Assuring Autonomy in Contested Environments

Protecting Information

Y. Wang, S. Nalluri, and M. Pajic, “Hyperproperties for Robotics: Motion Planning via HyperLTL”, IEEE International Conference on Robotics and Automation (ICRA), 2020, to appear

Hyper-properties for Cyber-Physical Systems

- CPS properties of interest commonly include properties such as
  - sensitivity to modeling errors,
  - probabilistic fairness, and
  - anomaly detectability

- These should capture a relationship between multiple simultaneous continuous-time runs

Example: System *Sensitivity* to Modeling Errors

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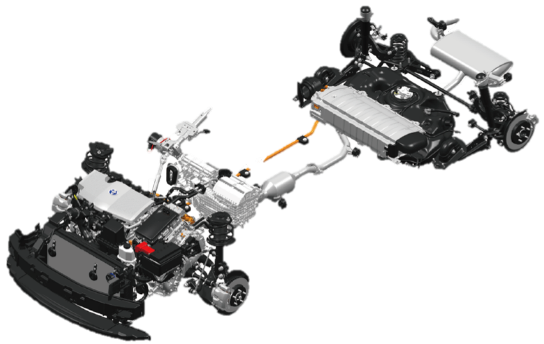
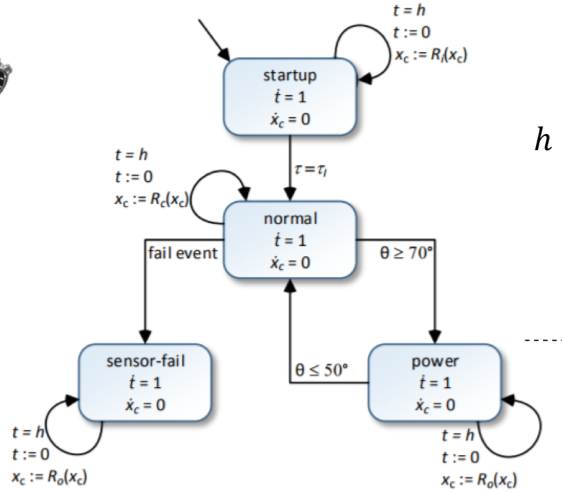
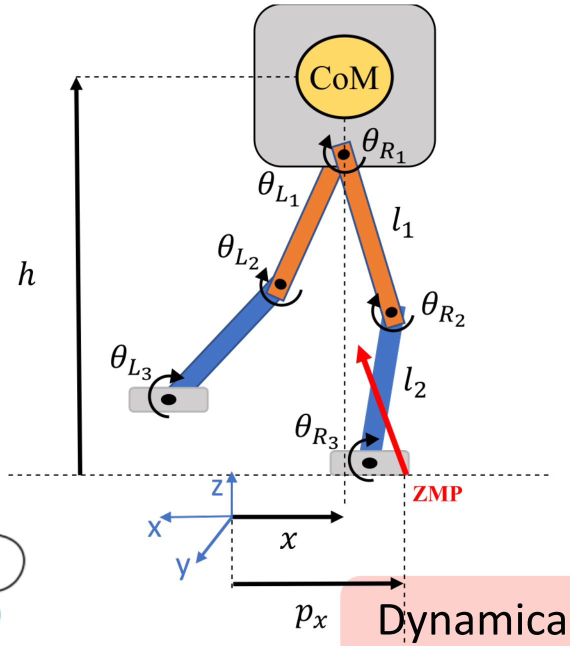
Toyota Powertrain Benchmark

