Synchronizing Tasks in Multi-Agent Systems with Max-Plus Algebra

Assured Autonomy in Contested Environments (AACE)
Program Review (Fall 2023)

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Objectives

“Synchronizing Tasks in Multi-Agent Systems with Max-Plus Algebra

- Formulate a general **multi-agent planning problem**
- Introduce a novel **mathematical theory** for solving the problem.
- **Survey** our current efforts to **extend** our **decentralized** methods to more general linear temporal logic (LTL) formal verification & synthesis problems.

Recent Publications

H. Riess, G. Henselman-Petrusek, M. Munger, R. Ghrist, Z. Bell, M. Zavlanos, (2023) “Network preference dynamics using lattice theory” (submitted).
H. Riess, M. Munger, & M. Zavlanos (2023). Max-Plus synchronization in decentralized trading systems. *2023 IEEE Conference on Decision and Control.*
H. Riess & R. Ghrist (2022). Diffusion of information on networked lattices by gossip. *2022 IEEE Conference on Decision and Control.*
Ghrist, R., & Riess, H. (2022). Cellular sheaves of lattices and the Tarski Laplacian. *Homology, Homotopy and Applications, 24*(1), 325-345.

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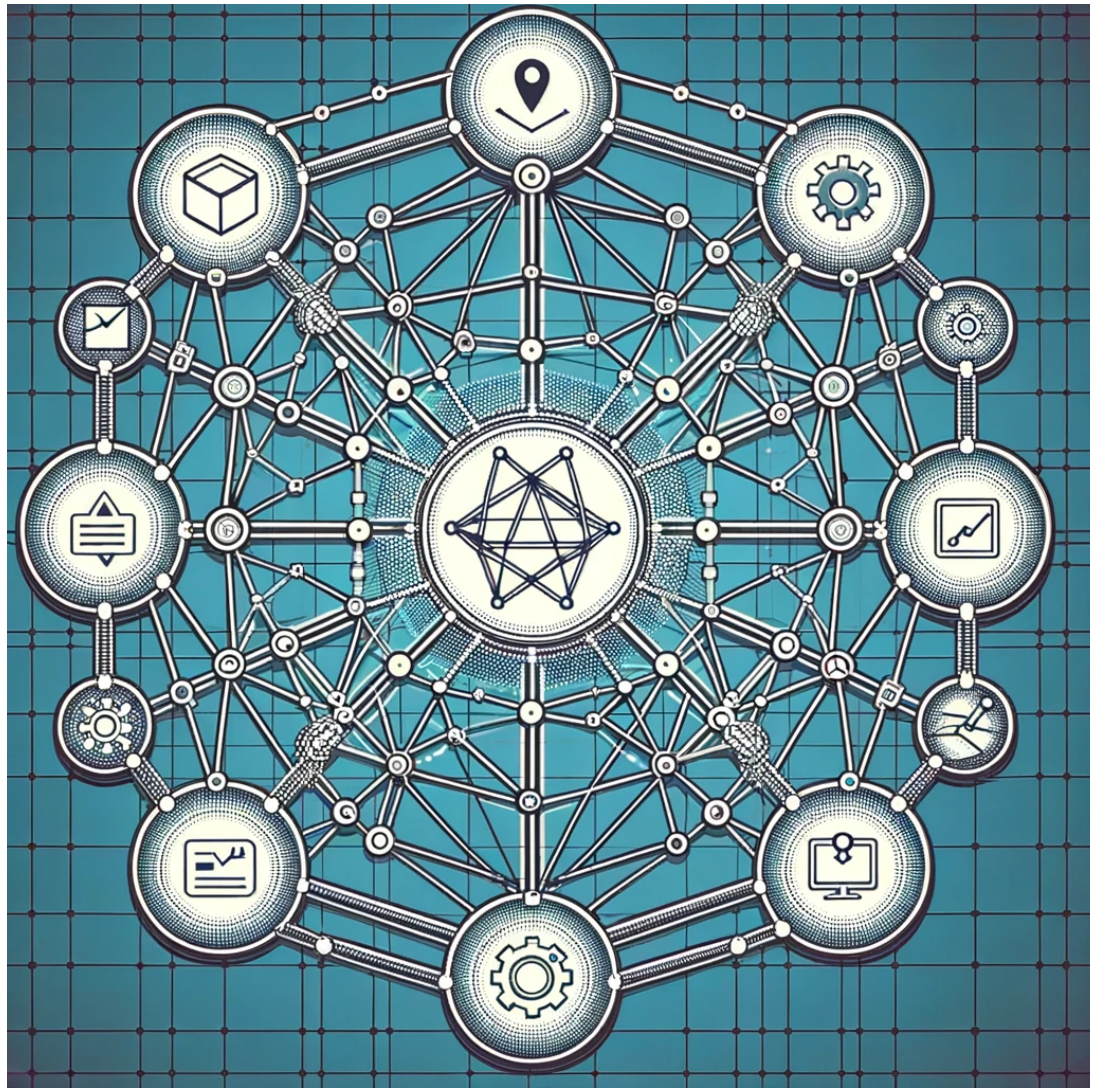
Motivation

Decentralized Autonomy & Coordination

- Heterogeneous autonomous agents
- Decentralized synthesis of task schedules
- E.g. intermittent connectivity control (Kantaros-Zavlanos 2016)
- Complex interactions between agents
- Synchrony v.s. Concurrency



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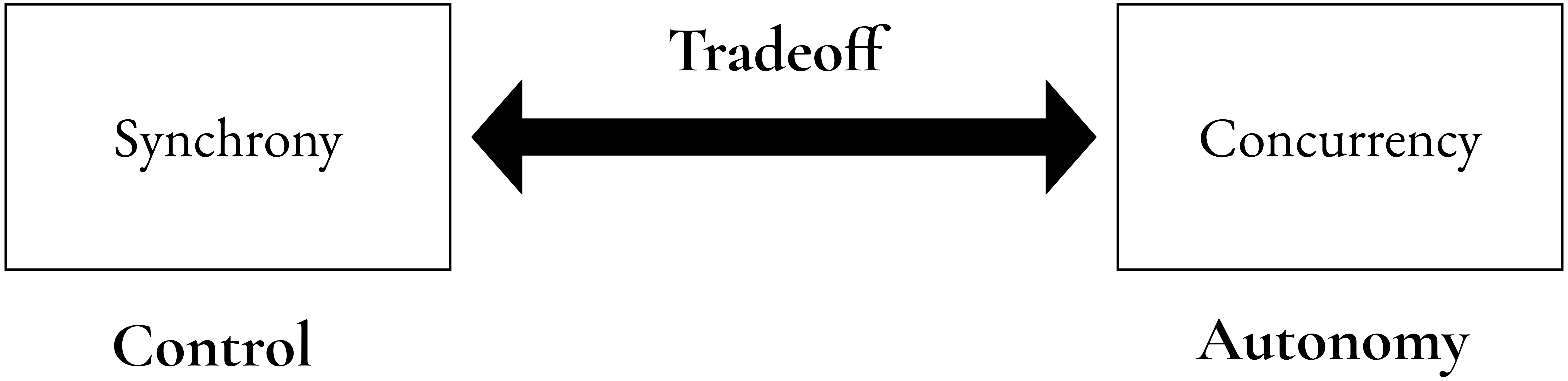
Multi-Agent Coordination

Motivation



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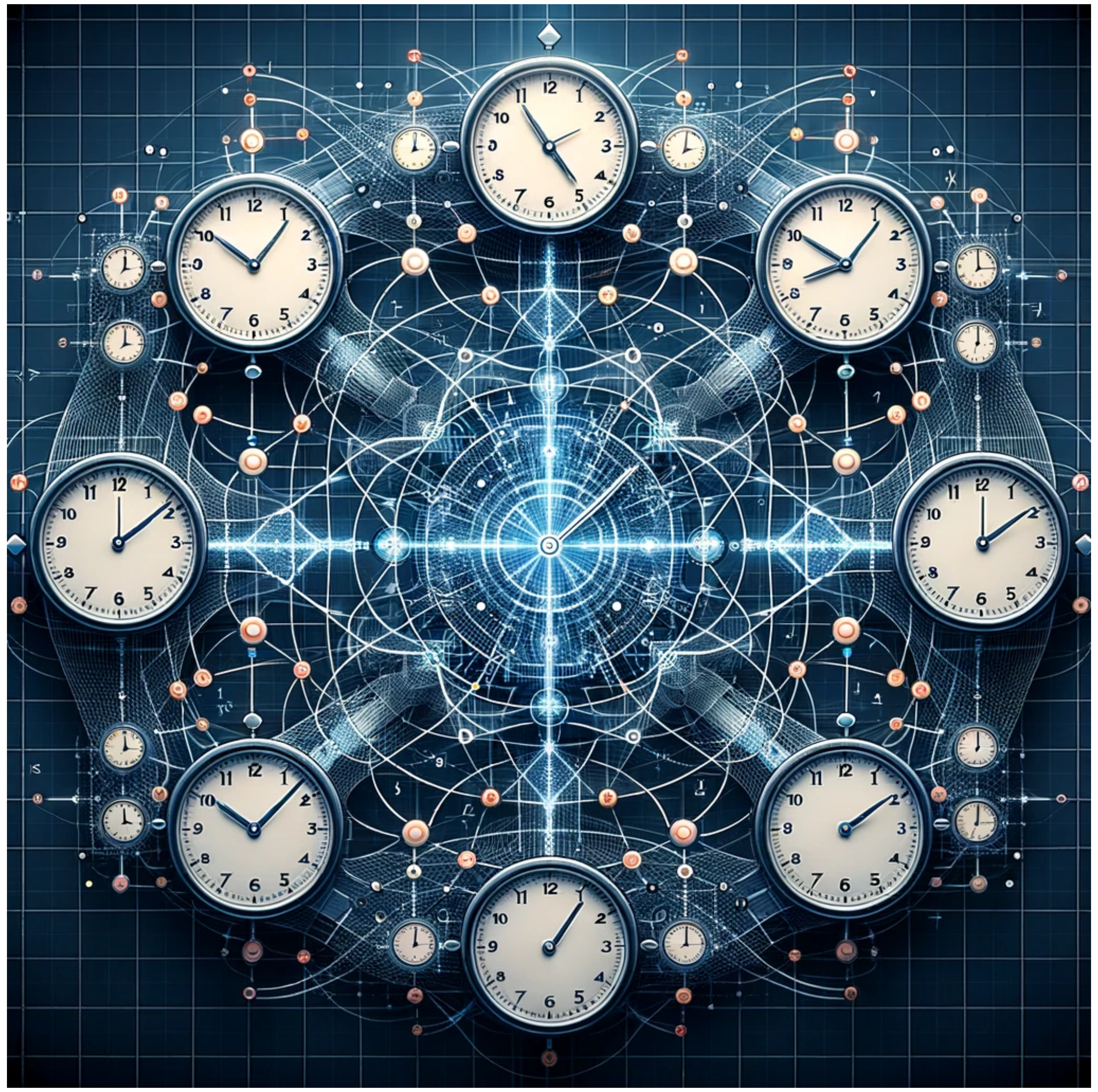
Synchronization

