Deep and Cooperative Learning for Approximate Dynamic Programming

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Outline

- Deep Neural Network Identifier for Approximate Optimal Control
  - Online System ID using DNN
  - Update the DNN in Real-Time
  - Approximate System Dynamics
  - Use System Model to Approximate Optimal Controller

- Cooperative Approximate Optimal Control
  - Leverage Multiple Agents Learning the Same Function

Deep Neural Network-based Approximate Optimal Tracking for Unknown Nonlinear Systems

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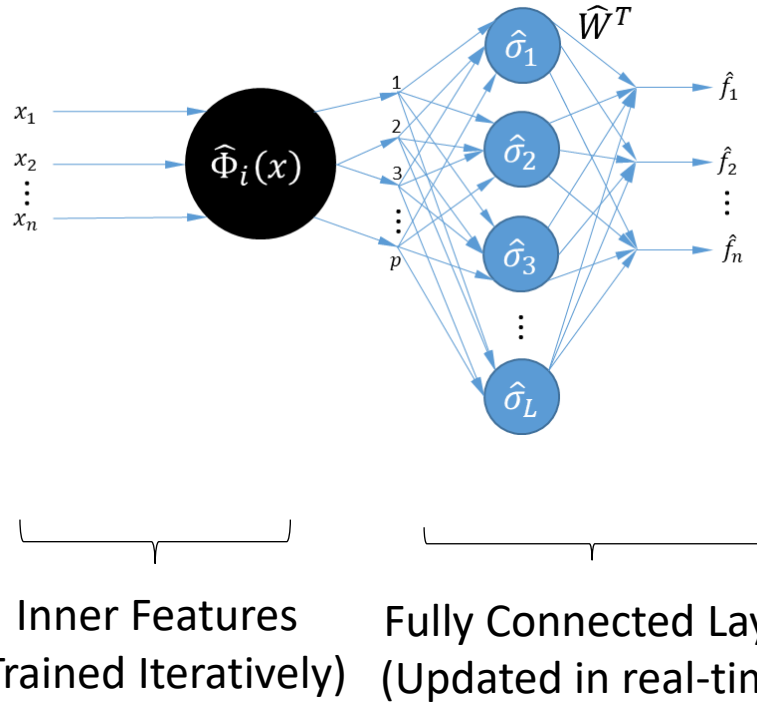
Real-Time DNN Approximation

- Multi-Timescale DNN
  - Online system identifier
  - Better than single hidden-layer
  - Inner features updates iteratively (switched)
  - Output-layer updates in real-time
  - Can we use a DNN to approximate an optimal control policy online?



Real-Time DNN Approximation

Dynamical System

Given a control affine nonlinear dynamical system:

$$\dot{x} = f(x) + g(x)u$$

DNN Approximation

Approximate the drift dynamics with a DNN:

Dynamics: $f(x) = \theta^T \phi^*\left(\Phi^*(x)\right) + \epsilon(x)$

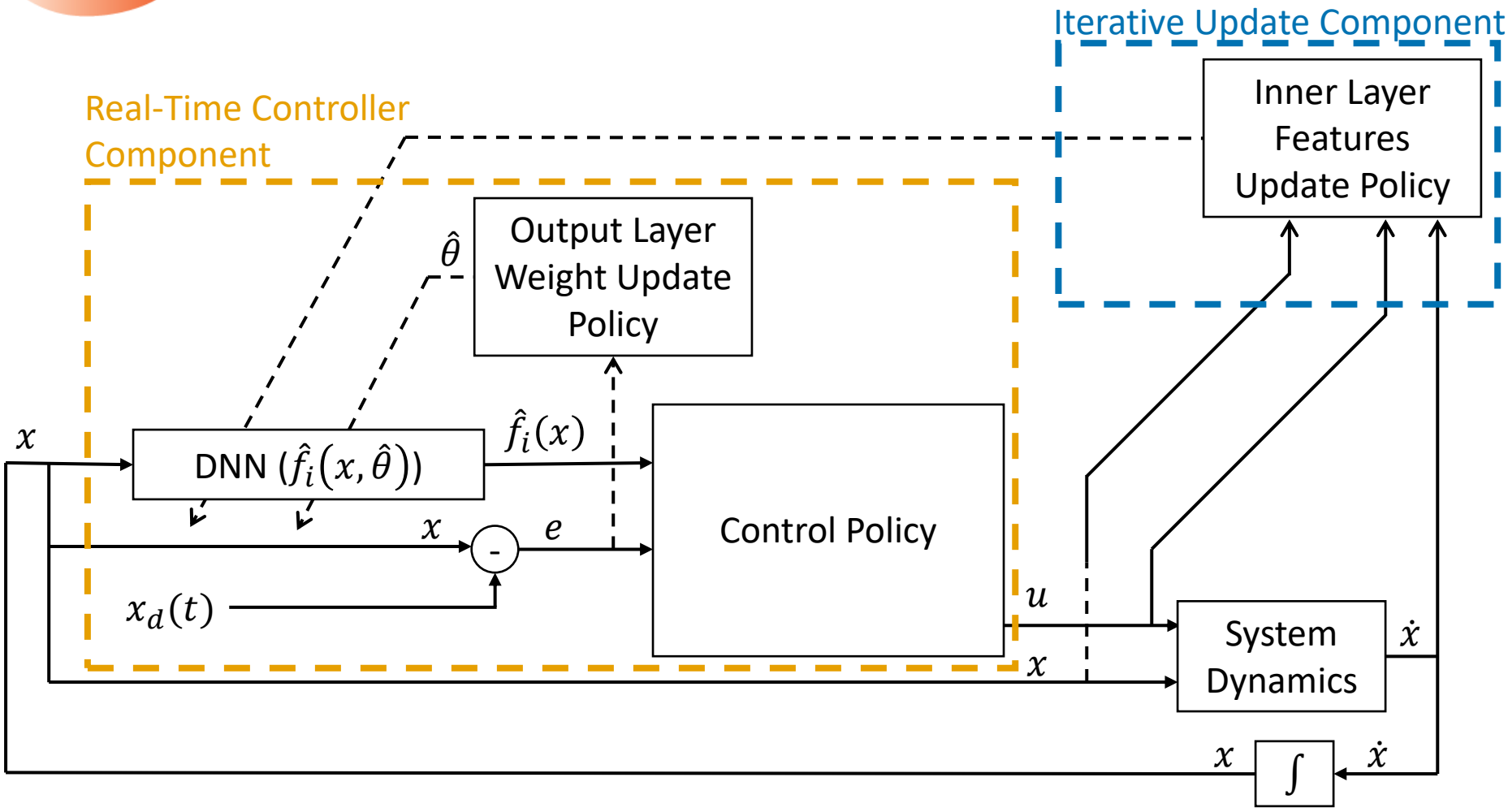
i^{th} DNN Approximation: $\hat{f}_i(x, \hat{\theta}) = \hat{\theta}^T \hat{\phi}_i\left(\hat{\Phi}_i(x)\right)$

Looks Like:

Reminds us of linearly parameterizable dynamics:

$$f(x) = \theta^T \phi^*\left(\Phi^*(x)\right) + \epsilon(x) \approx \theta^T Y(x) + \epsilon(x)$$

Real-Time DNN Approximation



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DNN-Identifier ADP

Dynamical System

Given a control affine nonlinear dynamical system:

$$\dot{\zeta} = F(\zeta) + G(\zeta)\mu$$

Control Objective

Design a controller, u, which minimizes a cost function:

$$J(\zeta, \mu) = \min_{u(\tau)\epsilon U} \int_0^\infty Q\big(\zeta(\tau)\big) + \mu(\tau)^T R\mu(\tau)\ d\tau$$

Cost-to-Go

Optimal value function:

$$V^*(x) = \min_{u(\tau)\epsilon U} \int_0^\infty Q\big(\zeta(\tau)\big) + \mu(\tau)^T R\mu(\tau)\ d\tau$$

Hamilton Jacobi Bellman Equation

Hamilton Jacobi Bellman (HJB) equation:

$$0 = Q(\zeta) + \mu^*(\zeta)^T R \mu^*(\zeta) + \nabla_\zeta V^*(\zeta)\big(F(\zeta) + G(\zeta)\mu^*(\zeta)\big)$$

Optimal Controller

From the HJB equation:

$$\mu^*(\zeta) = -\frac{1}{2} R^{-1} G(\zeta)^T \left(\nabla_\zeta V^*(\zeta)\right)^T$$

- Cannot solve HJB analytically.
- Approximate the Value Function (V^*)
  - Universal Function Approximation Property
  - Neural Networks

DNN-Identifier ADP

Optimal Value Function and Optimal Control Policy:

$$V^*(\zeta) = W^T\sigma(\zeta) + \varepsilon(\zeta) \qquad \mu^*(\zeta) = -\frac{1}{2}R^{-1}G(\zeta)^T\left(\nabla_\zeta\sigma(\zeta)^T W + \nabla_\zeta\varepsilon(\zeta)^T\right)$$

Unknown: Neural weights $\quad \widehat{W}_c, \widehat{W}_a \to W$

$\widehat{W}_c$: Critic weight
$\widehat{W}_a$: Actor weight

Value Function and Optimal Control Policy Approximation

$$\hat{V}\left(\zeta, \widehat{W}_c\right) = \widehat{W}_c{}^T\sigma(\zeta) \qquad \hat{\mu}\left(\zeta, \widehat{W}_a\right) = -\frac{1}{2}R^{-1}G(\zeta)^T\left(\nabla_\zeta\sigma(\zeta)^T\widehat{W}_a\right)$$

