Resilient Solutions to Underactuated Docking Operations

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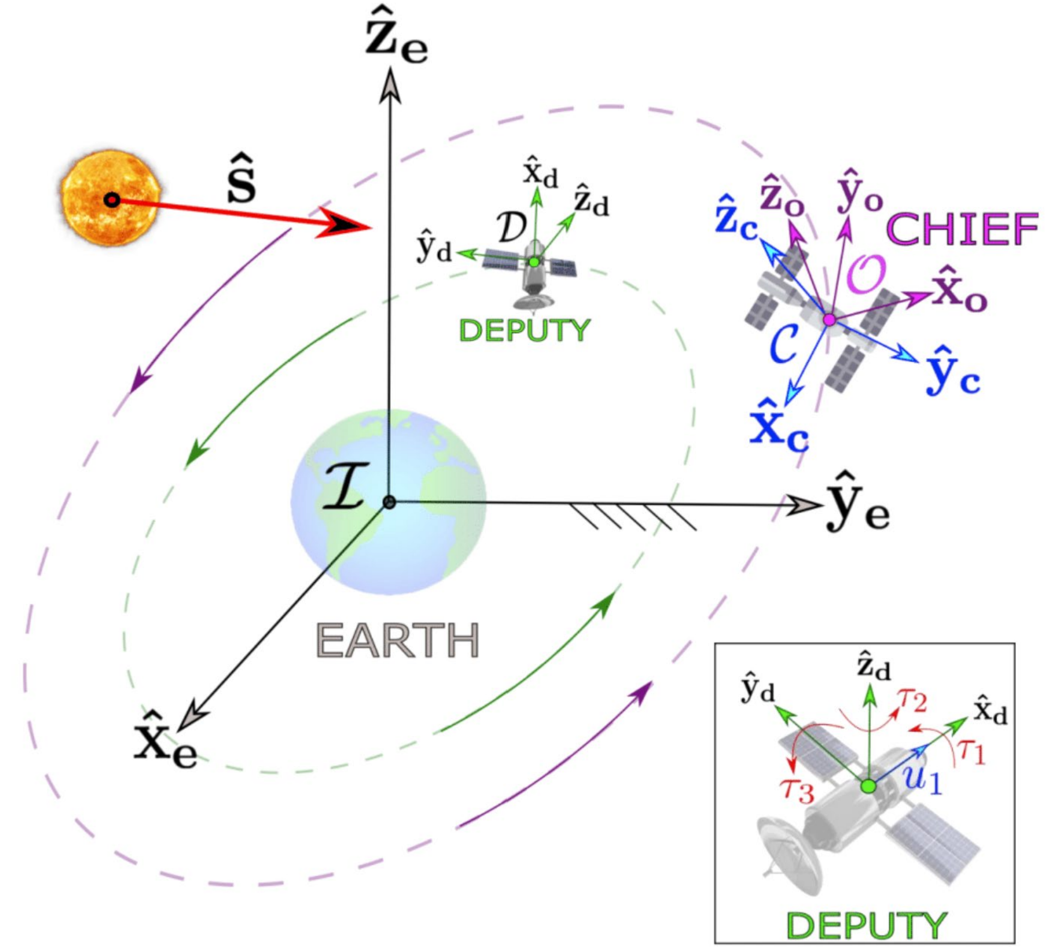
Motivation: Autonomous Docking Procedure On-Board Underactuated Deputy

- Autonomous rendezvous, proximity operations, and docking (ARPOD) are becoming increasingly important as space becomes increasingly crowded.
- Original 2D version was introduced as a challenge problem by Chris Petersen, Sean Phillips, Kerianne Hobbs, and Kendra Lang in 2020.
- 3D version of problem (where deputy is out-of-orbital plane) presents significant increase in dimensions and coupled dynamics.
- Legendre-Gauss-Radau (LGR) collocation was shown to be a promising approach.



