Randomized Greedy Algorithms for Sensor Selection in Large-Scale Satellite Constellations

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SYSTEMS GROUP

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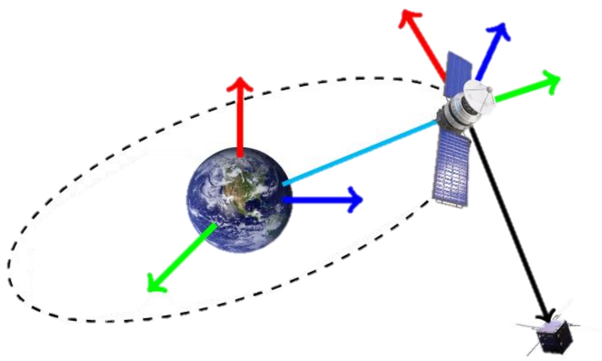
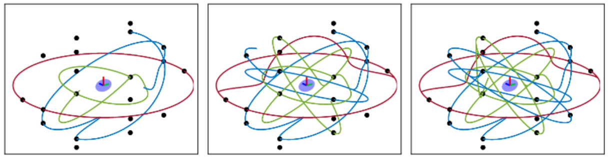
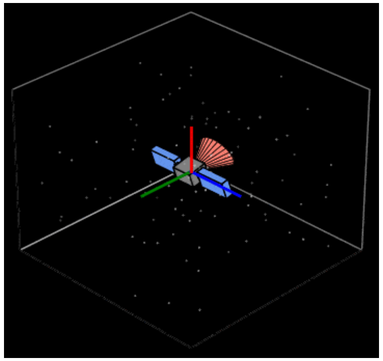
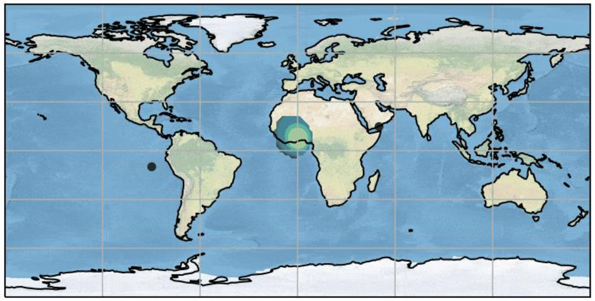
My Current Research on Space Systems



