INSTITUTE FOR COMPUTATIONAL ENGINEERING & SCIENCES
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The University of Texas at Austin
Electrical and Computer Engineering
Cockrell School of Engineering

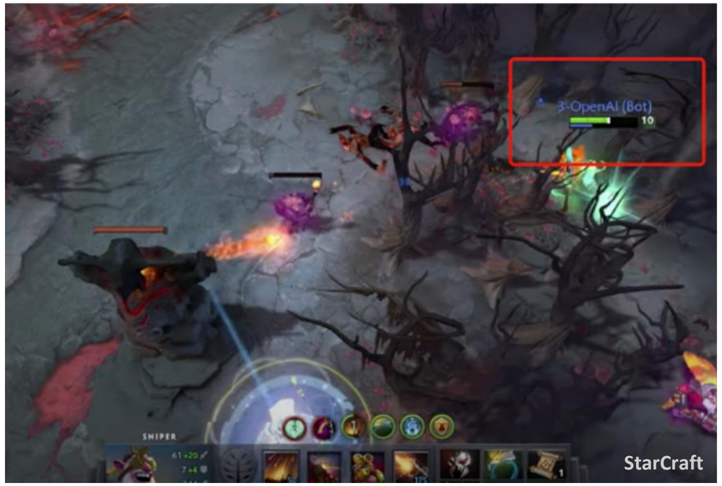
Online Policy Learning for Unknown and Varying Tasks in Adversarial Environments

Mahsa Ghasemi, Abolfazl Hashemi, Haris Vikalo, Ufuk Topcu

CoE Review

April 30th, 2021

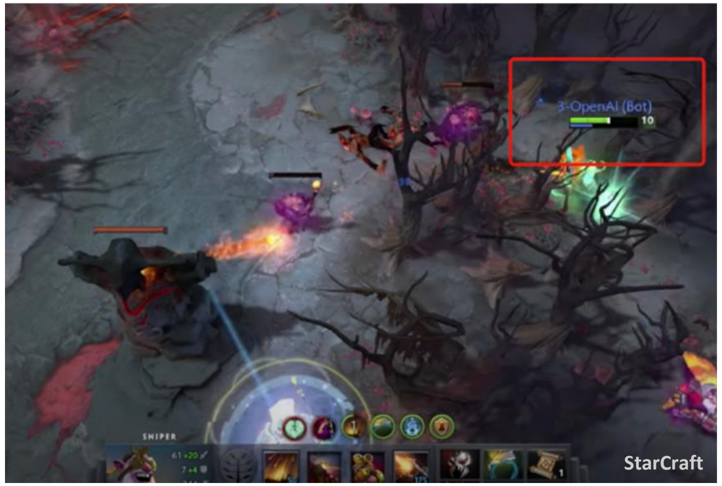
Sequential Decision Making



KUKA

Sequential Decision Making

Sequential Interaction with the environment



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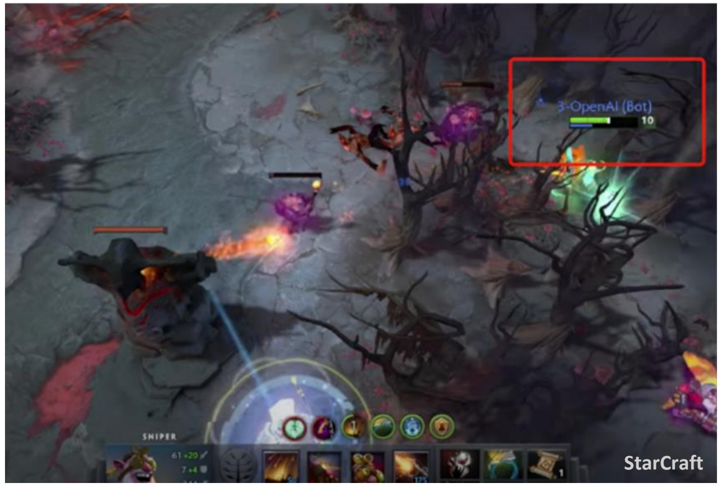

Learning from a fixed reward

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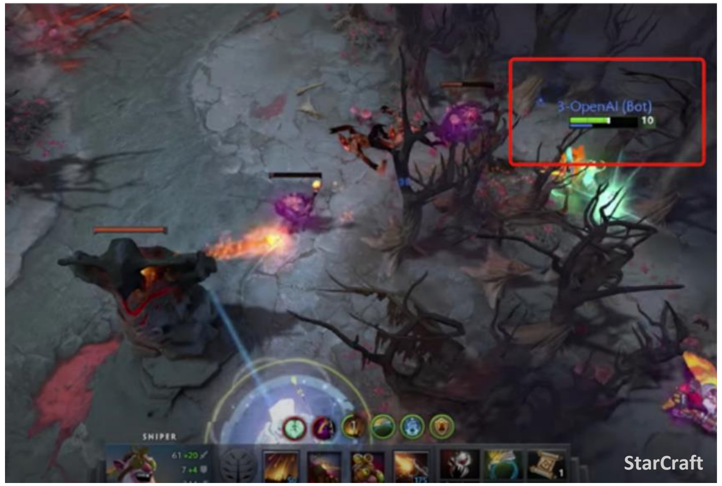
Offline: access to a lot of data