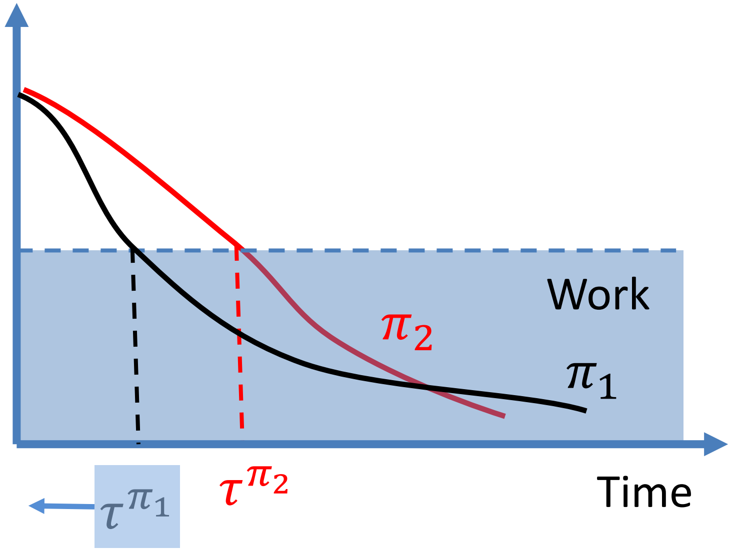


Embedded Controller



Walking Robot Benchmark With Reinforcement Learning





Dynamical response depends on system parameters

Y. Wang, M. Zarei, B. Bonakdarpour and M. Pajic, "Statistical Verification of Hyper-properties for Cyber-Physical Systems", *19th ACM SIGBED International Conference on Embedded Software* (**EMSOFT**), Oct 2019,
Best Paper Award Finalist

How does dynamical response change due to modeling errors or *wear-and-tear*?

- For example, start time change under probabilistic uncertainty?

Probabilistic hyperproperties*:* Sensitivity under probabilistic parameter change

$$\mathbf{Pr}_{\pi_1,\pi_2}(|\tau^{\pi_1} - \tau^{\pi_2}| \leq \delta) > 1 - \varepsilon$$

We need new logic to reason over ***multiple*** random paths!

HyperPSTL: Hyper Probabilistic Signal Temporal Logic

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STL → Add reference to different paths → HyperSTL → Add probabilistic quantifications → HyperPSTL → Add probabilistic arithmetic → (full) HyperPSTL

Syntax:

$$\varphi ::= a^{\pi} \mid \varphi^{\pi} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \mathcal{U}_{[t_1,t_2]}\varphi \mid p \bowtie p$$
$$p ::= \mathbb{P}^{\Pi}\varphi \mid \mathbb{P}^{\Pi}p \mid f(p,\dots,p)$$

- a ∈ AP, and AP is the finite set of *atomic propositions*,
- $t_1 < t_2$ with $t_1, t_2 \in \mathbb{Q}_{\infty}$,
- π is a path variable, and Π is a set of path variables,
- $\mathrm{fv}(\varphi) = \emptyset$
- $\mathbb{P}$ is the probability operator,
- $\bowtie \in \{<, >, =, \leq, \geq\}$,
- $f: \mathbb{R}^n \to \mathbb{R}$ is a n-ary elementary function, constants are viewed as 0-ary functions,

Semantics

For probabilistic arithmetic:

$$\mathcal{S} \vDash [\![p \bowtie p]\!]_{V_X} \Leftrightarrow \mathcal{S} \vDash [\![p]\!]_{V_X} \bowtie [\![p]\!]_{V_X}$$
$$\mathcal{S} \vDash [\![f(p,\dots,p)]\!]_{V_X} \Leftrightarrow \mathcal{S} \vDash f([\![p]\!]_{V_X},\dots,[\![p]\!]_{V_X})$$
$$\mathcal{S} \vDash [\![\mathbb{P}^{\Pi}(\varphi)]\!]_{V_X} \Leftrightarrow \mathcal{S} \vDash \mathbf{Pr}_{\boldsymbol{\sigma}\sim \mathrm{Path}^{|\Pi|}(X)}((\mathcal{S}, V_X[\Pi \to \boldsymbol{\sigma}]) \vDash \varphi)$$

$V_X[\Pi \to \boldsymbol{\sigma}]$ is a revision of the assignment V_X.

