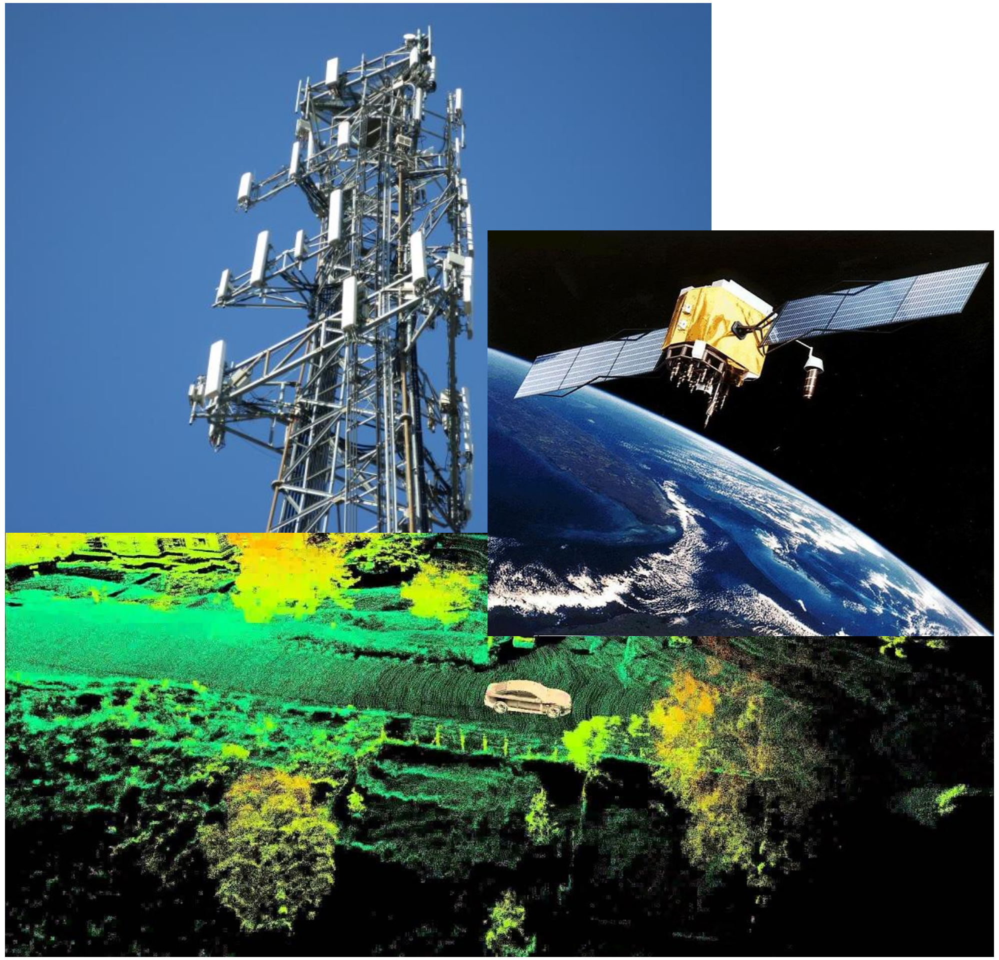
Path Planning in Environments with Intermittent State Feedback

