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AACE review

Adaptive Control and Integral Concurrent Learning for Differential Drag-Based Spacecraft Formations

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Motivations & contributions

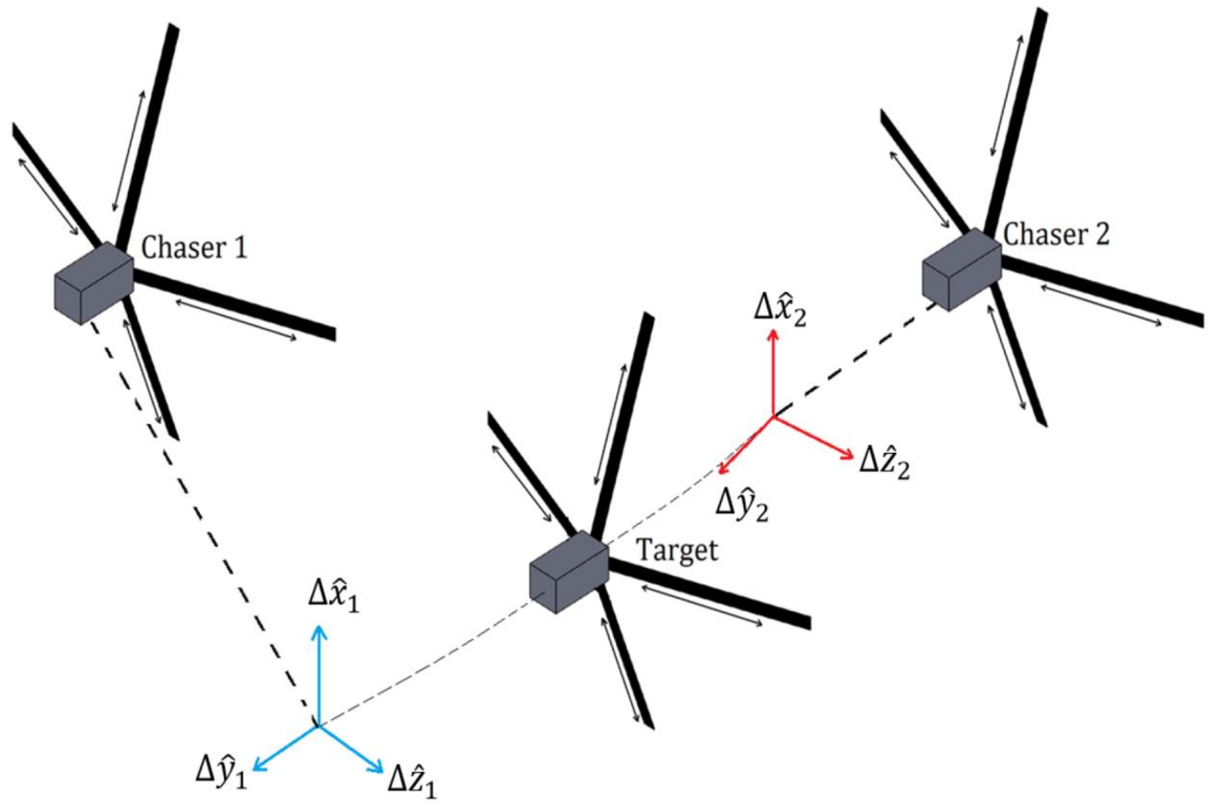
- Space is a contested warfighter operational scenario.

- Resident Space Object ID, on-orbit SSA, and multi-S/C coordination are key elements of the new scenario.

- **The ADAMUS efforts focus on solving the following problems (long term)**
  - Propellant-less relative maneuvering with respect to unknown space objects (cost effective & stealth).
  - Increase autonomy by eliminating dependence on atmospheric density forecasts for differential-drag based maneuvers.
  - Build the foundations to design complex multiple spacecraft missions. **RT4 (networked agent coordination)**

- **This talk shows (work so far)**
  - Simultaneous formation control and estimation of the target's drag parameter (may augment/substitute orbit determination). **RT2 (Adaptation)**

Relative Maneuvering & Online System ID



- *G. Chowdhary, T. Yucelen, M. Muhlegg, E. N. Johnson, "Concurrent learning adaptive control of linear systems with exponentially convergent bounds disturbances", International Journal of Control and Signal Processing 27 (4)(2013) 280-301.*

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Drag Acceleration

- Atmospheric drag acceleration experienced by a spacecraft

$$\ddot{\boldsymbol{r}}_{\boldsymbol{D}} = -\frac{\rho(t) S C_D}{2m} V_r^2 \, \hat{\boldsymbol{V}}_{\boldsymbol{r}}$$

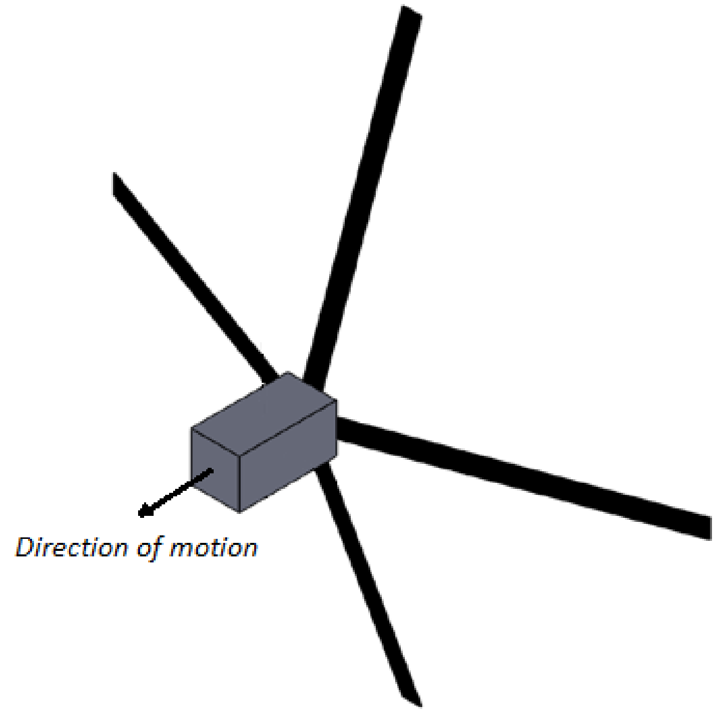
$\rho(t)$: Time-varying atmospheric density

C_D: Drag coefficient

S: Cross-sectional Area of the spacecraft

$\boldsymbol{V_r}$: Spacecraft-atmosphere relative velocity vector.

m : mass of the spacecraft



- There exist **several atmospheric models** with different levels of accuracy, one of the more complex is the semi-empirical NRLMSISE-00 model, which considers the spacecraft location, date and solar and geomagnetic activity indices. In addition to be computationally expensive, it relies on forecasts of some indices, **limiting autonomy**.