Motivation



- Agreement between agents in the discrete time domain
- Synchrony: groups of agents executing events at (approximately) the same time

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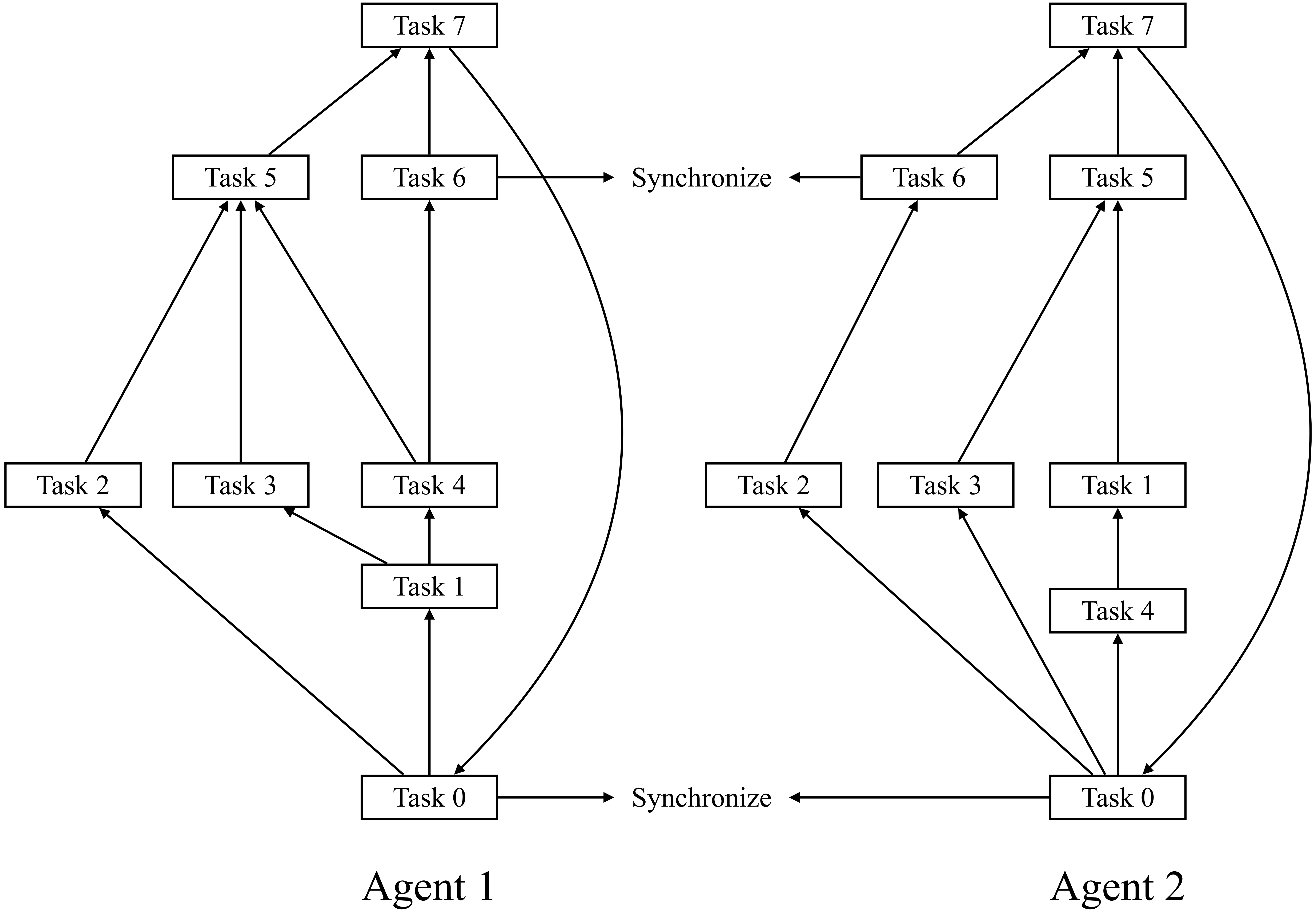
Multi-Agent Task Planning



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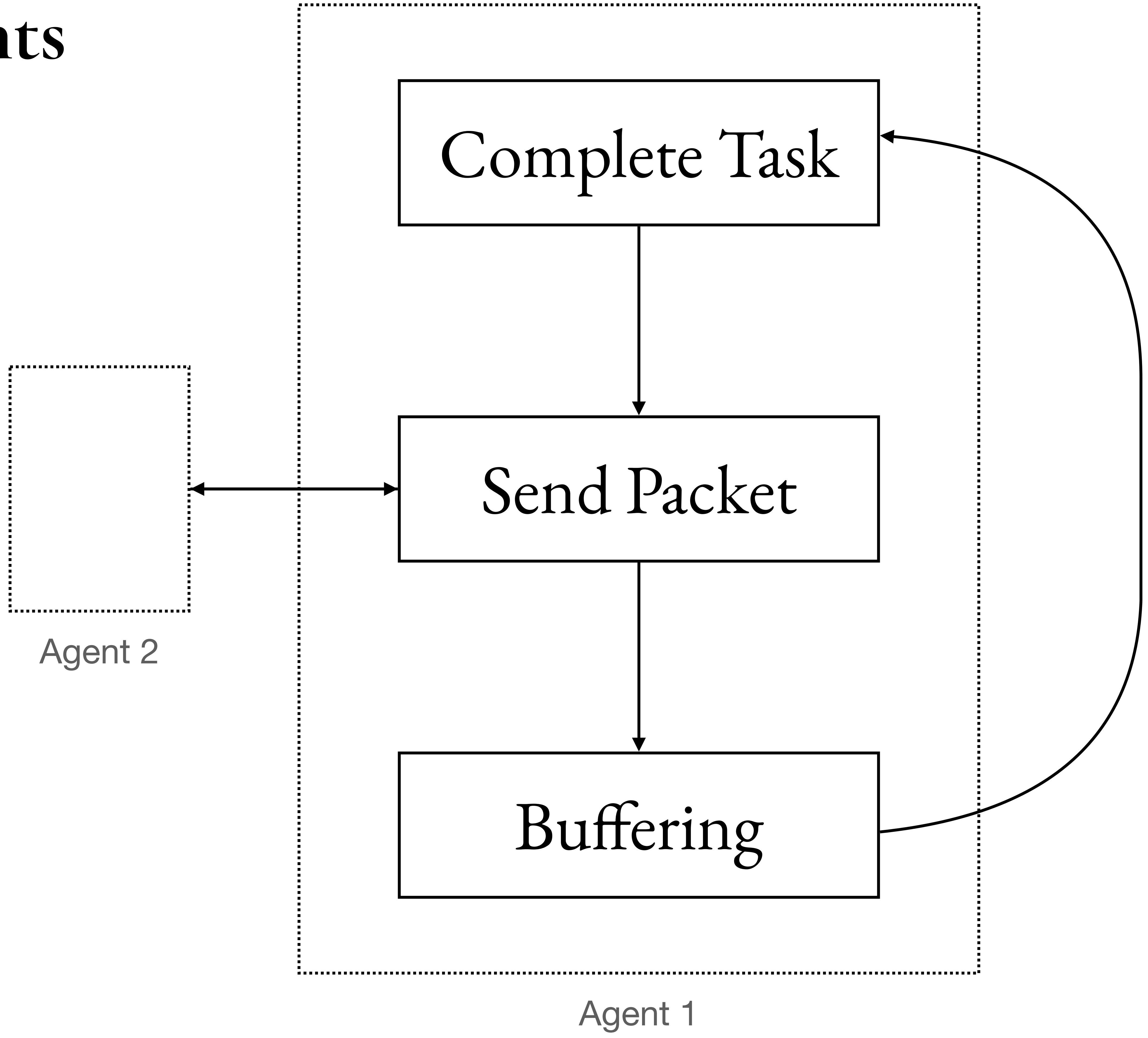
Multi-Agent Task Planning

Task specifications with temporal constraints

- Autonomous agents: $u, v \in \mathscr{A} = \{1,2,\ldots,N\}$
- Events: $i \in \{1,2,\ldots,M\}$ (e.g. actions, tasks)
- Events assigned to agents: $\mathscr{T} \subseteq \mathscr{N} \times \mathscr{A}$
- Transitions times between events: $\tau_{i,j}^{u} \geqslant 0$.
- Temporal constraints on the order of events
- Temporal constraints on coupled events
- Communication links between agents



Problem: Compute a global schedule satisfying constraints.