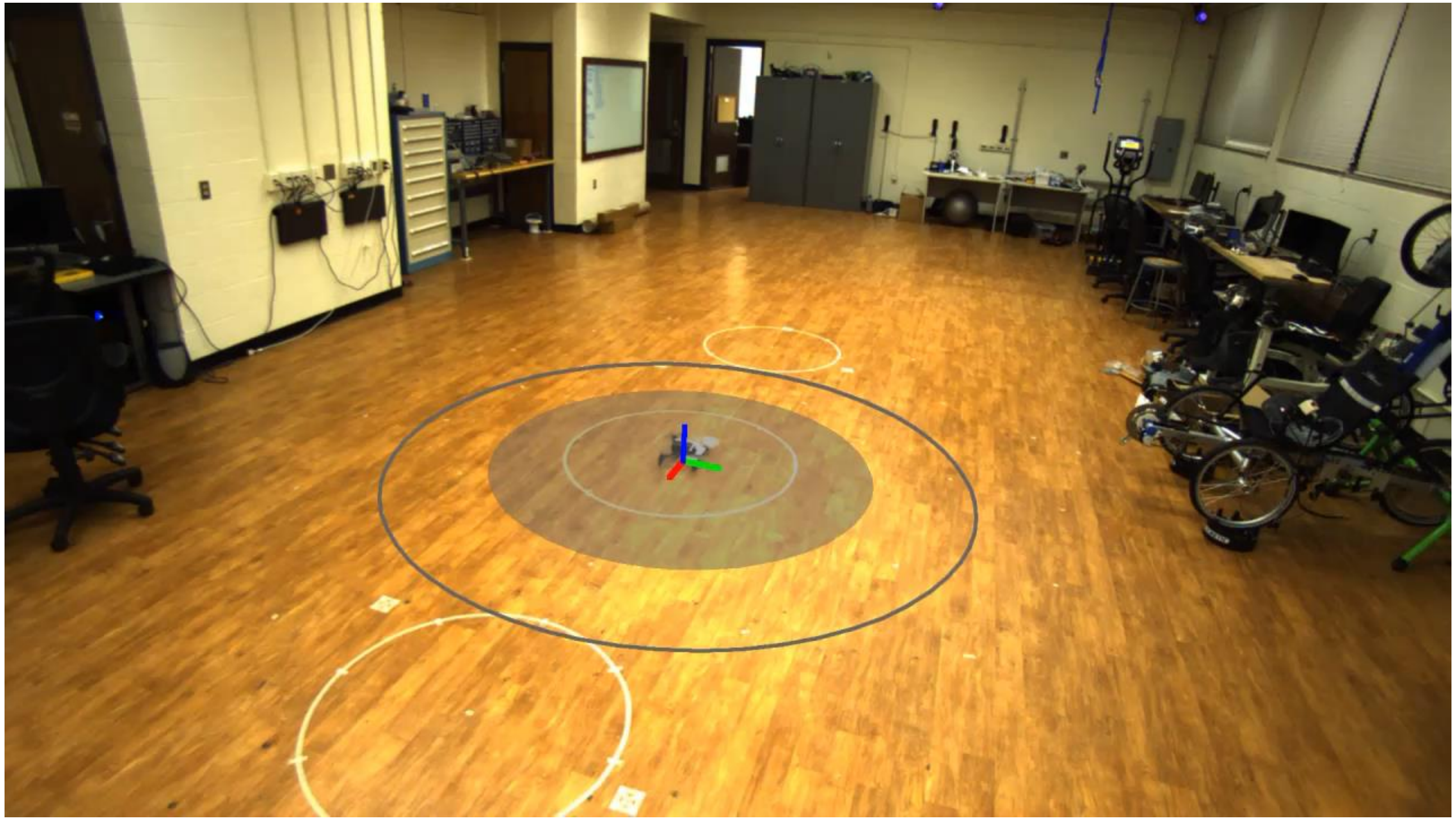
S. C. Edwards, D. M. Le, D. P. Guralnik and W. E. Dixon, "A Topologically Inspired Path-Following Method With Intermittent State Feedback," in IEEE Robotics and Automation Letters, vol. 6, no. 3, pp. 4449-4456, July 2021, doi: 10.1109/LRA.2021.3067295.

- Systems often operate in environments that include:
  - A2AD (Anti Access/Area Denial) Environments
  - Unknown Terrain
  - Complex Environments (Mountains, Extreme Weather, Marine, etc.)
- Limitations:
  - Sensing (GPS, Cameras, Lidar, etc.)
  - Global Communication



Problem Statement

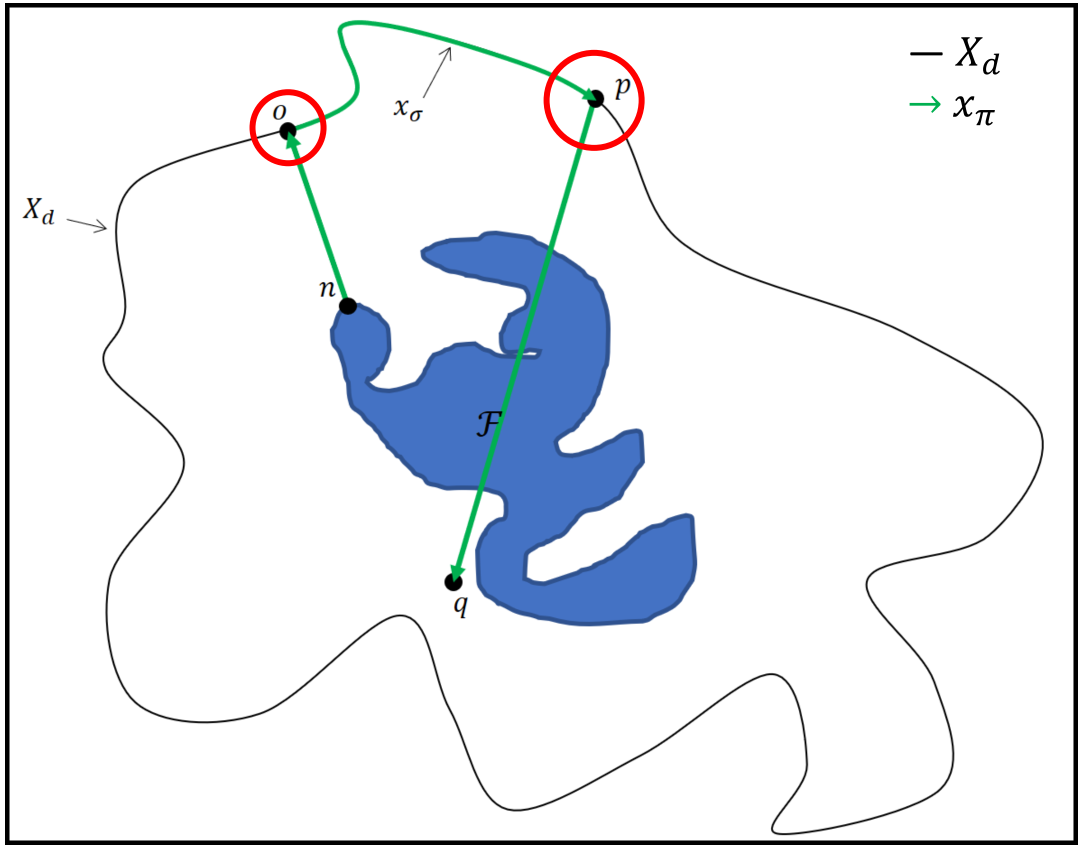
- Task: Agent is to follow path (X_d) where state feedback is denied
- Agent must switch between two tasks:
  - Following X_d
  - Obtaining state feedback to regulate tracking errors



