Distributed Cooperative MAS Navigation From Single-Agent Navigation Fields

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November 9, 2021

Recall from 2020...

Towards a Compositional Framework for Hybrid Differential Inclusions

Dan P. Guralnik
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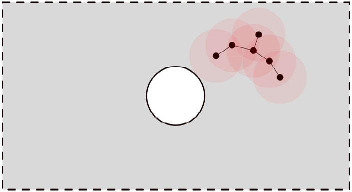
Jared Culbertson
AFRL/ACT3

April 14, 2020

Recall from 2020. . .

Motivation

Challenge: Autonomous generation of complex distributed cooperative behaviors requires reasoning over very large combinatorial structures.



For example, in networks where comms are constrained by distance,

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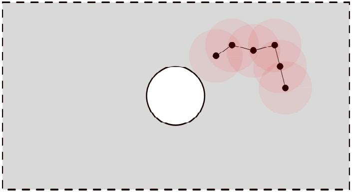

Switching between comms structures (e.g. spanning trees) is useful.

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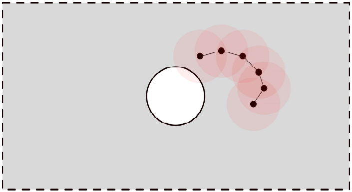

Coordinated motion under a fixed conroller...

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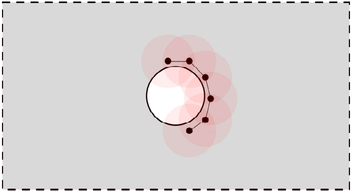

. . . may run into obstacles. . .

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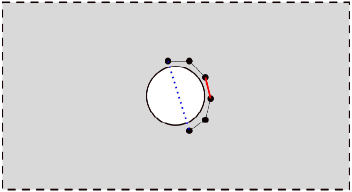

. . . suggesting a reassessment of the comms structure. . .

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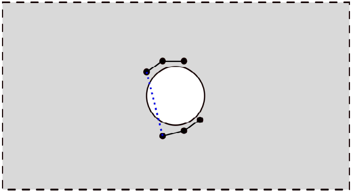

. . . including temporary disconnects with the aim of reconnecting soon thereafter. . .

Recall from 2020. . .

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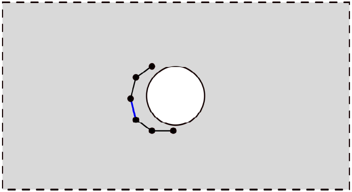

. . . using a different connectivity structure.

Recall from 2020. . .

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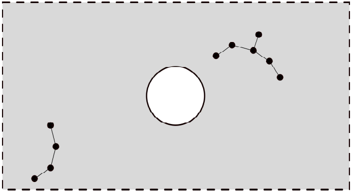

In the presence of additional resources. . .

Recall from 2020. . .

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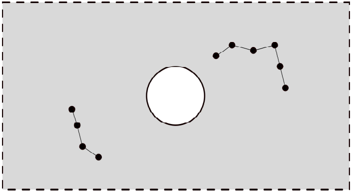

. . . a reactive control paradigm may provide alternative solutions. . .

Recall from 2020. . .

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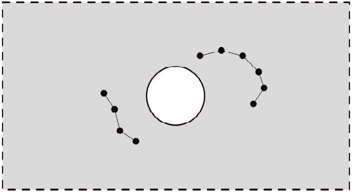

[agents move according to original plans]

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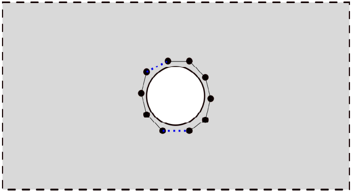

[rendevous generates new comms connections]

Recall from 2020. . .

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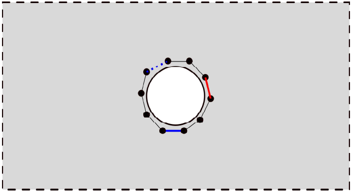

[less risky strategy becomes available]

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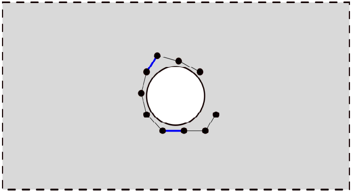

[resolution through edge-creation and edge-exchanges]

Recall from 2020. . .