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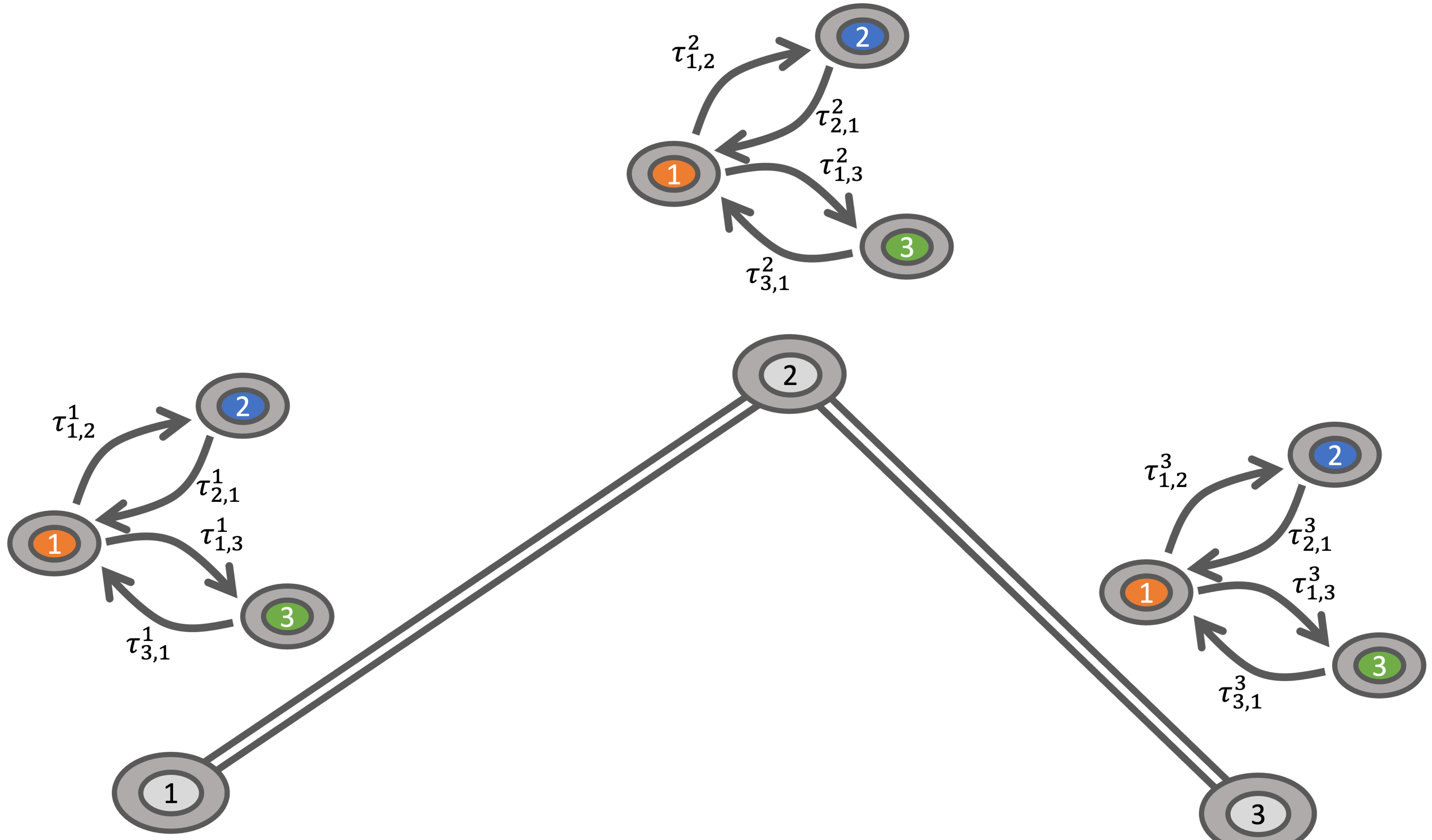
Multi-Agent Task Planning

Task specifications with temporal constraints

Communication between agents: **undirected** graph, $\mathcal{G} = (\mathcal{N}, \mathcal{E})$



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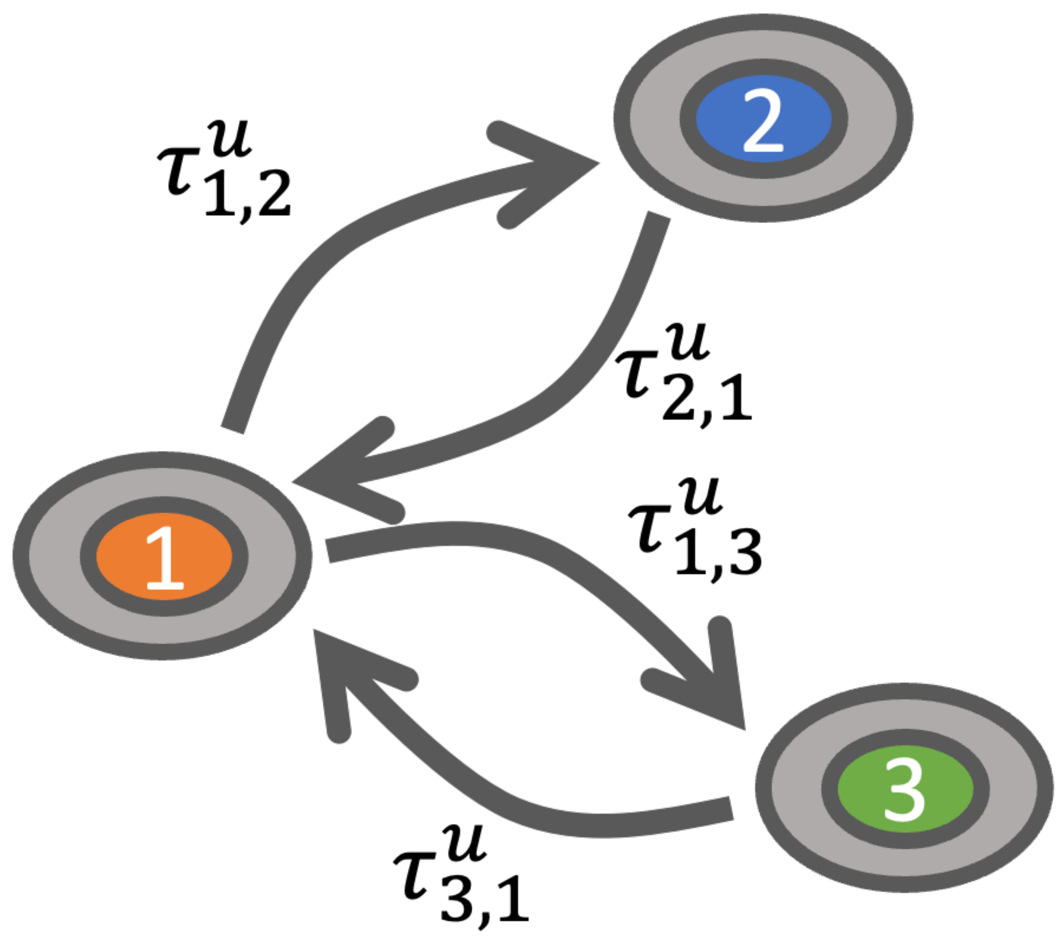
Multi-Agent Task Planning

Task specifications with temporal constraints

Transitions between tasks & durations: weighted **directed** graph



Task 1 can be performed only after Task 2 and Task 3 is performed;
Task 2 or Task 3 can only be performed after Task 1 is performed.

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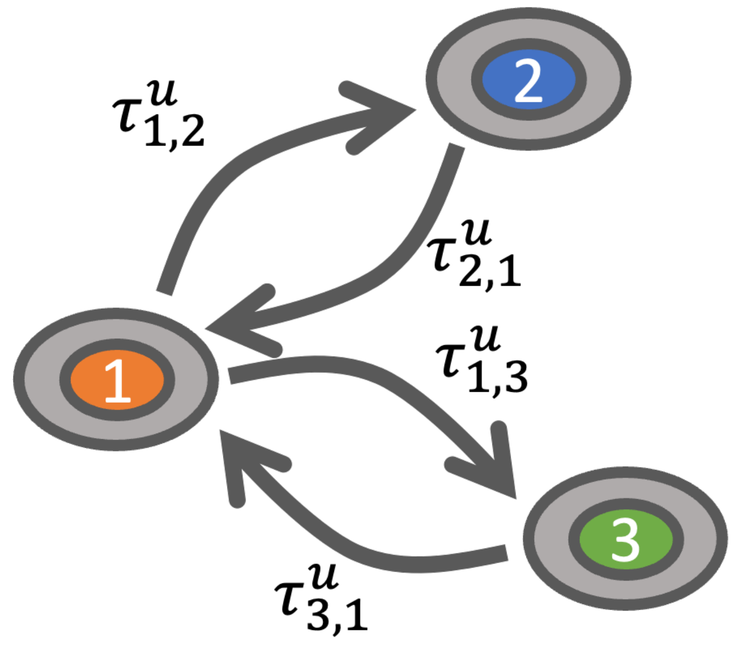
Multi-Agent Task Planning