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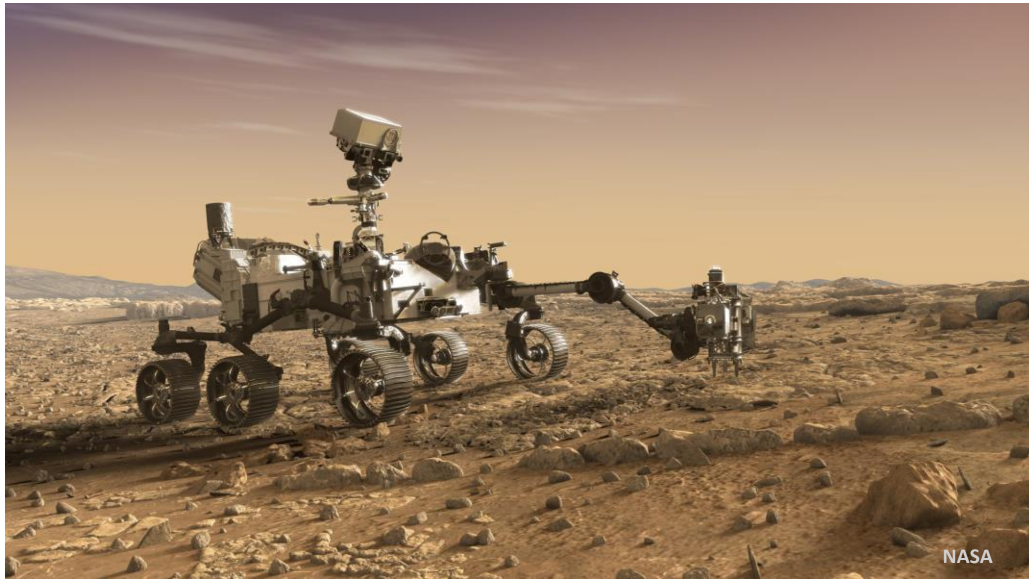
Sequential Decision Making with Varying Tasks



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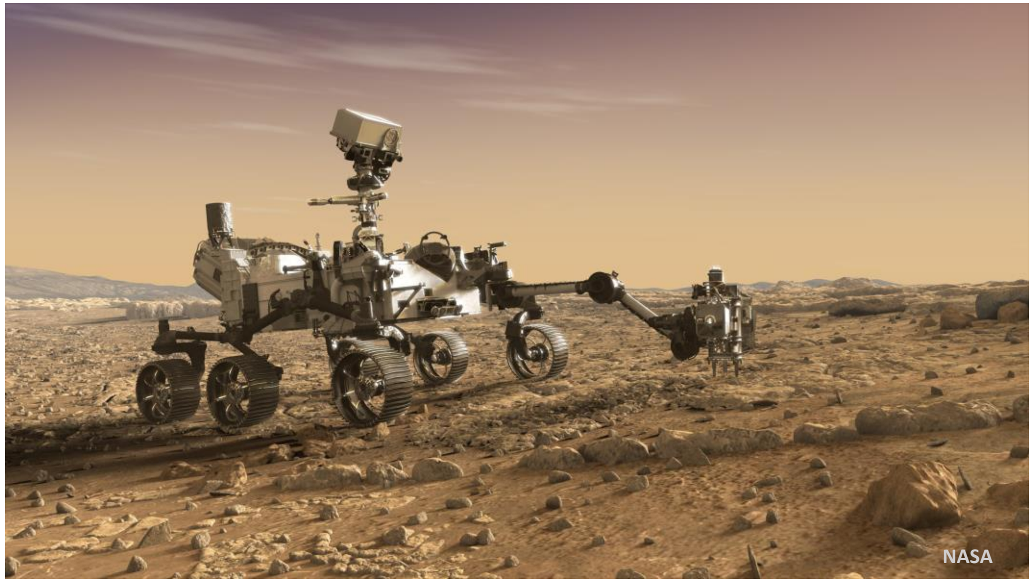
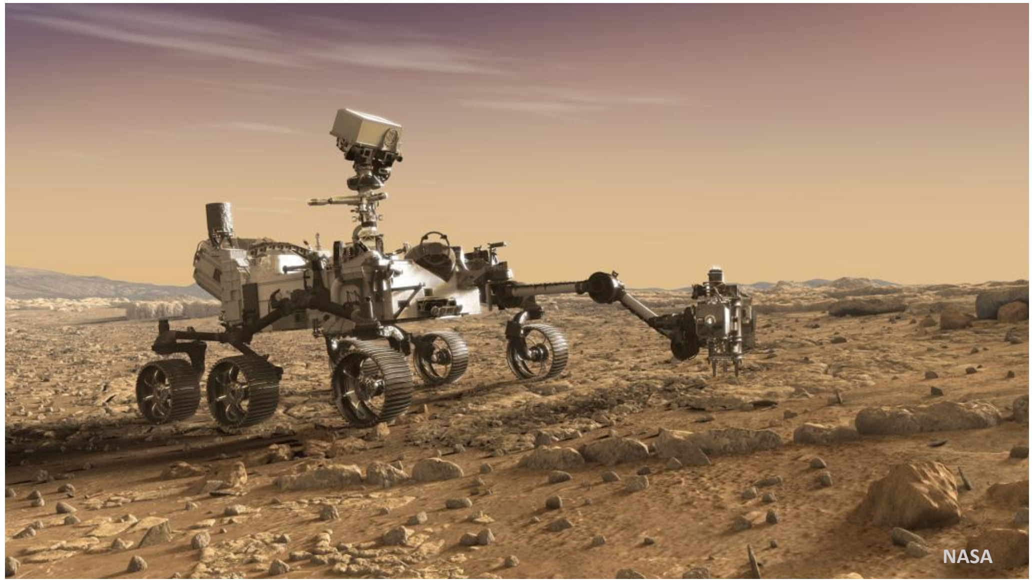