Hill's Frame and MRPs

- Reference frame $\mathcal{O}$ is fixed to the center of mass of the Chief (Hill's Frame)
- Frame $\mathcal{C}$ aligns with the principal axes of the Chief
- The Deputy has full attitude control and a unilateral thrust
- Let the control-input be defined as

$$u_d^{\mathcal{D}} = (\mathrm{f_x}, 0, 0)$$

- The entire state space is defined as

$$\xi := \left(\delta x, \delta y, \delta z, \delta\dot{x}, \delta\dot{y}, \delta\dot{z}, \zeta_1, \zeta_2, \zeta_3, \omega_1, \omega_2, \omega_3, \nu_x, \nu_y, \nu_z\right)^T$$

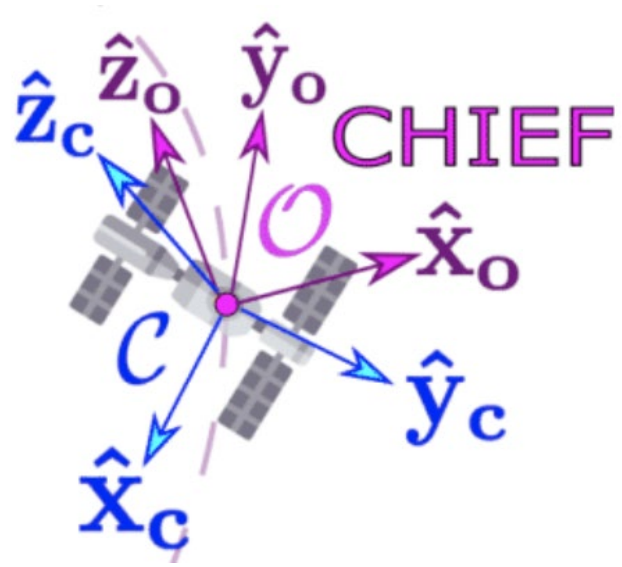
$$x := (\delta x, \delta y, \delta z, \delta\dot{x}, \delta\dot{y}, \delta\dot{z})^T \in R^6$$

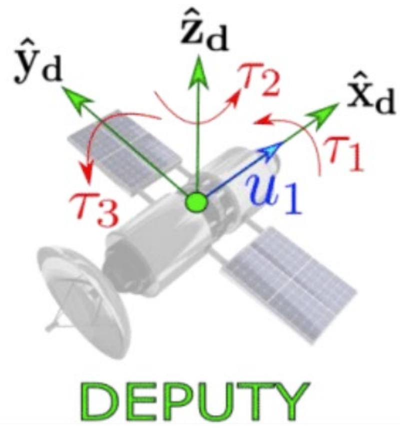
$$\dot{x} = Ax + B\mathcal{R}_{\mathcal{D}}^{\mathcal{O}} u_d^{\mathcal{D}}$$

- where

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\eta^2 & 0 & 0 & 0 & 2\eta & 0 \\ 0 & 0 & 0 & -2\eta & 0 & 0 \\ 0 & 0 & -\eta^2 & 0 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} \mathbf{0}_{3\times3} \\ \frac{1}{m}\mathcal{I}_3 \end{pmatrix}$$





The Dynamics are Coupled!

$$\frac{1}{m}\mathcal{R}_{\mathcal{D}}^{\mathcal{O}}(\zeta)u_d^{\mathcal{D}} = \frac{f_x}{m}\left(W_1(\zeta), W_2(\zeta), W_3(\zeta)\right)^T$$

Attitude and Translation Coupling

$$\dot{\zeta} = -\frac{1}{2}\left[\frac{1}{2}\left(1-\zeta^T\zeta\right)I_3 - \mathcal{S}(\zeta) + \zeta\zeta^T\right]\omega_{\mathcal{OD}}^{\mathcal{D}}$$

$$\dot{\omega}_1 = \omega_2 b(\zeta) - \omega_3 a(\zeta) + \frac{J_w}{J_1}\left(v_2[\omega_3 + b(\zeta)] - v_3[\omega_2 + a(\zeta)] - u_1\right) + \frac{1}{J_1}(J_2 - J_3)[\omega_2 + a(\zeta)][\omega_3 + b(\zeta)]$$

$$\dot{\omega}_2 = \omega_3 c(\zeta) - \omega_1 b(\zeta) + \frac{J_w}{J_2}\left(v_3[\omega_1 + c(\zeta)] - v_1[\omega_3 + b(\zeta)] - u_2\right) + \frac{1}{J_2}(J_3 - J_1)[\omega_3 + b(\zeta)][\omega_1 + c(\zeta)]$$

$$\dot{\omega}_3 = \omega_1 a(\zeta) - \omega_2 c(\zeta) + \frac{J_w}{J_3}\left(v_1[\omega_2 + a(\zeta)] - v_2[\omega_1 + c(\zeta)] - u_3\right) + \frac{1}{J_3}(J_1 - J_2)[\omega_1 + c(\zeta)][\omega_2 + a(\zeta)]$$

