Capturing a Non-Cooperative Tumbling Object: A Control Barrier Function Approach

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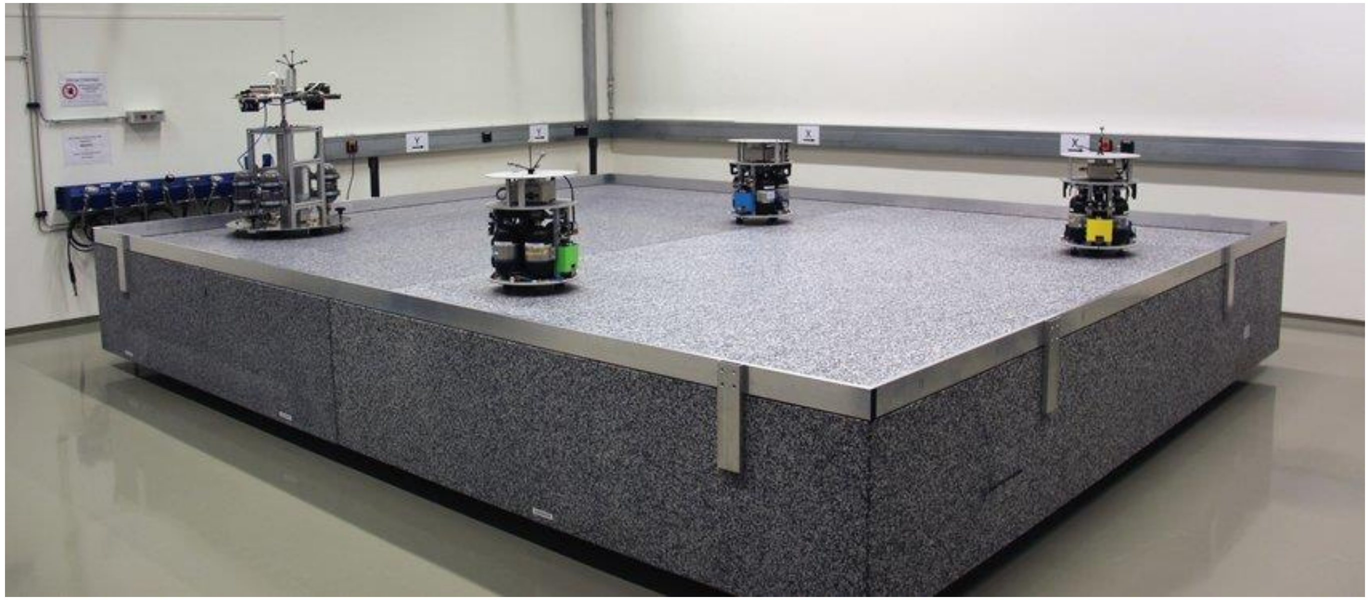
Problem Statement