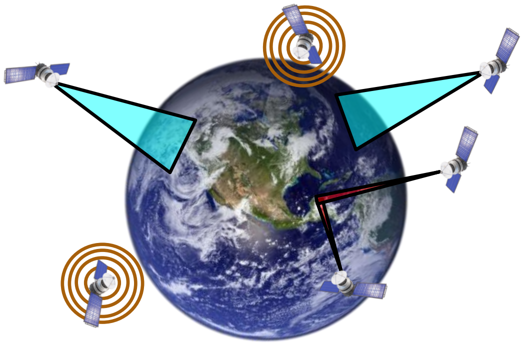


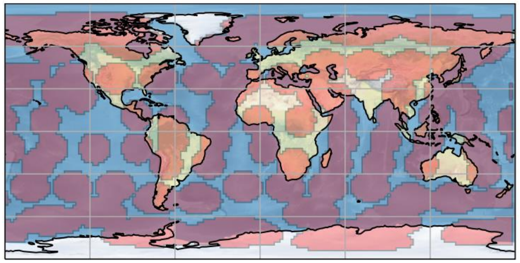
Satellite proximity operations





Planning for agile satellites





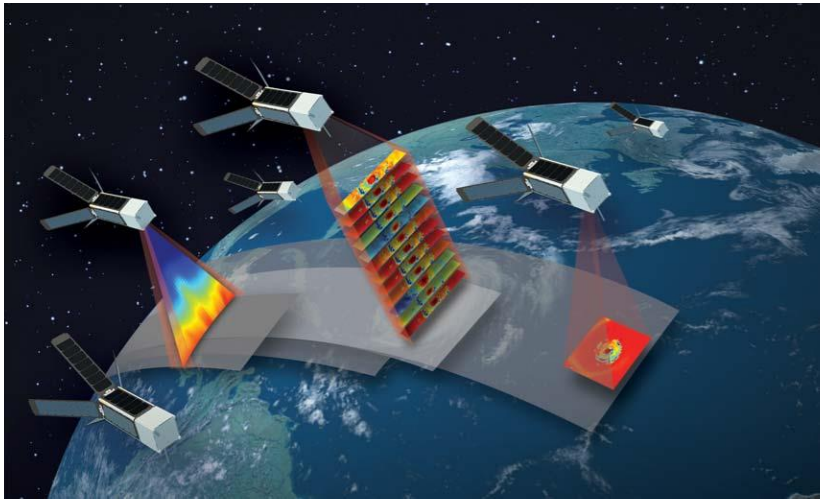
Sensor selection for constellations

Benefits of Large Constellations of Small Satellites

Cheaper, standardized parts

Redundancy in case of failure

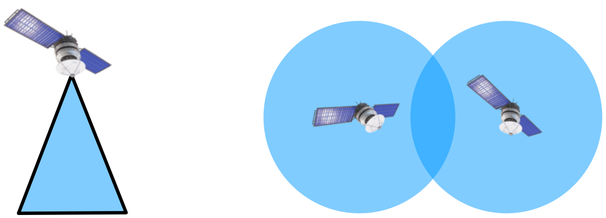
Greater temporal resolution through reduced revisit times



Nasa TROPICS

Basics of Submodular Set Functions

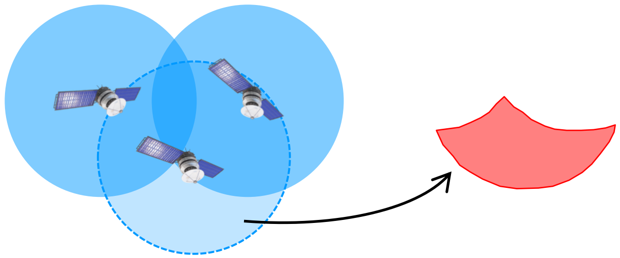
A set function $f: 2^{\mathcal{X}} \rightarrow \mathbb{R}$ is *submodular* if $f(\mathcal{S}) \leq f(\mathcal{T})$ for all $\mathcal{S} \subseteq \mathcal{T} \subseteq \mathcal{X}$



The quantity $f_j(\mathcal{S}) \triangleq f(\mathcal{S} \cup \{j\}) - f(\mathcal{S})$ is the *marginal gain* of adding $j \in \mathcal{X} \setminus \mathcal{S}$

The *weak submodularity constant* of f is

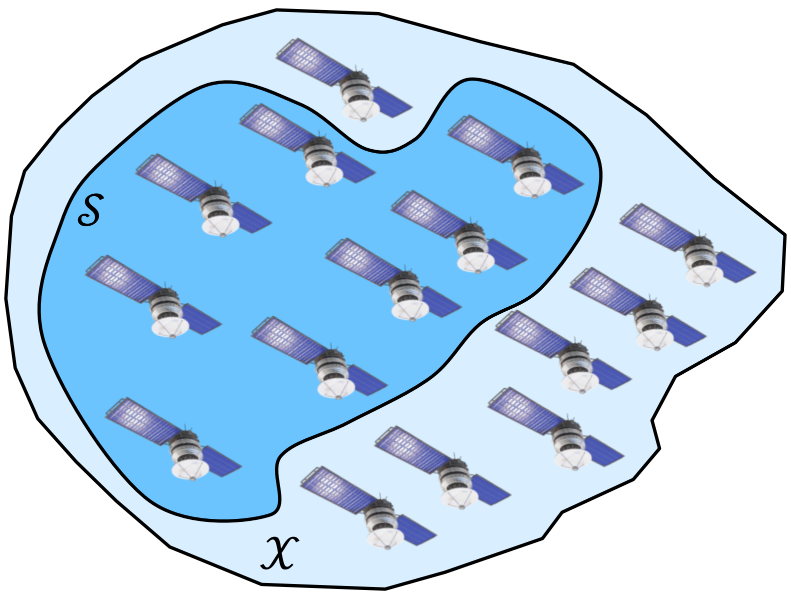
$$w_f \triangleq \min_{(\mathcal{S},\mathcal{T},i) \in \widetilde{\mathcal{X}}} f_i(\mathcal{T}) / f_i(\mathcal{S})$$