where

$$W_1(\zeta) = \frac{4}{(\zeta^2+1)^2}\left[-2\zeta_2^2 - 2\zeta_3^2 + \frac{(\zeta^2+1)^2}{4}\right]$$

$$W_2(\zeta) = \frac{4}{(\zeta^2+1)^2}\left[2\zeta_1\zeta_2 + \zeta_3(\zeta^2-1)\right]$$

$$W_3(\zeta) = \frac{4}{(\zeta^2+1)^2}\left[2\zeta_1\zeta_3 - \zeta_2(\zeta^2-1)\right]$$

and

$$a(\zeta) \coloneqq \frac{4\eta}{(\zeta^2+1)^2}\left[2\zeta_2\zeta_3 + \zeta_1(\zeta^2-1)\right]$$

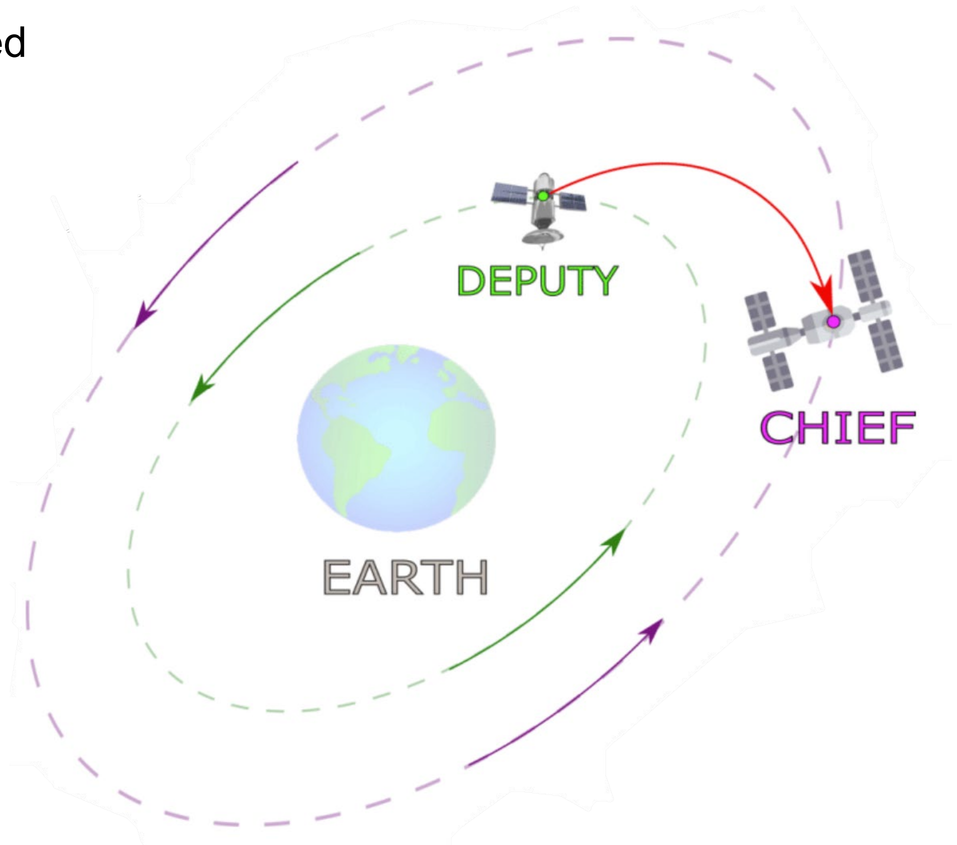
$$b(\zeta) \coloneqq \frac{4\eta}{(\zeta^2+1)^2}\left[\frac{(\zeta^2+1)^2}{4} - 2(\zeta_1^2 + \zeta_2^2)\right]$$

$$c(\zeta) \coloneqq \frac{4\eta}{(\zeta^2+1)^2}\left[2\zeta_1\zeta_3 - \zeta_2(\zeta^2-1)\right]$$

Problem Definition

Let the deputy be defined as $\xi \coloneqq \left(\delta x, \delta y, \delta z, \delta\dot{x}, \delta\dot{y}, \delta\dot{z}, \zeta_1, \zeta_2, \zeta_3, \omega_1, \omega_2, \omega_3, \nu_x, \nu_y, \nu_z\right)^T \in \mathbb{R}^{15}$

The underactuated ARPOD problem is **solved** if, given prescribed state constraint set $\mathcal{X} \subseteq \mathbb{R}^{15}$ and input constraint set $\mathcal{U} \subseteq \mathbb{R}^4$, a control trajectory $u(t): \mathbb{R}_{\geq 0} \to \mathcal{U}$ can be found such that $\xi \in \mathcal{X}\ \forall\, t \geq 0$ and $\xi \to 0$ as $t \to \infty$.