- *D. Guglielmo, S. Omar, R. Bevilacqua, et al., "Drag De-Orbit Device - A New Standard Re-Entry Actuator for CubeSats", Journal of Spacecraft and Rockets, Vol. 56, No. 1 (2019), pp. 129-145.*

Choosing an Atmospheric Density Model

- Variations of the atmospheric density in circular LEO are mostly due to day/night changes and J_2 perturbation.

- Previous work from different authors has shown that the density in this orbit regime has its principal Fourier components at zero and the (constant) orbit angular velocity Ω. Motivating the following model:

$$\rho(t) = D_1 + D_2 \sin(\Omega t) + D_3 \cos(\Omega t)$$

- D_1, D_2 and D_3 are constants.

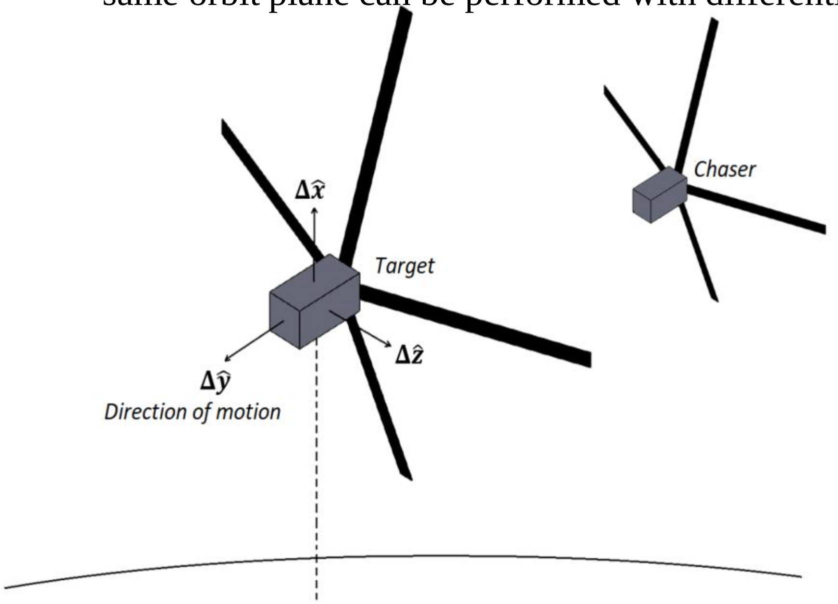
- This model is valid for short-term maneuvers (few days).

- The model is linearly parameterizable with respect to D_1, D_2 and D_3. This is a property that will be exploited in the controller development.

- *G. Gaias, J.-S. Ardaens, O. Montenbruck, "Model of J2 perturbed satellite relative motion with time-varying dierential drag", Celestial Mechanics and Dynamical Astronomy 123 (2015) 441-433.*

Spacecraft Relative Motion Dynamics

- The SS model is a linear representation of the relative motion between two spacecraft valid for relatively small distances (~tens of kilometers) in circular LEO.
- It considers the influence of the Earth's oblateness (J_2 perturbation).
- Given that the drag acceleration acts mostly opposite to the direction of motion, only maneuvers within the same orbit plane can be performed with differential drag.



$$\begin{bmatrix} \Delta\dot{x} \\ \Delta\ddot{x} \\ \Delta\dot{y} \\ \Delta\ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ b & 0 & 0 & a \\ 0 & 0 & 0 & 1 \\ 0 & -a & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta\dot{x} \\ \Delta y \\ \Delta\dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_y$$

$$\dot{\boldsymbol{X}} = A\boldsymbol{X} + \boldsymbol{B}u_y$$

- a and b are known constants and u_y is the differential drag.

- *S. A. Schweighart, R. J. Sedwick, High-fidelity linearized J2 model for satellite formation flight, Journal of Guidance, Control and Dynamics 25 (6)(2002) 1073-1080.*

- Adaptive controller designed using the Integral Concurrent Learning (ICL) technique for online parameter estimation.
- The use of an ICL-based adaptive controller results in exponential convergence of the states and the estimates provided a verifiable condition of finite excitation.

$$\dot{\boldsymbol{X}} = A\boldsymbol{X} + \boldsymbol{B}u_y$$

u_y is called the auxiliary control input and is defined as

$$u_y = \rho_t(t)C_{D,t}V_{r,t}^2 \frac{S_t}{2m_t} - \rho_i(t)C_{D,i}V_{r,i}^2\,\bar{u}$$

where the subscripts t and i represent the target and i^{th} chaser, respectively.
$\bar{u}$ is the actual control input that represents the area-to-mass ratio of the i^{th} chaser.