For AP and state formulas with $\mathrm{fv}(\varphi) = \emptyset$:

$$(\mathcal{S}, V_X) \vDash a^{\pi} \Leftrightarrow a \in L(V_X(\pi)(0))$$
$$(\mathcal{S}, V_X) \vDash \varphi^{\pi} \Leftrightarrow (\mathcal{S}, V_X(\pi)) \vDash \varphi$$
$$(\mathcal{S}, X) \vDash \varphi \Leftrightarrow \mathcal{S} \vDash [\![\varphi]\!]_{V_X}$$

For temporal operators:

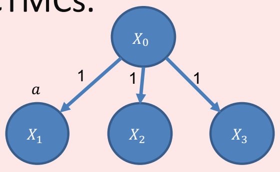
$$(\mathcal{S}, V_X) \vDash \varphi_1 \wedge \varphi_2 \Leftrightarrow (\mathcal{S}, V_X) \vDash \varphi_1 \text{ and } (\mathcal{S}, V_X) \vDash \varphi_2$$
$$(\mathcal{S}, V_X) \vDash \varphi_1 \mathcal{U}_{[t_1,t_2]}\varphi_2 \Leftrightarrow \exists t \in [t_1, t_2].$$
$$(\forall t' < t. (\mathcal{S}, V_X^{(t')}) \vDash \varphi_1) \wedge (\mathcal{S}, V_X^{(t)}) \vDash \varphi_2$$

where $V_X^{(t)}$ is the t-shift of the assignment V_X

Expressiveness:

Theorem: HyperPSTL strictly subsumes PSTL on CTMCs.

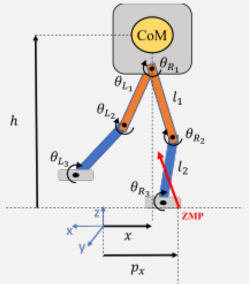
- CTMC has only 3 paths
- Satisfaction probability of any STL is $0, \frac{1}{3}, \frac{2}{3}, 1$, so $P(\varphi) = \frac{1}{9}$ is always false for any (φ)
- HyperPSTL $P^{(\pi_1,\pi_2)}(\boldsymbol{\mathcal{F}}(a^{\pi_1} \wedge a^{\pi_2})) = \frac{1}{9}$ is true



Sensitivity:

$$\mathbb{P}^{(\pi_1,\pi_2)}\left(\begin{array}{c}(\neg Q^{\pi_1} \wedge \neg Q^{\pi_2}) \\ \boldsymbol{\mathcal{U}}\left((Q^{\pi_1} \wedge \boldsymbol{\mathcal{F}}_{[0,\delta]}Q^{\pi_2}) \vee (Q^{\pi_2} \wedge \boldsymbol{\mathcal{F}}_{[0,\delta]}Q^{\pi_1})\right)\end{array}\right) > 1 - \varepsilon$$

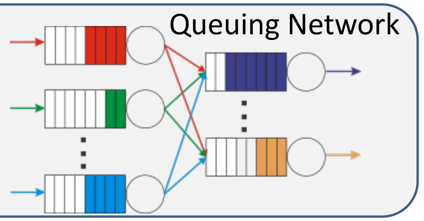
Toyota Powertrain Benchmark



Walking Robot Benchmark

Workload Fairness:

$$\mathbb{P}^{\pi_1}(|\mathbb{P}^{\pi_2}((\neg Q_i^{\pi_1} \wedge \neg Q_j^{\pi_2})\mathcal{U}(Q_i^{\pi_1} \wedge \hat{Y}_{[\tau,\infty)}Q_j^{\pi_2}))$$
$$- \mathbb{P}^{\pi_2}((\neg Q_i^{\pi_1} \wedge \neg Q_j^{\pi_2})\mathcal{U}(Q_j^{\pi_2} \wedge \hat{Y}_{[\tau,\infty)}Q_i^{\pi_1}))| \leq \delta)$$
$$\geq 1 - \varepsilon.$$