Methodology: LGR Collocation

- Methematical concept is the **interpolation** of state variables $x(t)$ and control input $u(t)$ at collocation points within the time interval $[t_0, t_f)$.

$$x(\psi_i) = \sum_{k=0}^{N} x_k P_k(\psi_i)$$

$$u(\psi_i) = \sum_{K=0}^{N} u_k P_k(\psi_i)$$

Radau points $\{\psi_1, \dots, \psi_i, \dots, \psi_N\}$

LeGendre polynomials $\{P_1, \dots, P_i, \dots, P_N\}$

$$\min_{\xi, \boldsymbol{u}} \sum_{k=0}^{N} \boldsymbol{U}_k^T \boldsymbol{U}_k$$

$$s.t. |u_d^{\mathcal{D}}| \leq [2,0,0] N$$

$$|\dot{\boldsymbol{\nu}}| \leq [181.3, 181.3, 181.3] \frac{rad}{s^2}$$

$$|\boldsymbol{\nu}| \leq [576.0, 576.0, 576.0] \frac{rad}{s}$$

$$|\boldsymbol{\omega}| \leq [2,2,2] \frac{deg}{s}$$

$$|\boldsymbol{v}| \leq [10,10,10] \frac{m}{s}$$

$$0 \leq \boldsymbol{\zeta} < 2\pi$$

$$10 \leq \rho(t_0) \leq 100 m$$

$$0 \leq \boldsymbol{\zeta}(t_0) \leq \pi$$

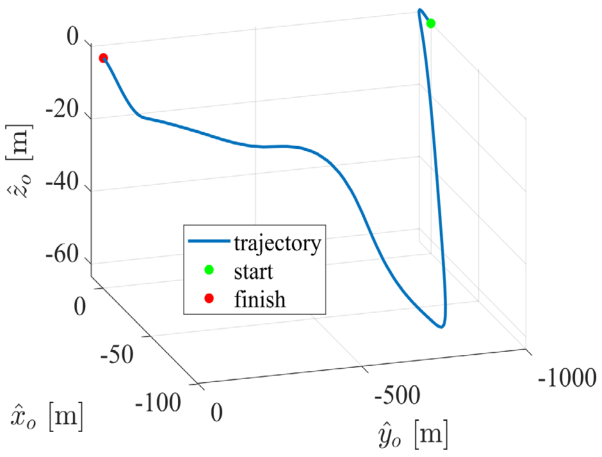
$$|\boldsymbol{\omega}(t_0)| \leq [1,1,1] \frac{rad}{s}$$

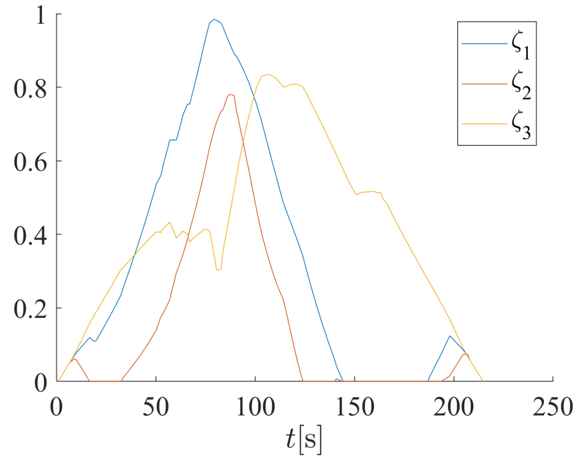
$$|\boldsymbol{\nu}(t_0)| \leq [1,1,1] \frac{rad}{s}$$