Bellman Error (BE): Residual from HJB

$$\hat{\delta}\left(\zeta, \widehat{W}_c, \widehat{W}_a\right) \triangleq Q(\zeta) + \hat{\mu}\left(\zeta, \widehat{W}_a\right)^T R\hat{\mu}\left(\zeta, \widehat{W}_a\right) + \nabla_\zeta\hat{V}\left(\zeta, \widehat{W}_c\right)\left(\hat{F}_i(\zeta) + G(\zeta)\hat{\mu}\left(\zeta, \widehat{W}_a\right)\right)$$

NN Update Laws

Instantaneous Bellman Error: Residual from HJB

$$\hat{\delta}(t) \triangleq \hat{\delta}\left(\zeta(t), \hat{W}_c(t), \hat{W}_a(t)\right)$$

Weight Update Laws using R-MBRL

$$\dot{\hat{W}}_c(t) = -\eta_{c1}\Gamma\frac{\omega(t)}{\rho(t)}\hat{\delta}(t) - \eta_{c2}\Gamma\frac{1}{N}\sum_{i=1}^{N}\frac{\omega_i(t)}{\rho_i(t)}\hat{\delta}_i(t)$$

Minimize the Bellman Error

$$\dot{\Gamma}(t) = \left(\lambda\Gamma(t) - \eta_{c1}\frac{\Gamma(t)\omega(t)\omega(t)^T\Gamma(t)}{\rho(t)} - \eta_{c2}\Gamma(t)\left(\frac{1}{N}\sum_{i=1}^{N}\frac{\omega_i(t)\omega_i^T(t)}{\rho_i(t)}\right)\Gamma(t)\right)\mathbf{1}_{\{}$$

Adjust the "learning rate"

$$\dot{\hat{W}}_a(t) = -\eta_{c1}\left(\hat{W}_a(t) - \hat{W}_c(t)\right) - \eta_{a2}\hat{W}_a(t) + \eta_{c1}\frac{G_\sigma(t)^T\hat{W}_a(t)\omega(t)^T}{4\rho(t)}\hat{W}_c(t)$$
$$+\eta_{c2}\left(\frac{1}{N}\sum_{i=1}^{N}\frac{G_{i\sigma}^T\hat{W}_a(t)\omega_i(t)^T}{4\rho_i(t)}\right)\hat{W}_c(t)$$

Follow the critic weights

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NN Update Laws

Instantaneous Bellman Error: Residual from HJB

$$\hat{\delta}(t) \triangleq \hat{\delta}\left(\zeta(t), \hat{W}_c(t), \hat{W}_a(t)\right)$$

Weight Update Laws using R-MBRL

$$\dot{\hat{W}}_c(t) = -\eta_{c1}\Gamma\frac{\omega(t)}{\rho(t)}\hat{\delta}(t) - \eta_{c2}\Gamma\frac{1}{N}\sum_{i=1}^{N}\frac{\omega_i(t)}{\rho_i(t)}\hat{\delta}_i(t)$$

On-Trajectory Point

$$\dot{\Gamma}(t) = \left(\lambda\Gamma(t) - \eta_{c1}\frac{\Gamma(t)\omega(t)\omega(t)^T\Gamma(t)}{\rho(t)} - \eta_{c2}\Gamma(t)\left(\frac{1}{N}\sum_{i=1}^{N}\frac{\omega_i(t)\omega_i^T(t)}{\rho_i(t)}\right)\Gamma(t)\right)\mathbf{1}_{\{\underline{\Gamma}\leq\|\Gamma\|\leq\bar{\Gamma}\}}$$

$$\dot{\hat{W}}_a(t) = -\eta_{c1}\left(\hat{W}_a(t) - \hat{W}_c(t)\right) - \eta_{a2}\hat{W}_a(t) + \eta_{c1}\frac{G_\sigma(t)^T\hat{W}_a(t)\omega(t)^T}{4\rho(t)}\hat{W}_c(t) + \eta_{c2}\left(\frac{1}{N}\sum_{i=1}^{N}\frac{G_{i\sigma}^T\hat{W}_a(t)\omega_i(t)^T}{4\rho_i(t)}\right)\hat{W}_c(t)$$

Off-Trajectory Points

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Theorem 1

Theorem 1:

Provided the necessary assumptions, gain conditions, and initial conditions hold, then the tracking error $e(t)$, weight estimation errors $\widetilde{W}_c(t)$and $\widetilde{W}_a(t)$, system ID DNN output weight estimation errors $\tilde{\theta}(t)$, state estimator error $\tilde{x}(t)$, and transient control policy $\mu(t)$ are uniformly ultimately bounded (UUB).