H-Y. Chen, et. al.,"A Switched Systems Approach to Path Following with Intermittent State Feedback", IEEE TRO, 2019.

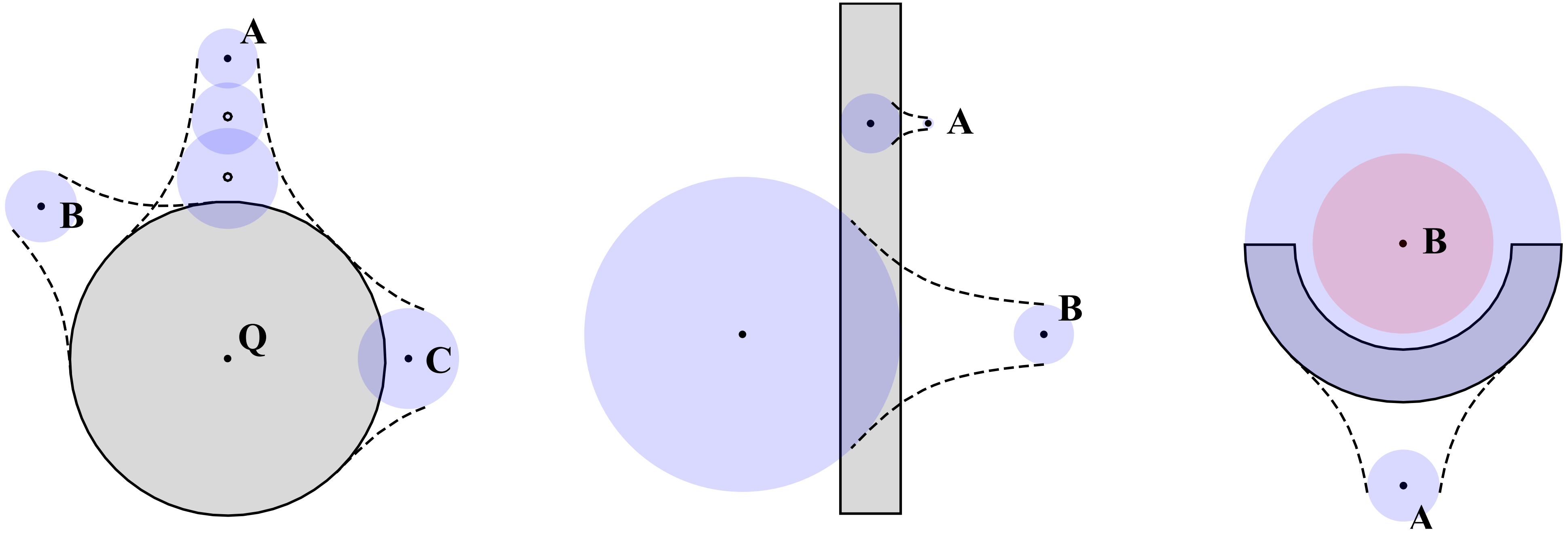
Planning Strategy

1. Depart from the feedback region to X_d.
2. Follow X_d until it is time to return to the feedback region.
3. Take a recommended return trajectory such to guarantee re-entry into the feedback region.