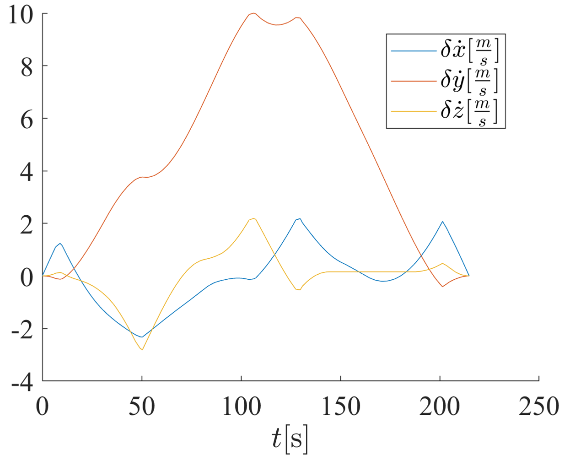
$$|\boldsymbol{\xi}(t_f)| \leq \mathbf{10}_{15}^{-8}$$

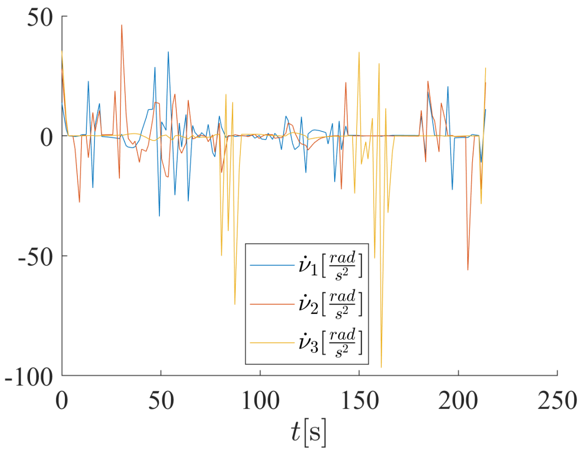
Where $\rho = \sqrt{x^2 + y^2 + z^2}$

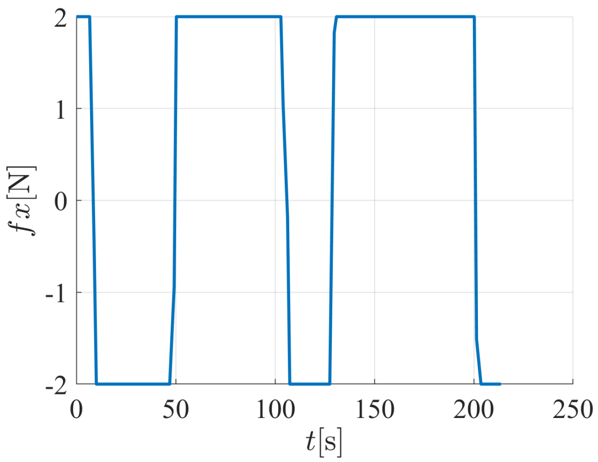
Min Time Collocation Results-Single Trajectory