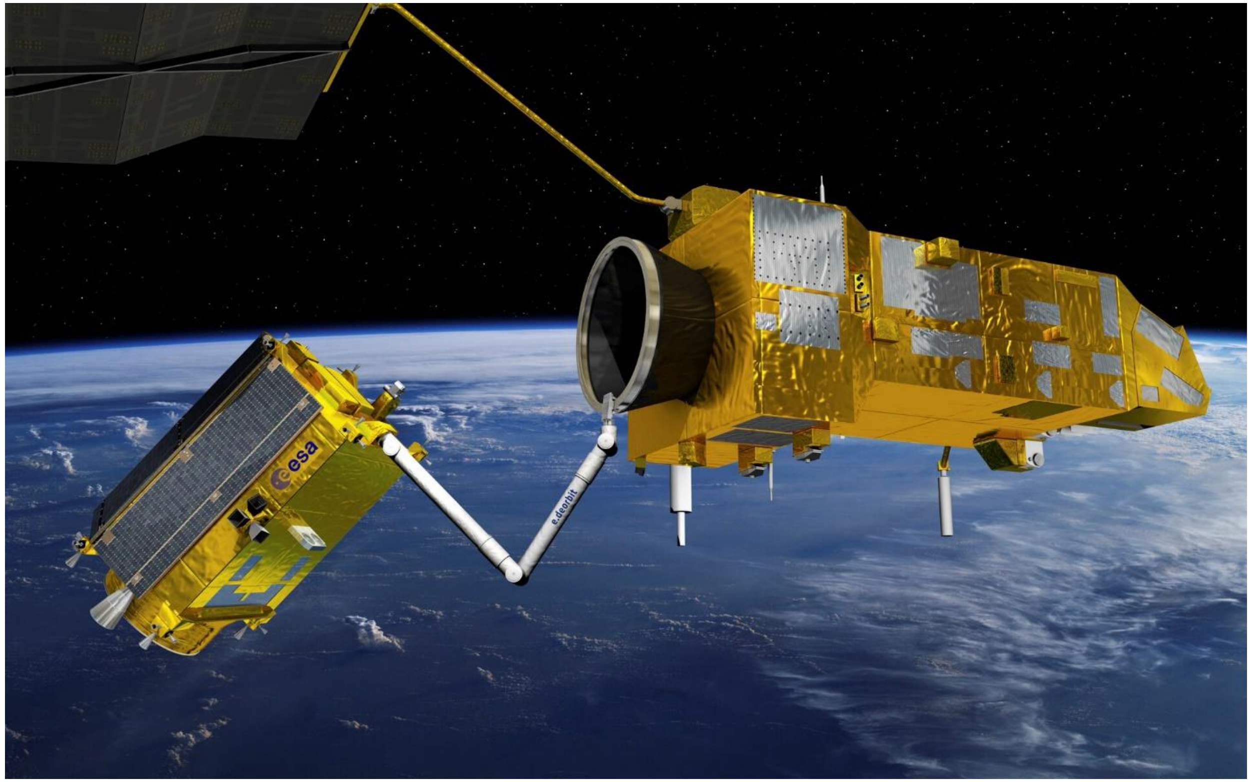
- Summer Intern at RV as a SMART Scholar
- Assigned to work in the ROC lab
- Satellite capture problem
  - rescue, inspection, refueling, upgrades, etc



Daitx, Henrique (2015) *Development of a combined attitude and position controller for a satellite simulator.* Master's, Cranfield University, UK

Problem Statement

- Non-cooperative (i.e., an unknown or tumbling object with a varying axis of rotation)
- Multiple redundant joint manipulators
- Safety Constraints
  - Collision avoidance Constraints
    - Client-Server
    - Manipulator-Server
    - Manipulator-Manipulator
  - Dynamic singularity avoidance Constraint
  - Vision constraints
    - Keep client agent is line of sight
    - Don’t look at the sun (saved for future work)
- Robustness
  - Uncertain client agent dynamics
  - Disturbances and perturbations
- Energy Considerations
  - Free-Floating VS Free-Flying
- Computational efficiency, possibly real time implementation
  - Computational resource constraint



Rendering of the European Space Agency’s proposed e.Deorbit mission for 2024
https://www.esa.int/ESA_Multimedia/Images/2017/01/e.Deorbit_s_robotic_arm

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Server Agent:

$$H^{(1)}(\delta^{(1)},\Theta)\ddot{q}^{(1)} + C^{(1)}(\delta^{(1)},\dot{\delta}^{(1)},\Theta,\dot{\Theta}) = Q$$

- $Q \triangleq J_Q u$ are the generalized forces, where $u \triangleq [f,\tau,\tau_\theta]$, and J_Q is a Jacobian
- $H^{(1)}$ is the known inertial matrix
- $C^{(1)}$ represents the known nonlinear terms
- $q^{(1)} \triangleq \left[r_{CM}^{(1)},\delta^{(1)},\Theta\right]$ is the generalized coordinate

Client Agent:

$$H^{(2)}(X^{(2)})\ddot{X}^{(2)} + C^{(2)}(X^{(2)},\dot{X}^{(2)}) + \tau_d(t) = 0$$

$$C^{(2)} \triangleq V(X^{(2)},\dot{X}^{(2)}) + F_S(\dot{X}^{(2)}) + G(X^{(2)})$$

- The above terms are uncertain, but have known bounds
- $X^{(2)}$ and $\dot{X}^{(2)}$ are measurable/calculable

