Updates on Research and Collaborations

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Center of Excellence for Assured Autonomy in Contested Environments
Fall 2022 Review
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Personnel & Collaborations with CoE PIs

- New lab alumni:

Dr. Kasra Yazdani
Now at Samsung

Dr. Katherine Hendrickson
Now at EpiSci

- Ongoing collaboration with Dawn Hustig-Schultz and Ricardo Sanfelice (UCSC)
  - Developed hybrid model of decentralized non-convex optimization
  - Paper at CDC ’21
  - One under review at ACC ‘23
  - Journal paper in preparation

- Ongoing collaboration with Ufuk Topcu (UT-Austin)
  - Looking at privacy in symbolic systems
  - Combining our privacy mechanisms with their multi-agent algorithms

Collaborations with Air Force Colleagues

- Applied optimization work to weapon-target assignment (WTA) problems
  - K. Hendrickson, P. Ganesh, K. Volle, P. Buzaud, K. Brink, and M.T. Hale, "Decentralized Weapon-Target Assignment under Asynchronous Communications".
  - **Accepted to Journal of Guidance, Control, and Dynamics**
  - Ongoing collaboration with Kevin Brink (RW) on discrete optimization
- Joint paper at AIAA SciTech with USAFA based on senior capstone
  - A. Broshkevitch, A. Hancock, A. Peters, M. Kim, M. Anderson, et al., "An Autonomous System for the Rapid Airfield Damage Repair Mission"

- Working with Zach Bell (RW) on feedback optimization
- Working with Ben Robinson (RY) on anomaly detection in multi-armed bandits
- Collaborating on MPC with Sean Phillips and Alex Soderlund (RV)
  - Currently focused on satellite docking
  - IFAC World Congress paper in preparation

- Engaging with AFRL every summer
  - William Warke went to RW for summer 2022 with Kevin Brink
  - Gabriel Behrendt went to RV for summer 2022 with Sean Phillips
  - Alexander Benvenuti went to RW for summer 2022 with Scott Nivison

Differential Privacy for Network Design and Analysis

Calvin Hawkins & Matthew Hale

Department of Mechanical and Aerospace Engineering University of Florida

1. "Differentially Private Formation Control: Privacy and Network Co-Design" Under review. https://arxiv.org/abs/2205.13406
2. "Node and Edge Differential Privacy for Graph Laplacian Spectra: Mechanisms and Scaling Laws" Under review. https://arxiv.org/abs/2104.00654

Motivation

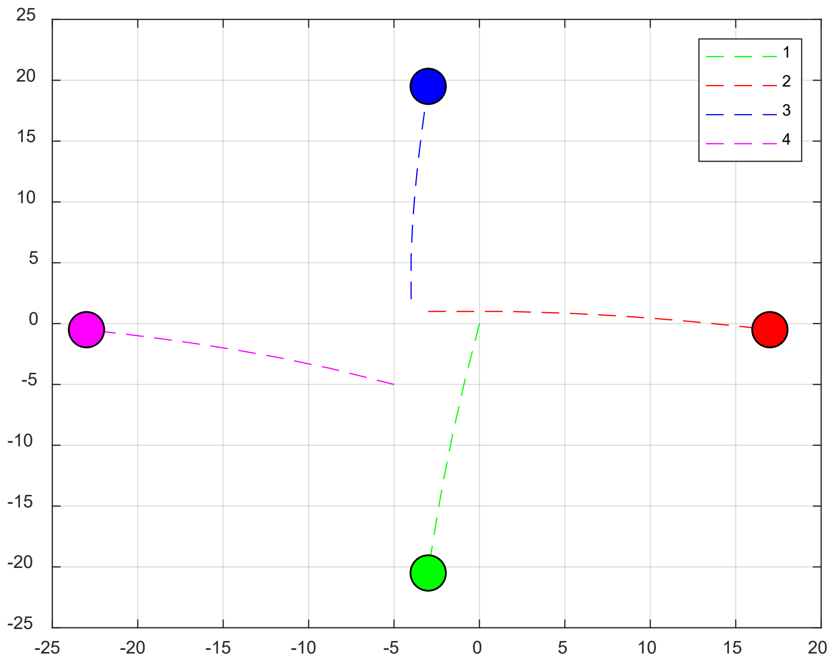
- Allow agents to collaborate while protecting their sensitive information.
- Examples:
  - Autonomous vehicles sharing location data
  - Social Networks sharing personal information
  - Data-driven control sharing sensitive state information
- Graph analyses may reveal sensitive information about individuals.

Two goals:
1. Develop tools to design networks sharing private information. (Part 1)
2. Develop tools for the private analysis of networks. (Part 2)

Part 1:
Privacy and Network Co-Design

Formation Control Background

- In this talk, formation control.
- Consider N agents where agent i has state $x_i(k) \in \mathbb{R}^d$.
- Network is modeled by a weighted, undirected graph G.
- If agents i and j communicate, maintain a distance of $\Delta_{ij} \in \mathbb{R}^d$.
- Without privacy, this is achieved by the formation control protocol.



$$x_i(k+1) = x_i(k) + \gamma \sum_{j \in N(i)} w_{ij}\left(x_j(k) - x_i(k) - \Delta_{ij}\right)$$

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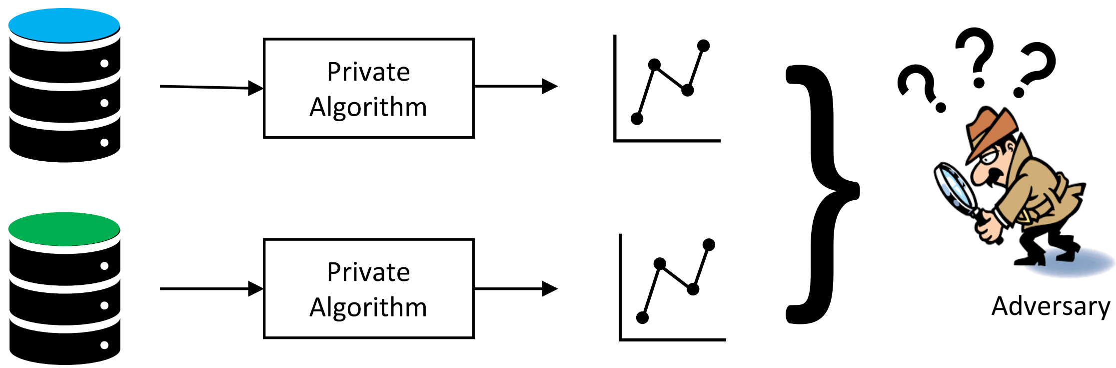
Differential Privacy Background

- Statistical notion of privacy that originated in the computer science literature.