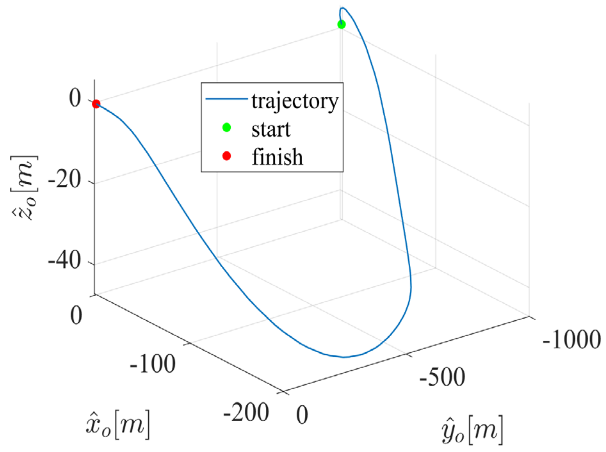


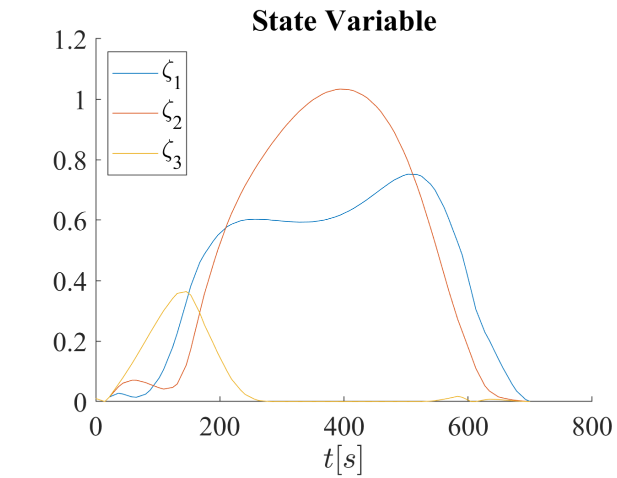


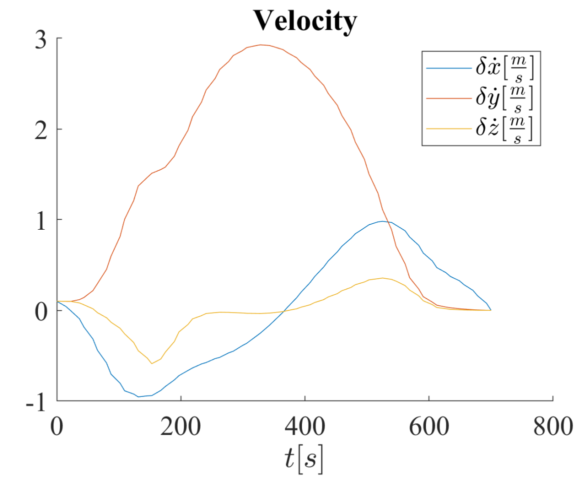


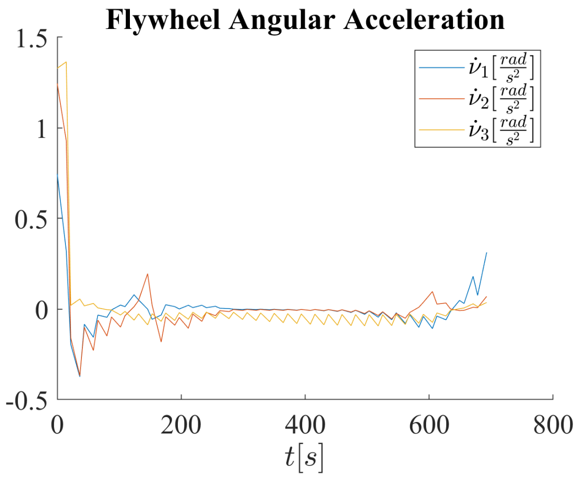
- 64 Radau Points
- Polynomial factor of 2
- Convergence tol: 10^{-8}
- Singular Arc

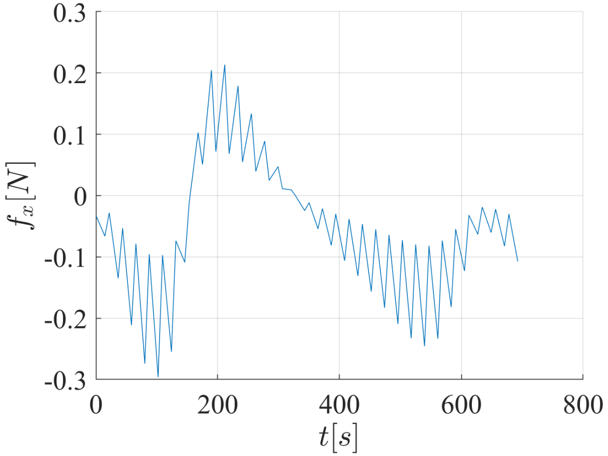
Min Ctrl-Input Collocation Results-Single Trajectory