Control Affine Representation of the Whole System:

$$\dot{Z} \in F(Z,u),$$

$$F \triangleq \begin{bmatrix} z_2^{(1)} \\ z_2^{(2)} \\ -\left(H^{(1)}\right)^{-1}C^{(1)} \\ -\left(H^{(2)}\right)^{-1}\left(C^{(2)} + \bar{\tau}_d\mathbb{B}\right) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \left(H^{(1)}\right)^{-1}Q \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Z \triangleq \left[q^{(1)},X^{(2)},\dot{q}^{(1)},\dot{X}^{(2)},\mathrm{mode},W\right]$$

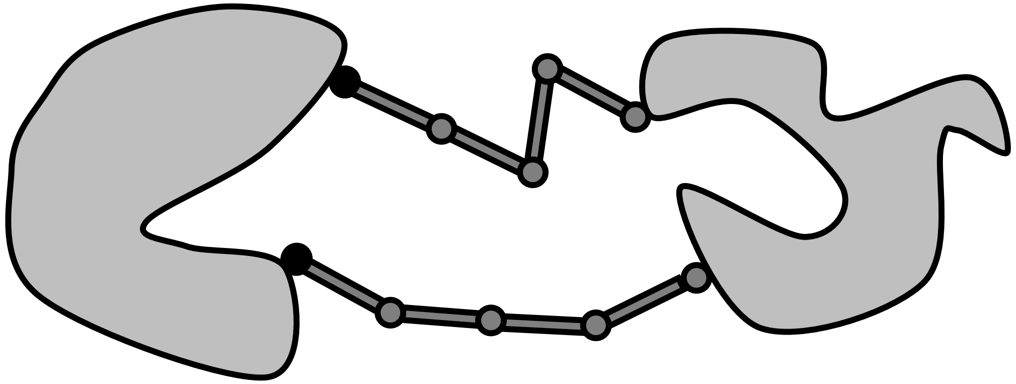
Higher Order Control Barrier Function

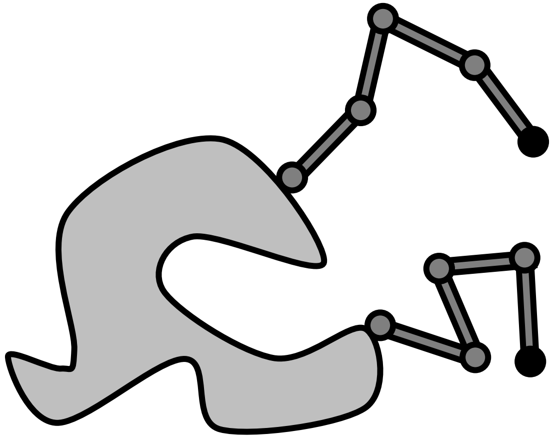
- Use Higher Order Control Barrier Functions to act as a filter on the nominal controller.
- Constrained Quadratic Program
    - Constraints are inequalities
- Constraints are derived according to HOCBFs
- Find control input that is as close to the nominal controller that satisfies the constraints
- Should get us a real time controller that drives the server agent to the desired capture state while satisfying safety constraints.

$$
\begin{aligned}
Q^*(Z) = \underset{Q \in \mathbb{R}^N}{\operatorname{argmin}} & \|Q - Q_{nom}\|^2 \\
\text{s.t.} \quad & Q \in \boldsymbol{K}_0 \text{ if mode} = 0 \\
& Q \in \boldsymbol{K}_1 \text{ if mode} = 1 \\
& Q \in \boldsymbol{K}_2 \text{ if mode} = 2 \\
& \|P_{1-3} Q\|^2 \leq f_{\max}^2 \\
& \|P_{4-6} Q\|^2 \leq \tau_{\max}^2 \\
& |P_{7-K} Q|^2 \leq \tau_{\theta,\max}^2
\end{aligned}
$$