- The auxiliary control input u_y is linearly parameterized as

$$u_y = \boldsymbol{Y}\boldsymbol{\Theta}$$

where $\boldsymbol{Y}$ is a measurable regression matrix and $\boldsymbol{\Theta}$ is the vector of uncertain parameters

$$\boldsymbol{\Theta} = \left[D_{1,i} C_{D,i} V_{r,i}^2 \quad D_{2,i} C_{D,i} V_{r,i}^2 \quad D_{3,i} C_{D,i} V_{r,i}^2 \quad D_{1,t} \frac{C_{D,t} S_t}{2m_t} V_{r,t}^2 \quad D_{2,t} \frac{C_{D,t} S_t}{2m_t} V_{r,t}^2 \quad D_{3,t} \frac{C_{D,t} S_t}{2m_t} V_{r,t}^2 \right]^T$$

- The control law is designed using the estimates ($\widehat{\boldsymbol{\Theta}}$) of $\boldsymbol{\Theta}$ as

$$\bar{u} = \left(\hat{\rho}_i(t) \hat{C}_{D,i} \widehat{V_{r,i}^2} \right)^{-1} \left(\hat{\rho}_t(t) \frac{\hat{C}_{D,t} \hat{S}_t}{2\hat{m}_t} \widehat{V_{r,t}^2} + \boldsymbol{K}\boldsymbol{X} \right)$$

where $\boldsymbol{K}$ is a vector of constant gains and $\boldsymbol{X}$ is the measurable state vector.

- The estimates are updated with the following ICL-based adaptive update law

$$\dot{\hat{\boldsymbol{\Theta}}} = proj\left(2\Gamma \boldsymbol{Y}^T \boldsymbol{B}^T P^T \boldsymbol{X} + \Gamma K_{ICL} \sum_{i=1}^{N_s} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{B}^T \left(\boldsymbol{X}(t_i) - \boldsymbol{X}(t_i - \Delta t) - \boldsymbol{\mathcal{U}}_{\boldsymbol{i}} - \boldsymbol{B}\boldsymbol{\mathcal{Y}}_i \widehat{\Theta}\right)\right)$$

where Δt represents an user-defined sampling time, Γ is the adaptation gain, K_{ICL} is a symmetric positive definite gain matrix, P is a symmetric positive definite matrix and

$$\boldsymbol{\mathcal{Y}}_{\mathrm{i}} = \int_{t-\Delta t}^{t} \boldsymbol{Y}(\sigma) d\sigma \quad , \quad \boldsymbol{\mathcal{U}}_{\mathrm{i}} = \int_{t-\Delta t}^{t} A\boldsymbol{X}(\sigma) d\sigma .$$

- Since $\boldsymbol{X}(t_i) - \boldsymbol{X}(t_i - \Delta t) - \boldsymbol{\mathcal{U}}_{\boldsymbol{i}} = \boldsymbol{B}\boldsymbol{\mathcal{Y}}_i \boldsymbol{\Theta}$. The adaptive update law can be rewritten as

$$\dot{\hat{\boldsymbol{\Theta}}} = proj\left(2\Gamma \boldsymbol{Y}^T \boldsymbol{B}^T P^T \boldsymbol{X} + \Gamma K_{ICL} \sum_{i=1}^{N_s} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{\mathcal{Y}}_i \tilde{\boldsymbol{\Theta}}\right)$$

- For the Lyapunov-based analysis, the candidate Lyapunov function is

$$V = \boldsymbol{X}^{\boldsymbol{T}} P \boldsymbol{X} + \frac{1}{2} \widetilde{\boldsymbol{\Theta}}^{\boldsymbol{T}} \Gamma \widetilde{\boldsymbol{\Theta}}$$

- After taking the time derivative and substituting the dynamics, control and adaptive update laws we get

$$\dot{V} = -\boldsymbol{X}^T Q_1 \boldsymbol{X} - \widetilde{\boldsymbol{\Theta}}^{\boldsymbol{T}} K_{ICL} \sum_{i=1}^{N_s} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{\mathcal{Y}}_i \widetilde{\boldsymbol{\Theta}}$$

where Q_1 is a positive definite matrix.

- The condition of finite excitation assumes there exists a time t=T >0 such that

$$\lambda_{min} \left\{ \sum_{i=1}^{N_S} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{\mathcal{Y}}_i \right\} > \bar{\lambda}$$

where $\bar{\lambda}$ is a user-defined positive threshold .

- Before $t = T$, $\mathrm{K_{ICL}} \sum_{i=1}^{N_s} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{\mathcal{Y}}_i$ is at least positive semi-definite. Therefore

$$\dot{V} \leq -\boldsymbol{X} Q_1 \boldsymbol{X}$$

which is a negative semi-definite result. By Barbalat's Lemma we can conclude that before $t = T$