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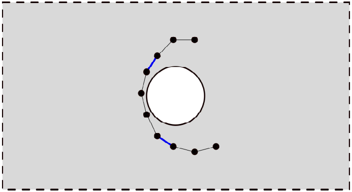

[continued motion as a group]

Recall from 2020. . .

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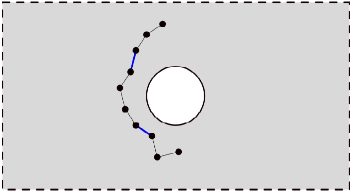


[they live happily ever after]

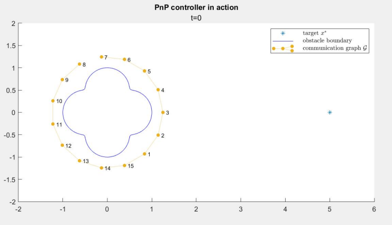
- Here, "very large combinatorial structure"=the space of all spanning trees over a varying set of agents.

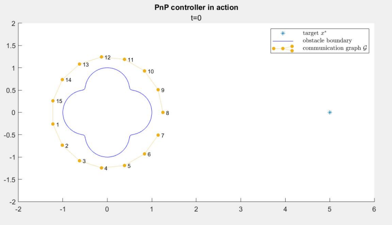
What low-level controllers could serve as building blocks (modes) for this kind of framework?

Provided:

- MAS with $\dot{x}_p = u_p$, $p \in \mathcal{V}$, in a compact domain $\Omega \subset \mathbb{R}^d$,
- *Obstacles (components of $\partial\Omega$) of general shape*,
- Distance-limited comms: $p, q \in \mathcal{V}$ *may* communicate $\Leftrightarrow$ $\|x_p - x_q\| \leq R$,
- Prescribed communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$,
- Available solution to single-agent navigation of Ω,

Task: the MAS follows a leader $\ell \in \mathcal{V}$, while $\|x_p - x_q\| \leq R$ for all $pq \in \mathcal{E}$.





A few possible objections.

- Why not just share target info and navigate individually?

  ⇝ *Agents may break the communication structure, jeopardizing the mission*

  ⇝ *Restricted agent access to target info, leader trajectory, or nav solution*

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  ⇝ *...but they are maturing, e.g.* [3, 4, 5, 6] *using only local sensing*

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Need a SYSTEMATIC & PRESCRIPTIVE *extension of* ARBITRARY *single-agent navigation solutions to distributed graph-maintaining MAS controllers ("Plug and Play")*

Our Notion of a Single-Agent Navigation Solution [12]:

Definition (Navigation Field)

Let $\Omega \subset \mathbb{R}^d$, $d \geq 2$ be a compact domain given by $\Omega \triangleq [\beta \geq 0]$, where β is a C^∞-smooth function of $\mathbb{R}^d$ with regular value 0. A navigation field on Ω is a locally Lipschitz-continuous map $\mathfrak{n}\colon \Omega \times \Omega \to \mathbb{R}^d$ satisfying the following conditions for every $y \in \mathrm{int}(\Omega)$:

1. $\langle \mathfrak{n}(y, z), \nabla_z \beta(z) \rangle > 0$ almost everywhere on $\partial\Omega$;
2. $z = y$ is the unique stable equilibrium of $\mathfrak{n}(y, -)$;
3. For almost all initial conditions $x(0) \in \Omega$, the solutions $x(t)$ of $\dot{x} = \mathfrak{n}(y, x)$ converge to y as $t \to \infty$;
4. There is a continuous positive function $\alpha : \mathrm{int}(\Omega) \to \mathbb{R}$ such that $\|\mathfrak{n}(y, z)\| \geq \alpha(y)\|y - z\|$ holds for all z in a neighborhood of y.

- All known solutions are of this form, many with $\alpha(y) \equiv 1$.
- Consistent with imposing Rantzer-type dual-Lyapunov conditions [13, 9].

Control Objective

MAIN IDEA: Replace consensus dynamics with the analogous navigation components.