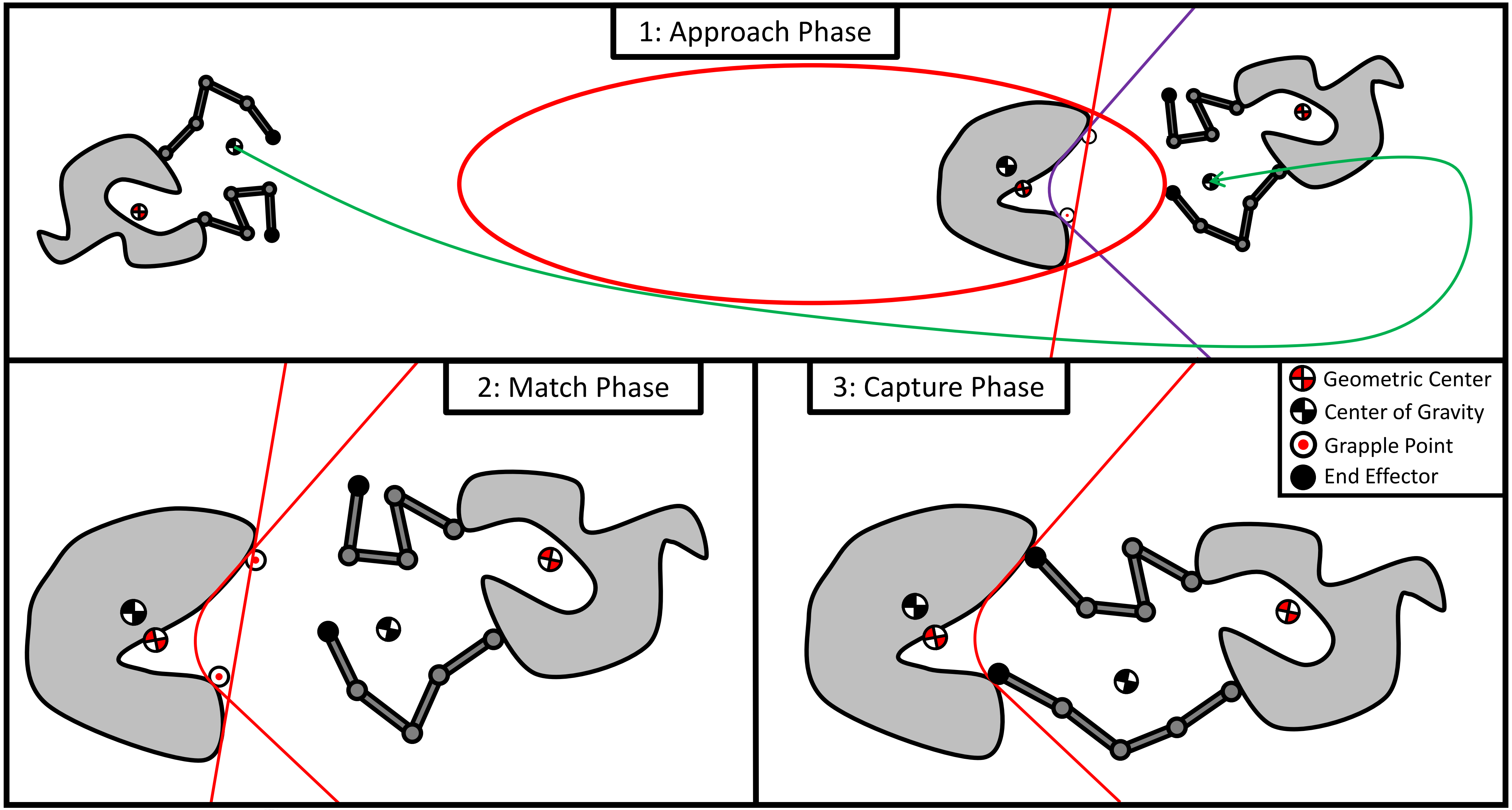
Nominal Controller

- There is a nominal controller that is designed to stabilize to a desired final state (i.e., capture)
- Does not care about safety