$$\lim_{t \to \infty} \|\boldsymbol{X}\| \to 0$$

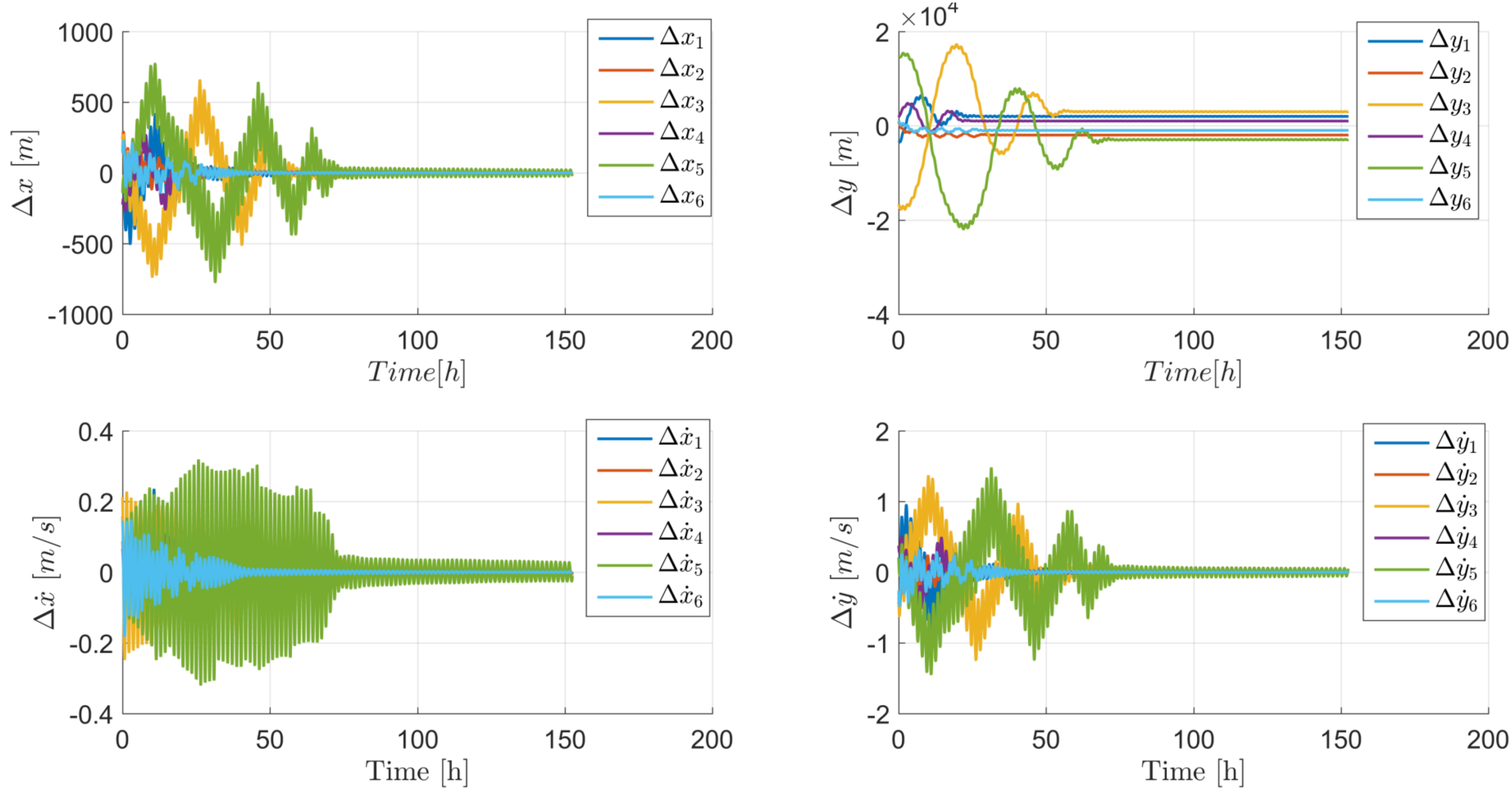
- After t=T, $\mathrm{K_{ICL}} \sum_{i=1}^{N_s} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{\mathcal{Y}}_i$ becomes positive definite, and $V(t)$ can be upper bounded as

$$V(t) \leq V(T) \exp\big(-\lambda(t - T)\big) \quad \forall t \geq T$$

- Then, the states $\boldsymbol{X}$ and the estimation error $\widetilde{\boldsymbol{\Theta}}$ is guaranteed to exponentially converge to zero after $t = T$.

Results – leader follower

- Numerical simulation with the physical properties of seven identical DMD-equipped CubeSats, full nonlinear individual dynamics, and NRLMSISE-00 atmospheric. Along-orbit formation maneuver with 1km inter-spacecraft separation.



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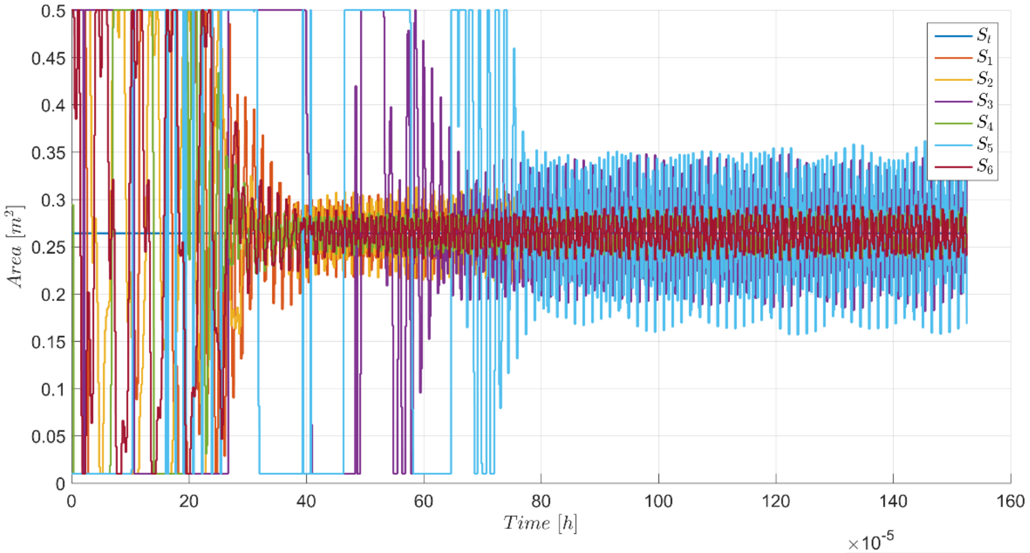
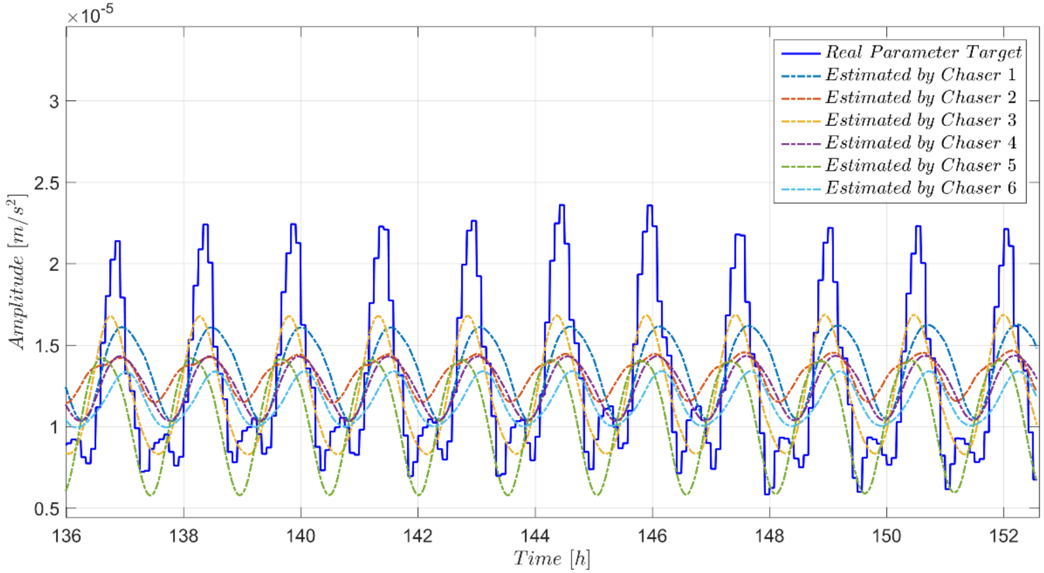
Results – leader follower