Task specifications with temporal constraints

Event graphs represented by matrices

$$\mathbf{A} = \begin{bmatrix} -\infty & \tau^u_{2,1} & \tau^u_{3,1} \\ \tau^u_{1,2} & -\infty & -\infty \\ \tau^u_{1,3} & -\infty & -\infty \end{bmatrix} \quad \longleftrightarrow \quad \Gamma(\mathbf{A})$$



If $\mathbf{x}_u \in \mathbb{R}^3_+$ is a vector encoding *starting times* of each task for Agent u, then

$$\begin{bmatrix} -\infty & \tau^u_{2,1} & \tau^u_{3,1} \\ \tau^u_{1,2} & -\infty & -\infty \\ \tau^u_{1,3} & -\infty & -\infty \end{bmatrix} \boxplus \begin{bmatrix} x^u_1 \\ x^u_2 \\ x^u_3 \end{bmatrix} = \begin{bmatrix} \max\{x^u_1 - \infty,\, x^u_2 + \tau^u_{2,1},\, x^u_3 + \tau^u_{3,1}\} \\ \max\{x^u_1 + \tau^u_{1,2},\, x^u_2 - \infty,\, x^u_3 - \infty\} \\ \max\{x^u_1 + \tau^u_{1,3},\, x^u_2 - \infty,\, x^u_3 - \infty\} \end{bmatrix} = \begin{bmatrix} \max\{x^u_2 + \tau^u_{2,1},\, x^u_3 + \tau^u_{3,1}\} \\ x^u_1 + \tau^u_{1,2} \\ x^u_1 + \tau^u_{1,3} \end{bmatrix}$$

is the time-vector each task is started next. Naturally leads to **max-plus algebra** approach to synchronization...

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Why Max-Plus Algebra?

Max-plus linear systems theory

- Discrete event systems (DESs): $\mathbf{x}(k+1) = \mathbf{A} \boxplus \mathbf{x}(k) = \mathbf{A}^{\boxplus k} \boxplus \mathbf{x}(0)$

- Concepts, properties, techniques from conventional linear system theory translate to max-plus systems theory

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}(k) \boxplus \mathbf{x}(k) \vee \mathbf{B}(k) \boxplus \mathbf{u}(k) \\ \mathbf{y}(k+1) &= \mathbf{C}(k) \boxplus \mathbf{x}(k+1) \end{aligned}$$

The "linear algebra of combinatorics" (Butkovič, 2003)

- $\mathbf{A} \boxplus \mathbf{x} = \mathbf{b}$, (minmal) set covering

- $\mathbf{x} = \mathbf{A} \boxplus \mathbf{x} \vee \mathbf{b}$, shortest path (Bellman-Ford)

- $\mathbf{A} \boxplus \mathbf{x} = \lambda + \mathbf{x}$, maximum cycle mean

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Max-Plus Algebra
Replace "plus" with "max" and "times" with "plus"

- Linear algebra over the max-plus semiring (also called a diod)

$$\mathbb{R}_{\max} = \left(\mathbb{R} \cup \{-\infty\}, \max, -\infty, +, 0\right)$$

- Max-plus matrix-vector multiplication generalizes linear transformations

$$[\mathbf{A} \boxplus \mathbf{x}]_i = \max_{j=1}^{n} [\mathbf{A}]_{i,j} + [\mathbf{x}]_j, \quad i = 1, 2, \ldots, m$$

  defining a max-plus linear transformation $A : \mathbb{R}^n_{\max} \to \mathbb{R}^m_{\max}$

- Linear with respect to **point-wise max** ($\vee$) & scalar addition:

$$\mathbf{A} \boxplus (\mathbf{x}_1 \vee \mathbf{x}_2) = \mathbf{A} \boxplus \mathbf{x}_1 \vee \mathbf{A} \boxplus \mathbf{x}_2$$
$$\mathbf{A} \boxplus (\mathbf{x} + h) = \mathbf{A} \boxplus \mathbf{x} + h$$

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Min-Plus Algebra

Replace “plus” with “min” and “times” with “plus”

- Linear algebra over the min-plus semiring (also called a diod)

$$\mathbb{R}_{\min} = \left(\mathbb{R} \cup \{\infty\}, \min, \infty, +, 0\right)$$

- Min-plus matrix-vector multiplication generalizes linear transformations

$$[\mathbf{B} \boxplus \mathbf{y}]_j = \min_{i=1}^{m} [\mathbf{B}]_{j,i} + [\mathbf{y}]_i, \quad j = 1,2,\ldots,n$$

defining a max-plus linear transformation $B : \mathbb{R}^m_{\min} \to \mathbb{R}^n_{\min}$

- Linear with respect to **point-wise min** ($\wedge$) & scalar addition:

$$\begin{aligned} \mathbf{B} \boxplus' (\mathbf{y}_1 \wedge \mathbf{y}_2) &= \mathbf{B} \boxplus' \mathbf{y}_1 \wedge \mathbf{B} \boxplus' \mathbf{y}_2 \\ \mathbf{B} \boxplus' (\mathbf{y} + h) &= \mathbf{B} \boxplus' \mathbf{y} + h \end{aligned}$$

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Residuation Theory

Pseudo-inverses of max-plus matrices are min-plus matrices

- Let $\bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$. Then, $\bar{\mathbb{R}}^n$ is an ordered **<u>lattice</u>** with $\mathbf{x} \vee \mathbf{y}$ (join) and $\mathbf{x} \vee \mathbf{y}$ (meet).
- Suppose $A : \mathbb{R}^n_{\max} \to \mathbb{R}^m_{\max}$ is a max-plus linear transformation. Then, there is a min-plus linear transformation $A^\sharp : \mathbb{R}^m_{\min} \to \mathbb{R}^n_{\min}$ given by

$$A^\sharp(\mathbf{y})_j = \max_{i=1}^{m} [\mathbf{y}]_i - [\mathbf{A}]_{j,i}, \quad j = 1,2,\ldots,n$$

such that $\mathbf{A}^\sharp \boxplus' (\mathbf{A} \boxplus \mathbf{x}) \geqslant \mathbf{x}$ and $\mathbf{A} \boxplus \left(\mathbf{A}^\sharp \boxplus' \mathbf{y}\right) \leqslant \mathbf{y}$, or, equivalently,

$$\mathbf{A} \boxplus \mathbf{x} \leqslant \mathbf{y} \quad \text{if and only if} \quad \mathbf{x} \leqslant \mathbf{A}^\sharp \boxplus' \mathbf{y}$$

- $A^\sharp(\mathbf{y}) = \mathbf{A}^\sharp \boxplus' \mathbf{y}$ where $\mathbf{A}^\sharp \in \mathbb{R}^{n\times n}_{\min}$ is a matrix defined $[\mathbf{A}^\sharp]_{i,j} = -[\mathbf{A}]_{j,i}$.
- The vector $\bar{\mathbf{x}} = \mathbf{A}^\sharp \boxplus' \mathbf{b}$ is the greatest solution to the equation $\mathbf{A} \boxplus \mathbf{x} = \mathbf{b}$.

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Multi-Agent Task Synchronization

Problem Formulation

Compute initial starting-times for each agent and for each task $[\mathbf{x}_u]_i$ *such that agents assigned to the same task start the task at approximately the same time after k occurrences.*

- $\mathcal{N}$, agents
- $\mathcal{A}$, events
- $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, (undirected) communication graph
- $\mathbf{A}_u \in \mathbb{R}_{\max}^{M \times M}$, max-plus matrices representing local temporal constraints; defined $[\mathbf{A}_u]_{i,j} = \tau_{j,i}^u$
- $\mathcal{T} \subseteq \mathcal{N} \times \mathcal{A}$, relation assigning agents to events
- $\mathcal{T}_{(u,v)} = \{i \in \mathcal{A} : (u, i), (v, i) \in \mathcal{T}\}$, tasks assigned to both $(u, v) \in \mathcal{E}$
- $M_{(u,v)} = |\mathcal{T}_{(u,v)}|$, number of tasks assigned to both $(u, v) \in \mathcal{E}$
- $\mathbf{x}_u \in \mathbb{R}_+^M$, vector of start times for each task for Agent u
- $\epsilon > 0$, synchronization threshold

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Multi-Agent Task Synchronization

Equilibrium Condition (Mathematical Formulation of the Problem)

Given $\{\mathbf{A}_u \in \mathbb{R}_{\max}^{M\times M}\}_{u\in\mathcal{N}}$, *find a vector* $\mathbf{x}(0) \in \left(\mathbb{R}_+^M\right)^N$ *such that*

$$\|\mathbf{P}_{(u,v)} \boxplus \mathbf{A}_u^k \boxplus \mathbf{x}_u - \mathbf{P}_{(u,v)} \boxplus \mathbf{A}_v^k \boxplus \mathbf{x}_u\|_\infty < \epsilon \quad \forall (u,v) \in \mathcal{E}$$

where $\mathbf{P}_{(u,v)} \in \mathbb{R}_{\max}^{M_{(u,v)} \times M}$ *is the projection onto* $\mathcal{T}_{(u,v)} \subseteq \mathcal{A}$ *for* $(u,v) \in \mathcal{E}$.