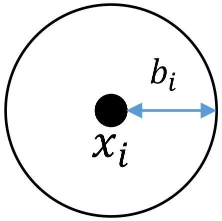


- Immune to post processing and robust to side information.
- Used by Apple, Google, Uber, and the 2020 Census.
- Agents can share trajectory data while protecting itself from other agents and eavesdroppers.

Differential Privacy Masks Differences

- Goal of Differential Privacy: Make "similar" pieces of data appear "approximately indistinguishable."
- Adjacency defines when pieces of data are similar.
  - For trajectories $x_i, x_i' \in \ell_p$

$$Adj(x_i, x_i') = \begin{cases} 1, & \|x_i - x_i{}'\|_{\ell_p} \leq b_i \\ 0, & otherwise \end{cases}$$



Definition (Differential Privacy): Let $\epsilon_i > 0$ and $\delta_i \in \left[0, \frac{1}{2}\right)$. A randomized mechanism M is (ϵ_i, δ_i) −differentially private for agent i if, for all adjacent x_i, x_i', we have

$$P[M(x_i) \in S] \leq e^{\epsilon_i} P[M(x_i') \in S] + \delta_i.$$

Sensitive data → Privacy mechanism tuned by ϵ_i, δ_i → Private data

Typical values:
$\epsilon_i \leq 3, \delta_i \leq 0.05$

Implementing Privacy

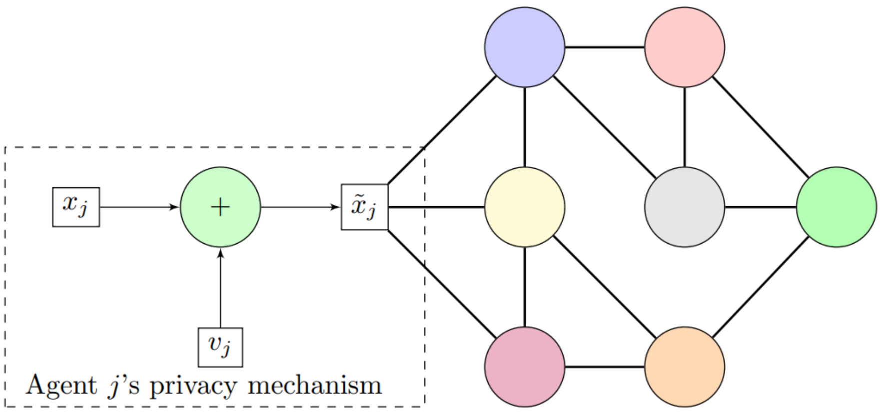
- Agent i must send $x_i(k)$ to its neighborhood $N(i)$ at each k.
- Instead, agent i will send a private version of its state to $N(i)$; $\tilde{x}_i(k)$.
- Differential privacy is implemented with the Gaussian Mechanism:

$$\tilde{x}_j(k) = x_j(k) + v_j(k),$$
$$v_j(k) \sim \mathcal{N}\left(0, \sigma_j^2 I_d\right).$$

Lemma: The Gaussian mechanism is (ϵ_i, δ_i) −differentially private for agent i if $\sigma_i \geq \kappa(\epsilon_i, \delta_i) b_i$, where

$$\kappa(\delta_i, \epsilon_i) = \frac{1}{2\epsilon_i}\left(K_{\delta_i} + \sqrt{K_{\delta_i}^2 + 2\epsilon_i}\right),$$

and $K_{\delta_i} = Q^{-1}(\delta_i)$.



Quantifying Performance

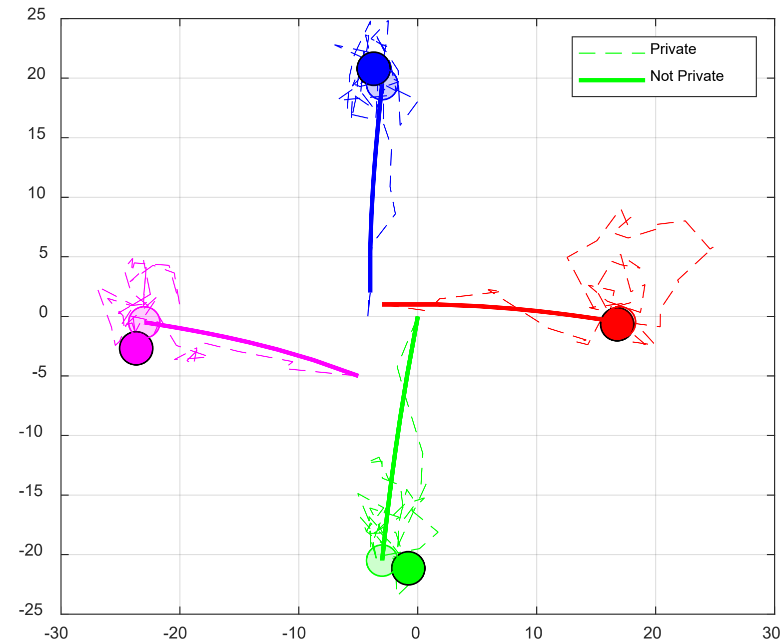
- With privacy the formation control protocol becomes

$$x_i(k+1) = x_i(k) + \gamma \sum_{j \in N(i)} w_{ij}(x_j(k) + v_j(k) - x_i(k) - \Delta_{ij}) + n_i(k).$$

- Let $e_i(k) = x_i(k) - \beta_i(k)$, where $\beta(k)$ is the state the non-private protocol converges to with initial condition $x(k)$.