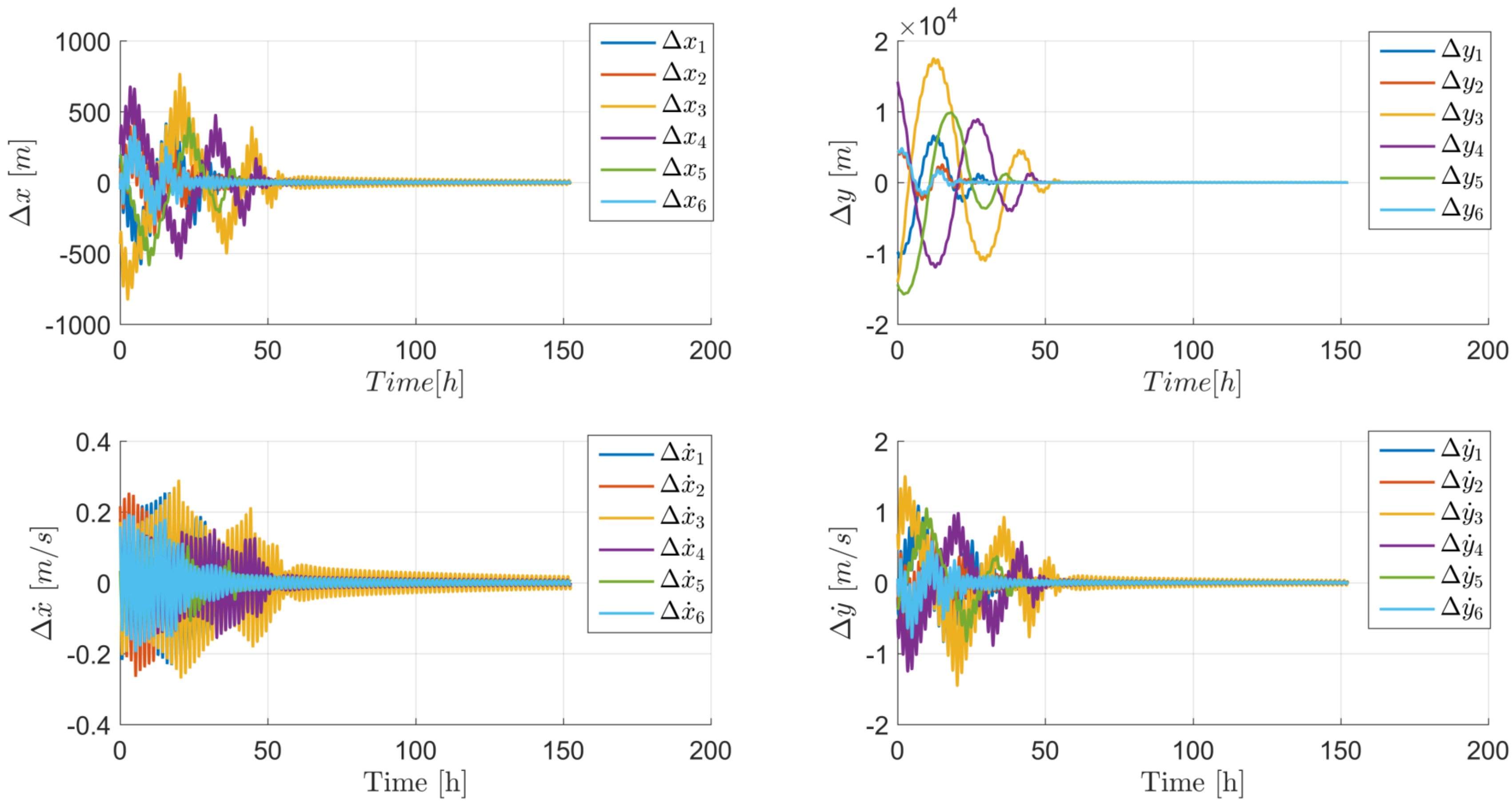


- Each maneuverable chaser estimates the unknown parameters of the target.

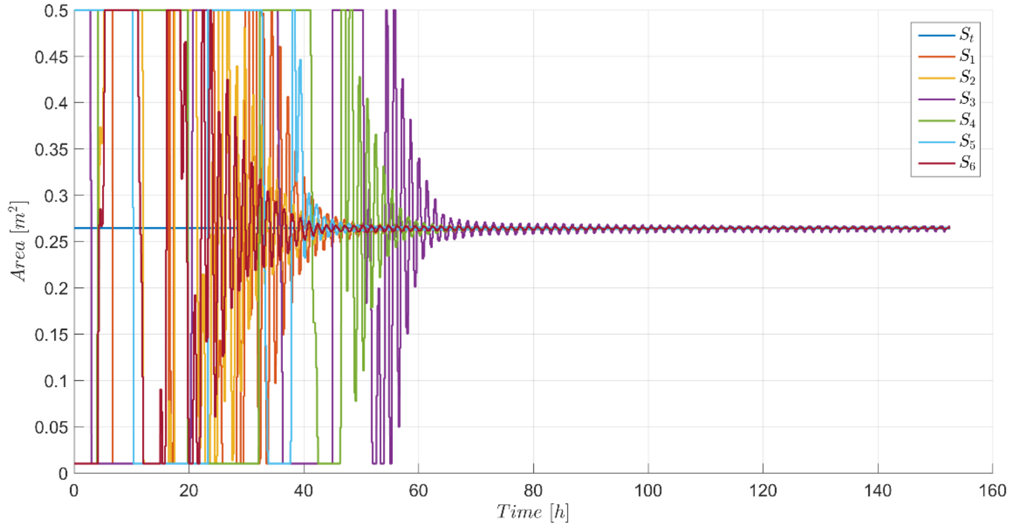
- Affecting estimation: nonlinear dynamics and density approximation model