- *The PnP field* is a superposition of navigation fields aimed at local targets,

$$u_p \triangleq \sum_{q\sim p} \xi_q^p \mathfrak{n}_q^p + v_p\,, \quad \mathfrak{n}_q^p(\mathbf{x}) \triangleq \mathfrak{n}(x_q, x_p) \text{ instead of } x_q - x_p. \tag{1}$$

- *Asymmetric Rescaling Factors,* $\xi_q^p(\mathbf{x}) \triangleq \xi(x_q, x_p)$ are TBD.
- *Task Component.* Guides the leader to the target with gain $\gamma > 0$,

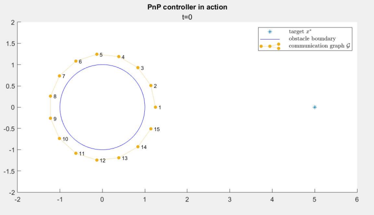
$$v_\ell(\mathbf{x}) \triangleq \gamma \mathfrak{n}(x^*, x_\ell) - \sum_{q\sim\ell} \xi_q^\ell \mathfrak{n}_q^\ell(\mathbf{x}), \quad v_p = 0 \text{ if } p \neq \ell. \tag{2}$$

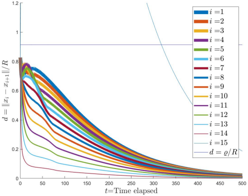
$\rightsquigarrow$ *may be replaced with a different leader task!*

Superposition of navigation fields leaves Ω invariant by design.

Control Objective

What if ξ_q^p where identically 1?





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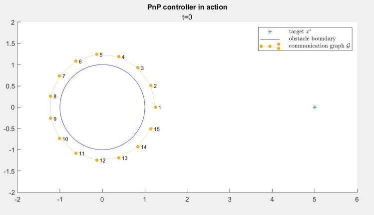
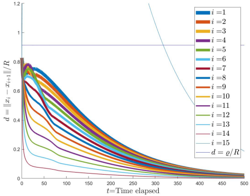
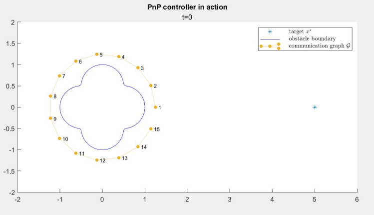
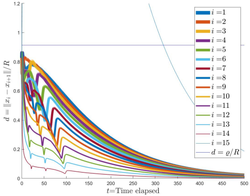




Compare with the clover-leaf, with a slow leader:





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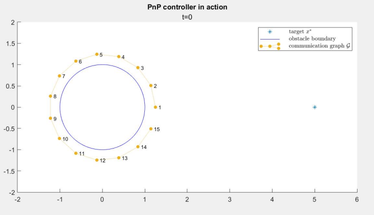
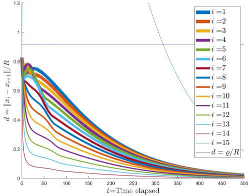
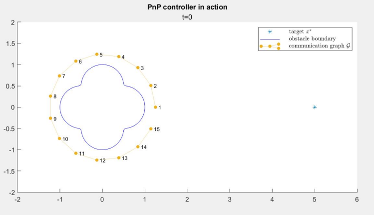
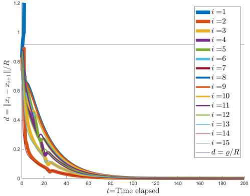




. . . and with the clover-leaf again, with a faster leader:





Configurations.

- *Configurations/Ensemble States*

$$\mathbf{x} \triangleq (x_p)_{p\in\mathcal{V}} \in (\mathbb{R}^d)^{\mathcal{V}}, \;\; \Delta\mathbf{x} \triangleq (x_q - x_p)_{pq\in\mathcal{E}} \in (\mathbb{R}^d)^{\mathcal{E}} \qquad (3)$$

$\rightsquigarrow$ *need to be careful about edge orientation, see our paper* [12]

- *s-Available edges* of a configuration $\mathbf{x}$, for $s > 0$, are

$$\mathcal{E}_s(\mathbf{x}) \triangleq \{pq \in \tbinom{\mathcal{V}}{2} : \|x_q - x_p\| \leq s\}. \qquad (4)$$

- *s-Valid Configurations for $\mathcal{G}$* are the ones in $\mathscr{C}_s(\mathcal{G})$, where

$$\mathscr{C}_s(\mathcal{G}) \triangleq \{\mathbf{x} \in \Omega^{\mathcal{V}} : \mathcal{E} \subseteq \mathcal{E}_s(\mathbf{x})\}. \qquad (5)$$

Weak Invariance Problem for Graph Maintenance:
For any $\varrho \in (0, R)$, construct controllers $\mathbf{u}$ such that every solution of $\dot{\mathbf{x}} = \mathbf{u}$ with initial ('safe') condition $\mathbf{x}(0) \in \mathscr{C}_\varrho(\mathcal{G})$ remains in $\mathscr{C}_R(\mathcal{G})$ for all time.