- To quantify performance at the network level, let

$$e_{ss} = \limsup_{k \to \infty} \frac{1}{N} \sum_{i=1}^{N} E[e_i(k)^2].$$



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Quantifying Performance

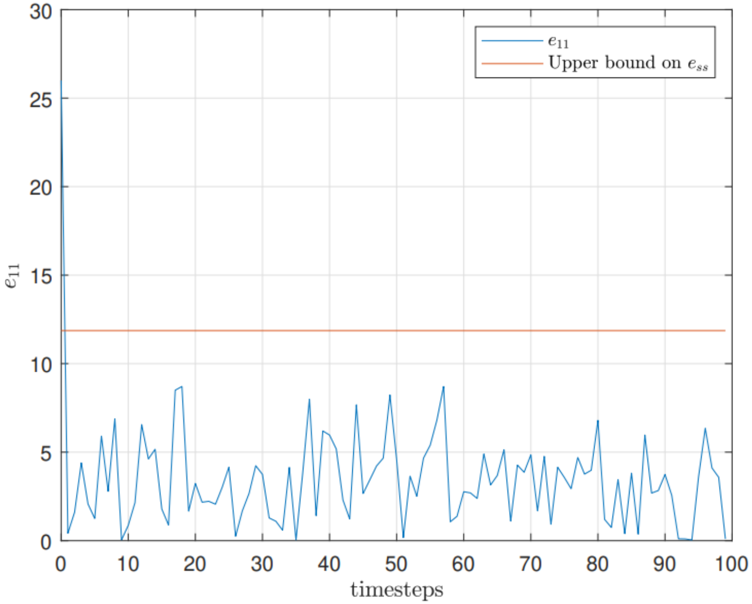
Theorem 1: Bounds on Steady-State Error

A network of N agents running the controller

$$x_i(k+1) = x_i(k) + \gamma \sum_{j \in N(i)} w_{ij}(\tilde{x}_j(k) - \tilde{x}_i(k) - \Delta_{ij})$$

is differentially private and has e_{ss} upper bounded by

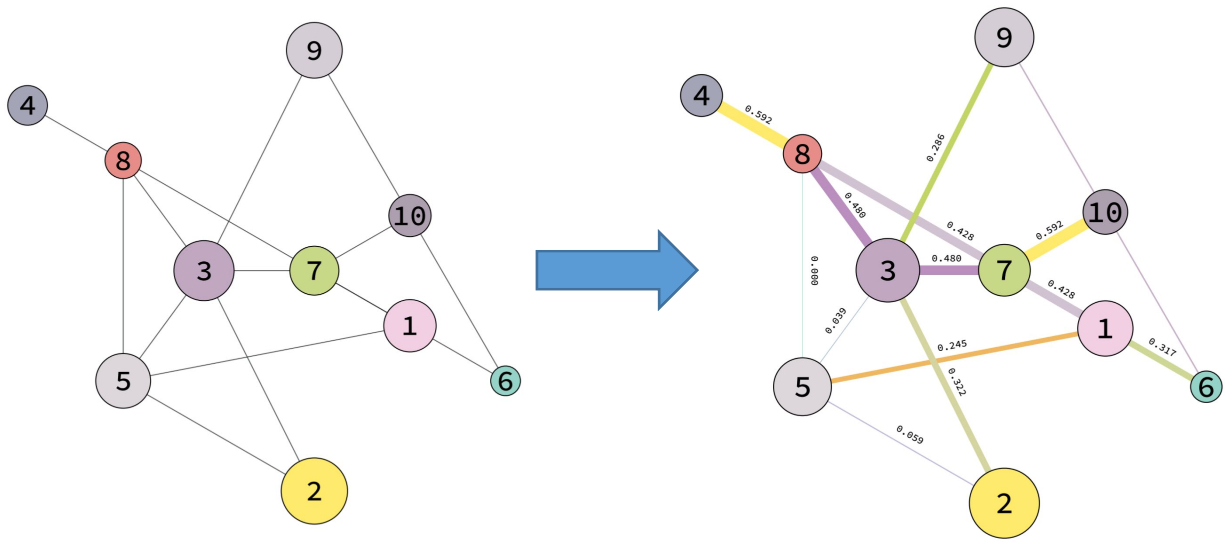
$$e_{ss} \leq \frac{\gamma d \sum_{i=1}^{N} \left(\sum_{j=1}^{N} {w_{ij}}^2 - \frac{\deg(i)^2}{N} \right) \kappa(\epsilon_i, \delta_i)^2 b_i^2}{N\, \lambda_2(G)(2 - \gamma \lambda_2(G))}$$



"Differentially Private Formation Control: Privacy and Network Co-Design" Under review. https://arxiv.org/abs/2205.13406

Privacy and Network Co-Design

- Goal: Formulate an optimization problem to design the communication topology and privacy parameters subject to constraints.
- Input: Design constraints and initial undirected, unweighted communication topology.
- Decision variables: $L(G)$, ϵ_i.



Privacy and Network Co-Design

- Objective function
  - Dense graph costs more: $\mathrm{T}r(L(G))$.
  - Agents want to be as private as possible: $\sum_{i\in[N]} \epsilon_i^2$.
  - Minimize $\Gamma\left(\{\epsilon_i\}_{i\in[N]},\ L(G)\right) = \vartheta Tr\left(L(G)\right) + \sum_{i\in[N]} \epsilon_i^2$.