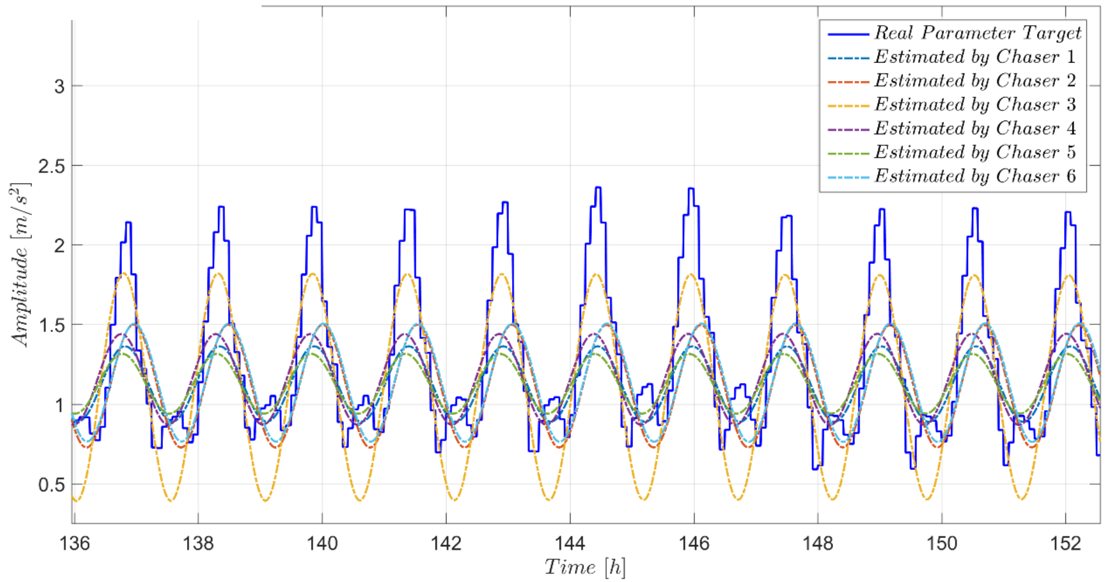
Results – rendezvous



Results – rendezvous



- Since all spacecraft are identical, the cross sectional areas converge to the same level.



Conclusion and Future Work

- Adaptive controllers have been proposed for autonomous spacecraft differential drag-based simultaneous relative estimation and maneuvers.
- The results are potentially implementable on-board small satellites.
- ICL adaptive controller including natural torques is under development in collaboration with the NCR.
- Future work includes networking the chasers and their estimations to enhance the performances.

Related Publications:

- C. Riano-Rios, R. Bevilacqua, W. E. Dixon, "Relative maneuvering for multiple spacecraft via differential drag using LQR and constrained least squares", 495 in: AAS Space Fight Mechanics Meeting, Maui, Hawaii, 2019, Paper No. AAS-19-346. **please, look here for collision avoidance**
- C. Riano-Rios, S. Omar, R. Bevilacqua, W. Dixon, "Spacecraft attitude regulation in low earth orbit using natural torques", in: 2019 IEEE 4th Colombian Conference on Automatic Control (CCAC), Medellin, Colombia, 2019.
- C. Riano-Rios, R. Bevilacqua, W. E. Dixon, "Adaptive control for differential drag-based rendezvous maneuvers with an unknown target", Acta Astronautica. To appear.
- C. Riano-Rios, R. Bevilacqua, W. E. Dixon, "Differential Drag-Based Multiple Spacecraft Maneuvering and On-Line Parameter Estimation Using Integral Concurrent Learning", Submitted to Acta Astronautica.
- Others in preparation…