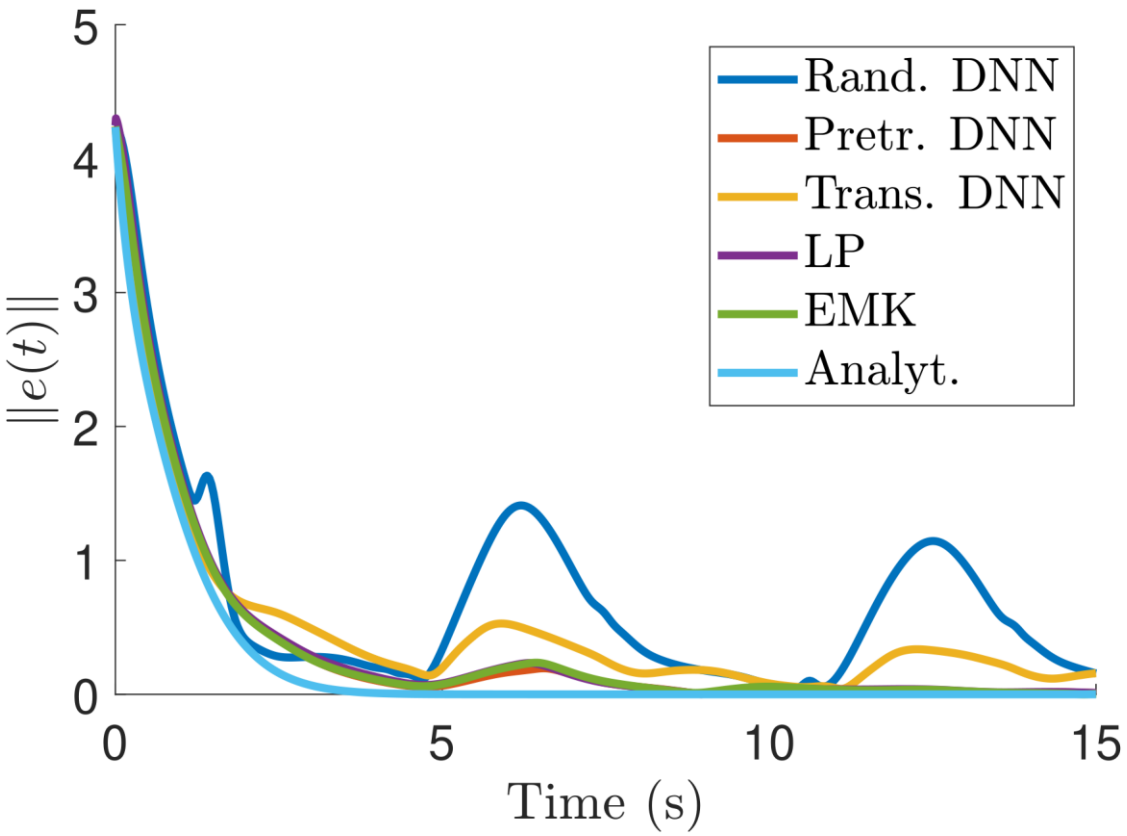
- While iteratively retraining (switching) DNN-based system ID terms:
  - $\|e\|$ converges to a neighborhood around zero
  - $\left\|\tilde{\theta}\right\|$, $\|\tilde{x}\|$ converge to a neighborhood around zero
  - $\left\|\widetilde{W}_c\right\|, \left\|\widetilde{W}_a\right\|$ converge to a neighborhood around zero
  - $\mu(t)$ is an approximation of the optimal control policy

Theorem 1

- Contribution of Theorem 1
  - Trajectory Tracking
  - Approximate Optimal Control Policy
  - Allows iteratively updating the DNN inner-layer features.

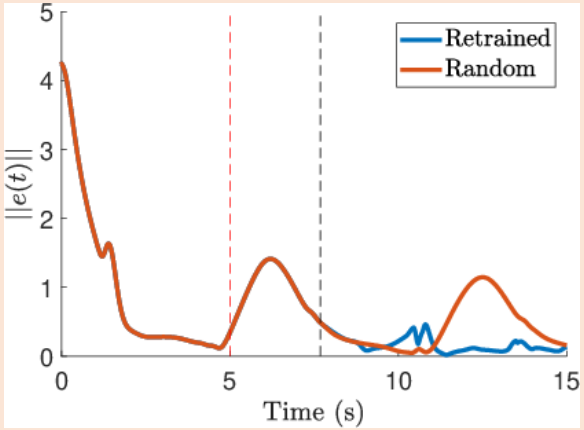
- Obstacles in Theorem 1
  - Updating the DNN introduces discontinuities
    - $\hat{\delta}(t) \triangleq \nabla_\zeta \hat{V}\left(\zeta, \widehat{W}_c\right)\left(\hat{F}_i(\zeta) + G(\zeta)\hat{\mu}\left(\zeta, \widehat{W}_a\right)\right) + \cdots$
    - $\dot{\widehat{W}}_c(t) = -\eta_{c1}\Gamma\frac{\omega(t)}{\rho(t)}\hat{\delta}(t) - \eta_{c2}\frac{1}{N}\sum_{i=1}^{N_e}\frac{\omega_i(t)}{\rho_i(t)}\hat{\delta}_i(t)$
    - $\dot{\widehat{W}}_c(t)$ is discontinuous
    - $\widehat{W}_c(t)$ is nonsmooth
    - Requires use of generalized solutions to show stability