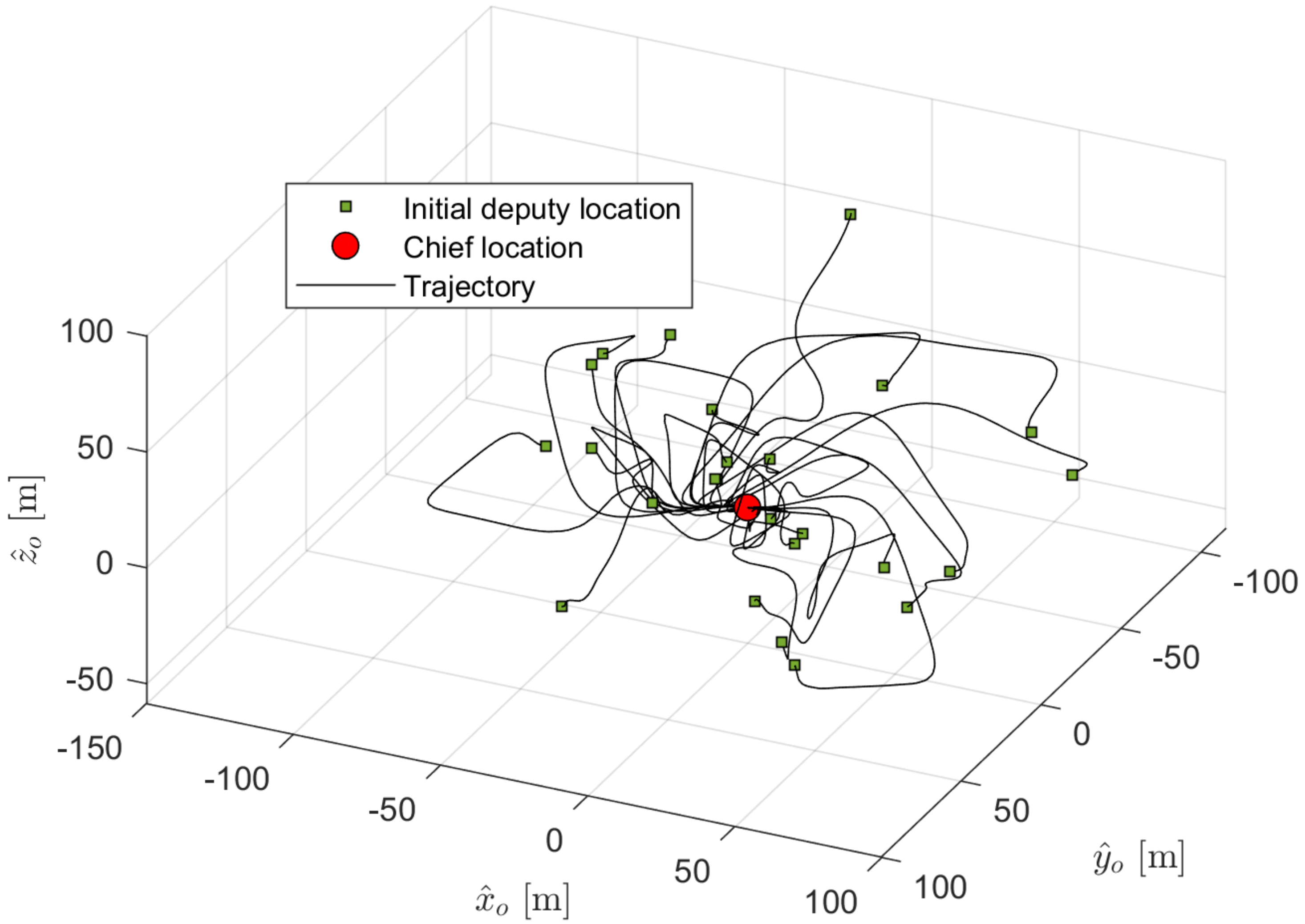






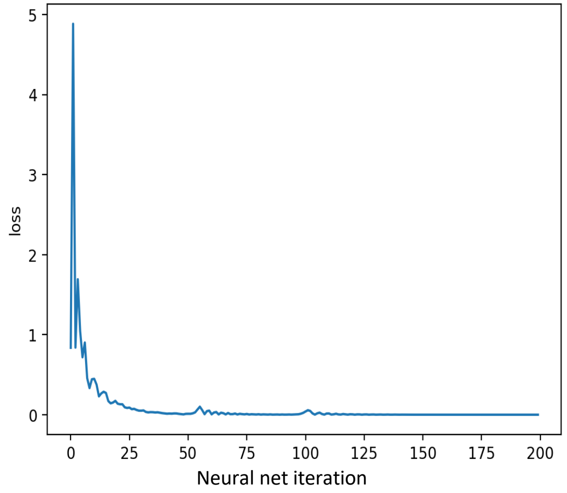


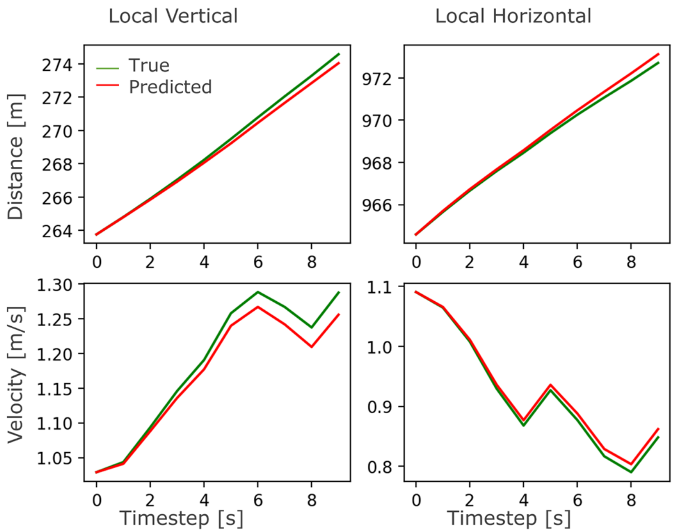
- 64 Radau Points
- Polynomial factor of 2
- Convergence tol: 10^{-8}
- Oscillations are present in prior works as well
  - Due to underactuation
  - Leverages non-linearities in the system
  - It's timing the thrust to match the right attitude