Submodular Maximization and the Greedy Algorithm

$$\max_{\mathcal{S} \subset \mathcal{X}} \quad f(\mathcal{S})$$

$$\text{s.t.} \quad |\mathcal{S}| \leq k$$



Standard GREEDY algorithm

Input: function $f =$, set $\mathcal{X} = \{ , , \dots , \}$, cardinality k

$\mathcal{S} \leftarrow \emptyset$

while $|\mathcal{S}| \leq k$ **do**

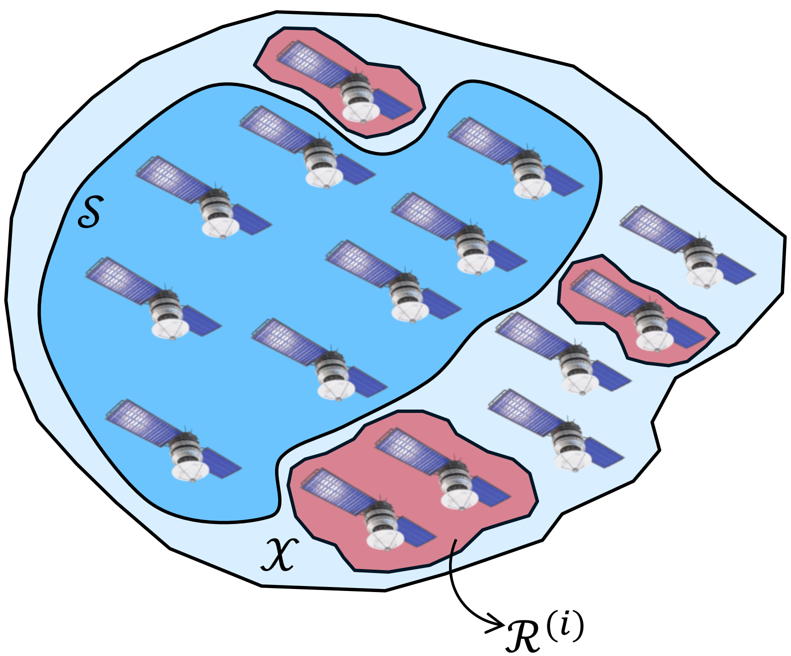
$\quad {}^* \in \arg\max_{\in \mathcal{X}} \{ : , : , \dots , : \}$

$\quad \mathcal{S} \leftarrow {}^*$

$\quad \mathcal{X} \leftarrow \mathcal{X} \setminus {}^*$

Expensive for large sensor networks!

Incorporation of Randomization into the Greedy Algorithm



Randomized GREEDY algorithm

Input: function f = , set $\mathcal{X} = \{ , , \ldots , \}$, cardinality k, subset sample cardinality r_i

$\mathcal{S} \leftarrow \emptyset$, $i \leftarrow 1$

while $\left|\mathcal{S}^{(i)}\right| \leq k$ **do**

$\mathcal{R}^{(i)} \leftarrow$ uniform sample of $\min(\mathrm{r_i}, |\mathcal{X}|)$ elements from $\mathcal{X}$

