Extremum Seeking for Less Conservative Dwell-time Conditions

Path Following with Stable and Unstable Modes Subject to Time-Varying Dwell-Time Conditions

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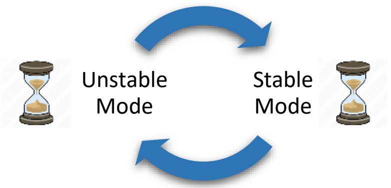
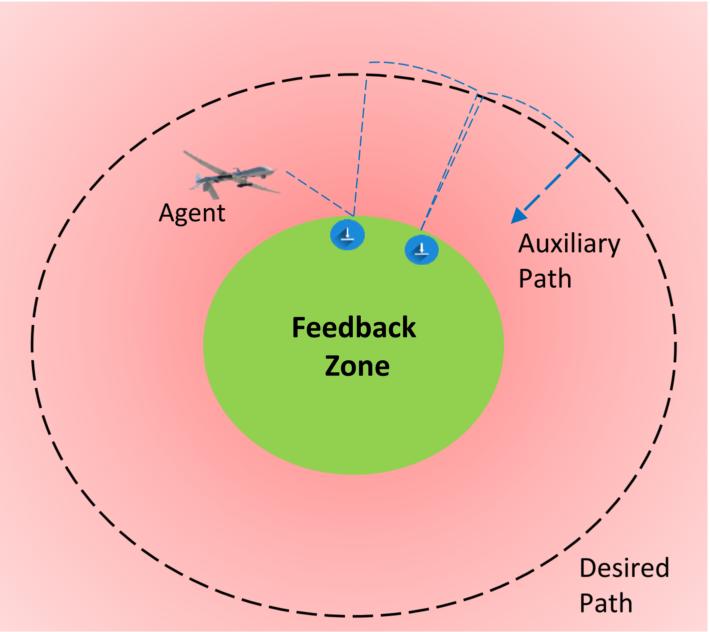
D. Le, H.-Y. Chen, A. R. Teel, and W. E. Dixon

Motivation

- Lyapunov-based switched system approaches
  - Switching between stable and unstable modes
  - Sufficient dwell-time criteria to ensure stability
  - **Drawback**: Dwell-time criteria are very conservative
    - Upon re-entry to the feedback zone, state information is collected and an ESC scheme is used to update the dwell-time





Control Design

- Error systems

$$e(t) = x(t) - x_\sigma(t) \quad \text{Tracking error}$$

$$\hat{e}(t) = \hat{x}(t) - x_\sigma(t) \quad \text{Estimate tracking error}$$

$$\tilde{e}(t) = x - \hat{x}(t) \quad \text{State estimate error}$$

where $\hat{x}$ is the state estimate and x_σ is the auxiliary path

- State Predictor

$$\dot{\hat{x}}(t) \triangleq \begin{cases} f(\hat{x}(t), t) + v(x(t), t) + v_r(\tilde{e}(t)) & p = a \\ f(\hat{x}(t), t) + v(\hat{x}(t), t) & p = u \end{cases}$$

State feedback available ($p = a$)

State feedback unavailable ($p = u$)

- Control Inputs

$$v(x(t), t) \triangleq \begin{cases} \dot{x}_\sigma(t) - \bar{d}\,\mathrm{sgn}(e(t)) - f(x(t), t) - ke(t) & p = a \\ \dot{x}_\sigma(t) - f(\hat{x}(t), t) - k\hat{e}(t) & p = u \end{cases}$$

$$v_r(\tilde{e}(t), t) \triangleq k_{\tilde{e}}\tilde{e}(t) + \bar{d}\,\mathrm{sgn}(\tilde{e}(t))$$

Switching Analysis

- Lyapunov Functions

$$V_e\left(e\left(t\right)\right) \triangleq \frac{1}{2}e^T\left(t\right)e\left(t\right)$$

$$V_{\hat{e}}\left(\hat{e}\left(t\right)\right) \triangleq \frac{1}{2}\hat{e}^T\left(t\right)\hat{e}\left(t\right)$$

$$V_{\tilde{e}}\left(\tilde{e}\left(t\right)\right) \triangleq \frac{1}{2}\tilde{e}^T\left(t\right)\tilde{e}\left(t\right)$$