Smaller ϵ_i → Stronger privacy

- Constraints
  - Performance: $e_{ss} \leq e_R \rightarrow \frac{\gamma^2 Tr(L(G)\Sigma_{\mathrm{V}}\mathrm{L(G)})}{N\ \lambda_2(G)(2-\gamma\lambda_2(G))} \leq e_R$.
  - Connectivity: $\lambda_2(G) \geq \lambda_{2_L}$.
  - Minimum level of privacy: $\epsilon_i \leq \epsilon_i^{max}$.

Privacy and Network Co-Design

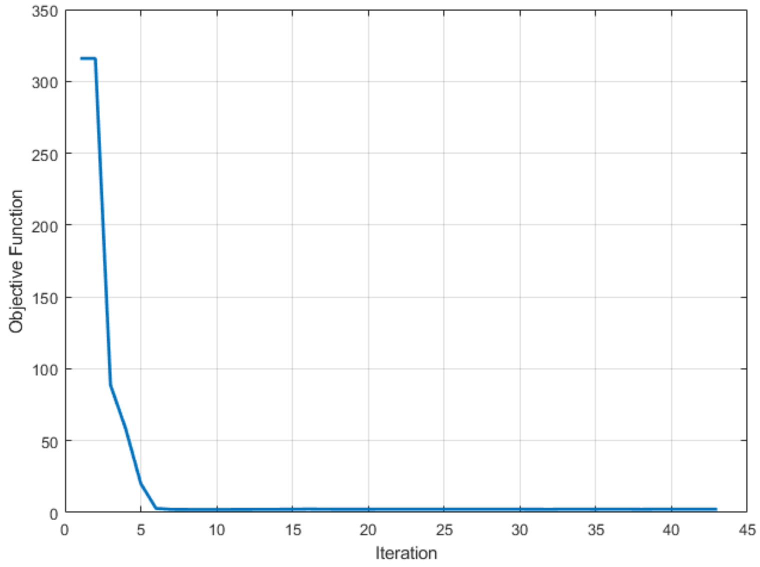
Problem 1: Privacy and Network Co-Design

$$\min_{L(\mathcal{G})\in\mathcal{L}(L_0),\{\epsilon_i\}_{i\in[N]}} \quad \vartheta \mathrm{Tr}(L(\mathcal{G})) + \sum_{i\in[N]} \epsilon_i^2$$

$$\text{subject to} \quad \frac{\gamma^2 d \mathrm{Tr}(L(G)\Sigma_v L(G))}{N\lambda_2(L)\left(2-\gamma\lambda_2(L)\right)} \leq e_R$$

$$\epsilon_i \leq \epsilon_i^{max} \quad \text{for all } i$$

$$\lambda_2(L(\mathcal{G})) \geq \lambda_{2_L}.$$

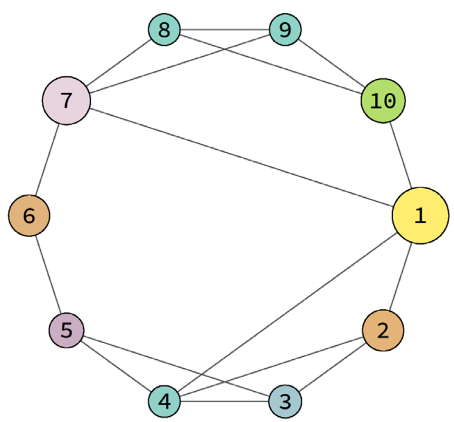


GitHub

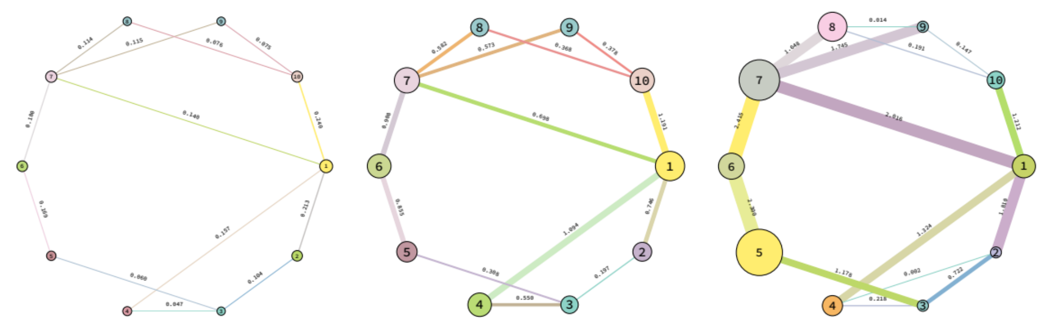
Co-Design Example

- Fix the input unweighted graph over $N = 10$ agents.
- A smaller a node is drawn, the more private it is. (Smaller epsilon)
- The thicker an edge is drawn, the more edge weight it has.
- Fix everything other than the required performance.

Input:



Sample output:

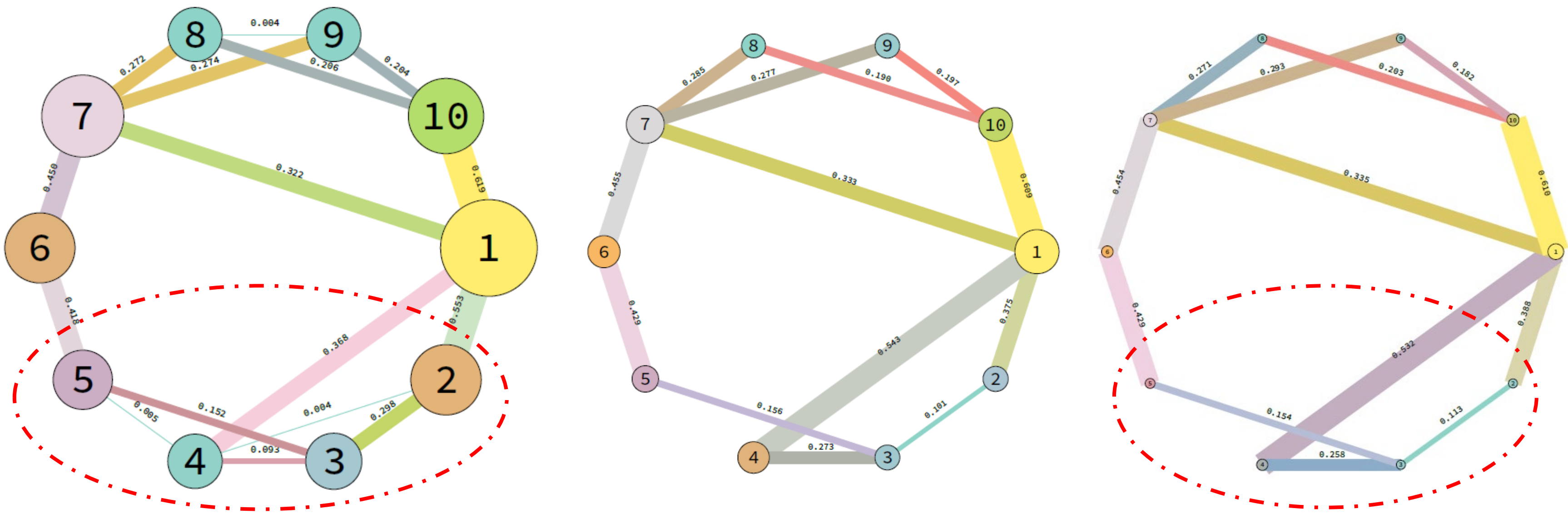


(a) $\lambda_{2L} = 0.1$ (b) $\lambda_{2L} = 0.5$ (c) $\lambda_{2L} = 1.0$

Co-Design Example: Tuning Performance

Fix $\gamma = \frac{1}{2n}, \vartheta = 10, \lambda_{2L} = 0.2, \delta_i = 0.05, b_i = 1.$

- $\epsilon_{max} = [0.4, 0.9, 0.55, 0.35, 0.8, 0.45, 0.7, 0.5, 0.52, 0.58].$
- Let $e_R \in \{2, 8, 64\}$.



$e_R = 2$ $\quad$ $e_R = 8$ $\quad$ $e_R = 64$

Performance requirements weaken.

1. C. Hawkins and M. Hale "Differentially Private Formation Control: Privacy and Network Co-Design," Under review: https://arxiv.org/pdf/2205.13406.pdf

Part 2:
Private Network Analysis

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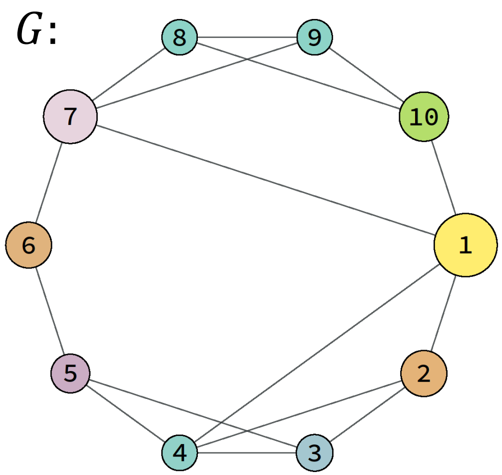
Background

- Graph analyses may reveal sensitive information about individuals.
- Numerous scalar-valued graph properties pose known privacy threats.
  - Counts of triangles.
  - Counts of subgraphs.
  - Lots more.
- In this talk, we will focus on the privacy of the spectrum of the graph Laplacian.

G:



Need privacy in these calculations!

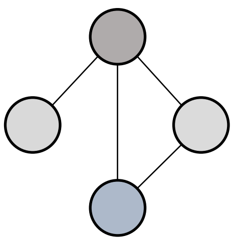
$$\lambda_1(L(G)), \ldots, \lambda_n(L(G))$$

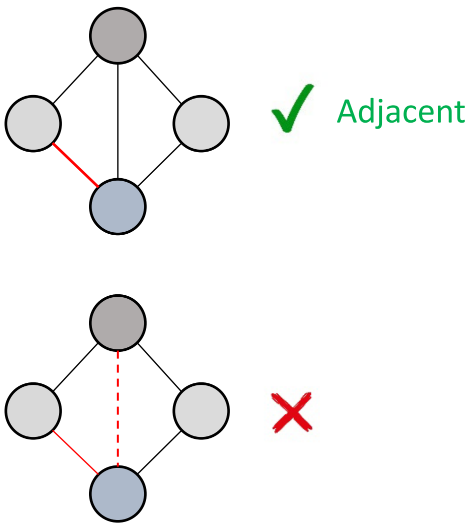
Implementing Privacy (Adjacency)

- "Similar" pieces of data appear "approximately indistinguishable."
- Similarity is defined over the edge set. Adjacency parameter A.

$$Adj(G,G') = \begin{cases} 1, & |E(G)\Delta E(G')| \leq A \\ 0, & otherwise \end{cases}$$

Edge Privacy:
$A = 1$





Definition (Edge Privacy):

Let $\epsilon > 0$ and $\delta \in \left[0, \frac{1}{2}\right)$. A randomized mechanism M is (ϵ, δ) −differentially private for agent i if, for all adjacent G, G', we have

$$P[M(G) \in S] \leq e^{\epsilon} P[M(G') \in S] + \delta$$

- Each eigenvalue is bounded on $[0, n]$.
- We use the bounded Laplace mechanism[1].

1. Holohan, Naoise, et al. "The bounded laplace mechanism in differential privacy." *arXiv preprint arXiv:1808.10410* (2018).