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Multi-Agent Task Synchronization

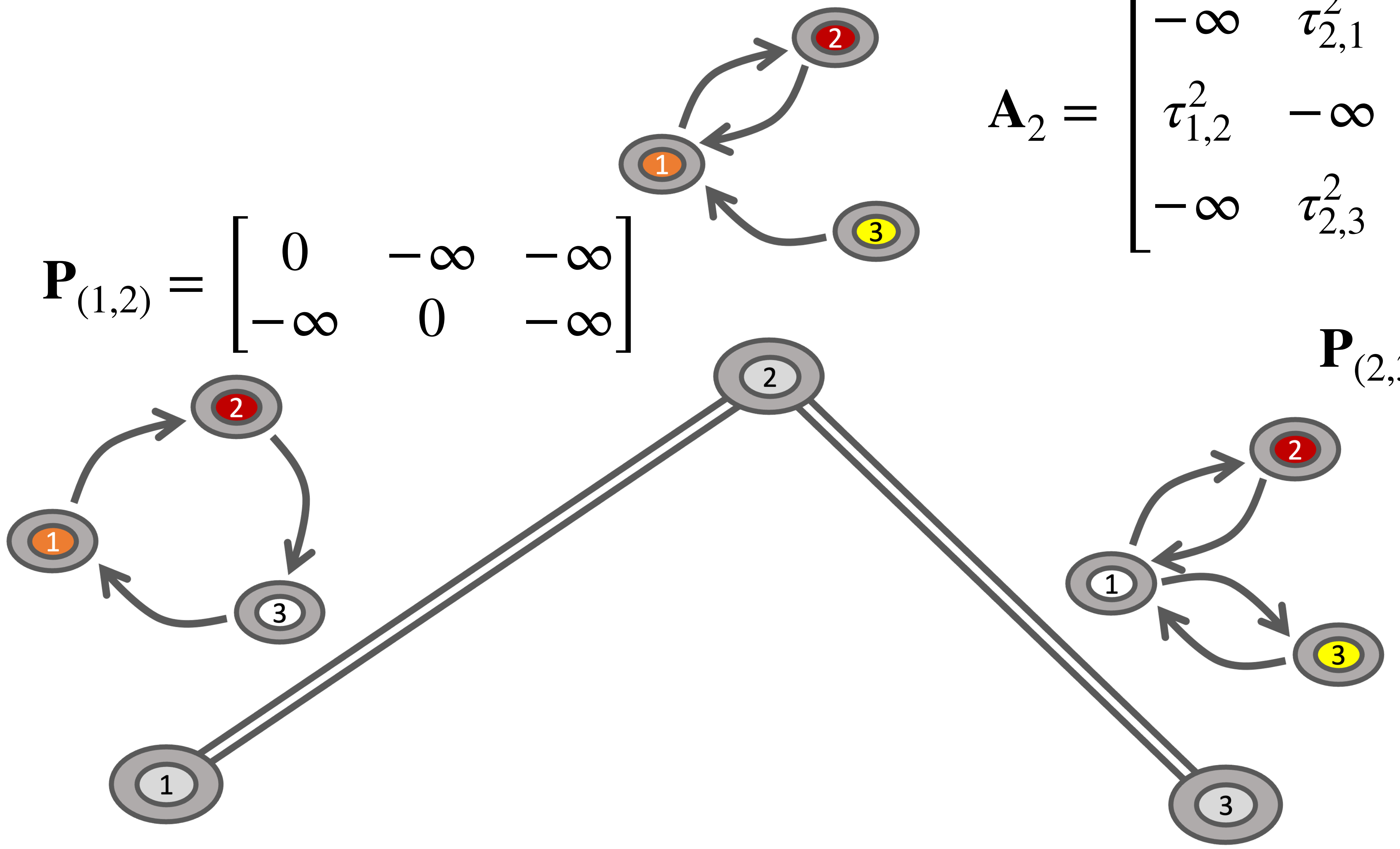
Example

$$\mathbf{A}_2 = \begin{bmatrix} -\infty & \tau^2_{2,1} & \tau^2_{3,1} \\ \tau^2_{1,2} & -\infty & -\infty \\ -\infty & \tau^2_{2,3} & -\infty \end{bmatrix}$$

$$\mathbf{P}_{(1,2)} = \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\infty \end{bmatrix}$$

$$\mathbf{P}_{(2,3)} = \begin{bmatrix} -\infty & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} -\infty & -\infty & \tau^1_{3,1} \\ \tau^1_{1,2} & -\infty & -\infty \\ -\infty & \tau^1_{2,3} & -\infty \end{bmatrix}$$



$$\mathbf{A}_3 = \begin{bmatrix} -\infty & \tau^3_{2,1} & \tau^3_{3,1} \\ \tau^3_{1,2} & -\infty & -\infty \\ \tau^3_{1,3} & -\infty & -\infty \end{bmatrix}$$

The Task 1 executed simultaneously by Agent 1, Agent 2, & Agent 3;
Task 2 by Agent 1 & Agent 2;
Task 3 by Agent 2 & Agent 3.

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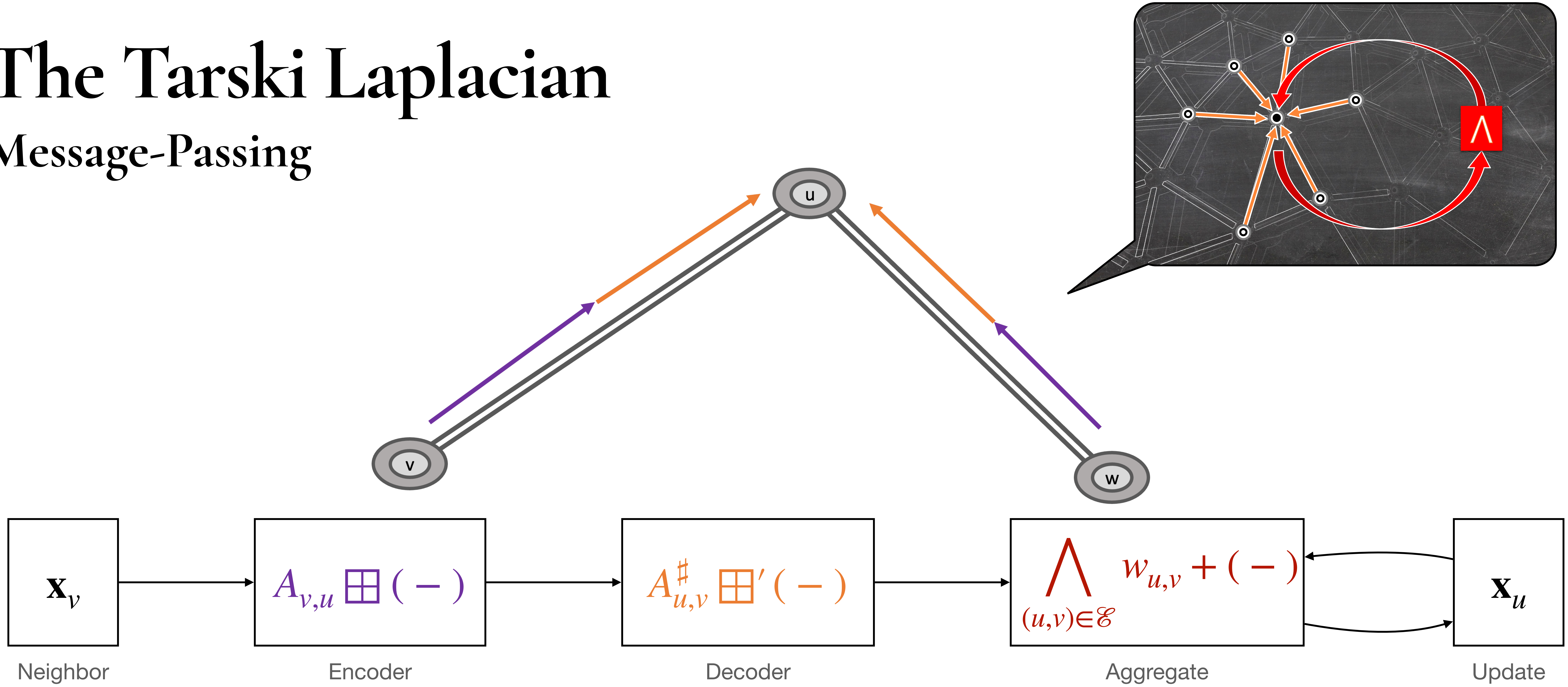
The Tarski Laplacian

Message-Passing



Graph Laplacian (ubiquitous), $[\mathbf{Lx}]_u = \frac{1}{\deg(u)} \sum_{(u,v)\in\mathscr{E}} w_{u,v}(x_u - x_v)$

Graph connection Laplacian (Signer-Wu, 2012), $(\mathscr{L}\mathbf{x})_u = \mathbf{x}_u - \frac{1}{\deg(u)} \sum_{(u,v)\in\mathscr{E}} w_{u,v}\mathbf{O}_{v,u}\mathbf{x}_v$

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The Tarski Laplacian

Definition (Riess-Zavlanos, 2023)
The Tarski Laplacian is a map $\mathscr{L} : (\mathbb{R}^M_{\max})^N \to (\mathbb{R}^M_{\max})^N$ *defined*

$$(\mathscr{L}\mathbf{x})_u = \underbrace{\bigwedge_{v \in \mathcal{N}_u} \underbrace{W(u,v)}_{\text{delay}} + \overbrace{\mathbf{A}_u^\sharp \boxplus' \Big(\underbrace{\mathbf{P}^\sharp_{(u,v)} \boxplus' \Big(\mathbf{P}_{(u,v)}}_{\text{projection}} \boxplus \underbrace{\mathbf{A}_v \boxplus \mathbf{x}_v}_{\text{next time}} \Big) \Big)}^{\text{residuation}}}_{\text{aggregation}}$$

where $\mathbf{W} \in \mathbb{R}^{N \times N}_{\min}$ *and* $\mathbf{A}_{u,v} \in \mathbb{R}^{M_{(u,v)} \times M_u}_{\max}$

Notation: in the task assignment problem, define $\mathbf{A}_{u,v} := \mathbf{P}_{(u,v)} \boxplus \mathbf{A}_u$.