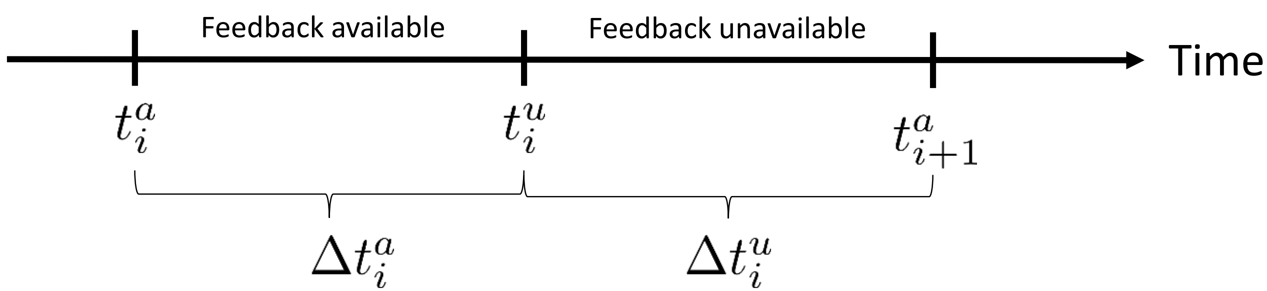
- Resulting error bounds
  - Extract convergence and divergence rates of error systems in each operating mode

$$\left\|\hat{e}(t)\right\| \leq C_{\hat{e}}\mathrm{e}^{-\min\{k,\lambda_a\}\left(t-t_i^{\mathcal{P}_a}\right)} \qquad p = a$$

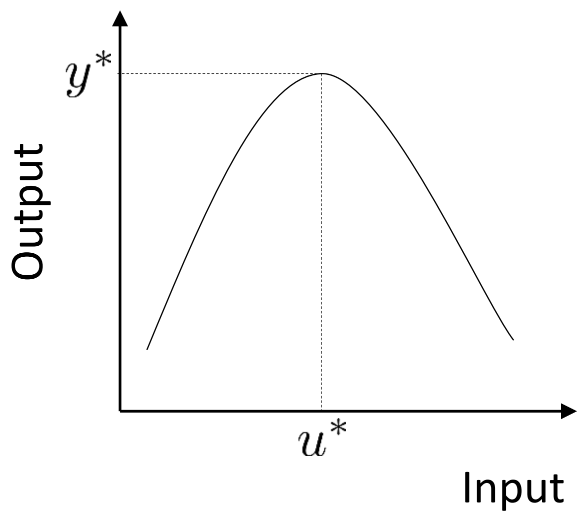
$$\left\|e\left(t\right)\right\| \leq \left[C_{\tilde{e}}\mathrm{e}^{2\lambda_u\left(t-t_i^{\mathcal{P}_u}\right)} - \frac{\bar{d}^2}{2\lambda_u}\right]^{1/2} + \left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\| \qquad p = u$$

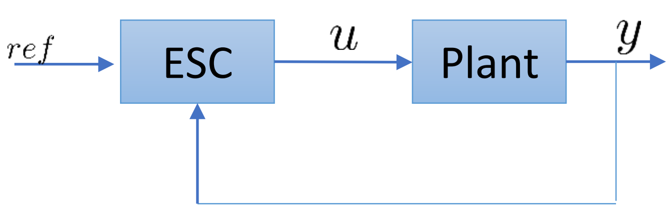
Switching Strategy

- Feedback Regions
  - Develop a minimum dwell-time condition to regulate the estimate tracking error to a minimum bound $\hat{e}_T$
- Feedback-denied regions
  - Develop a maximum dwell-time condition to ensure state estimate error is bounded by a maximum bound e_{max}
  - Collect data of the true error upon re-entry to the feedback region
  - Use ESC scheme to update the dwell-time condition
- Switching Instances



- Extremum Seeking Control (ESC)
  - Model-free optimization method
  - Exploits an unknown input-to-output mapping
  - ESC schemes drive an input to a steady state value such that the output is driven to a local (or global) extremum





Maximum Dwell-Time

- Consider the output

$$y\left(t_{i+1}^{\mathcal{P}_a}\right) \triangleq k_y\left(e_M - \phi - \left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\|\right)^2$$

Max error bound

Offset Parameter

True error upon Re-entry

  - A global extremum point corresponds to $e_M - \phi = \left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\|$

- Consider an input generated by the following ESC scheme

$$\begin{aligned}
\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right) + a\sin(wi),\\
\hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \hat{\Gamma}\left(t_{i}^{\mathcal{P}_a}\right) + \Delta\hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\Delta\hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq k\xi\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\xi\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \xi\left(t_{i}^{\mathcal{P}_a}\right) + \Delta\xi\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\Delta\xi\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \omega_l\left(y\left(t_{i+1}^{\mathcal{P}_a}\right) - \eta\left(t_{i+1}^{\mathcal{P}_a}\right)\right)a\sin(wi)\\
&\quad -\omega_l\xi\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\eta\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \eta\left(t_{i}^{\mathcal{P}_a}\right) + \Delta\eta\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\Delta\eta\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq -\omega_h\eta\left(t_{i+1}^{\mathcal{P}_a}\right) + \omega_h y\left(t_{i+1}^{\mathcal{P}_a}\right),
\end{aligned}$$