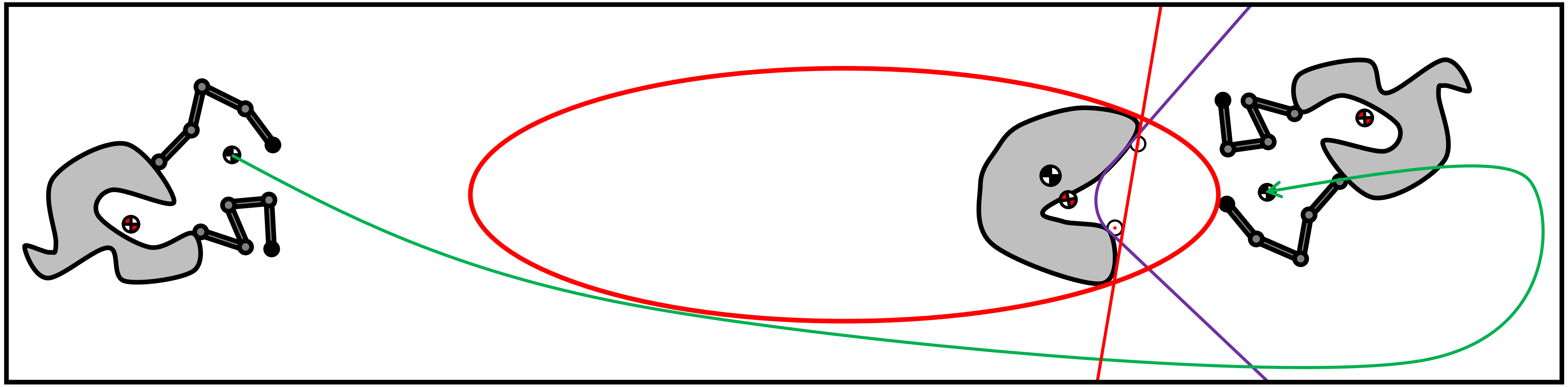




Proposed Strategy (Operating Modes)



Approach Phase: $\text{mode} = 0$



- Ellipsoid is fixed to the client agent, and is not only selected to prevent collisions, but also for trajectory shaping.
- Nominal controller will drive the server agent into the approach region (right of the purple surface)
  - This surface is fixed to the client agent.
- Server Agent is modelled as either a polyhedral, or a sphere

- $B_1 = \begin{cases} c - r_{GC}^{(1)}, & l = 0 \wedge m = 0 \\ \rho(\theta, \phi) - r_{GC}^{(1)}, & o/w \end{cases}$
- The control input does not show up, so a recursion of the CBF candidate above is required:
- $\tilde{B}_1 = \Gamma_1 + \gamma_1 + \epsilon_1$
- The uncertain dynamics of the client agent show up, so the regulation map is:
- $\bar{K}_1 = \{Q \in \mathbb{R}^{6+K} : \bar{\Gamma}_1(Z, Q) \leq -\tilde{\gamma}_1(Z)\}$