Edge-Potentials and Total Potentials, following [7].

- *Edge Tension Function.* For $r\colon [0,\infty) \to [0,\infty)$, $p,q \in \mathcal{V}$, define

$$w_{pq}(\mathbf{x}) \triangleq r(\|x_q - x_p\|) \tag{6}$$

  if $pq \in \mathcal{E}$ and $w_{pq} = 0$ otherwise.

- *Edge Potentials* are derived from the tension function via

$$V_{pq}(\mathbf{x}) \triangleq P(\|x_q - x_p\|),\ P(\rho) \triangleq \int_0^\rho r(s) s \mathbf{d}s. \tag{7}$$

$\rightsquigarrow$ ... when $r > 0$ is constant, V_{pq} is the usual spring potential

- *Total Potential.* All the edge potentials are collected to form

$$V_{\mathcal{G}}(\mathbf{x}) \triangleq \sum_{pq\in\mathcal{E}} V_{pq}(\mathbf{x}) = \tfrac{1}{2} \sum_{p\in\mathcal{V}} \sum_{q\sim p} P(\|x_q - x_p\|). \tag{8}$$

A Weak Invariance Principle (WIP)

Extending an argument from [7], we have: ⇝ *also works in hybrid settings* [14]

Theorem (Weak Invariance for Graph Maintenance)

Suppose r is monotone non-decreasing on $[0, R]$ and $|\mathcal{E}|\, P(\varrho) < P(R)$. Let $\mathbf{u} = \mathbf{u}(\mathbf{x})$ be a Lipschitz-continuous controller on $\Omega^{\mathcal{V}}$.
If $\dot{V}_{\mathcal{G}} \leq 0$ holds whenever $\|x_q - x_p\| \in [\varrho, R]$ for some $pq \in \mathcal{E}$, then every trajectory under $\mathbf{u}$ with $\mathbf{x}(0) \in \mathscr{C}_\varrho(\mathcal{G})$ remains in $\mathscr{C}_R(\mathcal{G})$ for all time.

Proof.

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Proof. Take $\mathbf{x}(t)$ a trajectory with $\mathbf{x}(0) \in \mathscr{C}_\varrho(\mathcal{G})$ exiting $\mathscr{C}_R(\mathcal{G})$. Let

$$t_1 \triangleq \inf\{t \in [0, \infty) \colon \mathbf{x}(t) \notin \mathscr{C}_R(\mathcal{G})\}, \; t_0 \triangleq \sup\{t \in [0, t_1) \colon \mathbf{x}(t) \in \mathscr{C}_\varrho(\mathcal{G})\}.$$

First, $\mathbf{x}(t_0) \in \mathscr{C}_\rho(\mathcal{G})$ implies $V_{\mathcal{G}}(t_0) \leq |\mathcal{E}|\, P(\varrho) < P(R)$. Next, for at least one $pq \in \mathcal{E}$ we have $\|x_q(t_1) - x_p(t_1)\| = R$, hence $P(R) \leq V_{\mathcal{G}}(t_1)$ and therefore also $V_{\mathcal{G}}(t_0) < V_{\mathcal{G}}(t_1)$.
However, by assumption we have $V_{\mathcal{G}}(t_1) - V_{\mathcal{G}}(t_0) = \int_{t_0}^{t_1} \dot{V}_{\mathcal{G}}(t)\mathrm{d}t \leq 0$, which contradicts the previous observation. $\square$

Consensus & Weighted Laplacians

Advantages of the Edge Potentials Design.

- *Edge Gradient.* This design comes to ensure the identities

$$\nabla_p V_{pq} = w_{pq}(x_p - x_q) = -\nabla_q V_{pq},\ \nabla_u V_{pq} = 0, \tag{9}$$

  for $pq \in \mathcal{E}$ and any $u \in \mathcal{V}$, $u \neq p, q$.