Maximum Dwell-Time

- While in the feedback-denied regions

Generated from ESC
Updated each time system re-enters feedback region

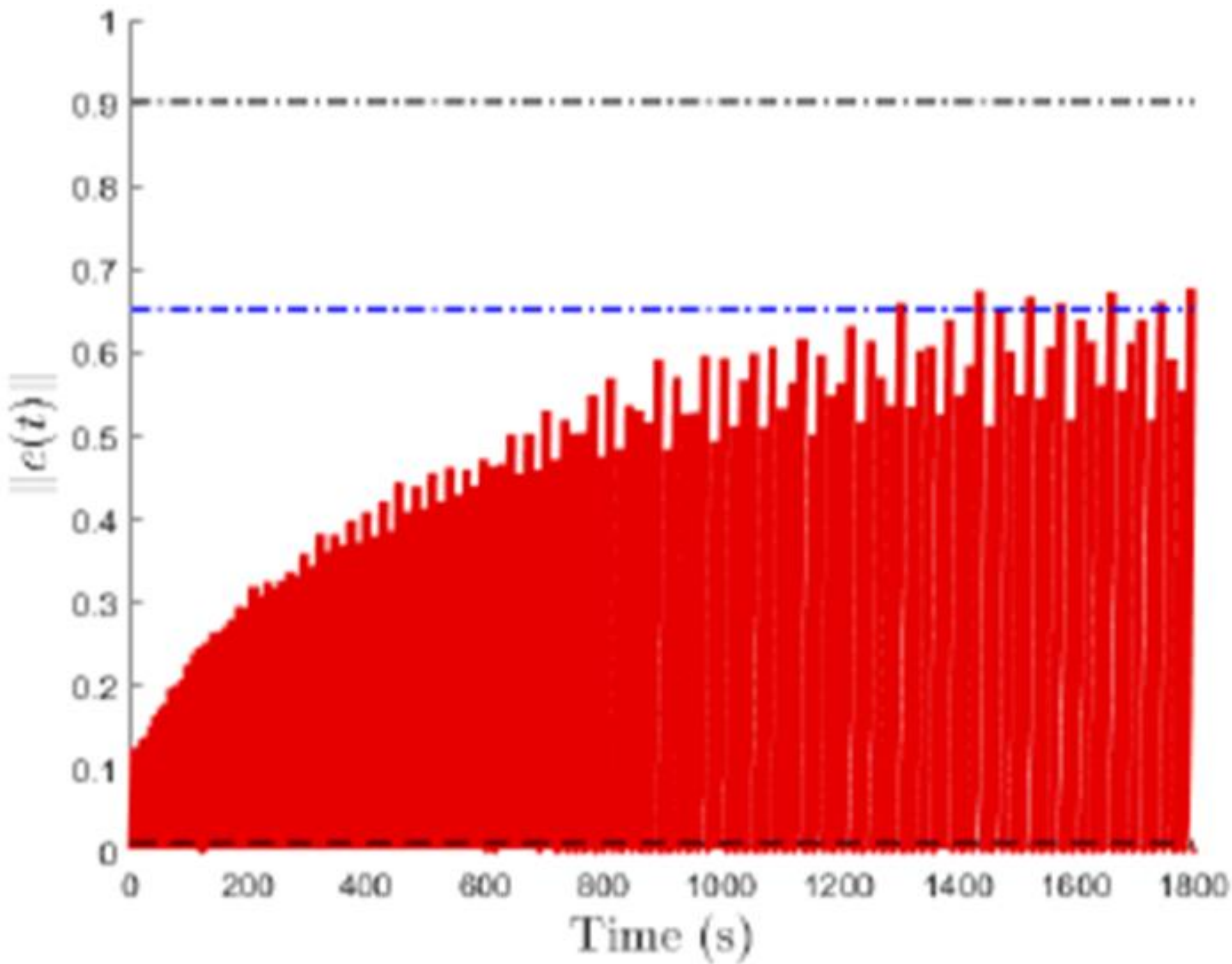
$$\left\| e\left(t_{i+1}^{\mathcal{P}^a}\right) \right\| \leq \left[C_{\tilde{e}} e^{2\lambda_u \Delta t_i^{\mathcal{P}_u}} - \frac{\bar{d}^2}{2\lambda_u} \right]^{1/2} + \left\| \hat{e}\left(t_i^{\mathcal{P}_u}\right) \right\| \leq \Gamma\left(t_{i+1}^{\mathcal{P}^a}\right)$$

- This results in the maximum dwell-time condition

$$\Delta t_i^{\mathcal{P}_u} \leq \frac{1}{2\lambda_u} \ln \left(\frac{\left(\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)^2 - \left\| \hat{e}\left(t_i^{\mathcal{P}_u}\right) \right\| \right)^2 + \frac{\bar{d}^2}{2\lambda_u}}{C_{\tilde{e}}} \right)$$

- ESC will drive the input $\Gamma \rightarrow \Gamma^*$ which will drive the output $y \rightarrow y^*$
- Time-varying dwell-time condition

Simulation Results



Simulation Results