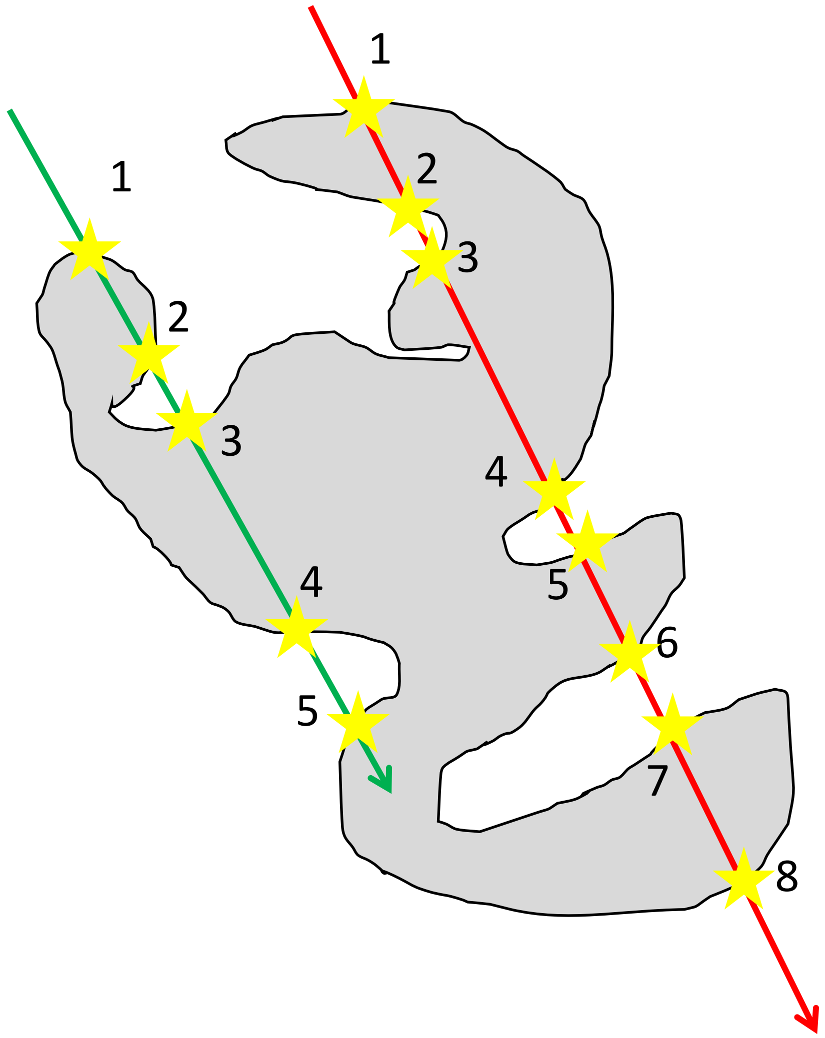
Previous Method

- Previous methods only considered circular feedback regions and circular paths to follow.
- This has issues…