- *Total Gradient.* Magically, it turns out that

$$\nabla V_{\mathcal{G}}(\mathbf{x}) = 2(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x}. \tag{10}$$

- *Weighted Vector Laplacian.* As an operator on $(\mathbb{R}^d)^{\mathcal{V}} \equiv \mathbb{R}^{\mathcal{V}} \otimes \mathbb{R}^d$,

$$((\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x})_p = \textstyle\sum_{q\sim p} w_{pq}(x_p - x_q). \tag{11}$$

  - $(\mathbf{L}_w \otimes \mathbf{I}_d)$ is positive-semidefinite;
  - $\ker(\mathbf{L}_w \otimes \mathbf{I}_d)$ is the consensus subspace, $\boldsymbol{\Delta}_{\mathcal{V}} \triangleq \{\mathbf{x} \colon \Delta\mathbf{x} = 0\}$, provided $\mathcal{G}$ is connected.

Consensus & Weighted Laplacians

Under the hood of, e.g. [15, 16, 7], as presented in [12]:

- Write $\mathbf{x} = \mathbf{x}_0 + \mathbf{x}^\perp$, $\mathbf{x}_0 \in \mathbf{\Delta}_\mathcal{V}$, $\mathbf{x}^\perp \in (\mathbf{\Delta}_\mathcal{V})^\perp$.
- Write the dynamics/controller as $\dot{\mathbf{x}} = \mathbf{u}$, $\mathbf{u} = -(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x} + \mathbf{v}$,
  - $-(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x}$ is the *consensus component*;
  - $\mathbf{v}$ is the *task component* of the controller.
- For $\|\Delta\mathbf{x}\|_\infty \in [\varrho, R]$, *a bunch of standard arguments* yields. . .

⇝ *an edge is at risk of breaking*

$$
\begin{aligned}
\dot{V}_\mathcal{G}(\mathbf{x}) &= \langle \dot{\mathbf{x}}, 2(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x} \rangle \\
&= \langle -(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x} + \mathbf{v}, 2(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x} \rangle \\
&= -2\|(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x}^\perp\|^2 + 2\langle \mathbf{v}, (\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x}^\perp \rangle \\
&\le -2\lambda_2(\mathcal{G}, w)^2 \|\mathbf{x}^\perp\|^2 + 2\|\mathbf{v}\| \|\mathbf{L}_w \otimes \mathbf{I}_d\| \|\mathbf{x}^\perp\| \\
&\le -\lambda_2(\mathcal{G}, w)^2 \|\Delta\mathbf{x}\|_\infty + 2\|\mathbf{v}\| \cdot \lambda_N(\mathcal{G}, w) \cdot \sqrt{N}\|\Delta\mathbf{x}\|_\infty \\
&\le -r(\varrho)^2 \lambda_2(\mathcal{G})^2 \varrho^2 + 2\|\mathbf{v}\| \cdot r(R)\lambda_N(\mathcal{G}) \cdot \sqrt{N}R \\
&\le -\lambda_2(\mathcal{G})^2 \varrho^2 r(\varrho)^2 + 2\|\mathbf{v}\| \cdot 2\Delta(\mathcal{G}) r(R) \cdot \sqrt{N}R.
\end{aligned}
\tag{12}
$$

Consensus & Weighted Laplacians

Applying the Weak Invariance Principle:

- For $\|\Delta\mathbf{x}\|_\infty \in [\varrho, R]$ we always have *$\rightsquigarrow$ an edge is at risk of breaking*

$$\dot{V}_{\mathcal{G}}(\mathbf{x}) \leq -\lambda_2(\mathcal{G})^2 \varrho^2 r(\varrho)^2 + 4\sqrt{N}\Delta(\mathcal{G}) R r(R) \|\mathbf{v}\| \qquad (13)$$

- In the case $\mathbf{v} = 0$, the graph will always be maintained.
- In the case $r(0) > 0$, an exponential rate of convergence to rendezvous is to be expected for sufficiently small $\|\mathbf{v}\|$.
- In the case when $\mathbf{v} \neq 0$, bounds on $\|\mathbf{v}\|$ may guarantee graph maintenance, with appropriate design of r.