Evolving environment and task

Sequential Decision Making with Varying Tasks



Evolving environment and task

Safety-critical operation

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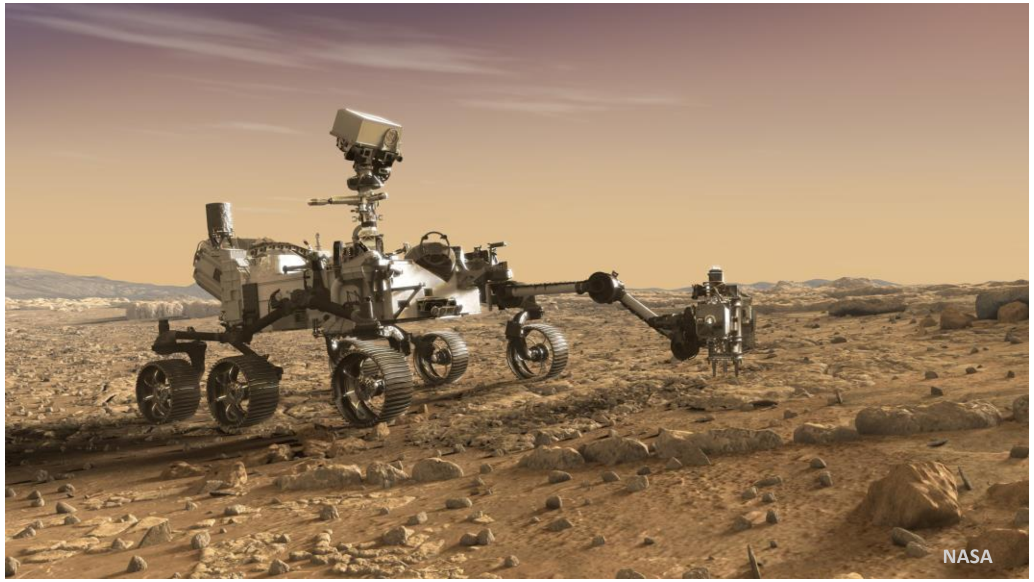