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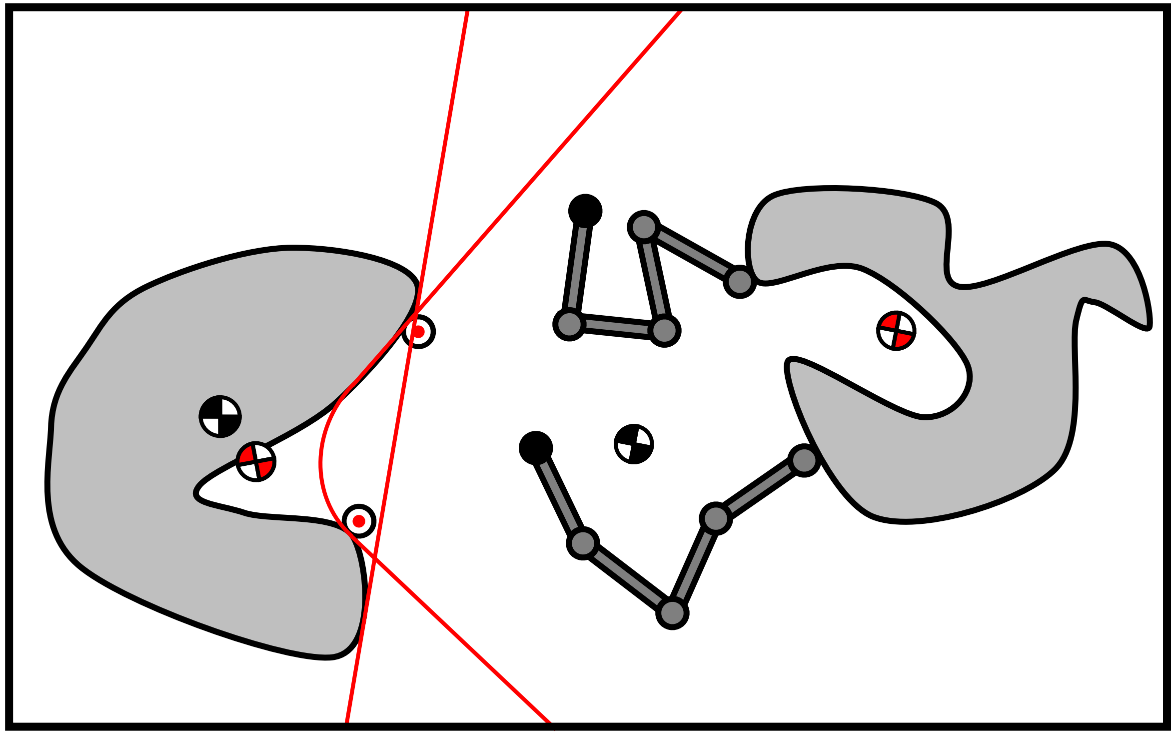
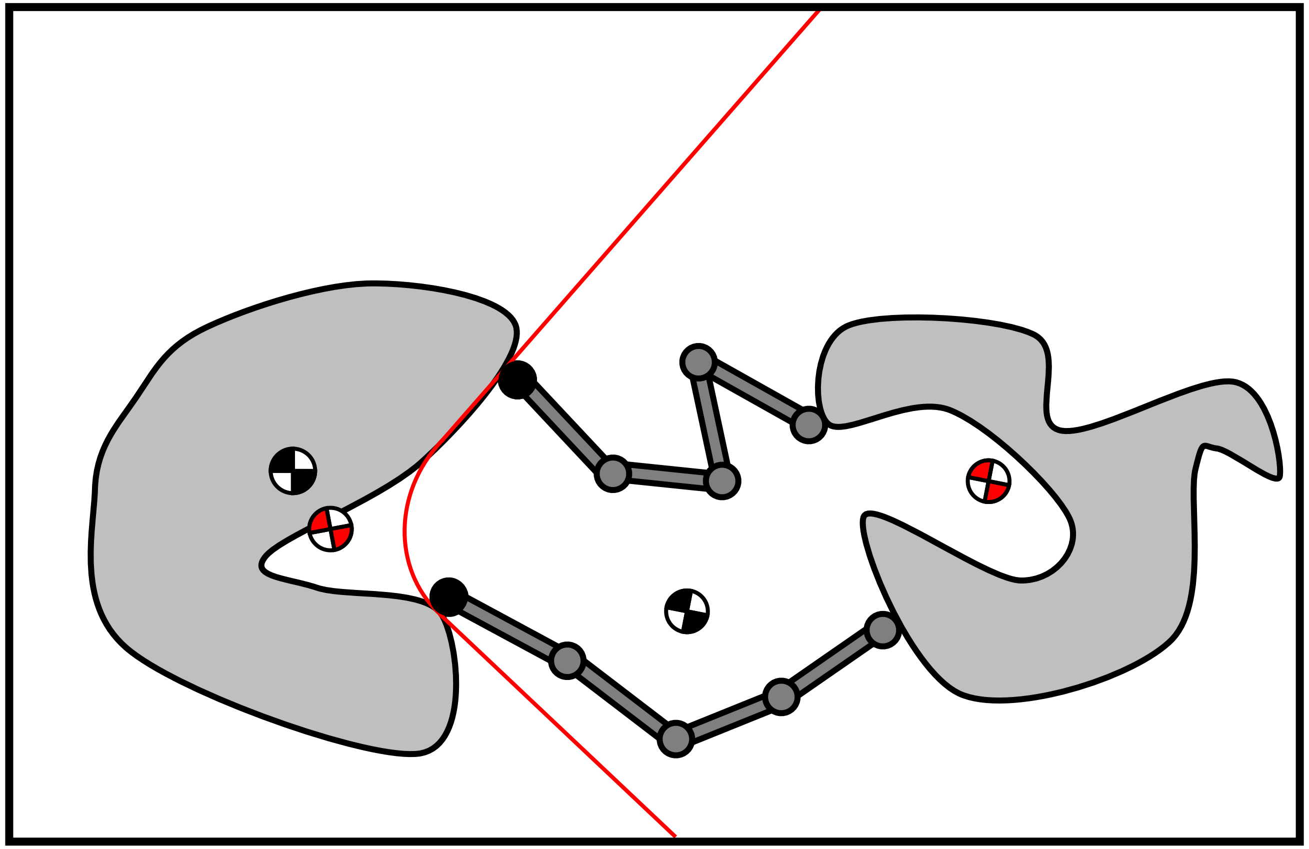
Match Phase: mode = 1

- Once the server agent falls into the approach region, a mode transition occurs, which erects a new safe set, which is better suited for this phase's goals.
- This phase allow for the relative velocities to regulate below a user specified threshold, thus setting the agent up for a safe capture in the next phase.
- The approach region and velocity matching barrier are promoted to CBF candidates.
- Like before, these CBF candidates have degree separation of two with the control input, and involves client agent uncertainties.