DNN ADP Simulation Results

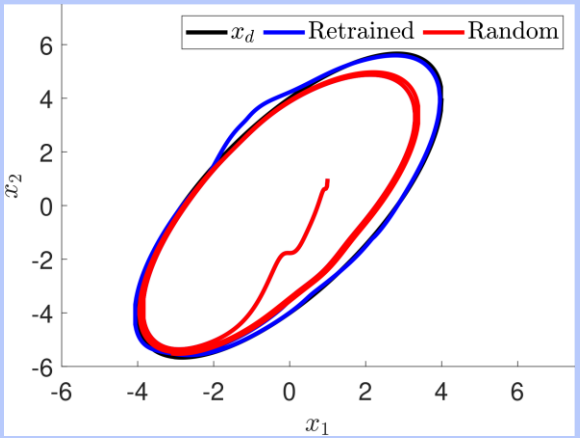


DNN ADP Simulation Results

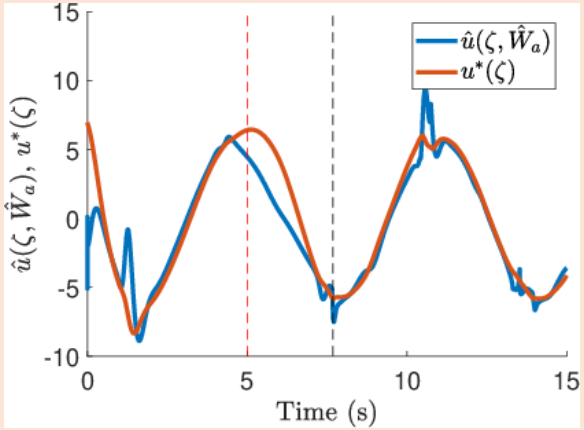
Norm of Error



Phase Plot



Optimal Control Comparison



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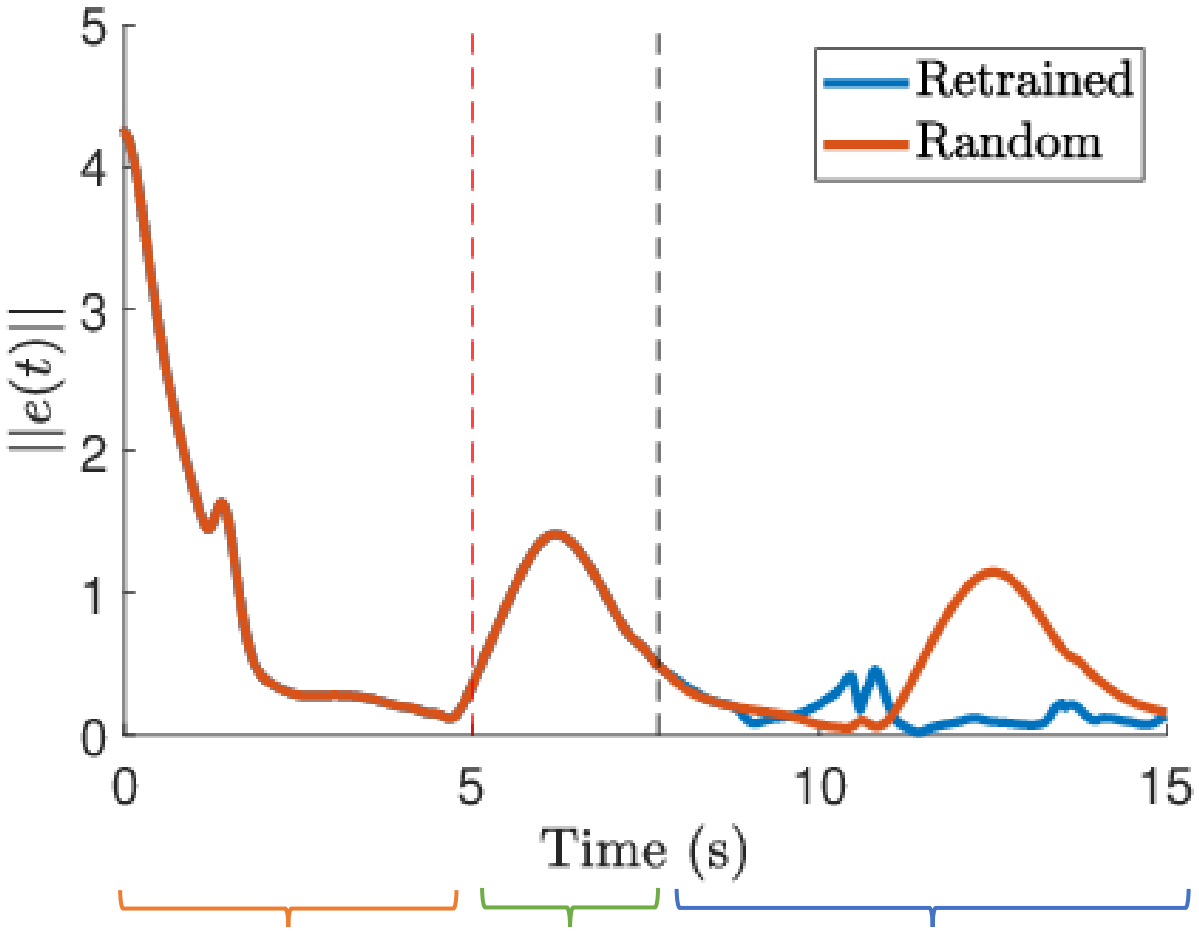
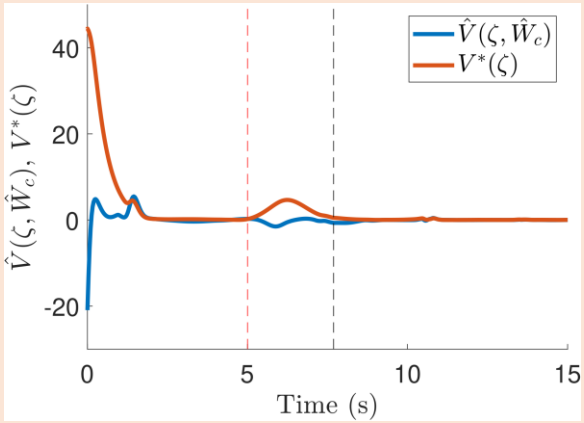
DNN ADP Simulation Results