$^* \in \arg\max_{\in \mathcal{R}^{(i)}} \{ : , : , \ldots , : \}$

$\mathcal{S}^{(i)} \leftarrow {}^*$

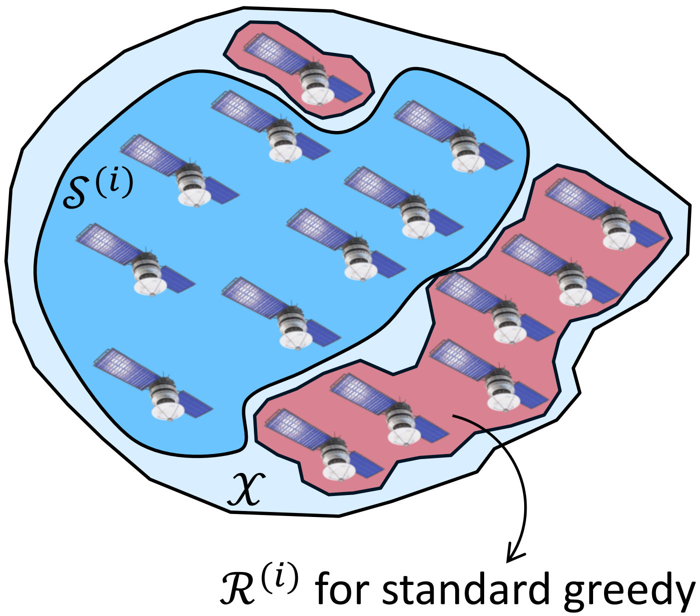
$\mathcal{X} \leftarrow \mathcal{X} \setminus {}^*$

$i \leftarrow i + 1$

Goal: Provide theoretical high-probability bounds on the performance for budget and performance-constrained models

Intuition Behind High-Probability Bounds

Now, focus on budgeted models, each satellite s_j has an associated cost c_j, greedy algorithm adds satellite with highest marginal-gain-to-cost ratio



Randomized greedy selects an element a fraction $\eta^{(i)}$ as good as standard greedy, i.e.,

$$\frac{f_{j_{rg}}(\mathcal{S}^{(i)})}{c_{j_{rg}}} \geq \eta^{(i+1)} \max_{j \in \mathcal{X} \setminus \mathcal{S}^{(i)}} \frac{f_j(\mathcal{S}^{(i)})}{c_j} = \eta^{(i+1)} \frac{f_{j_g}(\mathcal{S}^{(i)})}{c_{j_g}}$$

Idea: the sequence $\eta^{(1)}, \eta^{(2)}, \ldots$ forms a martingale, use concentration bounds (e.g. Azuma's inequality)

Modified Randomized Greedy (MRG)

$$\max_{\mathcal{S}\subset\mathcal{X}} \quad f(\mathcal{S})$$
$$\text{s.t.} \quad \Sigma_{j\in S} c_j \leq B$$

Maximize a submodular function subject to a budget constraint on the selection

Theorem 1. *Let $\eta = \eta^{(1)}, \eta^{(2)}, \ldots$ be a martingale satisfying the conditions of Azuma's inequality with $\mathbb{E}[\eta] \geq \mu$, for some $\mu \geq 0$. Then, for any confidence parameter $0 < \delta < 1$, MRG yields a set $\mathcal{S}_{mrg}$ such that*

$$\frac{f(\mathcal{S}_{mrg})}{f(\mathcal{S})} \geq \frac{1 - \exp(-1/w_f\,(\mu - c_{max}/B\sqrt{U/2\log 1/\delta}\,)}{2w_f^2}$$

holds with probability at least $1-\delta$, in which U is the smallest integer such that $\Sigma_{j=1}^{U}\bar{c}_j \geq B$, where $\bar{c}_1 \leq \bar{c}_2 \leq \cdots$ is the collection of ordered observation costs

Reductions of Bound for MRG

$$\frac{f(\mathcal{S}_{mrg})}{f(\mathcal{S})} \geq \frac{1 - \exp(-1/w_f\,(\mu - c_{max}/B\,\sqrt{U/2\log 1/\delta}\,)}{2w_f^2}$$

If $c_{max}\sqrt{U} \ll B$, then the bound reduces to

$$\frac{f(\mathcal{S}_{mrg})}{f(\mathcal{S})} \geq \frac{1 - \exp(-\mu/w_f)}{2w_f^2}$$