We are looking for something similar, but with the additional guarantee of Ω remaining invariant (obstacle-avoidance)

The "Plug'n Play" MAS Controller

Now define the PnP field "for real":

- *The PnP field* is a superposition of navigation fields aimed at local targets,

$$u_p \triangleq \sum_{q\sim p} \xi_q^p \mathfrak{n}_q^p + v_p \,, \quad \mathfrak{n}_q^p(\mathbf{x}) \triangleq \mathfrak{n}(x_q, x_p). \tag{14}$$

- *Asymmetric Rescaling Factors,* $\xi_q^p(\mathbf{x}) \triangleq \xi(x_q, x_p)$ given by

$$\xi(y,z) \triangleq \frac{r(\|y-z\|)\|y-z\|^2}{\langle \mathfrak{n}(y,z), y-z \rangle}. \tag{15}$$

- *Task Component.* Guides the leader to the target with gain $\gamma > 0$,

$$v_\ell(\mathbf{x}) \triangleq \gamma \mathfrak{n}(x^*, x_\ell) - \sum_{q\sim\ell} \xi_q^\ell \mathfrak{n}_q^\ell(\mathbf{x}), \quad v_p = 0 \text{ if } p \neq \ell. \tag{16}$$

So why do we need the asymmetric rescaling from (15)?

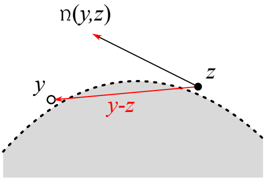
- To deploy the WIP, must relate $\mathbf{u}(\mathbf{x})$ to $(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x}$.

Definition

Let $\delta \in (0,1]$. A navigation field $\mathfrak{n}$ on Ω is (R,δ)-good, if for all $y, z \in \Omega$ with $\|y - z\| \leq R$ one has

$$\langle \mathfrak{n}(y,z), y - z \rangle \geq \delta \|\mathfrak{n}(y,z)\| \|y - z\|. \tag{17}$$

- $\mathfrak{n}$ is "well-aligned" with the radial field for nearby targets: $\cos\angle(\mathfrak{n}(y,z), y - z) \geq \delta$.
- Smaller R leads to larger δ...
- Trade-off between obstacle curvature and communication radius?



The "Plug'n Play" MAS Controller

Relating u to $(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x}$. Consider the orthogonal decomposition

$$\mathfrak{n}(y,z) \triangleq \mathfrak{p}(y,z) + \mathfrak{o}(y,z),\ \mathfrak{p}(y,z) \in \mathrm{Sp}(y-z),\ \mathfrak{o}(y,z) \perp (y-z). \qquad (18)$$

- When $\|y-z\| \leq R$, this and (15) result in

$$\xi(y,z)\mathfrak{n}(y,z) = r(\|y-z\|)(y-z) + \xi(y,z)\mathfrak{o}(y,z). \qquad (19)$$

- If $\mathfrak{n}$ is (R,δ)-good, then $\mathfrak{o}(y,z)$ satisfies

$$\|y-z\| \leq R \implies \|\mathfrak{o}(y,z)\| \leq \sqrt{1-\delta^2}\|\mathfrak{n}(y,z)\|. \qquad (20)$$

- Overall, the PnP field takes the form:

$$\mathbf{u} = -(\mathbf{L}_w \otimes \mathbf{I}_d)\mathbf{x} + \mathfrak{O} + \mathbf{v}, \quad \mathfrak{O}_p \triangleq \sum_{q\sim p} \xi_q^p \mathfrak{o}_q^p, \quad \mathfrak{o}_q^p \triangleq \mathfrak{o}(x_q, x_p). \qquad (21)$$

Now we may apply the WIP!

WIP for the PnP controller. Modifying (13) for our case,

$$\dot{V}_{\mathcal{G}}(\mathbf{x}) \leq -\lambda_2(\mathcal{G})^2 \varrho^2 r(\varrho)^2 + 4\sqrt{N}\Delta(\mathcal{G}) R r(R) \left(\|\mathfrak{O}\| + \|\mathbf{v}\|\right)$$

$$\vdots$$

$$\begin{aligned} &\leq -\lambda_2(\mathcal{G})^2 \varrho^2 r(\varrho)^2 \\ &\quad + 4N\Delta(\mathcal{G})^2 \frac{\sqrt{1-\delta^2}}{\delta} R^2 r(R)^2 \left(1 + \frac{d_\ell}{\Delta(\mathcal{G})} \frac{1}{\sqrt{N}}\right) \\ &\quad + 4\sqrt{N}\Delta(\mathcal{G}) R r(R) \cdot \gamma \|\mathfrak{n}(x^*, x_\ell)\|. \end{aligned} \tag{22}$$