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Safety-critical operation

Limited feedback from the environment

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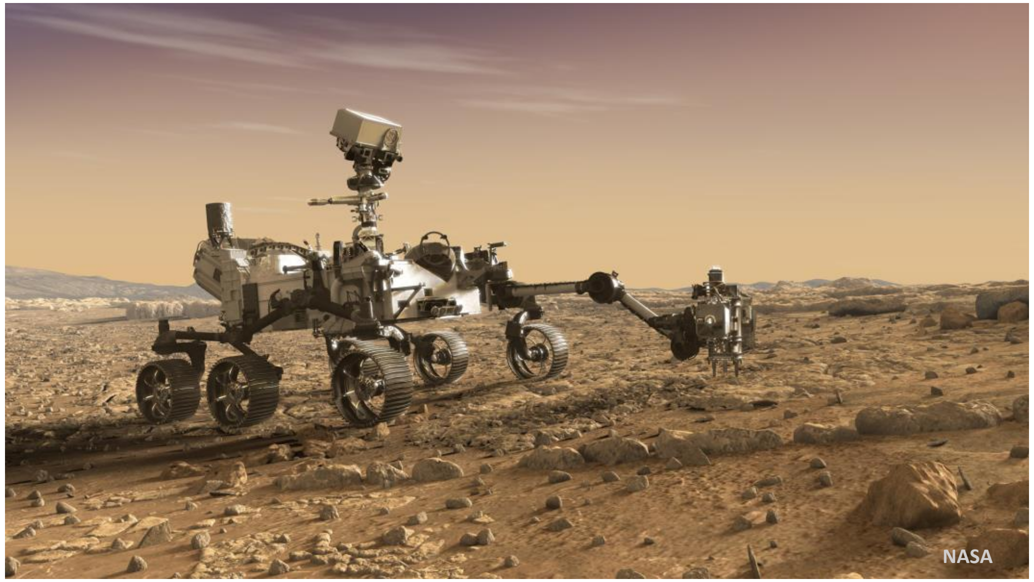

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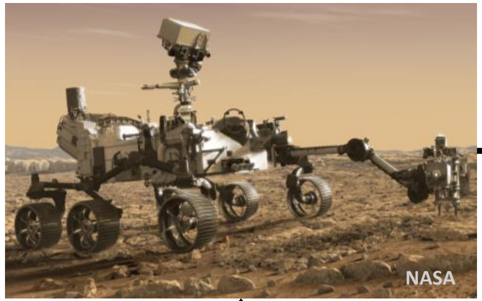
How can we design online algorithms with high probability guarantees for varying tasks?

Online Learning with Implicit Exploration for Varying Tasks

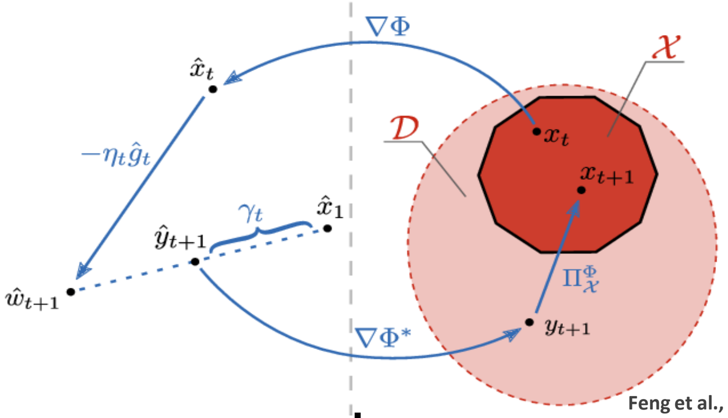


Play an action

Limited, time-varying feedback

Reward estimator

Estimated reward



Feng et al., 2020

Online mirror descent (OMD)

Occupancy measure

Policy update

New policy

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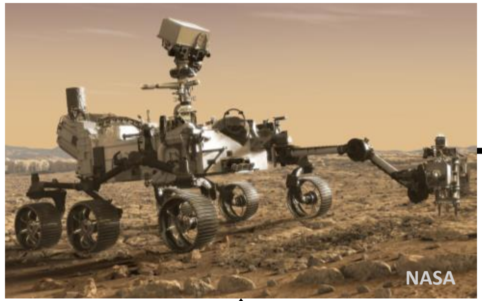

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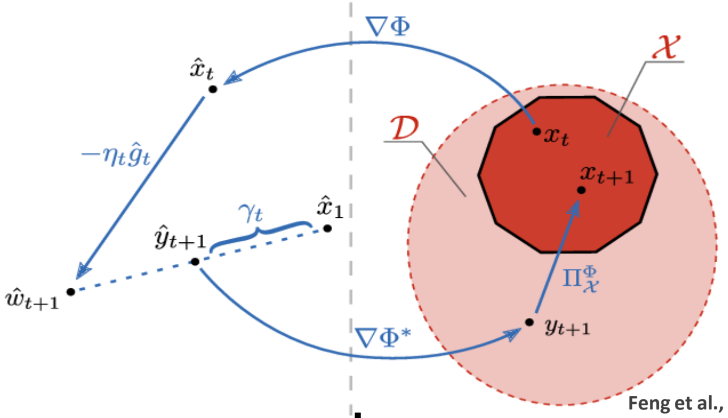

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Contributions:

- A novel optimistically-biased reward estimator for implicit exploration
- Policy search using online mirror descent (OMD)
- Minimax optimal regret bound with high probability