If f is submodular and $r_i = |\mathcal{X}|$, the bound further reduces to

$$\frac{f(\mathcal{S}_{mrg})}{f(\mathcal{S})} \geq \frac{1}{2}(1 - e^{-1})$$

Dual Randomized Greedy (DRG)

$$\min_{\mathcal{S} \subset \mathcal{X}} \quad \Sigma_{j \in \mathcal{S}} c_j$$
$$\text{s.t.} \quad f(\mathcal{S}) \geq A$$

Minimize the selection cost subject to a performance constraint

Theorem 2. *Let* $\eta = \eta^{(1)}, \eta^{(2)}, \ldots$ *be a martingale satisfying the conditions of Azuma's inequality with* $\mathbb{E}[\eta] \geq \mu$, *for some* $\mu \geq 0$. *Then, for any confidence parameter* $0 < \delta < 1$, *DRG yields a set* $\mathcal{S}_{drg}$ *such that*

$$\frac{c(\mathcal{S}_{drg})}{c(\mathcal{S}^*)} \leq \frac{w_f}{\mu}\left[1 + (L-1)\log w_f + \log M/m\right] + \frac{\sqrt{1/2 \log 1/\delta \, c^2(\mathcal{S}_{drg})}}{\mu c(\mathcal{S}^*)}$$

Holds with probability at least $1 - \delta$, *where* $L \leq |\mathcal{X}|$ *is the number of iterations required by DRG and* $c^2(\mathcal{S}_{drg}) \triangleq \Sigma_{j \in \mathcal{S}_{drg}} c_j^2$.

Reductions of Bound for DRG

$$\frac{c(\mathcal{S}_{drg})}{c(\mathcal{S})} \leq \frac{w_f}{\mu}\left[1 + (L-1)\log w_f + \log M/m\right] + \frac{\sqrt{1/2 \log 1/\delta \, c^2(\mathcal{S}_{drg})}}{\mu c(\mathcal{S}^*)}$$

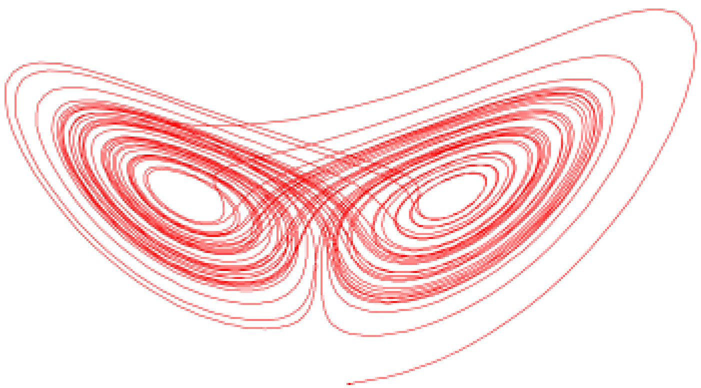
If $r_i = |\mathcal{X}|$, then the bound reduces to

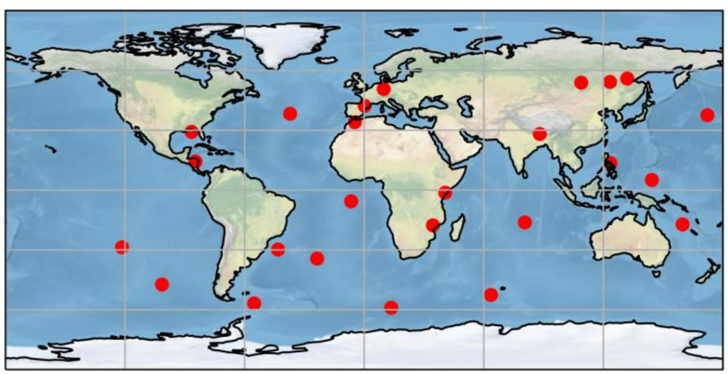
$$\frac{c(\mathcal{S}_{drg})}{c(\mathcal{S})} \leq w_f\left[1 + (L-1)\log w_f + \log M/m\right]$$