HyperSMC tool for SMC of HyperPSTL with desired confidence/significance level

https://gitlab.oit.duke.edu/cpsl/hypersmc

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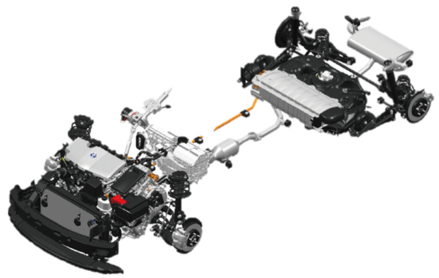
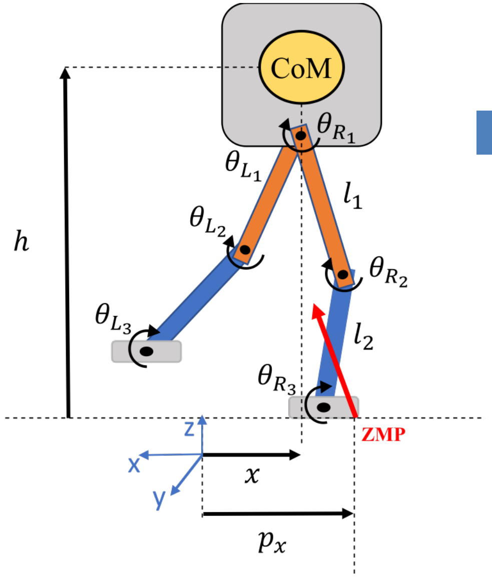
Example: Sensitivity Verification for real-world CPS

$$\mathbb{P}^{(\pi_1,\pi_2)}\left((\neg Q^{\pi_1} \wedge \neg Q^{\pi_2})\ \boldsymbol{\mathcal{U}}\left(\left(Q^{\pi_1} \wedge \boldsymbol{\mathcal{F}}_{[0,\delta]} Q^{\pi_2}\right) \vee \left(Q^{\pi_2} \wedge \boldsymbol{\mathcal{F}}_{[0,\delta]} Q^{\pi_1}\right)\right)\right) > 1-\varepsilon$$





Walking Robot Benchmark With Reinforcement Learning Controller

δ	ε	α	Acc.	Sam.	Time (s)	Ans.
2.4	0.02	0.01	**1.00**	7.4e+01	3.0e-01	False
2.4	0.02	0.05	**0.99**	4.4e+01	1.4e-01	False
2.4	0.12	0.01	**1.00**	4.2e+01	1.2e-01	True
2.4	0.12	0.05	**1.00**	2.1e+01	7.0e-02	True
2.4	0.2	0.01	**1.00**	1.3e+01	4.0e-02	True
3.0	0.02	0.01	**1.00**	1.1e+01	2.4e-02	False
3.0	0.02	0.05	**1.00**	6.5e+00	1.1e-02	False
3.0	0.12	0.05	**0.98**	7.0e+01	4.3e-01	False
3.0	0.2	0.01	**1.00**	1.6e+02	5.5e-01	True
3.0	0.2	0.05	**0.98**	1.0e+02	2.9e-01	True

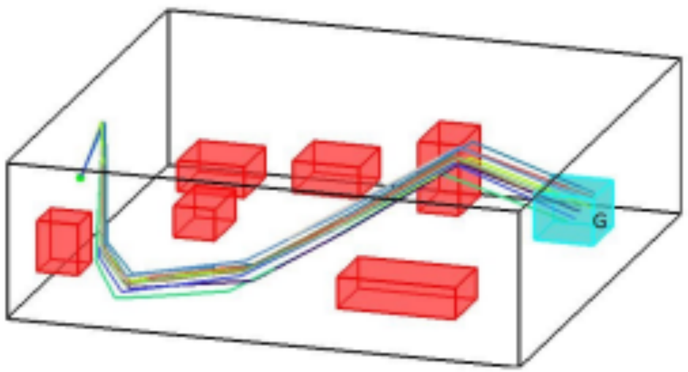
Toyota Powertrain Benchmark

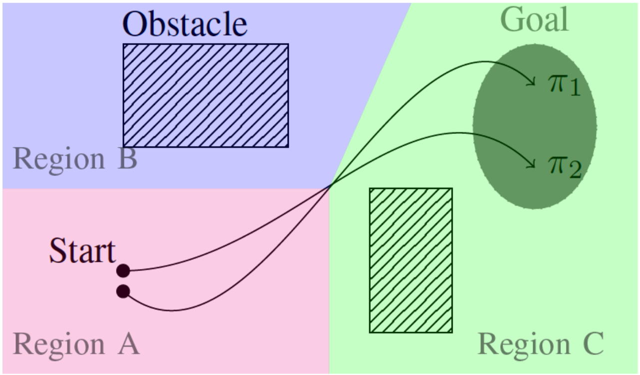
δ	ε	α	Acc.	Sam.	Time (s)	Ans.
0.15	0.95	0.05	**1.00**	5.9e+01	8.1e+00	True
0.15	0.95	0.01	**1.00**	9.0e+01	1.3e+01	True
0.15	0.99	0.05	**0.99**	6.6e+01	9.1e+00	False
0.15	0.99	0.01	**1.00**	9.7e+01	1.4e+01	False
0.20	0.95	0.05	**0.98**	5.9e+01	8.1e+00	True
0.20	0.95	0.01	**1.00**	9.0e+01	1.2e+01	True
0.20	0.99	0.05	**1.00**	3.0e+02	4.2e+01	True
0.20	0.99	0.01	**0.99**	4.6e+02	1.8e+02	True