For $N \geq 4$ we have:

$$\begin{aligned} \dot{V}_{\mathcal{G}}(\mathbf{x}) &\leq -\lambda_2(\mathcal{G})^2 \varrho^2 r(\varrho)^2 \\ &\quad + 6N\Delta(\mathcal{G})^2 \underbrace{\frac{\sqrt{1-\delta^2}}{\delta}}_{\triangleq \delta^*} R^2 r(R)^2 \\ &\quad + 4\sqrt{N}\Delta(\mathcal{G}) R r(R) \cdot \gamma \|\mathfrak{n}(x^*, x_\ell)\|. \end{aligned} \tag{23}$$

...All we need for a WIP is...

$$6\delta^* N\Delta(\mathcal{G})^2 R^2 r(R)^2 + 4\gamma\sqrt{N}\Delta(\mathcal{G}) R r(R) \|\mathfrak{n}(x^*, x_\ell)\| \leq \lambda_2(\mathcal{G})^2 \varrho^2 r(\varrho)^2 \quad (\ddagger)$$

- Evidently, δ close to 1 and γ small enough will do the trick, with careful design of the tension, r.

The "Plug'n Play" MAS Controller

. . . All we need for a WIP is. . .

$$6\delta^* N\Delta(\mathcal{G})^2 R^2 r(R)^2 + 4\gamma\sqrt{N}\Delta(\mathcal{G})Rr(R)\|\mathfrak{n}(x^*, x_\ell)\| \leq \lambda_2(\mathcal{G})^2 \varrho^2 r(\varrho)^2 \quad (\ddagger)$$

- Evidently, δ close to 1 and γ small enough will do the trick, with careful design of the tension, r.
- *BAD NEWS: we are not free to select a small δ^*. It is a geometric property of our task!*
- You cannot satisfy $(\ddagger)$ with a cycle around a circular obstacle.
- Communication radius needs to be very small compared to obstacles. . .
- . . . but the number of agents cannot be too large either: $\lambda_2(\mathcal{G}) = 4\sin^2\frac{\pi}{2N}$ for a chain.

The "Plug'n Play" MAS Controller

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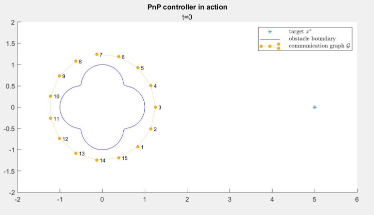
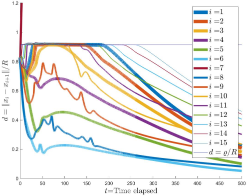
- . . . but the number of agents cannot be too large either: $\lambda_2(\mathcal{G}) = 4\sin^2\frac{\pi}{2N}$ for a chain.
- *GOOD NEWS: $(\ddagger)$ is extremely conservative. The PnP controller works much better in practice!*

Case Study [12]: const. tension when safe

Soft springs between agents:





Case Study [12]: const. tension when safe

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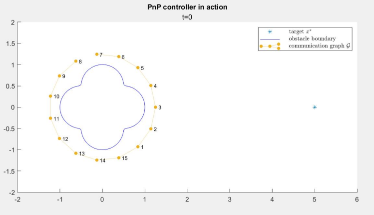
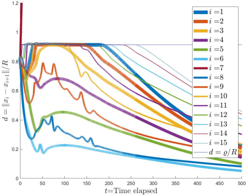
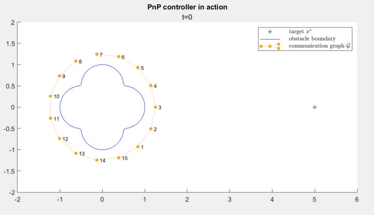
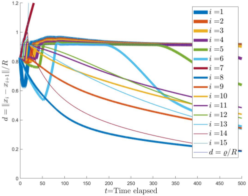