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The Tarski Laplacian

Definition (Riess-Zavlanos, 2023)

The Tarski Laplacian is a map $\mathscr{L} : \left(\mathbb{R}^M_{\max}\right)^N \to \left(\mathbb{R}^M_{\max}\right)^N$ *defined*

$$\mathscr{L}(\mathbf{x})_u = \bigwedge_{(u,v)\in\mathscr{E}} [\mathbf{W}]_{u,v} + \mathbf{A}^{\sharp}_{u,v} \boxplus' \left(\mathbf{A}_{v,u} \boxplus \mathbf{x}_v\right)$$

where $\mathbf{W} \in \mathbb{R}^{N\times N}_{\min}$ *and* $\mathbf{A}_{u,v} \in \mathbb{R}^{M_{(u,v)} \times M_u}_{\max}$.

Iterative map $\mathscr{F}(\mathbf{x}) = \mathscr{L}(\mathbf{x}) \wedge \mathbf{x}$ and dynamics $\mathbf{x}(t+1) = \mathscr{F}\left(\mathbf{x}\right)$

Theorem 1 (Riess-Zavlanos)

Suppose $\mathbf{x}, \mathbf{y} \in \mathrm{Fix}(\mathscr{F}) \subseteq \left(\mathbb{R}^M_{\max}\right)^N$. *Then,* $\mathbf{x} \vee \mathbf{y} \in \mathrm{Fix}(\mathscr{F})$ *and* $\mathbf{x} + h \in \mathrm{Fix}(\mathscr{F})$.

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The Tarski Laplacian

Definition (Riess-Zavlanos, 2022)
The Tarski Laplacian is a map $\mathscr{L} : \left(\mathbb{R}^{M}_{\max}\right)^{N} \to \left(\mathbb{R}^{M}_{\max}\right)^{N}$ *defined*

$$\mathscr{L}(\mathbf{x})_u = \bigwedge_{(u,v)\in\mathscr{E}} [\mathbf{W}]_{u,v} + \mathbf{A}^{\sharp}_{u,v} \boxplus' \left(\mathbf{A}_{v,u} \boxplus \mathbf{x}_v\right)$$

where $\mathbf{W} \in \mathbb{R}^{N\times N}_{\min}$ *and* $\mathbf{A}_{u,v} \in \mathbb{R}^{M_{(u,v)} \times M_u}_{\max}$.

Iterative map $\mathscr{F}(\mathbf{x}) = \mathscr{L}(\mathbf{x}) \wedge \mathbf{x}$ and dynamics $\mathbf{x}(t+1) = \mathscr{F}\left(\mathbf{x}(t)\right)$

Theorem 2 (Riess-Zavlanos)

$\|\mathbf{A}_{u,v} \boxplus \mathbf{x}_u - \mathbf{A}_{v,u} \boxplus \mathbf{x}_v\|_\infty \leq [\mathbf{W}]_{u,v}$ *for all* $(u, v) \in \mathscr{E}$ *if and only if* $\mathbf{x} \in \mathrm{Fix}(\mathscr{F})$. *Then,* $\mathbf{x} \in \mathrm{Fix}(\mathscr{F})$ *if and only if* $\mathbf{x}$ *satisfies the Equilibrium Condition.*

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Convergence Analysis

Two convergence regimes: $\alpha > 0$, $\alpha = 0$

- Compute fixed points via the **heat equation**

$$\mathbf{x}(k+1) = \mathscr{L}\left(\mathbf{x}(k)\right) \wedge \mathbf{x}(k)$$

with initial condition $\mathbf{x}(0) \in \left(\mathbb{R}^D\right)^N$

- Gradients vanish

$$\begin{aligned} \alpha(k) &= \|\mathbf{x}(k) - \mathbf{x}(k+1)\|_\infty \\ &= \|\mathbf{x}(k) - \mathbf{x}(k) \wedge \mathscr{L}\left(\mathbf{x}(k)\right)\|_\infty \end{aligned}$$

Theorem 3 (Riess-Zavlanos)

Let $\mathscr{F} = \mathscr{L} \wedge I$. The operator $\mathscr{F} : \left(\mathbb{R}^D\right)^N \to \left(\mathbb{R}^D\right)^N$ is non-expansive in the ℓ_∞-norm. Furthermore, $\alpha(k) \to \alpha$ as $k \to \infty$ for some $\alpha \geqslant 0$.

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Recap

Theoretical Results

Theorem 1

Suppose $\mathbf{x}, \mathbf{y} \in \mathrm{Fix}(\mathcal{F}) \subseteq \left(\mathbb{R}^M_{\max}\right)^N$. *Then,* $\mathbf{x} \vee \mathbf{y} \in \mathrm{Fix}(\mathcal{F})$ *and* $\mathbf{x} + h \in \mathrm{Fix}(\mathcal{F})$.

Theorem 2

$\|\mathbf{A}_{u,v} \boxplus \mathbf{x}_u - \mathbf{A}_{v,u} \boxplus \mathbf{x}_v\|_\infty \leq [\mathbf{W}]_{u,v}$ *for all* $(u, v) \in \mathcal{E}$ *if and only if* $\mathbf{x} \in \mathrm{Fix}(\mathcal{F})$.

Theorem 3

The operator $\mathcal{F} : \left(\mathbb{R}^D\right)^N \to \left(\mathbb{R}^D\right)^N$ *is non-expansive in the* ℓ_∞*-norm. Furthermore,* $\alpha(k) \to \alpha$ *as* $k \to \infty$ *for some* $\alpha \geqslant 0$.

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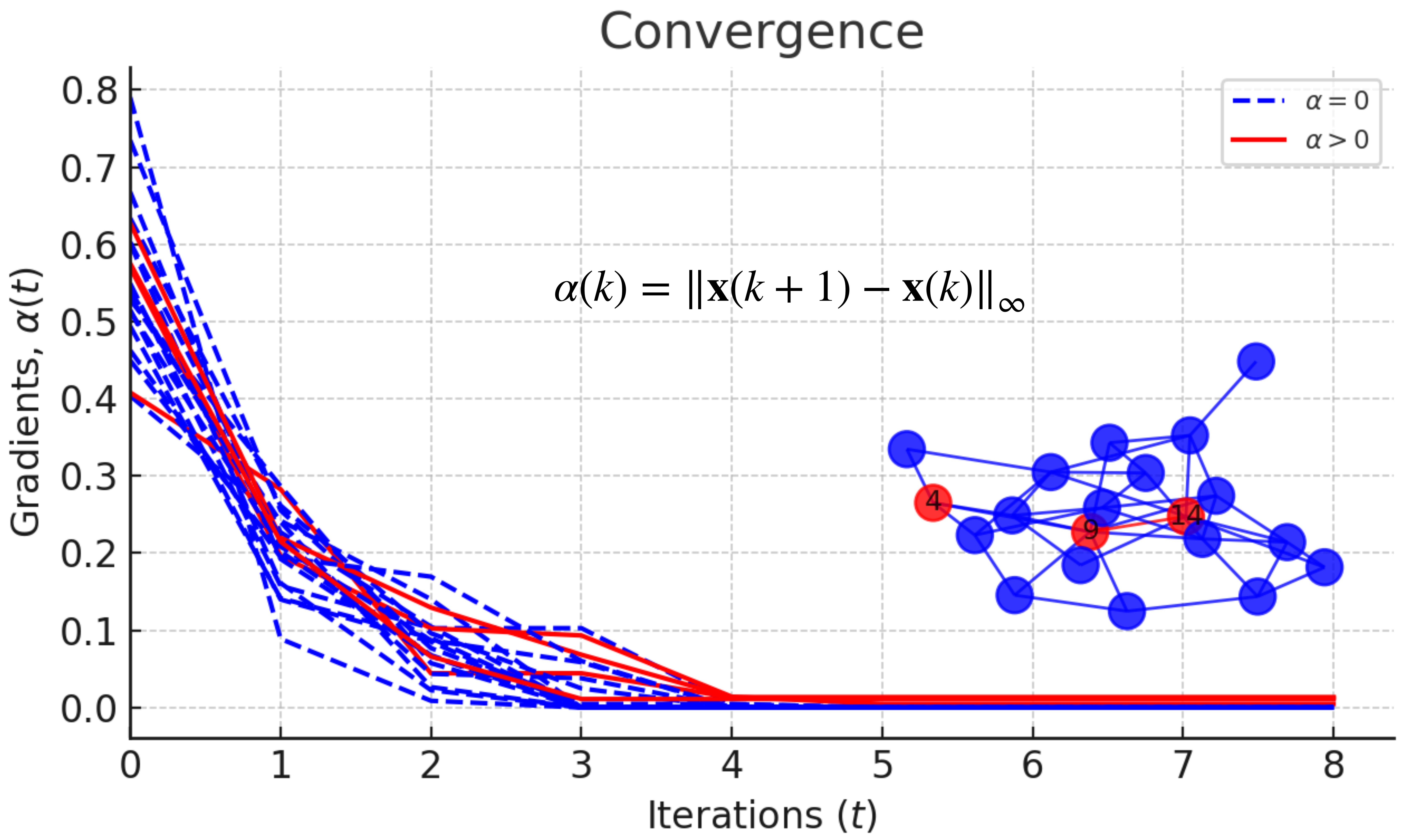
Convergence