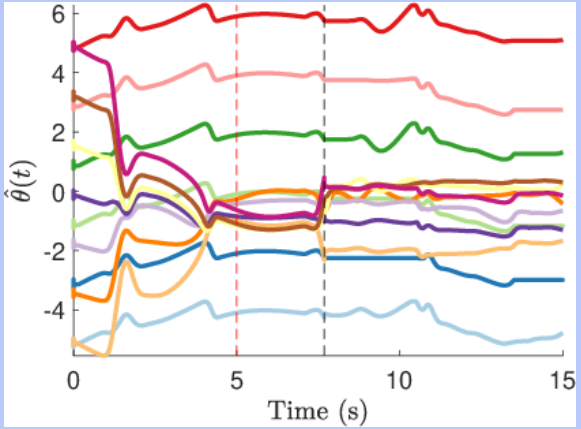


DNN ADP Simulation Results

Value Function Comparison



DNN Output Weights



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Cooperative Model-Based Reinforcement Learning for Approximate Optimal Tracking

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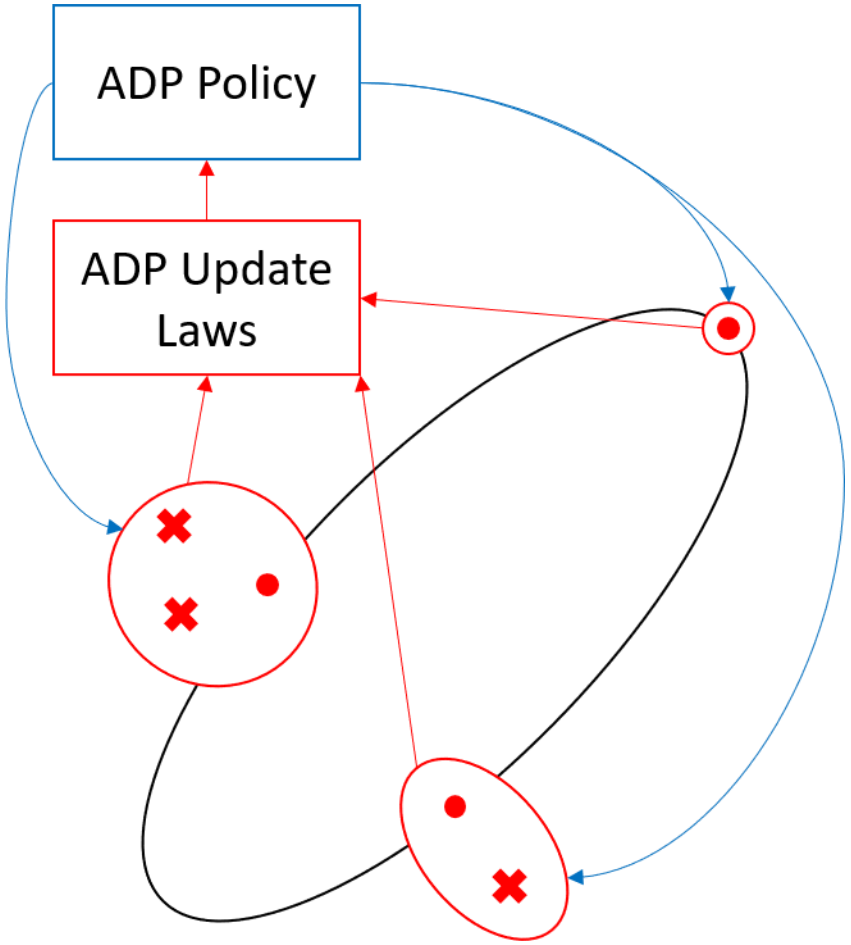
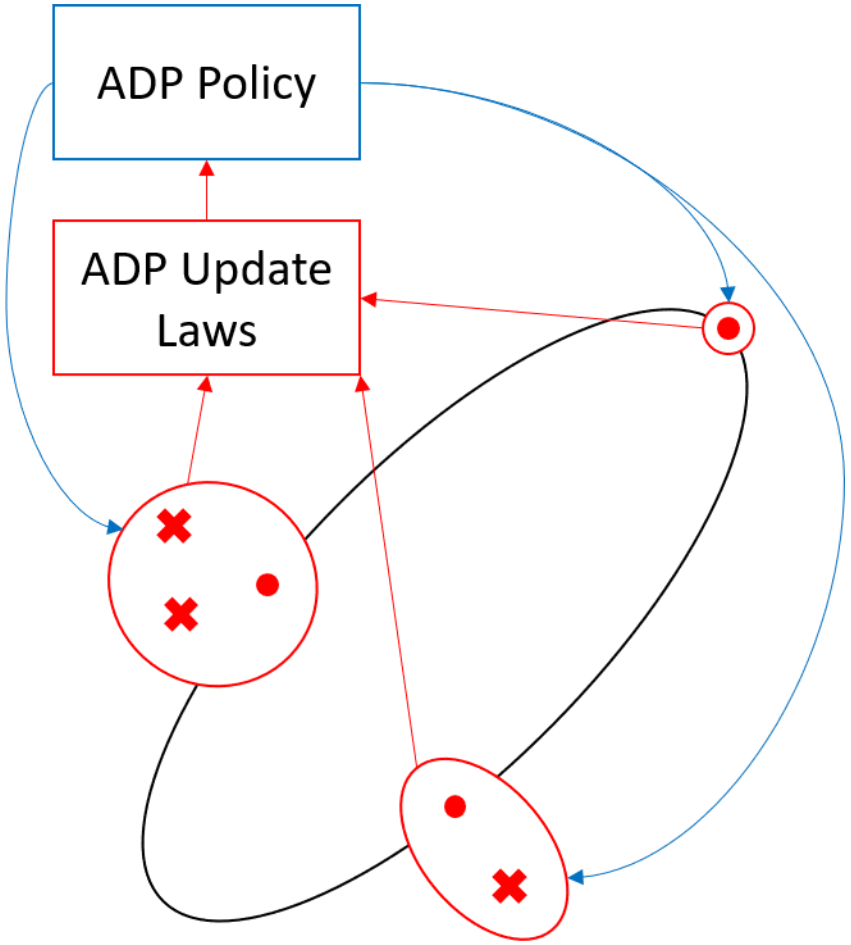
Cooperative BE Extrapolation

- Developed Method
  - Homogeneous agents
    - i.e., identical dynamics