End of Presentation

Backup Slides

COLLISIONS

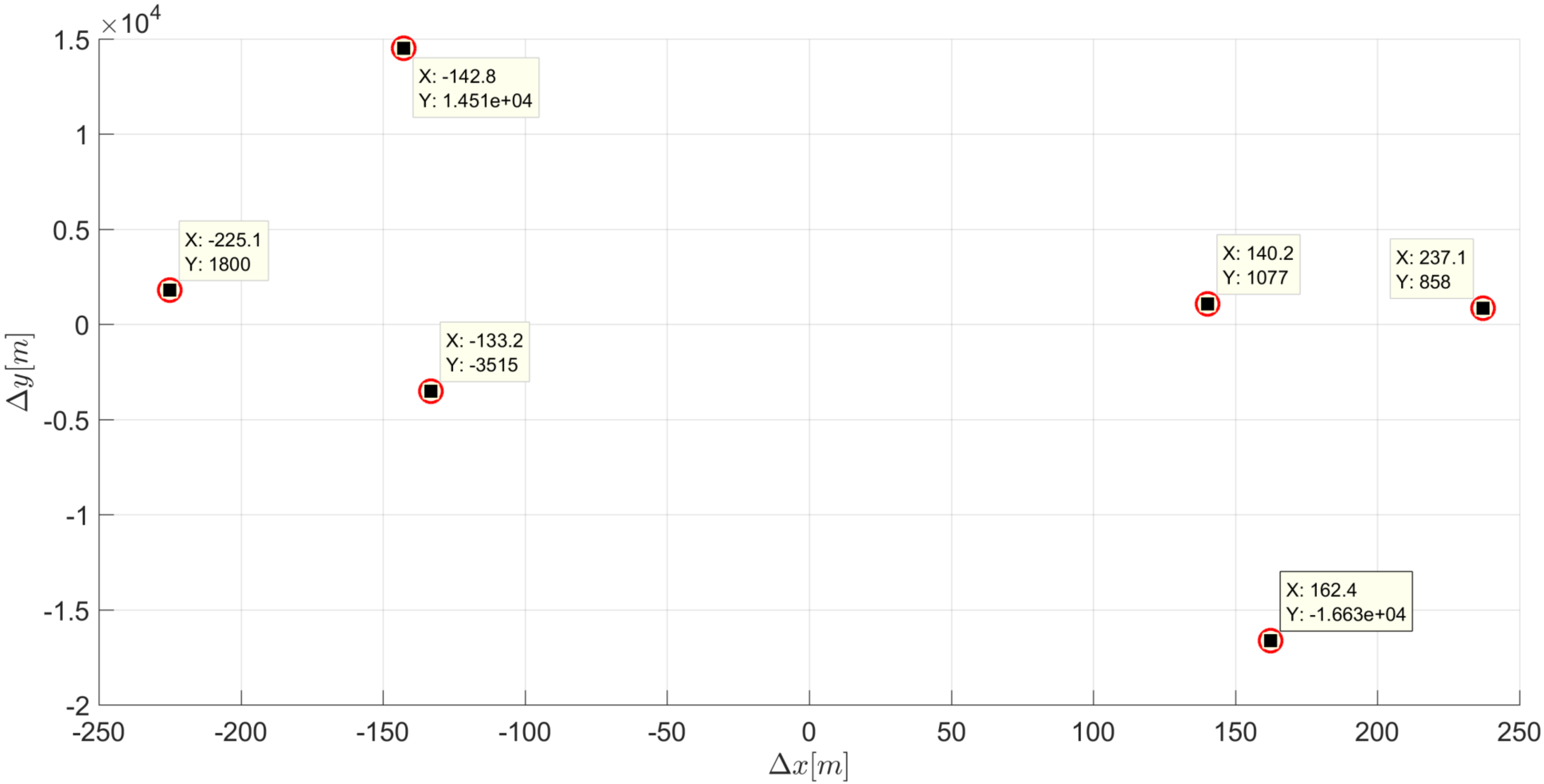
“The presence of multiple spacecraft maneuvering at relatively small distances increases the risk of possible collisions, especially for rendezvous maneuvers. Having the same controller driving each chaser to the rendezvous state with respect to the target yields a similar behavior in the relative path that a chaser follows to reach it, and given the state feedback term in the control law it is expected that the control effort is reduced as the chaser approaches the target. Therefore, if a rendezvous maneuver is required, some chasers could follow similar paths and will be maneuvering in close proximity to the target for a

significant portion of the maneuver, increasing the collision risk.

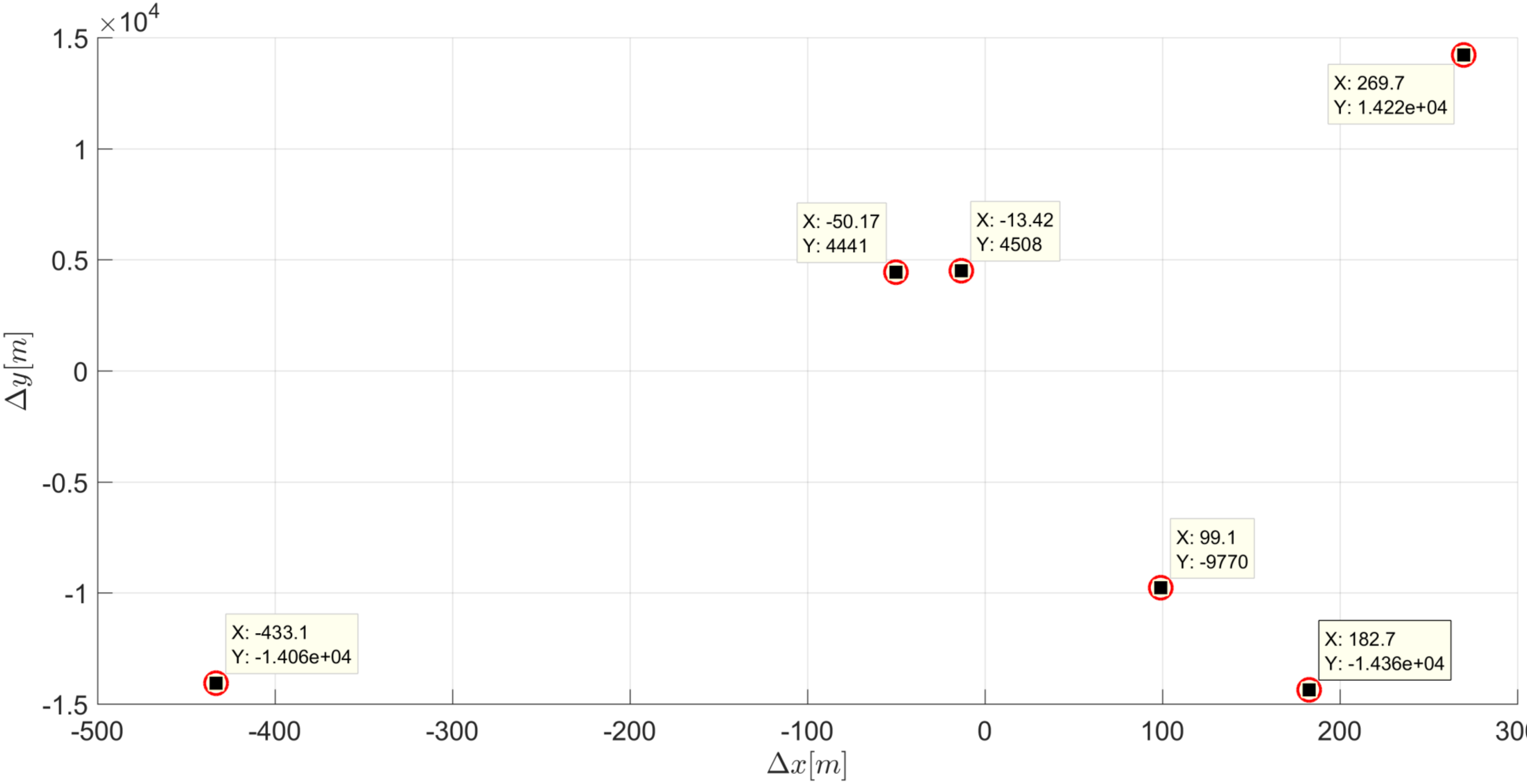
To reduce the collision risk, this undesired behavior could be addressed by introducing an along-orbit formation as an intermediate step…”e stage where the desired separations represent \parking" positions. Then, once the positions of the chasers are stable along the same orbit, these separations can be sequentially reduced to drive each chaser to the rendezvous state in a more controlled way when in close proximity to the

target”