SMC procedure extended to support verification of learning enabled controllers

[1] M. Zarei, Y. Wang, and M. Pajic, "Statistical Verification of Learning-Based Cyber-Physical Systems", *23rd ACM International Conference on Hybrid Systems: Computation and Control* (**HSCC**), 2020, **to appear**.

Hyperproperties for Motion Planning





Privacy-aware Motion Planning

$$\exists\pi_1\exists\pi_2.(\pi_1 \text{ and } \pi_1 \text{ are different paths}) \wedge (\pi_1 \text{ and } \pi_1 \text{ give identical observation}) \wedge (\pi_1 \text{ and } \pi_1 \text{ reach goal}).$$

$$\exists\pi_1\exists\pi_2.(\mathrm{sec}(\pi_1) \neq \mathrm{sec}(\pi_2)) \wedge (\mathrm{obs}(\pi_1) = \mathrm{obs}(\pi_2))$$

Optimality of Synthesized Plans

$$\exists\pi.\Big((\pi \text{ reaches goal}) \wedge (\forall\pi'.((\pi' \text{ reaches goal}) \Rightarrow (\pi \text{ reaches goal}))\Big)$$

$$\exists\pi_1\forall\pi_2.\ (\mathbf{s}_0{}^{\pi_1} \wedge \mathbf{s}_0{}^{\pi_2}) \wedge (\Diamond_T(g^{\pi_2} \Rightarrow \Diamond_T g^{\pi_1}));$$

$$\exists\pi_1\forall\pi_2.\ (\mathbf{s}_0{}^{\pi_1} \wedge \mathbf{s}_0{}^{\pi_2}) \wedge (\Diamond_T(g^{\pi_1} \Rightarrow \Diamond_T g^{\pi_2}))$$

Robustness of Synthesized Plans

$$\exists\pi\forall\pi'.(\pi \text{ is derived by disturbing } \pi') \wedge (\pi \text{ and } \pi' \text{ reach goal}).$$

$$\exists\pi_1\forall\pi_2.\ \mathrm{cls}_{\mathbf{s}_0}(\pi_1,\pi_2) \wedge \mathrm{cls}_{\mathtt{A}}(\pi_1,\pi_2) \Rightarrow (\varphi^{\pi_1} \wedge \varphi^{\pi_2})$$

HyperLTL

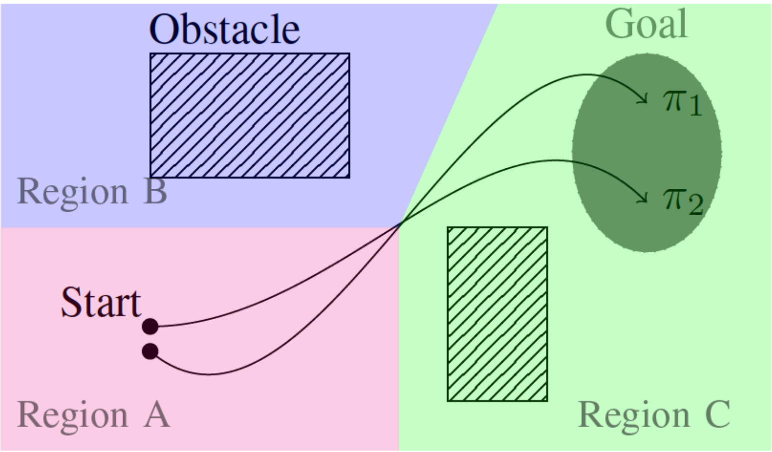
As a HyperLTL objective may contain multiple path variables, its satisfaction involves assigning concrete (infinite) paths to all these path

