k \Xi_{j,\sigma} \Delta_{j,\sigma} + \varepsilon_\sigma(x_d)) - k_s \|e\|_1 \\ &+ \max \sum_{j=0}^k \{ e^T \Lambda_{j,\sigma} \text{vec}(\tilde{V}_j) - \text{vec}(\tilde{V}_j)^T \Lambda_{j,\sigma}^T e \} \\ &- \rho(\|e\|) \|e\|^2 - k_1 \|e\|^2 \end{aligned}$$

- Further upper-bounding yields

$$\dot{\bar{V}}_{\sigma}(z, t) \stackrel{a.e.}{\leq} -k_1 \|e\|^2$$

- Using the LaSalle-Yoshizawa theorem for switched systems yields

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0$$

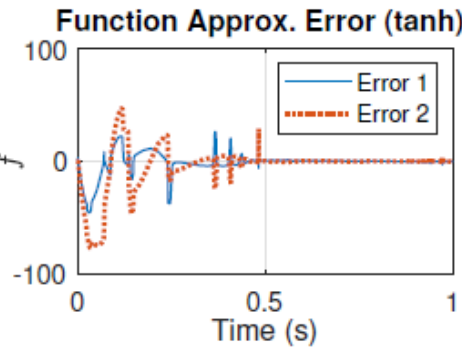
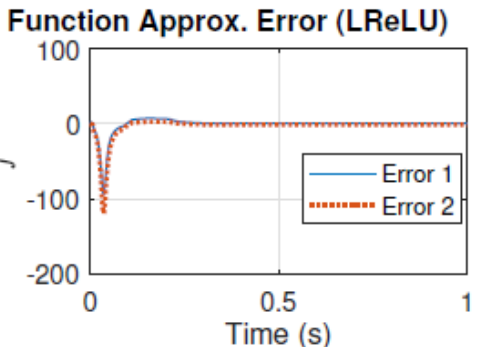
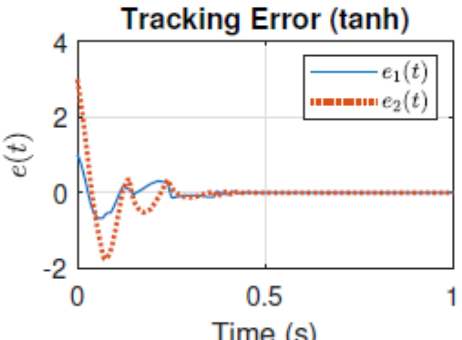
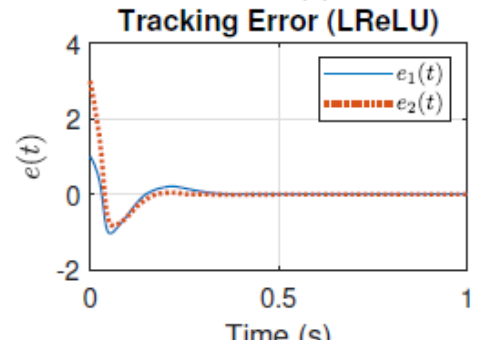
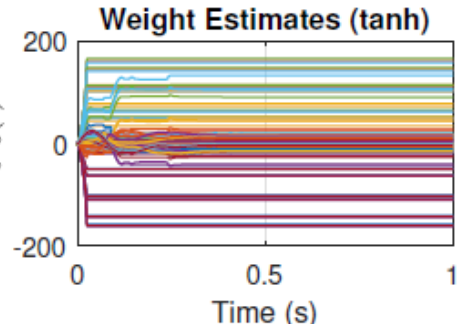
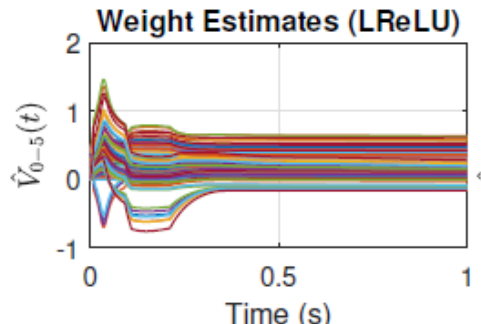


Simulation Results

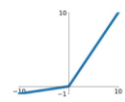
- System

$$f(x) = \begin{bmatrix} x_1 x_2 \tanh(x_2) + \operatorname{sech}^2(x_1) \\ \operatorname{sech}^2(x_1 + x_2) - \operatorname{sech}^2(x_2) \end{bmatrix}$$

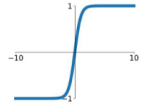
- Each DNN has 6 hidden layers with 7 neurons



Leaky ReLU
 $\max(0.1x, x)$



tanh
 $\tanh(x)$



Activation Function	$\ e_{RMS}\ $	$\ \tilde{f}_{RMS}\ $
LReLU	0.3045	5.4326
Hyperbolic Tangent	2.0898	42.1838



Conclusions and Future Work

- The first result on Lyapunov-derived adaptation laws for all layers
- Leaky ReLU-based DNN provided seven-fold improvement in tracking and function estimation performance
- Future work will involve extension for residual networks

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NCR Links

Website: <https://ncr.mae.ufl.edu/>

YouTube: <https://www.youtube.com/user/NCRatUF/videos>

Thank you!