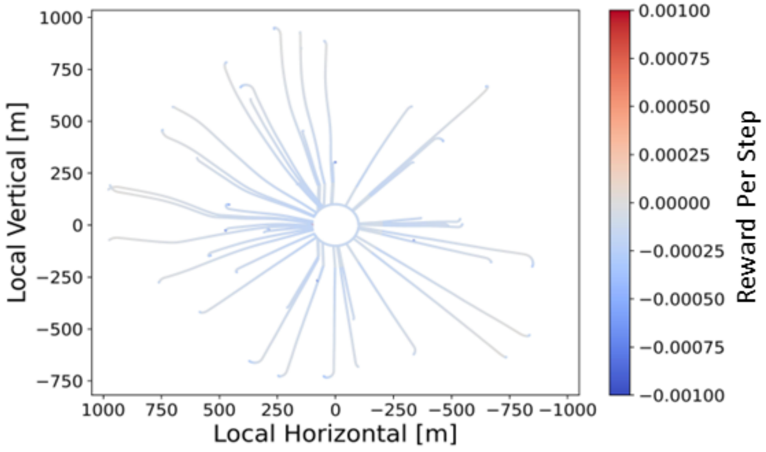
Initial deputy location
Chief location
Trajectory
100
50
0
-50
$\hat{z}_o$ [m]
-150
-100
-50
0
50
100
$\hat{x}_o$ [m]
-100
-50
0
50
100
$\hat{y}_o$ [m]

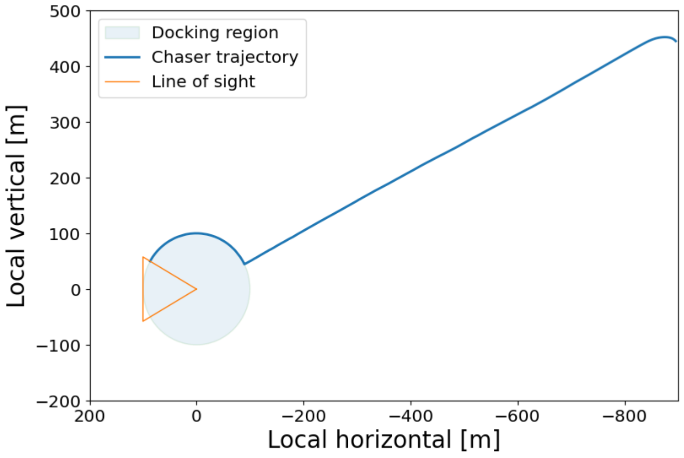
Future Work

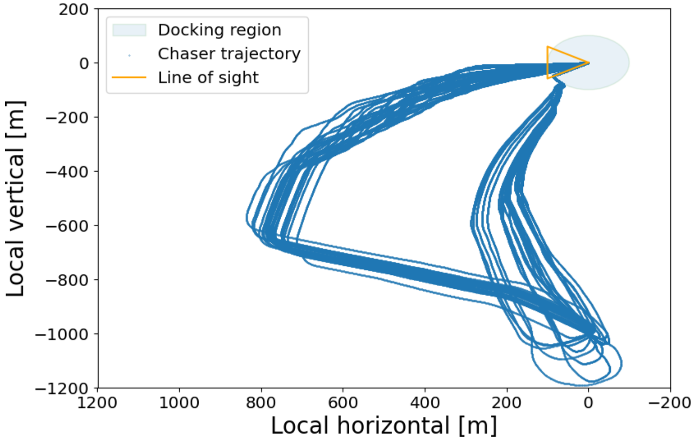




- Model Based Reinforcement Learning
  - Dynamics discovery
  - Real time planning\replanning
  - Zeroth order MPC can handle non-differentiable objectives and complex constraints







Questions