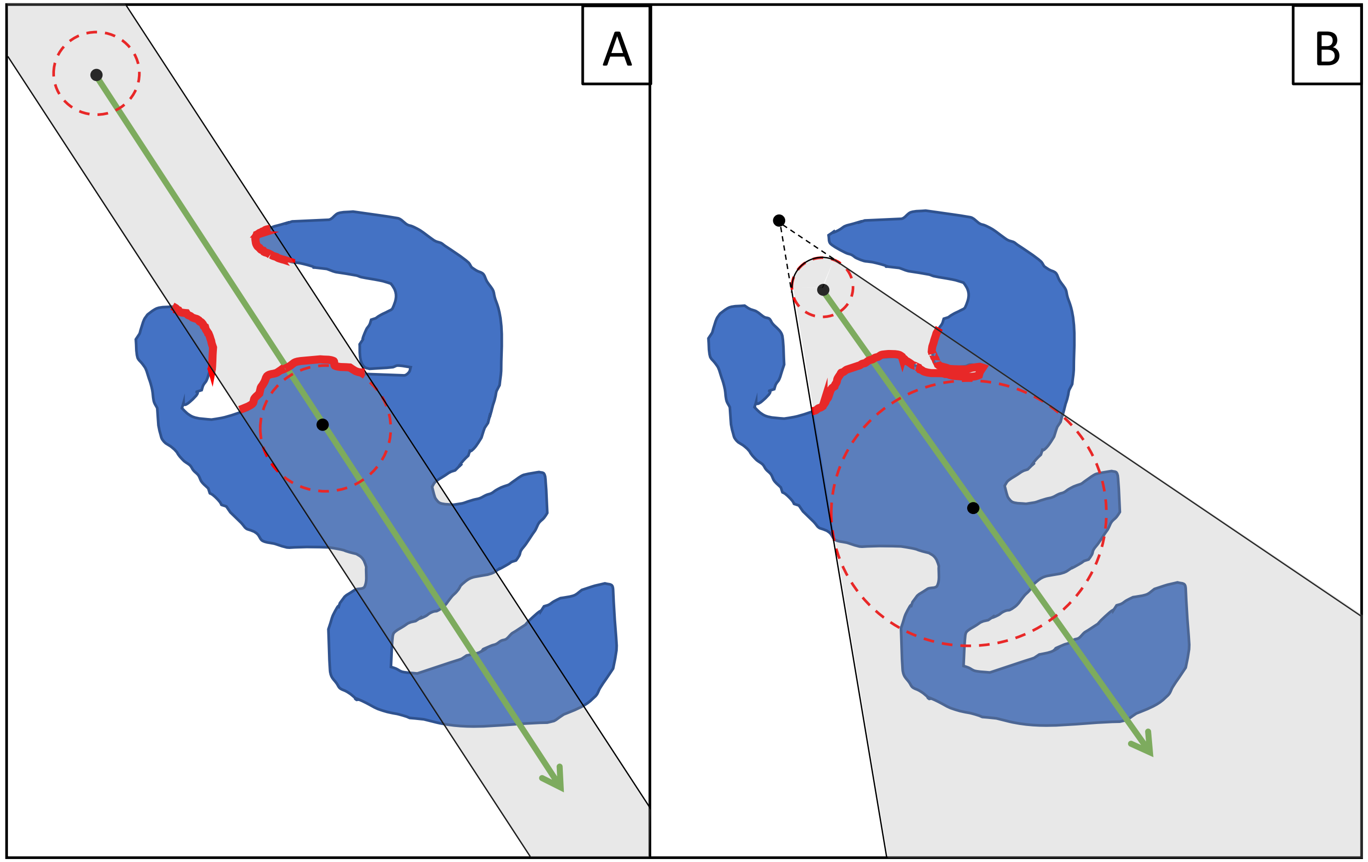
Jordan Curves

- Jordan Curve (C) separates plane into two regions
- A curve originating in the exterior with an odd number of intersections with C terminates in the interior
- An even number of intersections terminates in the exterior.