$$V : \Pi \to (2^{\mathsf{AP}})^{\omega}$$

The satisfaction relation for the HyperLTL path formulas is then defined for V by

$$
\begin{array}{lcl}
V \models \mathsf{a}^{\pi} & \Leftrightarrow & \mathsf{a} \in V(\pi)(0) \\
V \models \neg\varphi & \Leftrightarrow & V \not\models \varphi \\
V \models \varphi_1 \wedge \varphi_2 & \Leftrightarrow & V \models \varphi_1 \text{ and } V \models \varphi_2 \\
V \models \bigcirc\varphi & \Leftrightarrow & V^{(1)} \models \varphi \\
V \models \varphi_1 \,\mathcal{U}_T\, \varphi_2 & \Leftrightarrow & \exists t \leq T.\ \big(V^{(t)} \models \varphi_2 \text{ and} \\
 & & \quad (\forall t' < t.\ V^{(t')} \models \varphi_1)\big) \\
V \models \exists\pi.\ \psi & \Leftrightarrow & \text{there exists } \sigma \in (2^{\mathsf{AP}})^{\omega}, \\
 & & \quad \text{such that } V[\pi \mapsto \sigma] \models \psi \\
V \models \forall\pi.\ \psi & \Leftrightarrow & \text{for all } \sigma \in (2^{\mathsf{AP}})^{\omega}, \\
 & & \quad V[\pi \mapsto \sigma] \models \psi \text{ holds}
\end{array}
$$

HyperLTL for Motion Planning



Initial-state opacity for fixed strategy

$$\exists \pi_1 \exists \pi_2 . (\mathbf{s}_0{}^{\pi_1} \wedge (\neg \mathbf{s}_0{}^{\pi_2})) \wedge (\Box_T (\mathbf{a}^{\pi_1} = \mathbf{a}^{\pi_2})) \wedge ((\Diamond_T g^{\pi_1}) \wedge (\Diamond_T g^{\pi_2}))$$

Current-state opacity

$$\exists \pi_1 \exists \pi_2 . (\mathbf{s}_0{}^{\pi_1} \wedge \mathbf{s}_0{}^{\pi_2}) \wedge \wedge (\neg \Box_T (\mathbf{a}^{\pi_1} = \mathbf{a}^{\pi_2})) \wedge (\Box_T (o^{\pi_1} = o^{\pi_2}))$$

Shortest Path

$$\exists \pi_2 \forall \pi_1 .\ (\mathbf{s}_0{}^{\pi_1} \wedge \mathbf{s}_0{}^{\pi_2}) \wedge (\Diamond (g^{\pi_2} \Rightarrow \Diamond g^{\pi_1}))$$

Longest Path

$$\exists \pi_2 \forall \pi_1 .\ (\mathbf{s}_0{}^{\pi_1} \wedge \mathbf{s}_0{}^{\pi_2}) \wedge (\Diamond (g^{\pi_1} \Rightarrow \Diamond g^{\pi_2}))$$

Initial-state robustness

$$\exists \pi_1 \forall \pi_2 . (\mathbf{s}_0{}^{\pi_1} \wedge S_0^{\pi_2}) \wedge (\varphi^{\pi_1} \wedge \varphi^{\pi_2}) \wedge (\Box_T (\mathbf{a}^{\pi_1} = \mathbf{a}^{\pi_2}))$$

Action robustness

$$\exists \pi_1 \forall \pi_2 .\ (\mathbf{s}_0{}^{\pi_1} \wedge \mathbf{s}_0{}^{\pi_2}) \wedge (\varphi^{\pi_1} \wedge \varphi^{\pi_2})$$