Privacy Mechanism

Definition (Bounded Laplace mechanism for λ_i[1]). Let $b > 0$ and $D = [0, n]$. Then the bounded Laplace mechanism W_{λ_i} is given by its probability density function $f_{W_{\lambda_i}}$ as

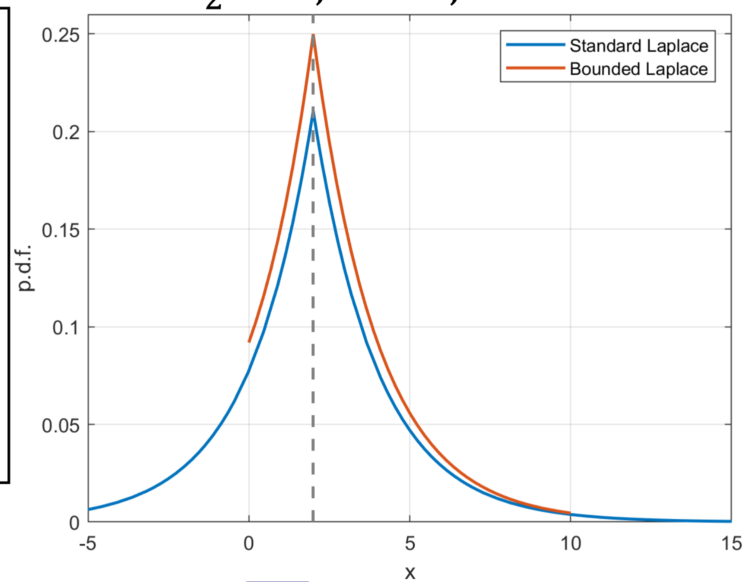
$$f_{W_{\lambda_i}}(x) = \begin{cases} 0, & x \notin D \\ \dfrac{1}{C(\lambda_i, b)} \dfrac{1}{2b} e^{-\frac{|x-\lambda_i|}{b}}, & x \in D \end{cases}$$

Where $C(\lambda_i, b)$ is a normalizing term.

Theorem 2: Let $\varepsilon > 0$, $\delta \in (0,1)$ and adjacency parameter A. The bounded Laplace mechanism is (ε, δ)-differentially private if

$$b \geq \frac{2A}{\varepsilon - \log\left(\dfrac{1 - \frac{1}{2} e^{-\frac{2A}{b}}\left(1 + e^{-\frac{n}{b}-1}\right)}{1 - \frac{1}{2}\left(1 + e^{-\frac{n}{b}}\right)}\right) - \log(1 - \delta)}$$

$\lambda_2 = 2, b = 2, n = 10$

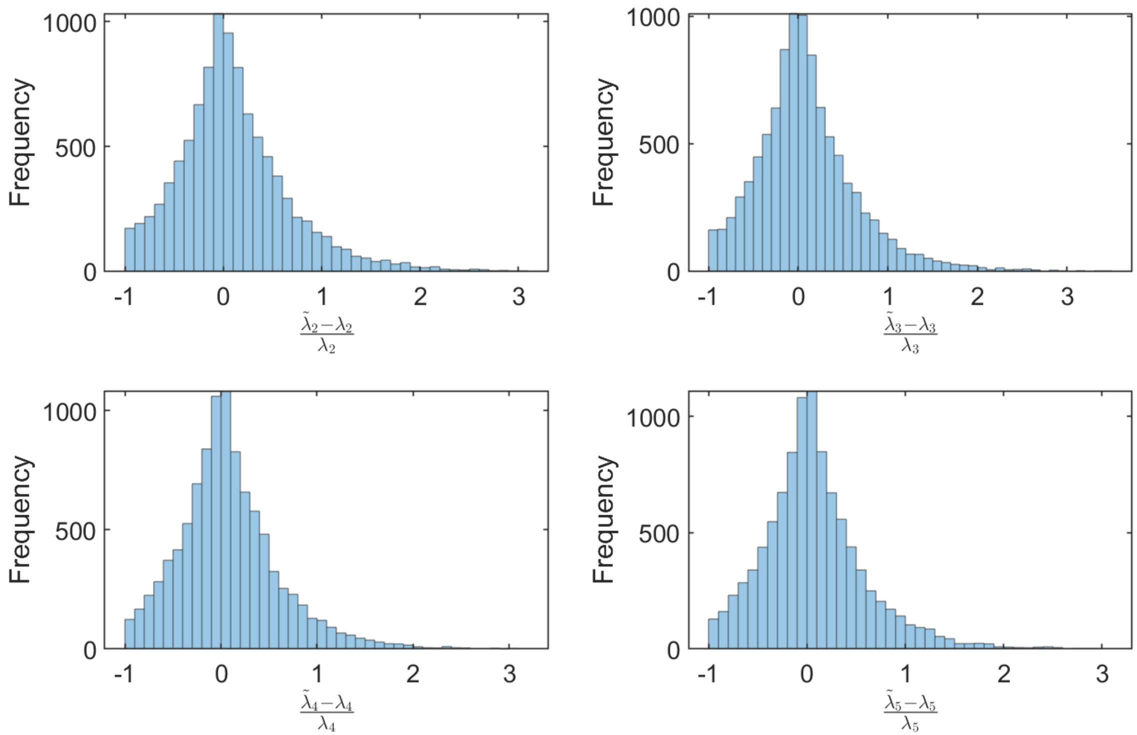


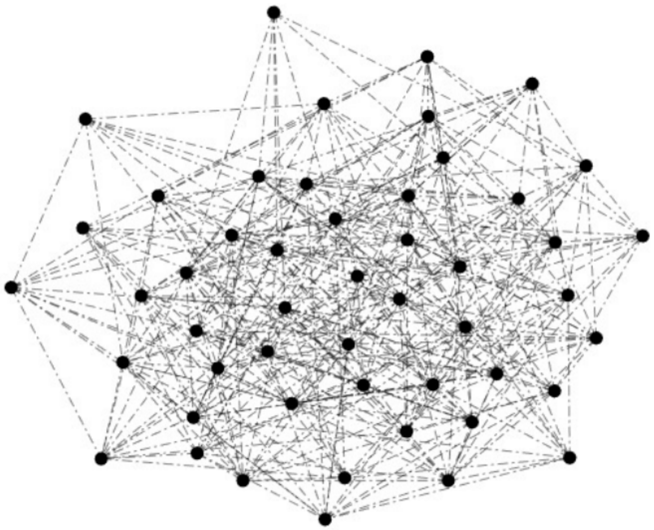
Private Analysis: Accuracy

Theorem 3:

$$E[\tilde{\lambda}_i - \lambda_i] = \frac{1}{2C(\lambda_i, b)}\left(2\lambda_i + be^{-\frac{\lambda_i}{b}} - (n+b)e^{-\frac{n-\lambda_i}{b}}\right) - \lambda_i$$

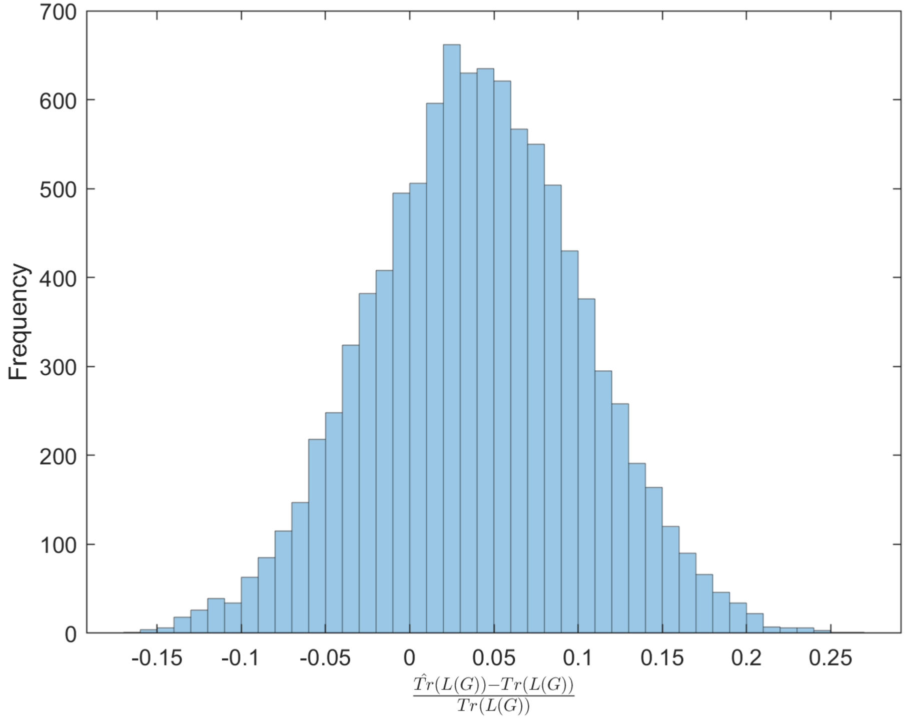
- Fix a graph G on 50 nodes, $\epsilon = 0.6, \delta = 0.05, A = 2$.
- Generate 10^4 private $\tilde{\lambda}_i$ for $i \in \{2, \ldots, 5\}$.

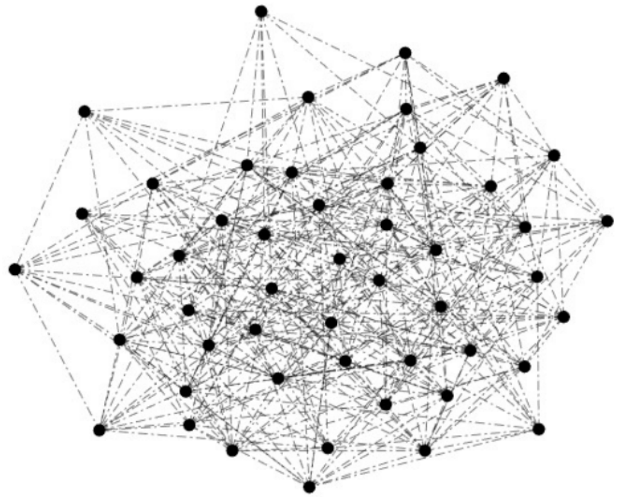




Private Analysis: Trace

- $Tr(L(G)) = \sum_{i=1}^{n} \lambda_i(L(G))$.
- Fix a graph G, $\epsilon = 0.4, \delta = 0.05, A = 2$.
- Generate 10^4 private spectra $\{\tilde{\lambda}_i\}_{i=1}^{n}$.
- Estimate the trace as $\widehat{Tr}(L(G)) = \sum_{i=1}^{n} \tilde{\lambda}_i(L(G))$