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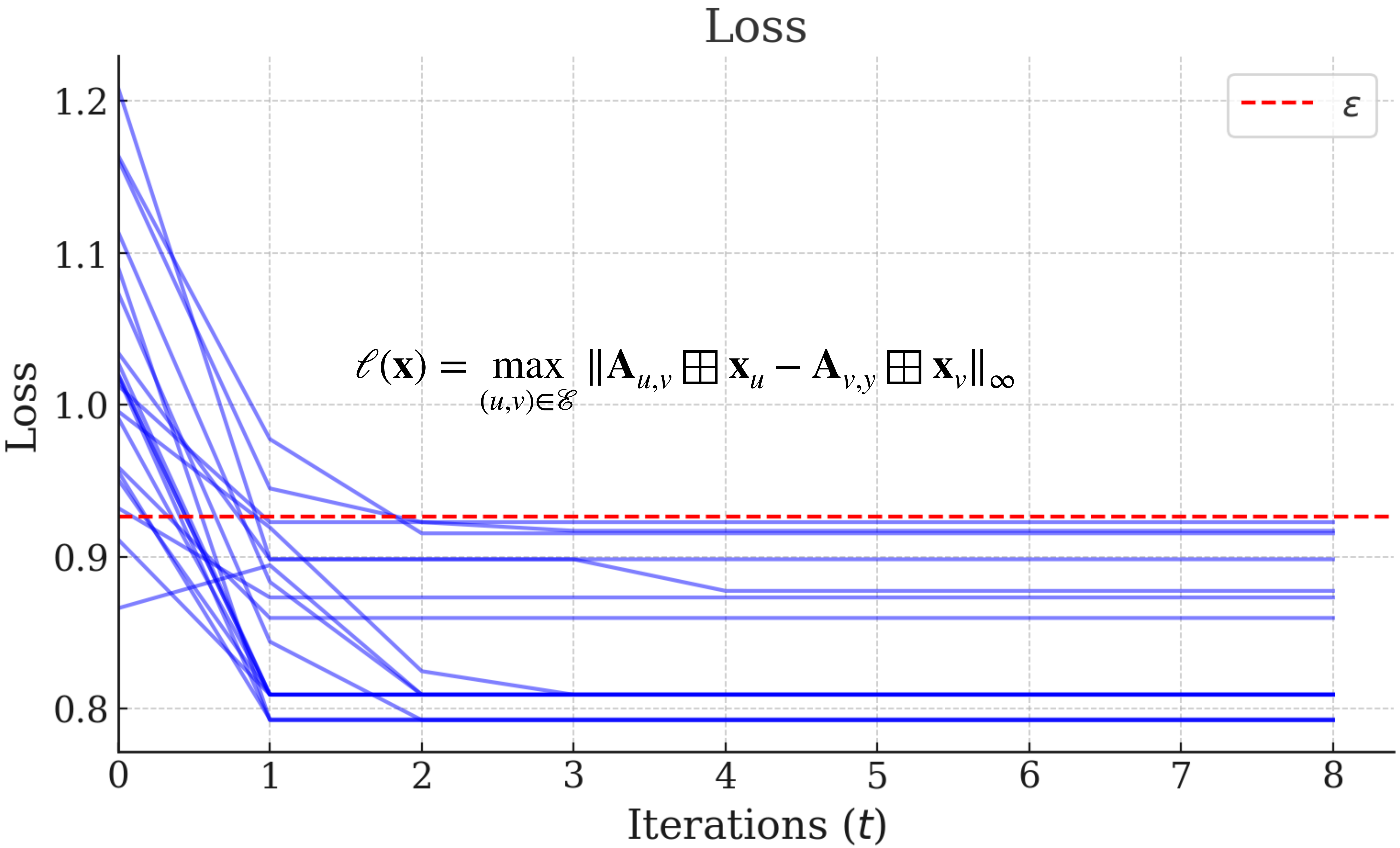
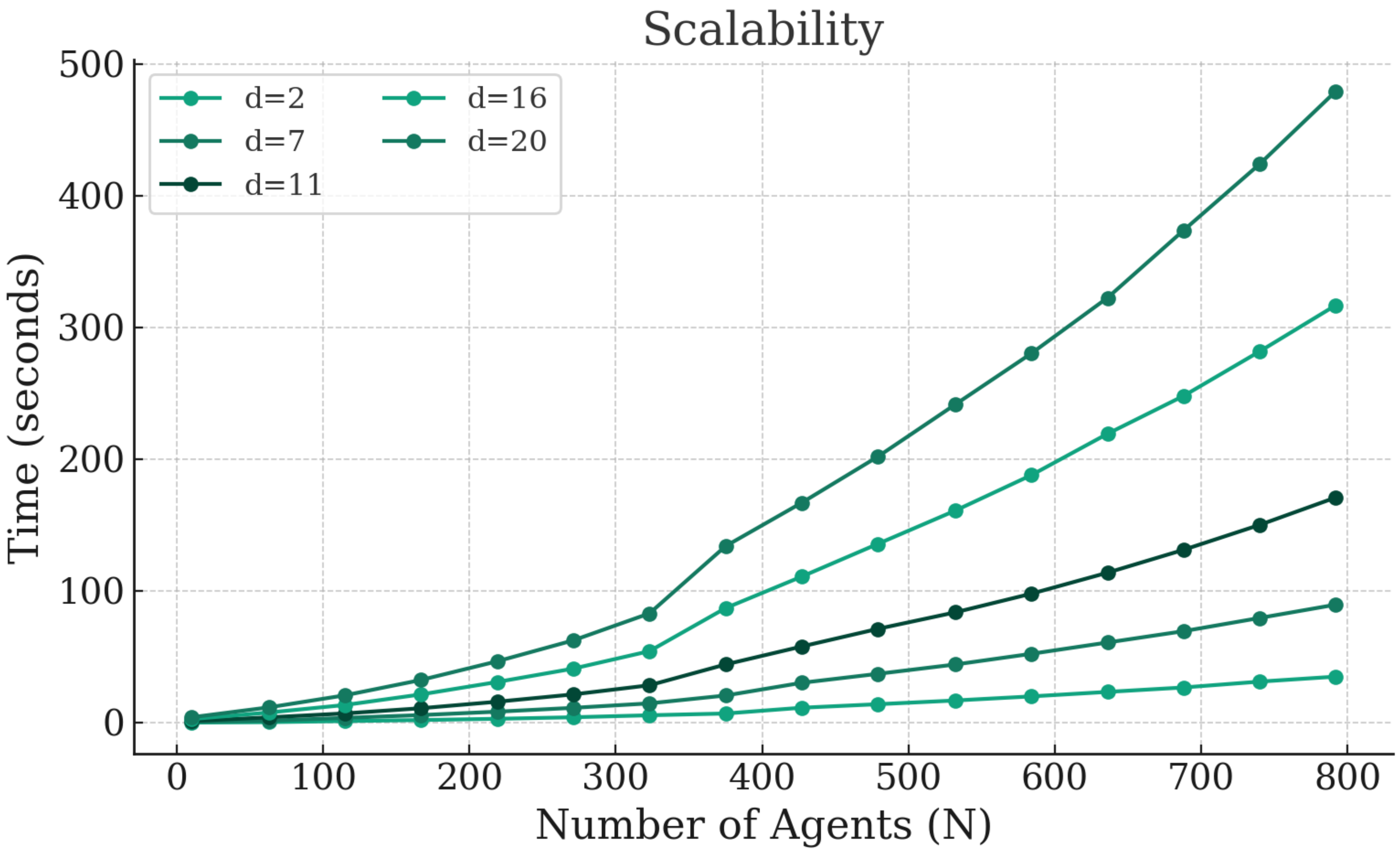
Convergence



Scalability



Runtime of the tropical Tarski Laplacian in the number of agents and the dimension (number of events).

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Towards Decentralized Formal Verification

Linear Temporal Logic (LTL)

LTL is a langue for model-checking for formal verification and control synthesis.

$$\pi \mid \varphi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \varphi_1 \cup \varphi_2 \mid \bigcirc\varphi \mid \square\varphi \mid \lozenge\varphi$$

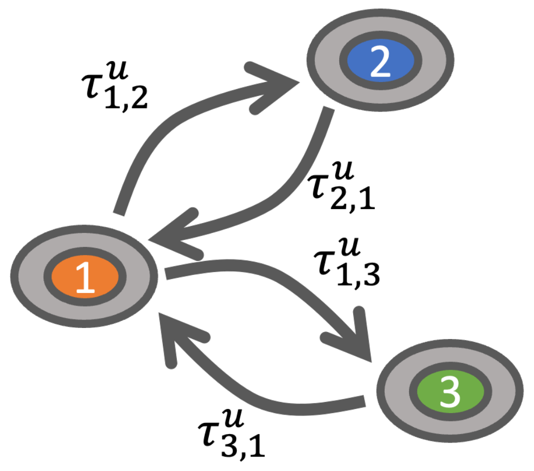
Our Framework

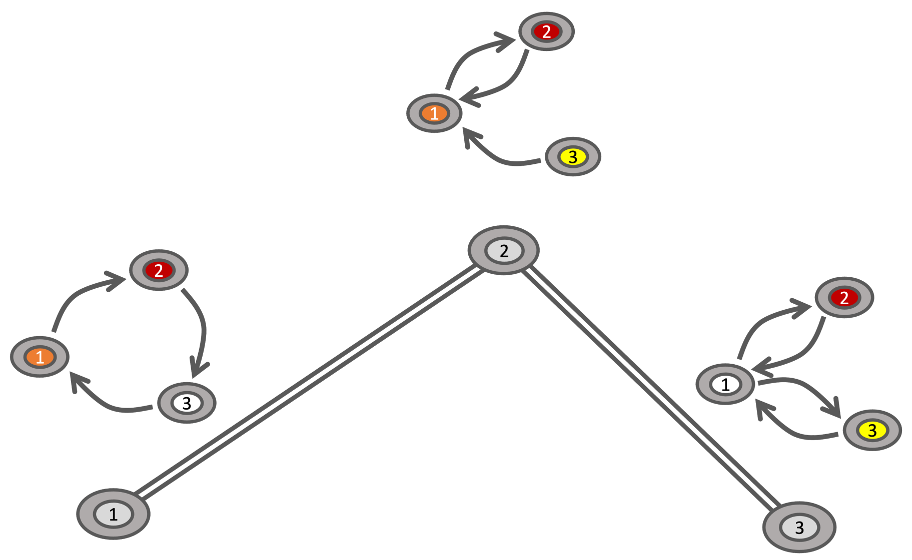
- Atomic propositions: $\Phi = \{\pi_i^v\}_{i\in\mathcal{A}, v\in\mathcal{N}}$
- Constraints on the order of tasks

$$\varphi_v = \bigwedge_{i\in\mathcal{A}} \neg\pi_v^i \cup \left(\bigwedge_{j\in\mathcal{P}_v(i)} \pi_v^j \right)$$

- Constraints on synchronization of tasks

$$\psi_i = \bigcirc^k \left(\bigwedge_{w\in\mathcal{A}_i} \pi_w^i \right)$$





Towards Decentralized Model Checking

What's Next

Robust & adaptive schedule synchrony

- Time-invariant temporal constraints
- Delays and feedback through control inputs
- Dynamic topology & multi-hop synchronization.