Adversarial Markov Decision Process (A-MDP)

finite state space

finite action space

$$\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \ell_t)$$

probabilistic transition function

time-varying loss function

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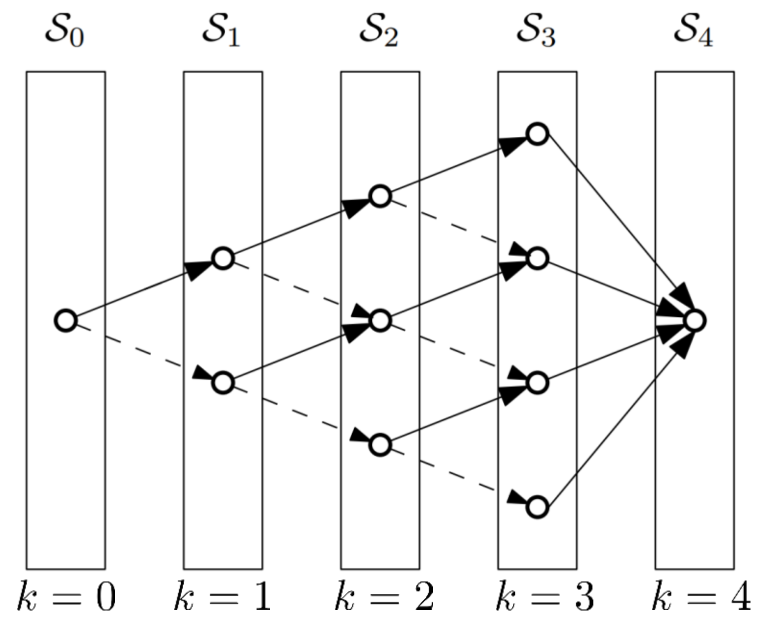
Loop-free episodic A-MDP:

- States are partitioned into layers
- Transition only exists from one layer to the next



Neu et al., 2020

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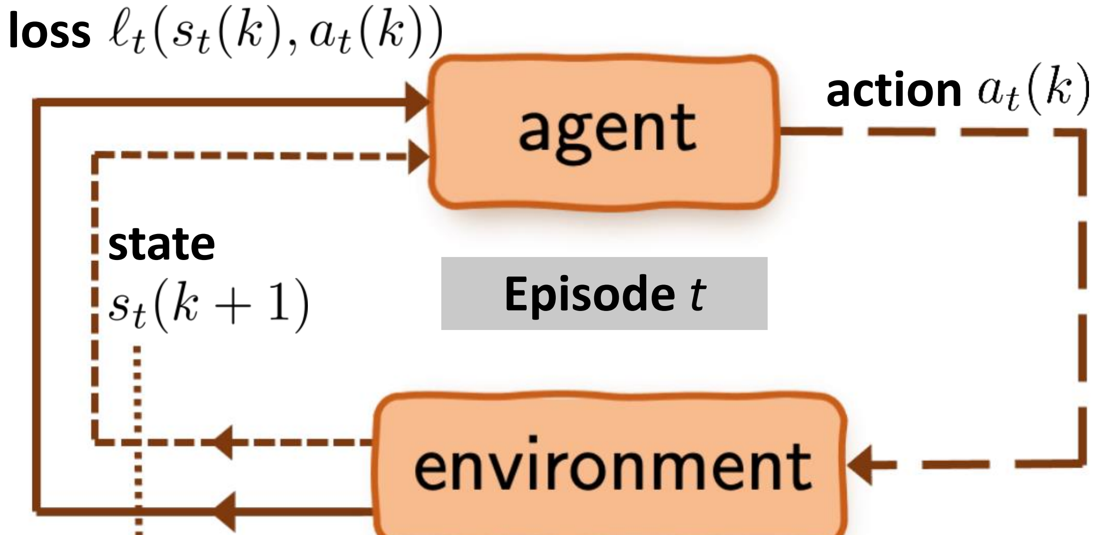

Chen et al., 2019

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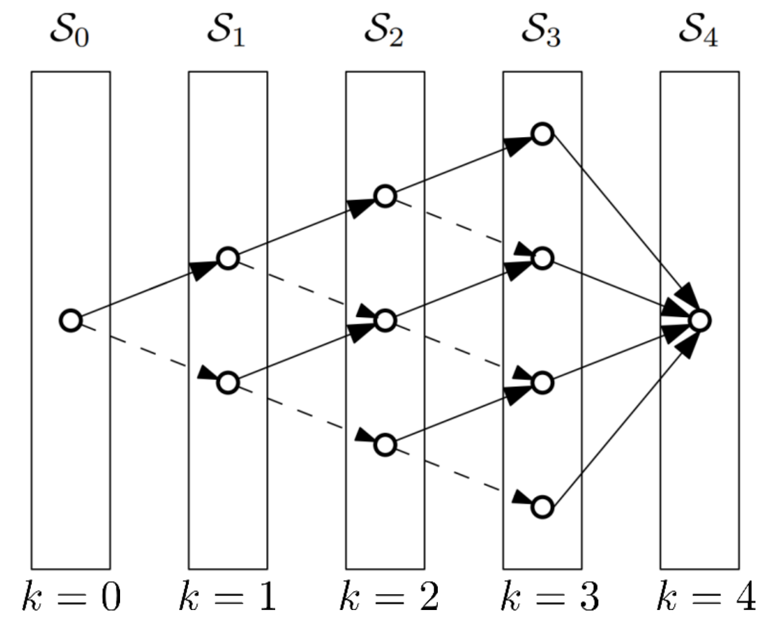