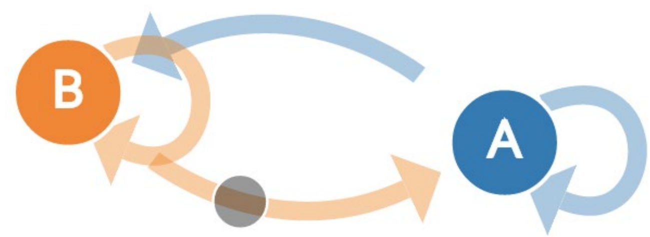




True Value: $Tr(L(G)) = 736$
Average error: 3.97%

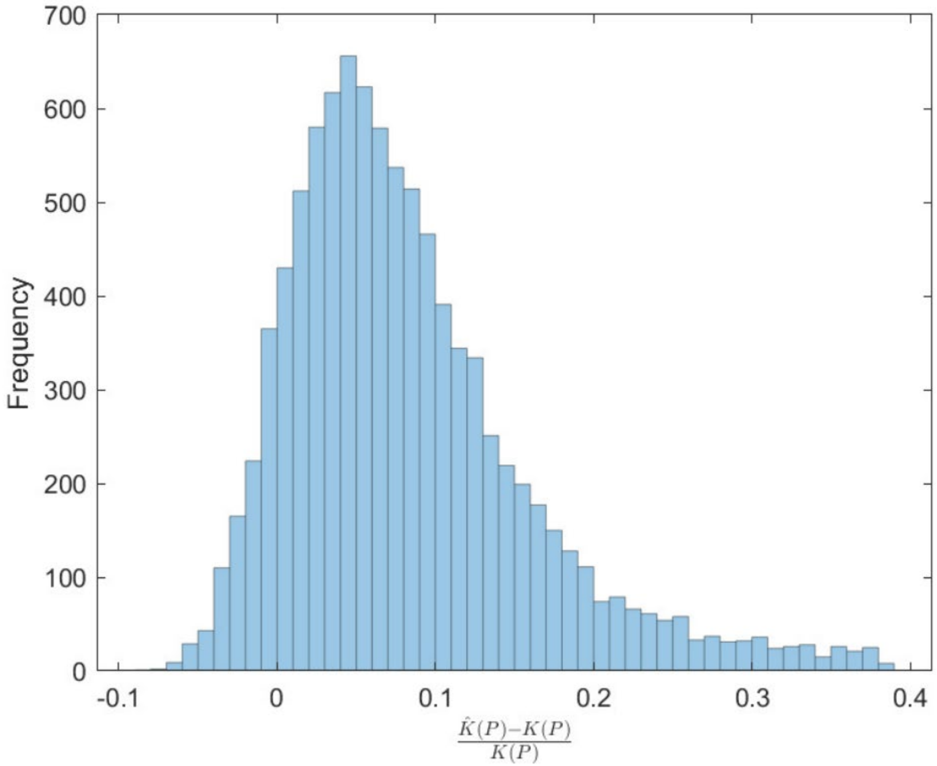
Private Analysis: Kemeny's Consant

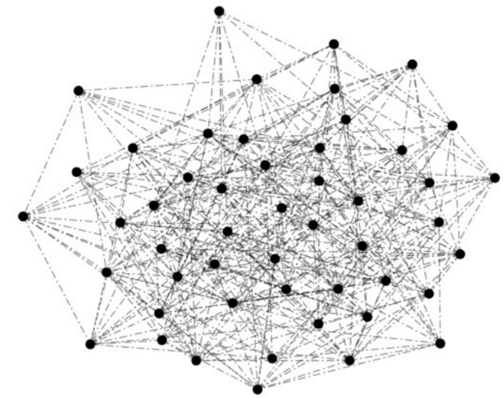
- Kemeny's constant = expected time steps from state i to a random state sampled from the stationary distribution of a Markov chain.
- In network control, discrete time consensus is governed by $P = I - \gamma L(G)$
- P can be analyzed as the transition matrix of Markov chain.
  - Error of consensus is in terms of the Kemeny constant.
- Estimate with the private spectrum $\{\tilde{\lambda}_i\}_{i=1}^{n}$.
- Kemeny's consant: $K(P) = \sum_{i=2}^{n} \frac{1}{1-\lambda_i(P)}$
  - $K(P) = \frac{1}{\gamma} \sum_{i=2}^{n} \frac{1}{\lambda_i(L(G))}$
  - Private estimate $\widehat{K}(P) = \frac{1}{\gamma} \sum_{i=2}^{n} \frac{1}{\tilde{\lambda}_i(L(G))}$



Private Analysis: Kemeny's Consant

- Fix a graph G, $\epsilon = 1.0, \delta = 0.05, A = 2, \gamma = \frac{1}{n}$.
- Generate 10^4 private spectra $\{\tilde{\lambda}_i\}_{i=1}^{n}$
- Estimate Kemeny's constant as $\widehat{K}(P) = \frac{1}{\gamma}\sum_{i=2}^{n}\frac{1}{\tilde{\lambda}_i(L(G))}$





True Value: $K(P) = 102.70$
Average error: 0.55%

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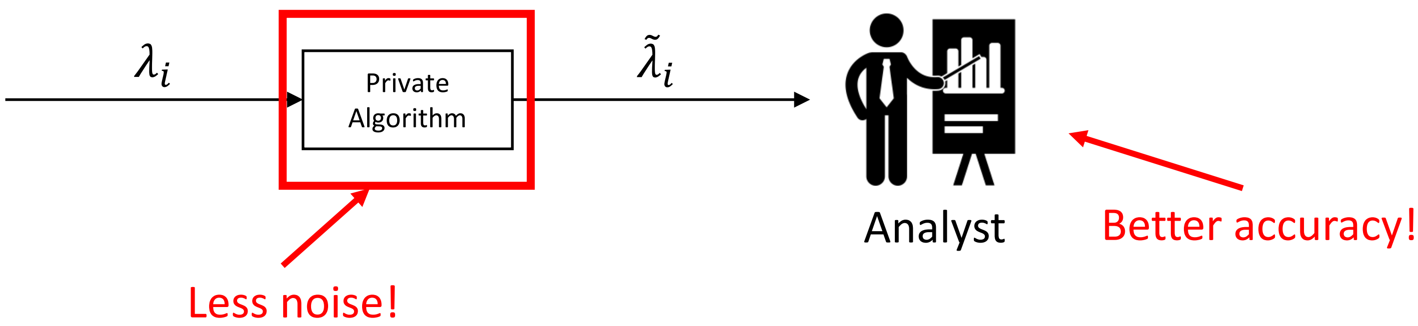
Future Work

- The development of new privacy mechanisms for other graph properties.
- Applications to basic reproduction number of an epidemic model.
- Privacy in multi-agent MDPs and reinforcement learning.



Thank you

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