  - Each agent computes one on- and some off-trajectory BE extrapolation points.
  - BE extrapolation is used in update laws.
  - Divides computational expense of BE extrapolation.



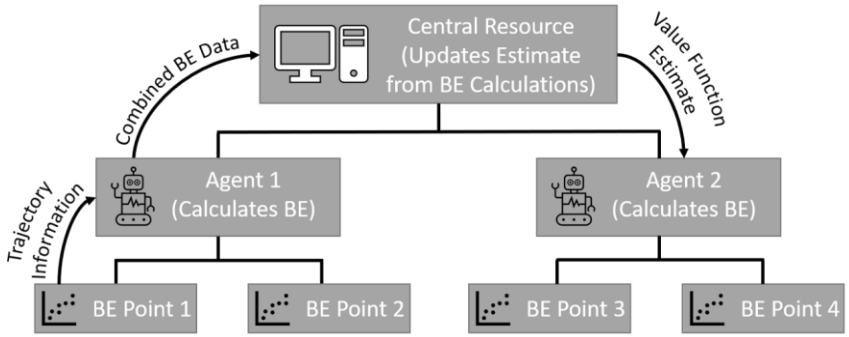
Cooperative BE Extrapolation

- Why Cooperative?
  - Share data between agents
  - Train DNN system ID
    - Collect data faster
    - More agents means more data
  - Value Function Appx.
    - Without excessive computational loads on one agent