If f is submodular, the bound further reduces to

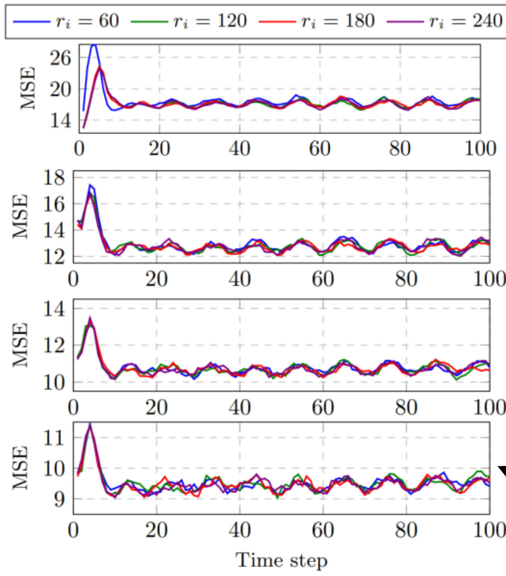
$$\frac{c(\mathcal{S}_{drg})}{c(\mathcal{S})} \leq [1 + \log M/m]$$

Applying MRG: Atmospheric Sensing Estimation





Randomly sample 25 points, model conditions at each by chaotic Lorenz-63 system



Reduce computation time with minimal performance change

	$B = 25$	$B = 50$	$B = 75$	$B = 100$
$r_i = 60$	1.52	1.59	1.59	1.58
$r_i = 120$	2.93	2.95	3.03	3.01
$r_i = 180$	3.96	3.96	3.98	3.96
$r_i = 240$	4.33	4.33	4.39	4.34

Applying DRG: Ground Coverage

Enforce a minimum coverage fraction (CF) must be obtained at each time step

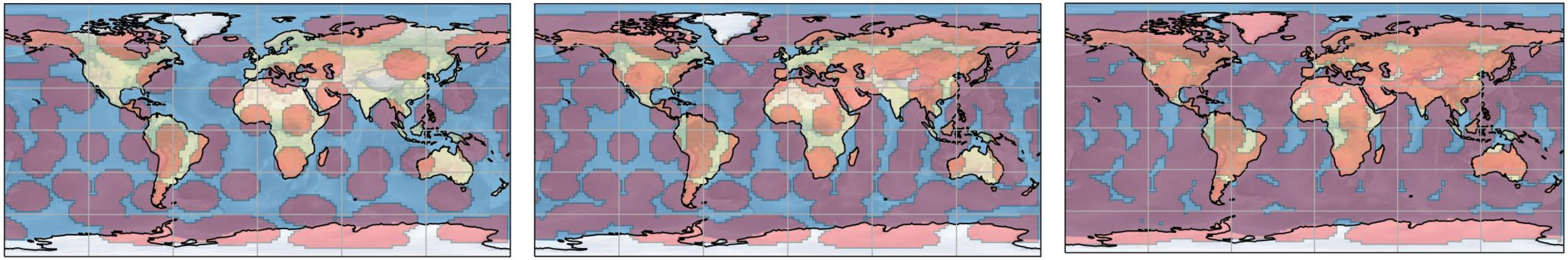
Randomization reduces computation time with minimal change in budget

Average budget cost

	$CF = 0.5$	$CF = 0.7$	$CF = 0.9$
$r_i = 60$	63.08	99.29	161.74
$r_i = 120$	62.64	99.30	161.47
$r_i = 180$	62.69	99.24	161.34
$r_i = 240$	62.89	99.10	161.24

Average computation time

	$CF = 0.5$	$CF = 0.7$	$CF = 0.9$
$r_i = 60$	0.53	0.86	1.37
$r_i = 120$	1.08	1.67	2.64
$r_i = 180$	1.60	2.50	3.64
$r_i = 240$	2.00	2.80	3.96



Ground coverage at time step 50 for varying minimum coverage areas, $r_i = 120$

Conclusion

Studied randomized greedy algorithms for budget and performance-constrained submodular optimization problems

Provided theoretical high-probability bounds on their performance, showed how to recover non-randomized versions

Future work will extend these results to robust submodular optimization problems

Thank you!