IC leader-follower formation



IC rendezvous



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Stability Analysis for ICL Adaptive controller

- Candidate Lyapunov function is

$$V = \boldsymbol{X}^{\boldsymbol{T}} P \boldsymbol{X} + \frac{1}{2} \widetilde{\boldsymbol{\Theta}}^{\boldsymbol{T}} \Gamma \widetilde{\boldsymbol{\Theta}}$$

- For analysis purposes, express u_y as

$$u_y = u_{FB} + u_{AD}$$

where u_{FB} and u_{AD} are feedback and adaptive terms respectively.

$$u_{FB} = -\boldsymbol{K}\boldsymbol{X}$$

where $\boldsymbol{K}$ is a constant gain vector obtained from solving an LQR problem that minimizes the cost function

$$J = \int_0^\infty \boldsymbol{X}^T Q \boldsymbol{X} + R u_{FB}^2$$

Stability Analysis for ICL Adaptive controller

- The solution of the LQR problem produces the symmetric positive definite matrix P by solving the Algebraic Riccati Equation. P is the matrix used in the Lyapunov Candidate Function.

$$V = \boldsymbol{X}^{\boldsymbol{T}} P \boldsymbol{X} + \frac{1}{2} \widetilde{\boldsymbol{\Theta}}^{\boldsymbol{T}} \Gamma \widetilde{\boldsymbol{\Theta}}$$

- Since $u_y = \boldsymbol{Y}\widetilde{\boldsymbol{\Theta}} + \boldsymbol{Y}\widehat{\boldsymbol{\Theta}},$ then

$$u_{AD} = \boldsymbol{Y}\widetilde{\boldsymbol{\Theta}} + \boldsymbol{Y}\widehat{\boldsymbol{\Theta}} + \mathbf{KX}$$

and

$$\boldsymbol{Y}\widehat{\boldsymbol{\Theta}} = \hat{\rho}_t(t)\hat{C}_{D,t}\hat{V}_{r,t}^2 \frac{S_t}{2m_t} - \hat{\rho}_i(t)\hat{C}_{D,i}\hat{V}_{r,i}^2 \bar{u}$$

- The Lyapunov derivative can be written as

$$\dot{V} = \boldsymbol{X}^T \left(PA^* + {A^*}^T P \right) \boldsymbol{X} + 2\boldsymbol{X}^T P \boldsymbol{B} u_{AD} - \widetilde{\boldsymbol{\Theta}}^T \Gamma^{-1} \dot{\widehat{\boldsymbol{\Theta}}}$$

where $A^* = A - \boldsymbol{BK}$ is Hurwitz since $\boldsymbol{K}$ is obtained by solving the LQR problem

Stability Analysis for ICL Adaptive controller

- Substituting u_{AD} and $\dot{\hat{\boldsymbol{\Theta}}}$ in the Lyapunov derivative yields

$$\dot{V} = -\boldsymbol{X}^T Q_1 \boldsymbol{X} + 2\boldsymbol{X}^T P\boldsymbol{B}\left(\boldsymbol{Y}\widetilde{\boldsymbol{\Theta}} + \hat{\rho}_t(t)\hat{C}_{D,t}\hat{V}_{r,t}^2 \frac{S_t}{2m_t} - \hat{\rho}_i(t)\hat{C}_{D,i}\hat{V}_{r,i}^2\,\bar{u} + \mathbf{KX}\right) +$$
$$-\widetilde{\boldsymbol{\Theta}}^T\Gamma^{-1}\left(2\Gamma\boldsymbol{Y}^T\boldsymbol{B}^T P^T\boldsymbol{X} + \Gamma K_{ICL}\sum_{i=1}^{N_s}\boldsymbol{\mathcal{Y}}_i^T\boldsymbol{\mathcal{Y}}_i\widetilde{\boldsymbol{\Theta}}\right)$$

where Q_1 is the symmetric positive definite matrix that satisfies the Lyapunov equation

$$PA^* + {A^*}^T P = -Q_1$$

- Substituting the control law $\bar{u}$ we get

$$\dot{V} = -\boldsymbol{X}^T Q_1 \boldsymbol{X} + -\widetilde{\boldsymbol{\Theta}}^T K_{ICL}\sum_{i=1}^{N_s}\boldsymbol{\mathcal{Y}}_i^T\boldsymbol{\mathcal{Y}}_i\widetilde{\boldsymbol{\Theta}}$$

Stability Analysis for ICL Adaptive controller

- Before t=T, $\dot{V}$ can be upper bounded by

$$\dot{V} \leq -\boldsymbol{X}^{\boldsymbol{T}} Q_1 \boldsymbol{X}$$

using Barbalat's Lemma results in asymptotic regulation of the state vector $\boldsymbol{X}$.

- After t=T,

$$\dot{V} = -\boldsymbol{X}^T Q_1 \boldsymbol{X} + -\widetilde{\boldsymbol{\Theta}}^T K_{ICL} \sum_{i=1}^{N_s} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{\mathcal{Y}}_i \widetilde{\boldsymbol{\Theta}}$$

From this result, if can be shown that after t=T, V decreases exponentially as

$$V(t) \leq V(T) \exp\big(-\lambda(t-T)\big) \quad \forall t \geq T$$

where

$$\lambda = \min\{\lambda_{min}\{Q_1\}, \lambda_{min}\{K_{ICL} \sum_{i=1}^{N_s} \boldsymbol{\mathcal{Y}}_i^T \boldsymbol{\mathcal{Y}}_i\}\}$$