Symbolic Synthesis from HyperLTL

Computing Required Time-Horizon

- A HyperLTL objective contains multiple paths => unlike with LTL formulas, the required time horizon may be different among the utilized path variables
- $H(\varphi, \pi)$ – the required time horizon for a path variable π in a HyperLTL objective φ

$$H(\mathsf{a}^{\pi}, \pi') = \begin{cases} 0 & \text{if } \pi' = \pi \\ -\infty & \text{otherwise.} \end{cases}$$

$$H(\bigcirc\varphi, \pi) = H(\varphi, \pi) + 1,$$

$$H(\varphi_1\, \mathcal{U}_T\, \varphi_2, \pi) = \max\{H(\varphi_1, \pi), H(\varphi_2, \pi)\} + T$$

$$H(\neg\varphi, \pi) = H(\varphi, \pi),$$

$$H(\varphi_1 \wedge \varphi_2, \pi) = \max\{H(\varphi_1, \pi), H(\varphi_2, \pi)\},$$

$$H(\exists\pi'.\ \psi, \pi) = H(\psi, \pi), \quad H(\forall\pi'.\ \psi, \pi) = H(\psi, \pi)$$

Example:

$$H(\exists\pi_2\forall\pi_1.\ (\mathsf{s_0}^{\pi_1} \wedge \mathsf{s_0}^{\pi_2}) \wedge (\Diamond_T(g^{\pi_2} \Rightarrow \Diamond_T g^{\pi_1})), \pi_1) = H(\Diamond_T(g^{\pi_2} \Rightarrow \Diamond_T g^{\pi_1}), \pi_1) =$$

$$H(\Diamond_T g^{\pi_1}, \pi_1) + T = 2T$$

Model Conversion for SMT-Based Synthesis

- Start from a Discrete-Transition System (DTS) $\mathcal{M}$
- A general HyperLTLf objective φ

$$\varphi = Q_1\pi_1 \ldots Q_n\pi_n \text{ where } Q_i \in \{\exists, \forall\} \text{ for } i \in \{1, \ldots, n\}$$

$$P_i = \bigwedge_{t \in [H_i]} (\mathbf{s}_i(t) = \mathrm{T}_{\mathcal{M}}(\mathbf{s}_i(t-1), \mathbf{a}_i(t-1)))$$

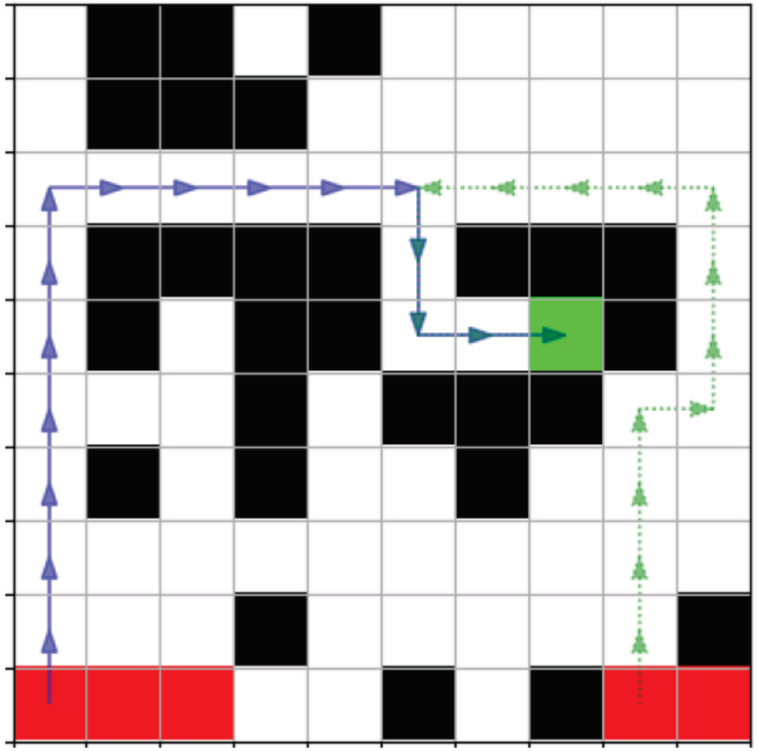
$$[\mathsf{Q}_i\pi_i] = \mathsf{Q}_i\mathbf{s}_i(0)\mathsf{Q}_i\mathbf{a}_i(0)\ldots\mathsf{Q}_i\mathbf{a}_i(H_1 - 1)$$

Resulting in a first-order formula:

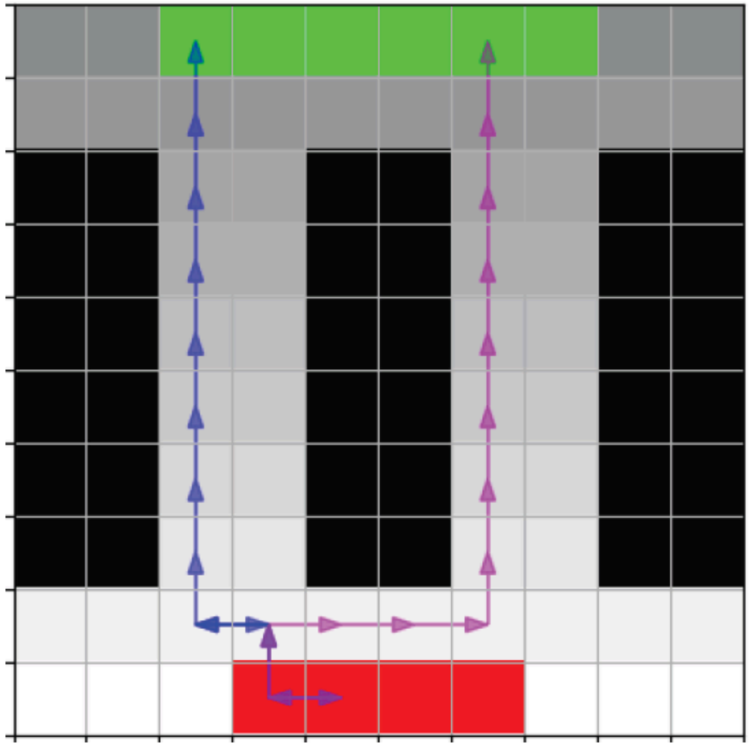
$$[\mathsf{Q}_1\pi_1]\ldots[\mathsf{Q}_n\pi_n].(\bigwedge_{i \in [n]} P_i) \wedge \varphi$$

Can be solved with an SMT solver (e.g., Z3)

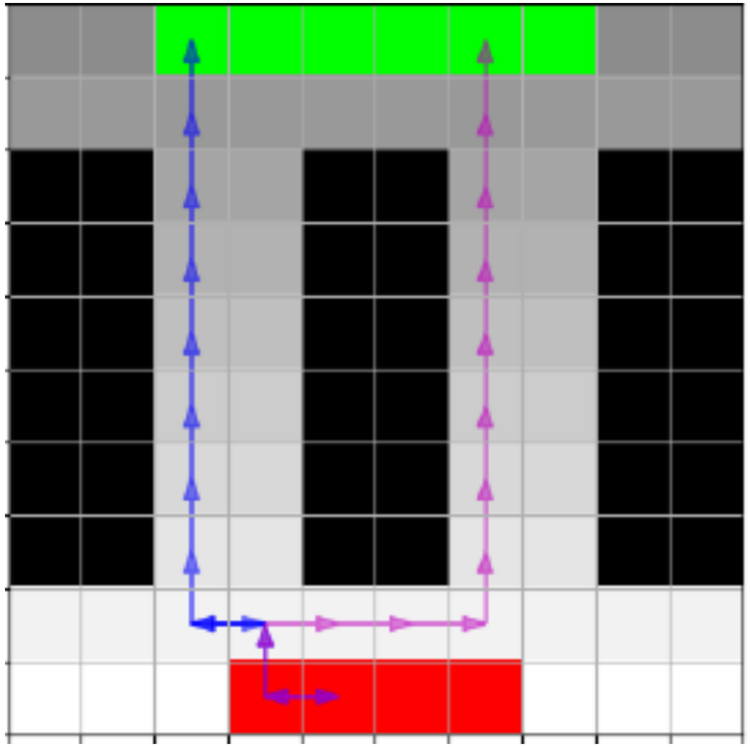
Symbolic Synthesis from HyperLTL [ICRA'20]



Shortest path



Current-state opacity



Initial-state opacity

MPHyper tool – Motion Planning from HyperLTL: https://gitlab.oit.duke.edu/cpsl/mp_hyper

Thank you