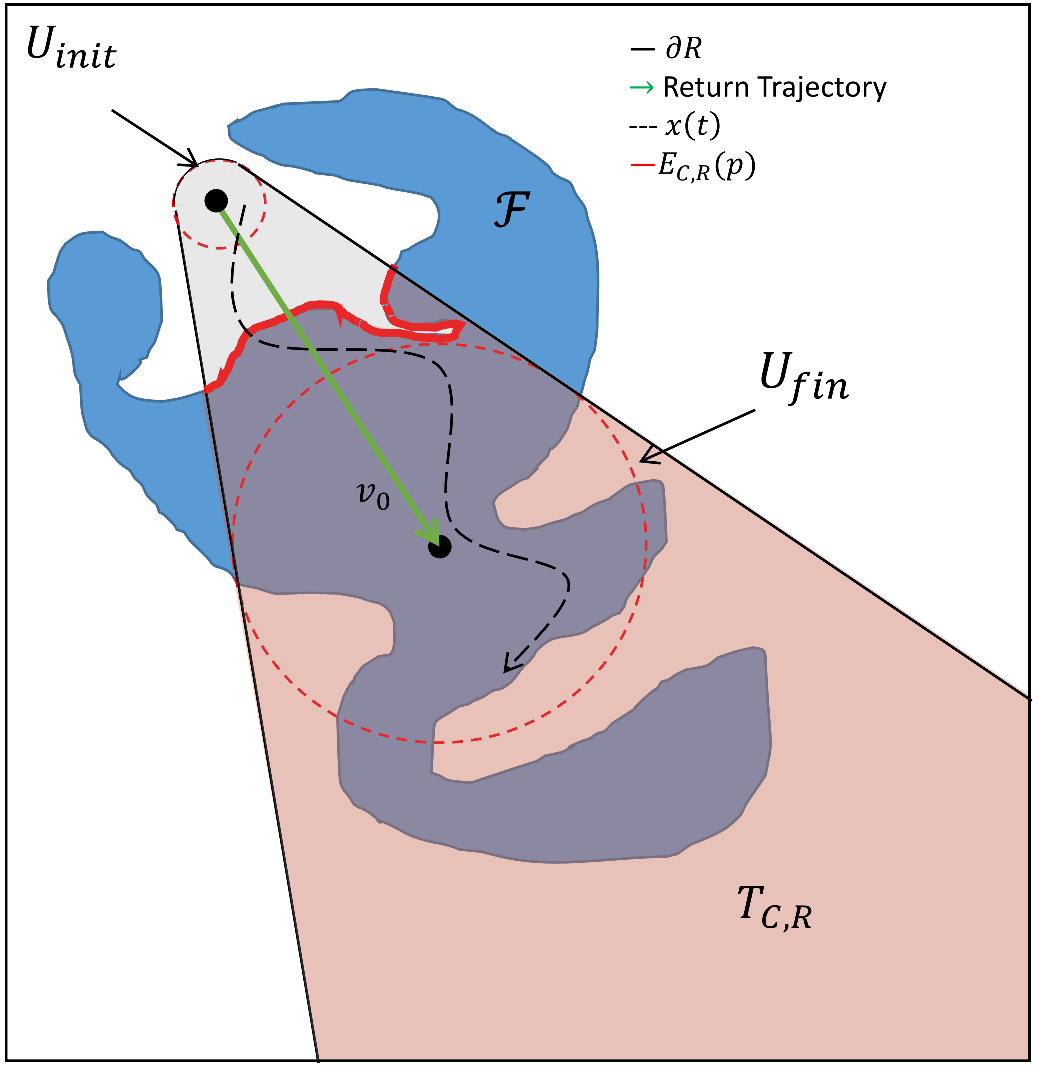
Bounding Regions

- Bounding regions are used to upper bound potential trajectories
- Required for separation of the initial position state and terminal state

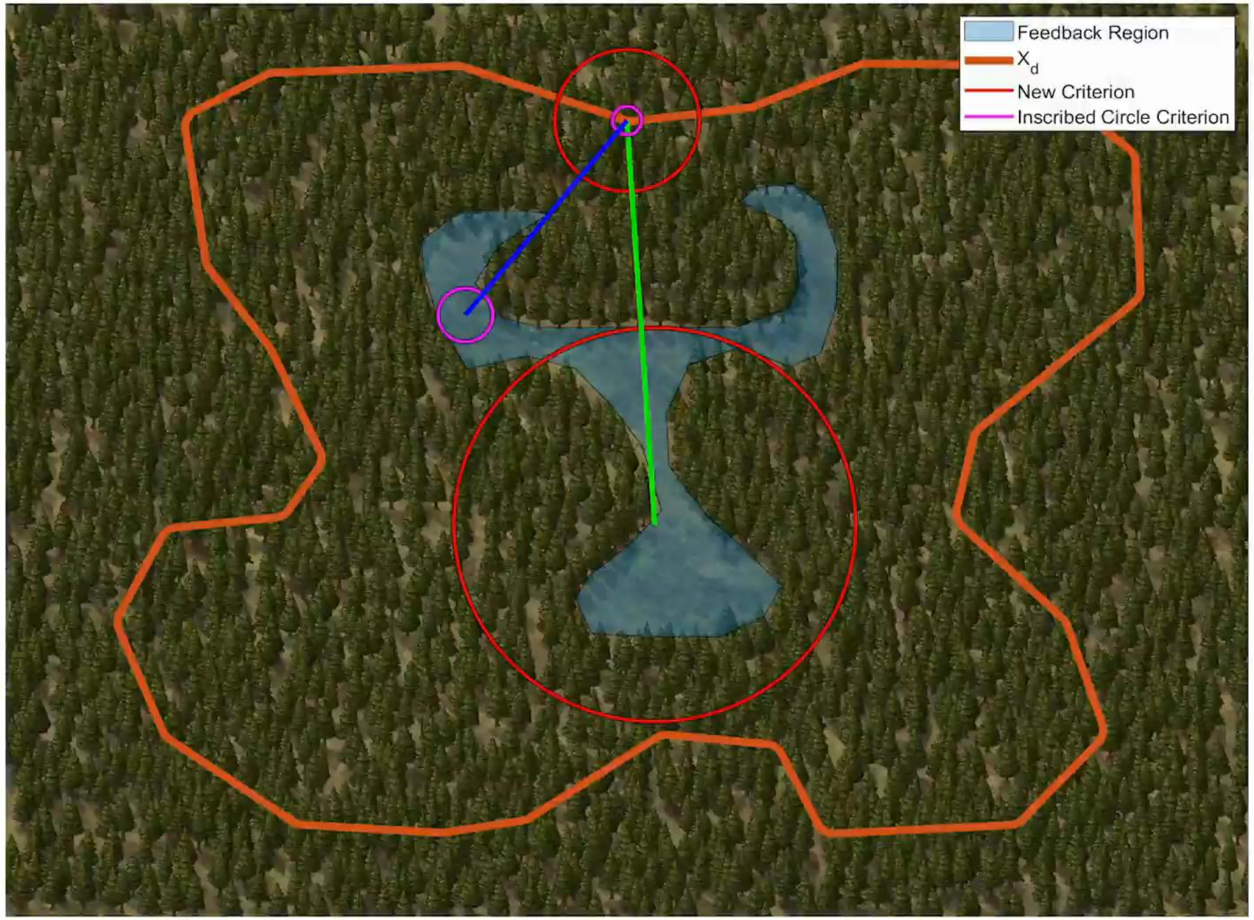


Target Regions

- Using Jordan Curves, "Target Regions" ($T_{C,R}$) are developed to guarantee an agent's re-entry.
- The return target is the center of the ball inscribed in $T_{C,R}$.
- This generalizes to higher dimensions.