Asynchronous schedule updates

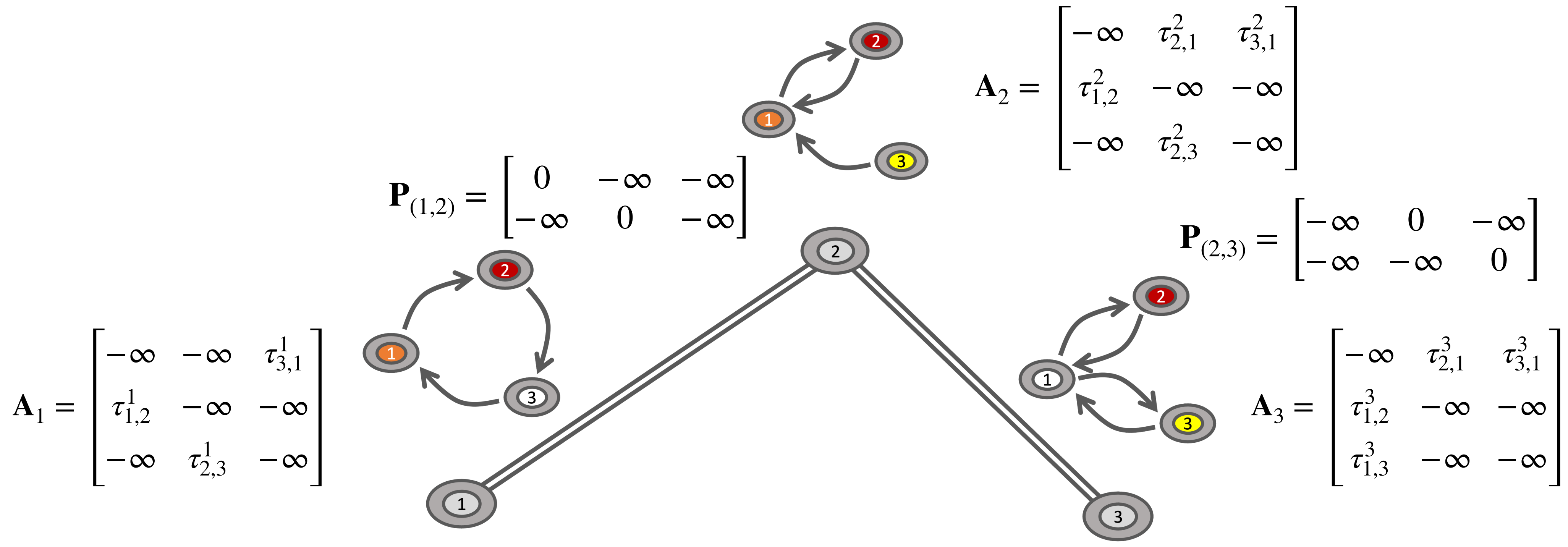
Expanding the language

- Expanding constraints on the order of events
- Expanding constraints on synchronization of events.

Higher order LTL: $\bigwedge_{i \in \mathscr{A}} \Diamond \left(\bigwedge_{v \in \mathscr{A}_i} \pi_v^i \right)$ v.s. $\bigcirc^k \left(\bigwedge_{w \in \mathscr{A}_i} \pi_w^i \right)$

Lattice Graph Diffusion ⇒ Model Checking

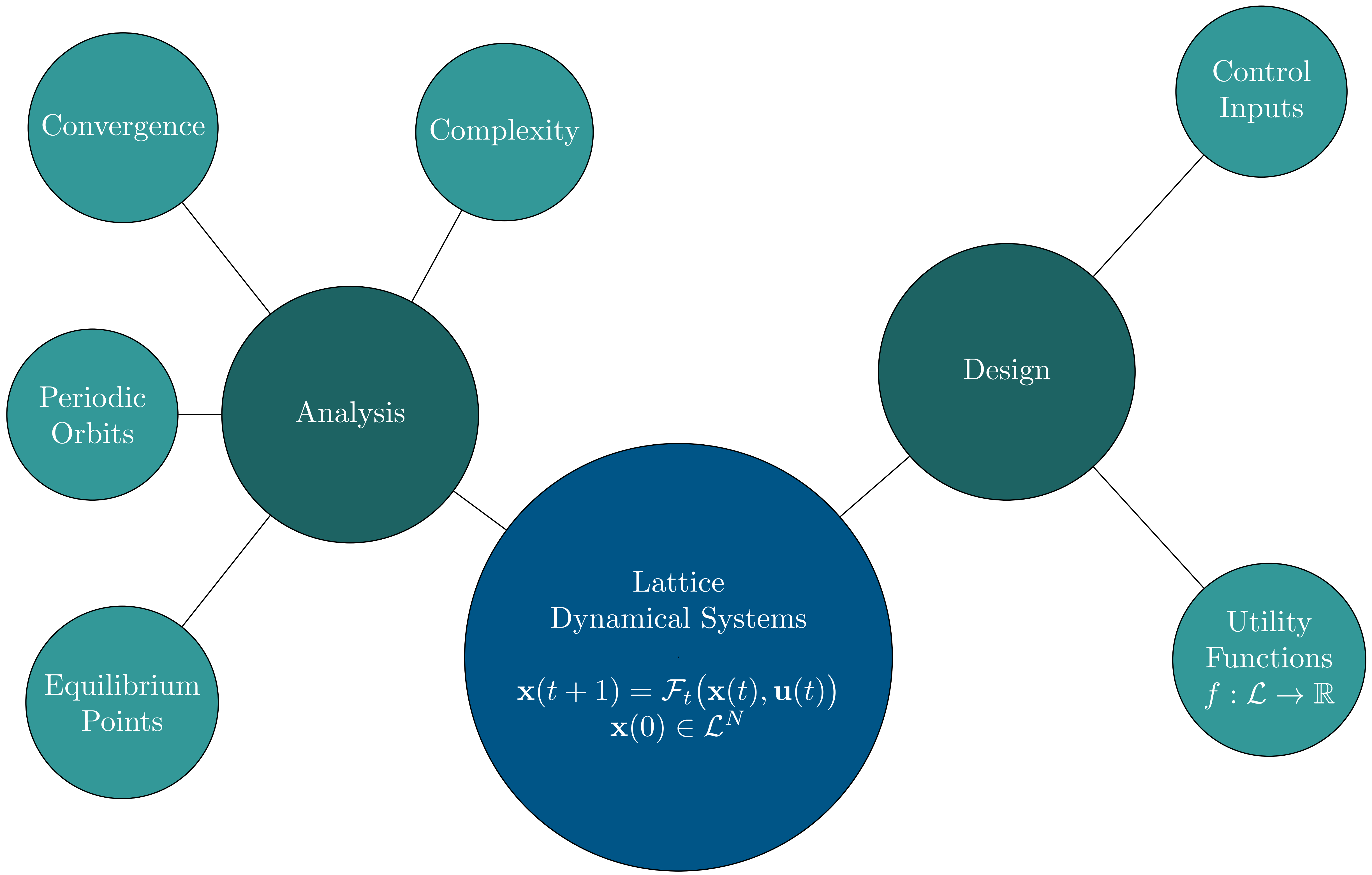
$$(\mathscr{L}\mathbf{x})_u = \underbrace{\bigwedge_{v \in \mathscr{N}_u} \underbrace{W(u,v)}_{\text{delay}} + \overbrace{\mathbf{A}_u^\sharp \boxplus' \left(\underbrace{\mathbf{P}_{(u,v)}^\sharp \boxplus' \left(\mathbf{P}_{(u,v)}}_{\text{projection}} \boxplus \underbrace{\mathbf{A}_v \boxplus \mathbf{x}_v}_{\text{next time}} \right) \right)}^{\text{residuation}}}_{\text{aggregation}}$$



Beyond Schedules

Lattice Graph Diffusion ⇒ Model Checking

- Knowledge diffusion
- Preference dynamics



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Conclusion

“Synchronizing Tasks in Multi-Agent Systems with Max-Plus Algebra

- Formulated a general **multi-agent planning problem**
- Introduced a novel **mathematical theory** for solving the problem.
- **Surveyed** our current efforts to **extend** our **decentralized** methods to more general linear temporal logic (LTL) formal verification & synthesis problems.

Recent Publications

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Thank you. Questions?

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