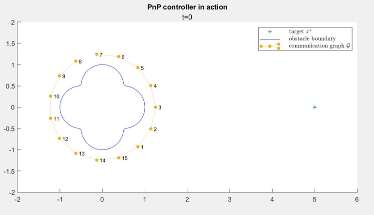
Compare with softer springs:

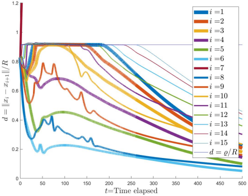




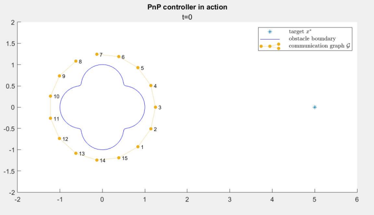
Case Study [12]: const. tension when safe

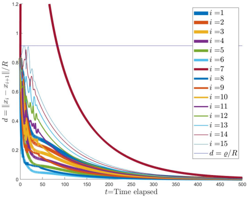
Soft springs between agents:





Compare with stiff springs:





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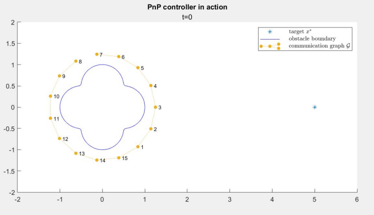
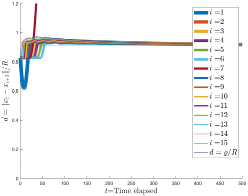
Case Study: zero tension when safe

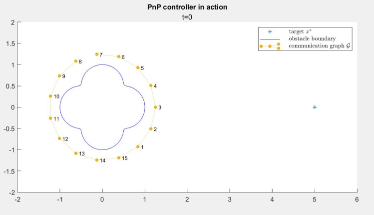
Leader at front of obstacle (closer to the target):

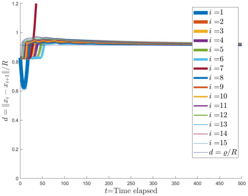




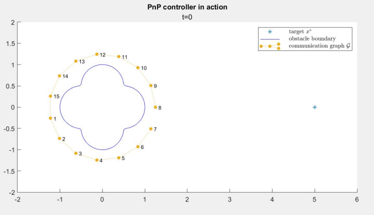
Case Study: zero tension when safe

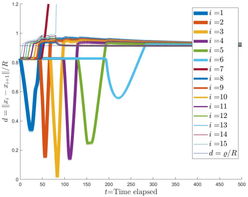
Leader at front of obstacle (closer to the target):





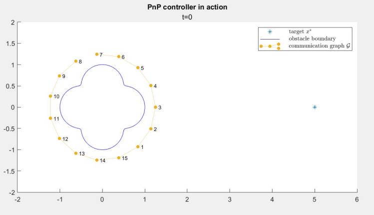
Compare with leader behind the obstacle (low PnP gain):

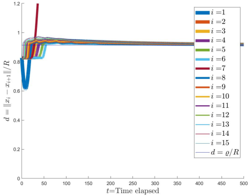




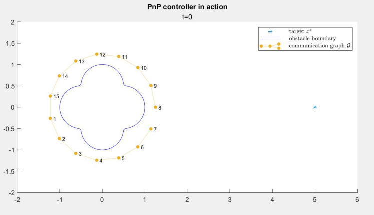
Case Study: zero tension when safe

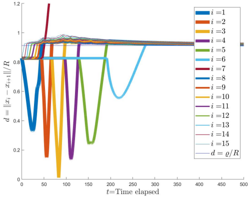
Leader at front of obstacle (closer to the target):





. . . and leader behind the obstacle, with higher PnP gain:





Last Thoughts

To be continued...

- *General behavior (non-zero tension).* When the graph gets disconnected, its components seem to be driven to rendezvous. Is there a theorem here?
- *Topological constraints.* Accounting for cycles winding around obstacles or trapping them (in 3D)
- *Better WIP bounds,* especially in the zero-tension case?
- *Expand the range of tasks.* Using generalized dual-Lyapunov functions à-la Rantzer [13]?
- *Hybrid Open System of PnP-controlled MAS tasks.* A categorical framework for disconnecting and reconnecting, adding and removing agents, etc.

Thank You for Your Attention!

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