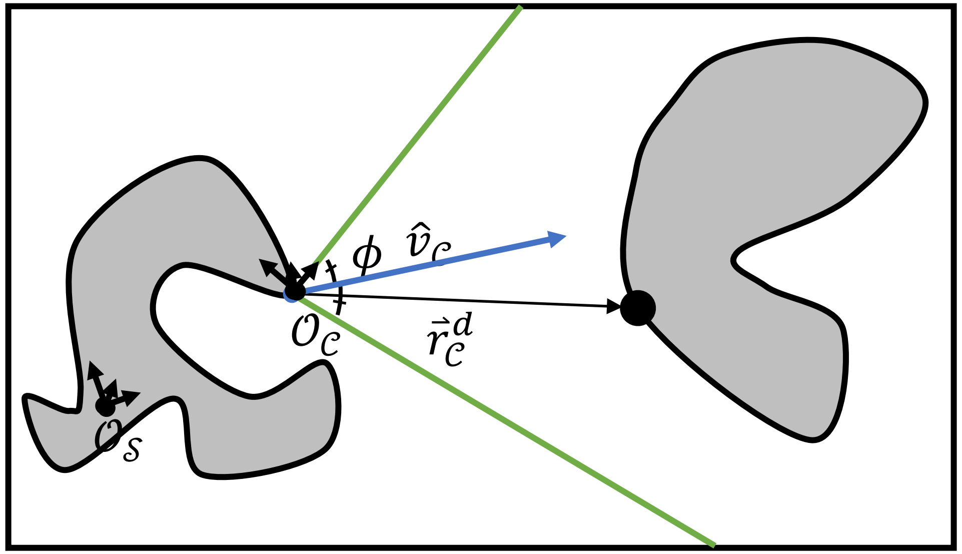
Capture Phase: $\text{mode} = 2$

- Once the velocities are matched, the next mode transition occurs.
- In this phase, the velocity matching barrier is dropped, thus allowing the server agent to reach out and capture the client agent.
- In this situation, it may be desirable to have your control effort directed towards the manipulators, rather than the base:
  - Weighted pseudo-inverse of the server agent's Jacobian
  - $J_S^+ \triangleq W^{-1} J_S^T \left(J W^{-1} J^T\right)^{-1}$
  - W is a user selected weight matrix



General System Constraints

- These constraints are present during all the mission phases.
- Base Actuation Constraints
  - There is a maximum thrust and torque that can be applied to the server agent's base.
- Vision Constraint
  - Must keep the client agent within FoV of the server agent so to have state measurements of the client agent.
- Manipulator Actuation Constraints
  - There is a range of allowable joint angles for each joint
  - There is a maximum torque that each manipulator can have as an input
- Manipulator-Manipulator Collision Avoidance
  - With multiple manipulator arms, collision between them must be avoided
- Manipulator-Base Collision Avoidance
  - The manipulators are not allowed to collide with the server's base
- Dynamic Singularity Avoidance
  - There are configurations the server agent can be, where certain movements are physically prohibited, and therefore must be avoided.
  - The manipulability index $\mu \triangleq \sqrt{\det(J_S J_S^T)}$ is used as a constraint
    - When it is zero, J_S loses rank, and is full rank otherwise



Concluding Results

- Each operating mode (or phase) has its own safe set, where each safe set is pre-asymptotically stable.
- We do not get asymptotic stability since the safe set is not compact due to how the approach region constraints are designed.
- However, the nominal controller does drive the system to a particular state, thus, in practice, prohibiting the system from flowing without bound.
- It remains to be shown that this controller will operate in real time on a Raspberry Pi, but based on CBF based QP controllers being implemented on other systems using similar devices (e.g,. Ames's group with the BeagleBone Black), it seems plausible.

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\end{aligned}
$$

Future Work

- Simulations
- Conduct experiments in the ROC lab
- Multiple Impedance Controller as the nominal controller
- Investigate the "don't look at the sun" constraint