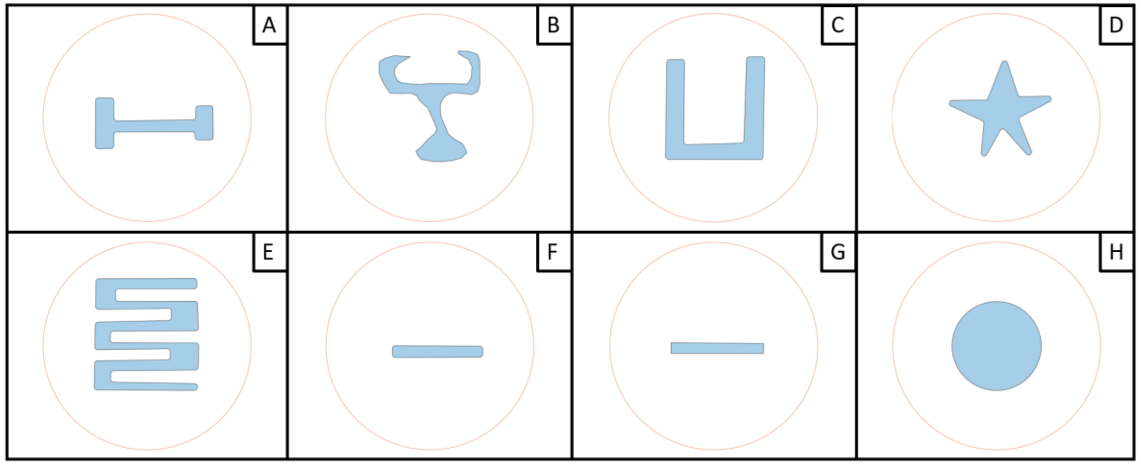


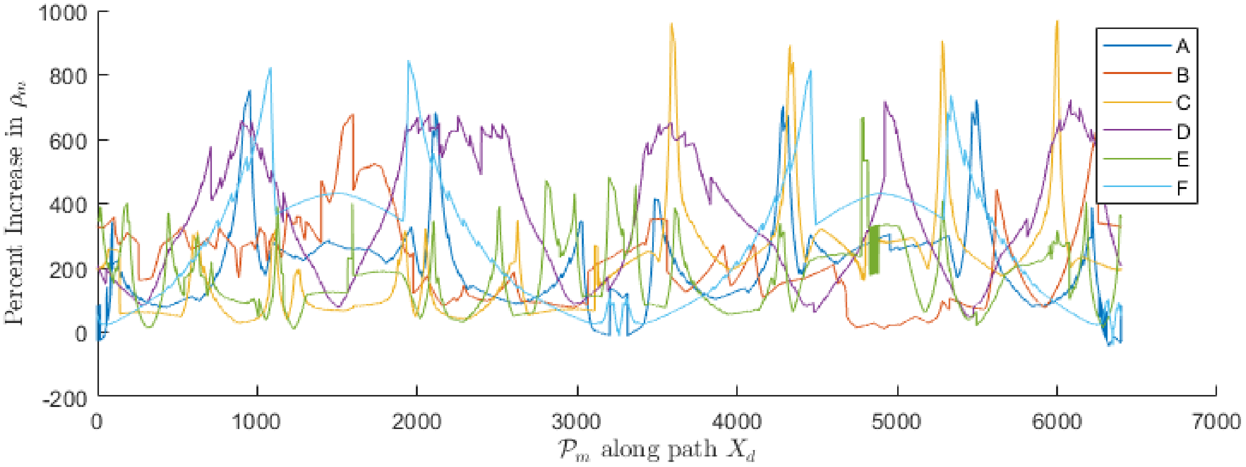
Maximum Allowed Uncertainty Regions (MAURs)



Results

- With a total of 46,855 data points, the mean-average increase in the MAUR was found to be 233%.
- The largest increase was found to be 969%. This was observed in the “horseshoe” geometry (C).