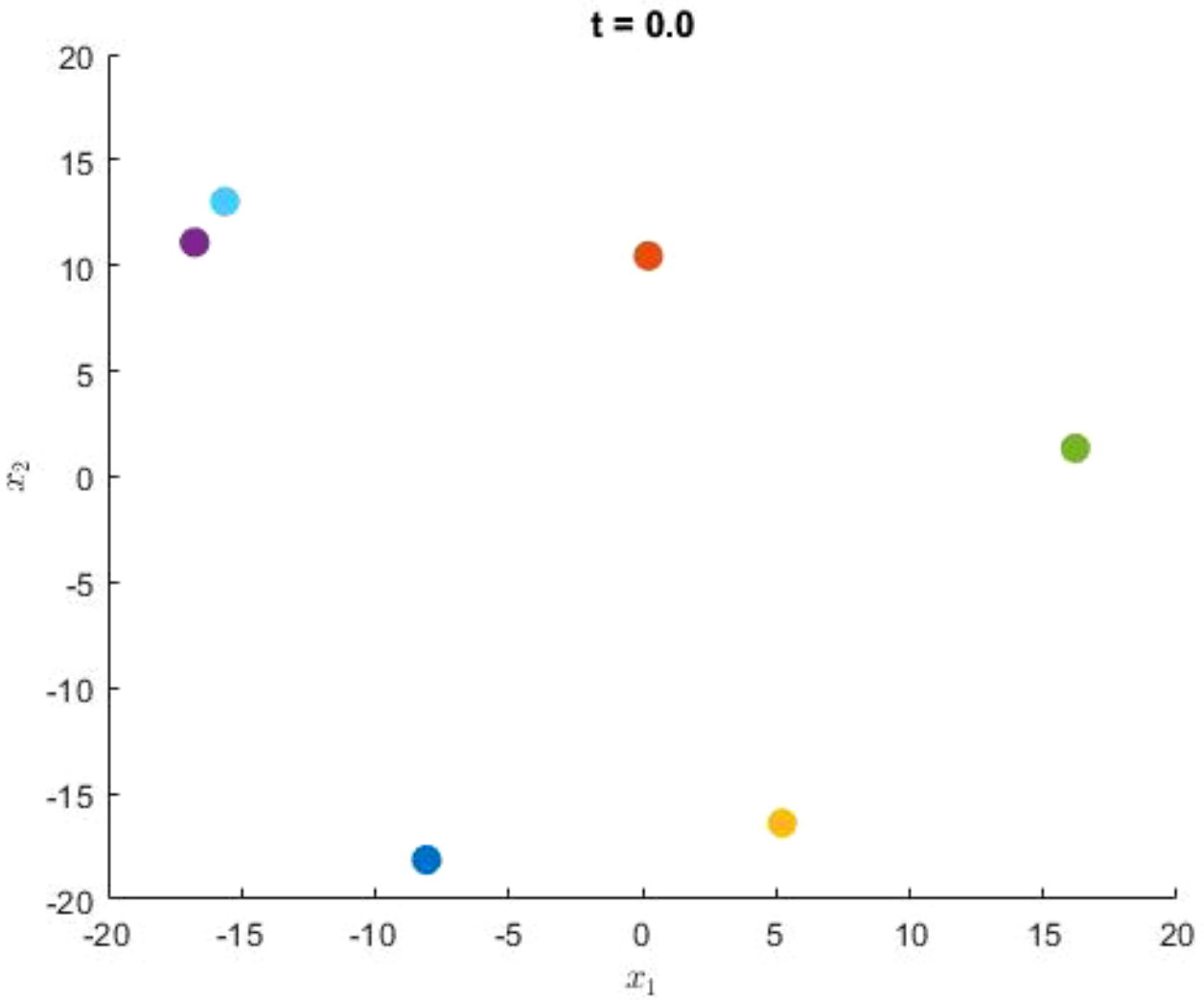


Cooperative BE Extrapolation

- Developed Method
  - Alternative view
  - Each agent calculates BE trajectory data.
  - Agent communicates BE data to central resource
  - Central resource communicates ADP policy to agents



Simulation



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Conclusion

- Future Work
  - DNN ADP
    - Perform DNN-based value function approximation
      - Currently single-layer NN
  - Cooperative ADP
    - Extend to heterogeneous agents
    - How do we use share data between agents to cooperatively accomplish a global objective?

Questions?

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