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Agent's Policy Representation via Occupancy Measure

Looking for a time-varying stochastic policy $\pi_t : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$

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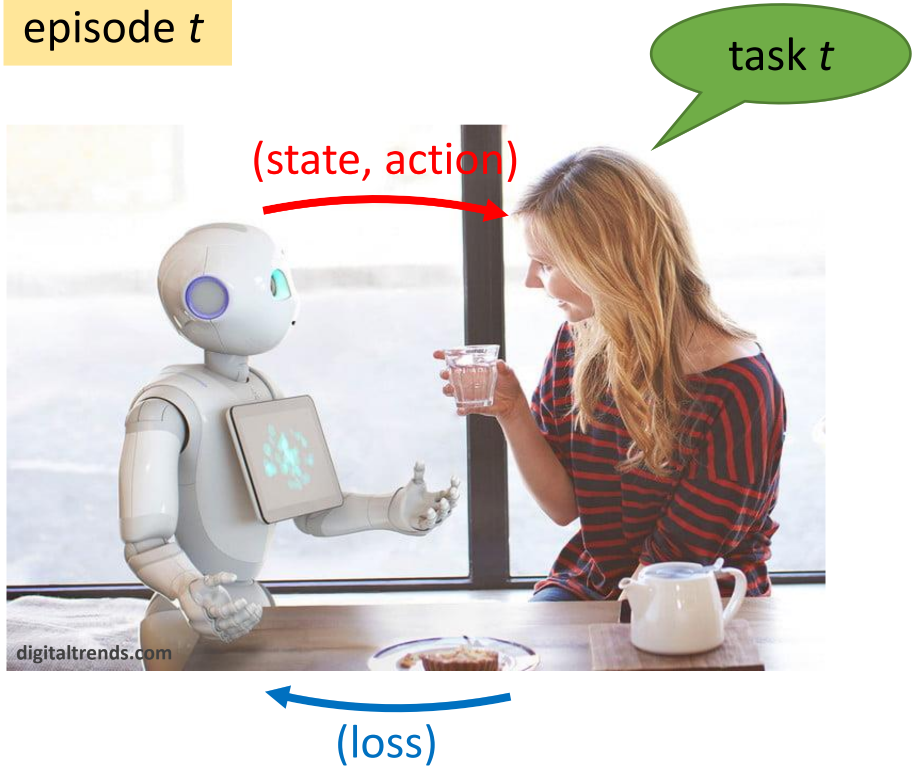
Stochastic stationary policy given an occupancy measure

$$\pi^{\rho}(a|s) = \frac{\rho(s, a)}{\sum_{a' \in \mathcal{A}} \rho(s, a')}, \quad \forall (s, a) \in \mathcal{S} \times \mathcal{A}$$

Regret Minimization

episode t



Regret Minimization

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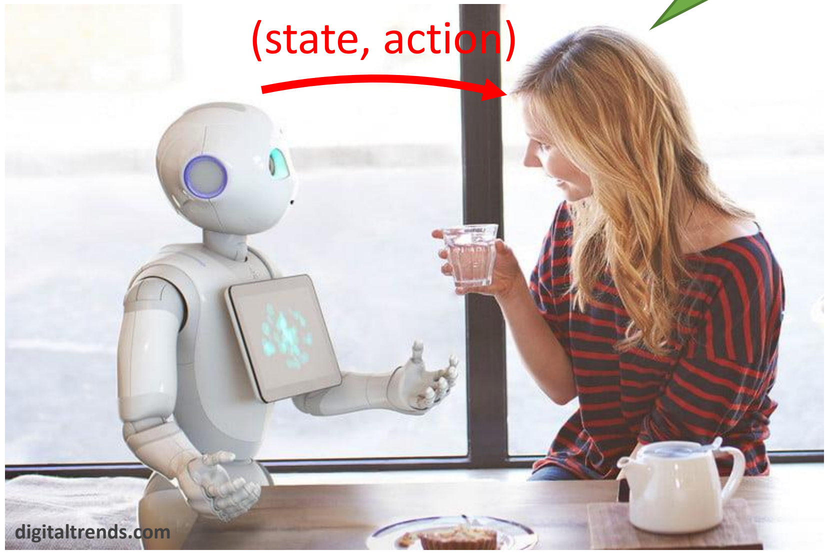
task t

(state, action)



(loss)

Unknown and time-varying loss function (A-MDP)

$$\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \ell_t)$$

Bandit feedback

$$\ell_t(s_t(k), a_t(k))$$

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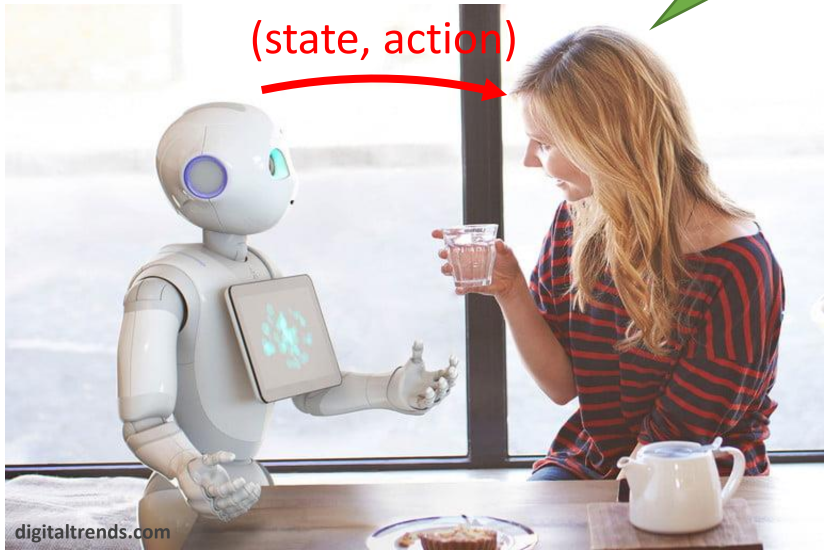

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Learn a policy with sublinear regret:

$$\mathcal{R}_T := \max_{\pi} \mathcal{L}_T - \mathcal{L}_T(\pi)$$

best fixed policy in hindsight

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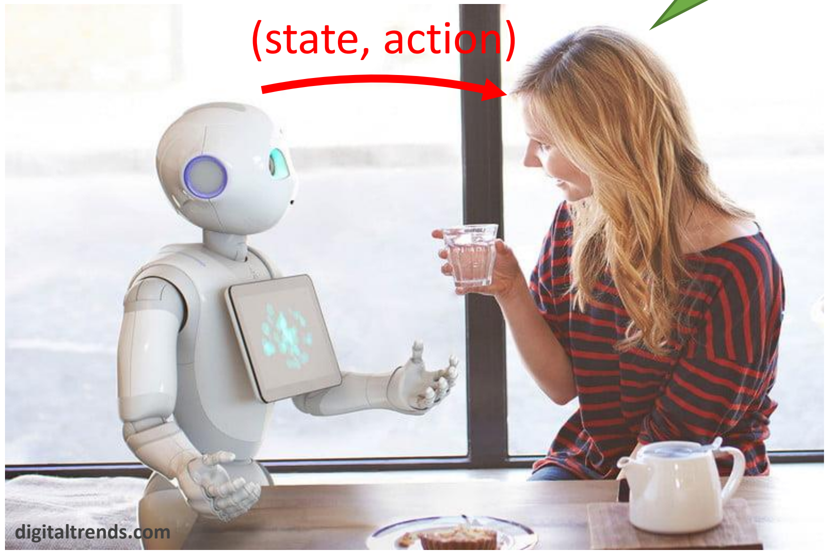
episode *t*



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Question: Can we obtain low regret with high probability?

Optimistic Loss Estimator

Bandit feedback → Estimating the loss of all state-action pairs

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A novel optimistically biased estimator for the loss function:

history at current episode

$$\hat{\boldsymbol{\ell}}_t(s,a) = \frac{\ell_t(s,a)}{\boldsymbol{\rho}_t(s,a) + \gamma}\mathbb{I}\{(s,a) \in \mathbf{h}(t)\}$$

exploration parameter

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exploration parameter

Optimistically biased

$$\mathbb{E}\left[\hat{\boldsymbol{\ell}}_t(s,a)|\mathbf{h}(t-1)\right] \leq \ell_t(s,a)$$

→ Implicit exploration

Policy Optimization via Online Mirror Descent

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An OMD algorithm utilizing the proposed loss estimator:

$$\boldsymbol{\rho}_{t+1} = \arg \min_{\rho \in \Delta(\mathcal{M})} \Big\{ \overset{\text{learning rate}}{\eta} \underbrace{\langle \rho, \hat{\boldsymbol{\ell}}_t \rangle}_{\text{loss}} + \underbrace{\overset{\text{unnormalized KL divergence}}{D}(\rho \| \boldsymbol{\rho}_t)}_{\text{policy change}} \Big\}$$

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Constrained optimization → Two-step procedure

$$\tilde{\boldsymbol{\rho}}_{t+1} = \arg \min_{\rho} \left\{ \eta \langle \rho, \hat{\boldsymbol{\ell}}_t \rangle + D(\rho \| \boldsymbol{\rho}_t) \right\}$$

$$\boldsymbol{\rho}_{t+1} = \arg \min_{\rho \in \Delta(\mathcal{M})} \{ D(\rho \| \tilde{\boldsymbol{\rho}}_{t+1}) \}$$

No-Regret Learning with High-Probability

Result: Establishing sublinear regret bounds both on expectation and with high-probability

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Theorem: (high-probability regret bound)

Let $\delta \in (0,1)$. If

$$\eta = \gamma = \sqrt{L \frac{\log(|\mathcal{S}||\mathcal{A}|/L)}{2T|\mathcal{S}||\mathcal{A}|}},$$

with probability at least $1-\delta$,

$$\text{regret} \leq \mathcal{O}(\sqrt{LT|\mathcal{A}||\mathcal{S}|\log(|\mathcal{S}||\mathcal{A}|/L)}\log\tfrac{1}{\delta}).$$

episode length number of episodes number of states number of actions

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Minimax optimal regret (up to logarithmic terms)

No-Regret Learning for Uniformly Ergodic MDPs

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Theorem: (high-probability regret bound for uniformly ergodic A-MDP)

Let $\delta \in (0,1)$. With probability at least $1-\delta$,

$$\text{regret} \leq CT^{\frac{2}{3}}\tau^{\frac{2}{3}}|\mathcal{S}|^{\frac{2}{3}}|\mathcal{A}|^{\frac{2}{3}}\sqrt{\log(|\mathcal{S}||\mathcal{A}|)\log T\log\tfrac{1}{\delta}} + C'\tau\log T.$$

time horizon   mixing time   number of states   number of actions

Conclusion and Future Work

- Studied the problem of learning unknown and varying tasks in adversarial environments
- Proposed an online learning framework that achieves a minimax optimal regret bound with high probability
- Extended our framework to the class of general A-MDPs

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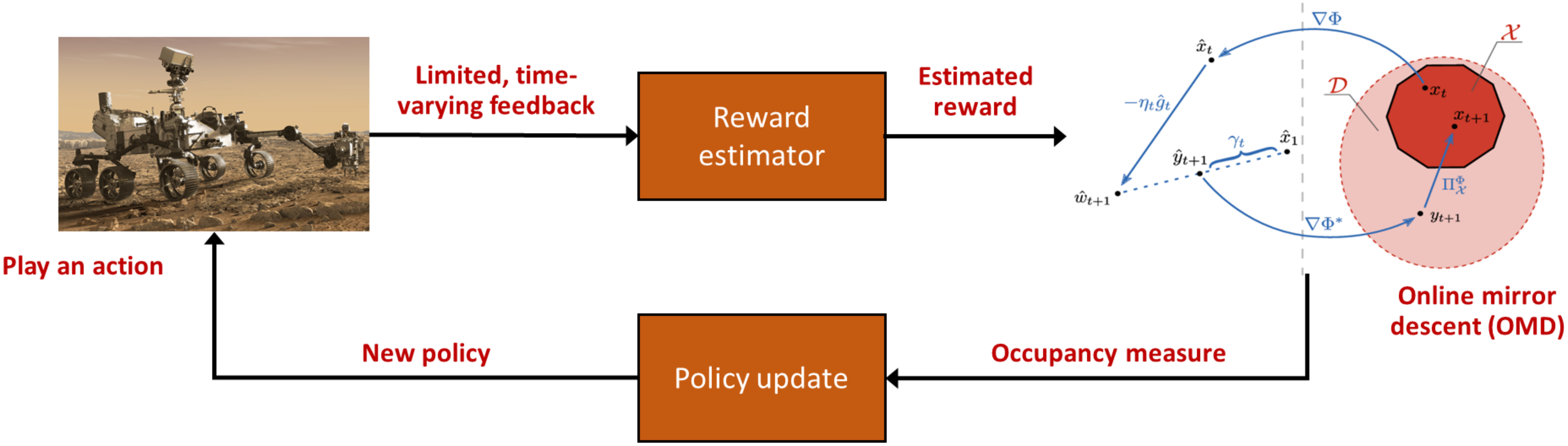
Future Directions

- Structure-aware and game-theoretic online learning
- Parameter-free and anytime algorithms
- Unknown, time-varying dynamics and large-scale state spaces



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