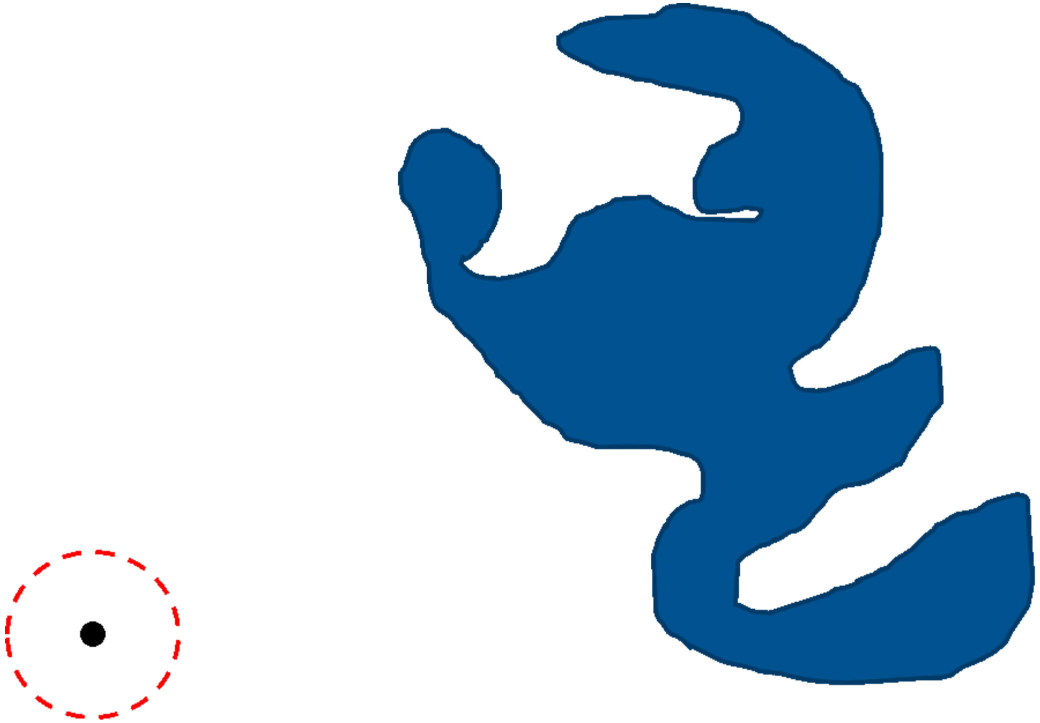


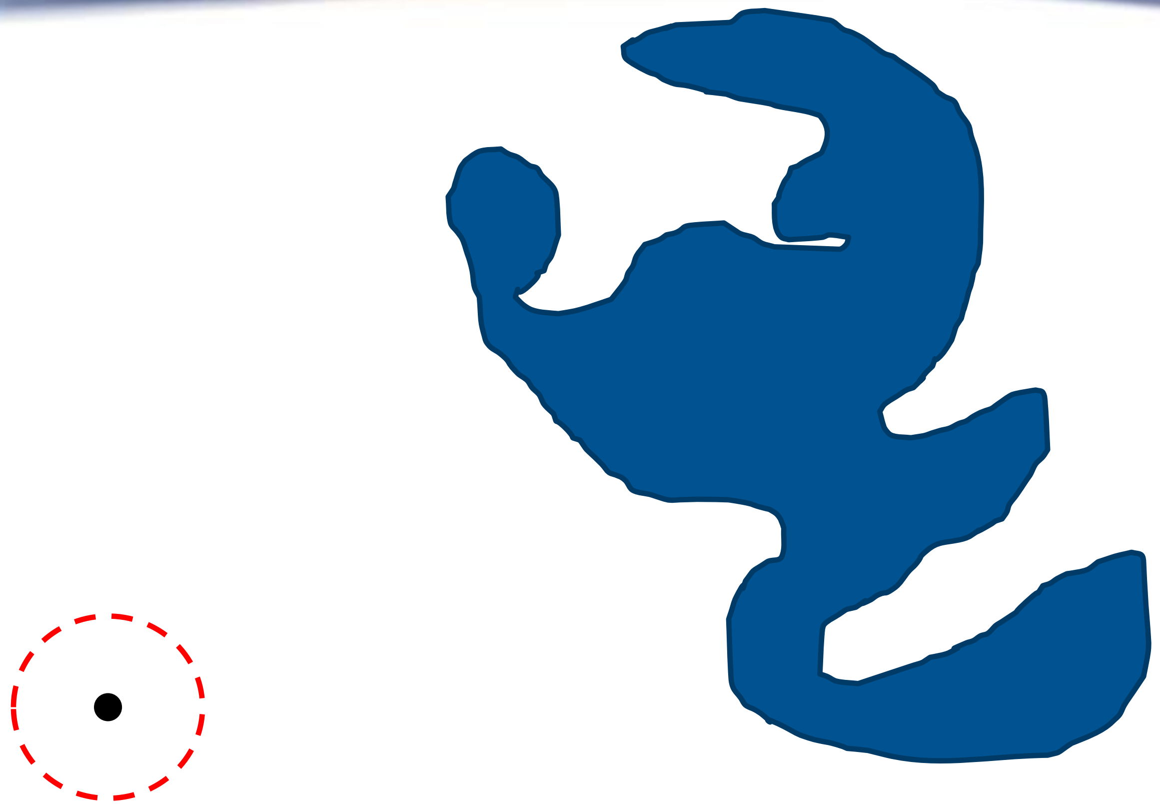
MAUR Computation Methods

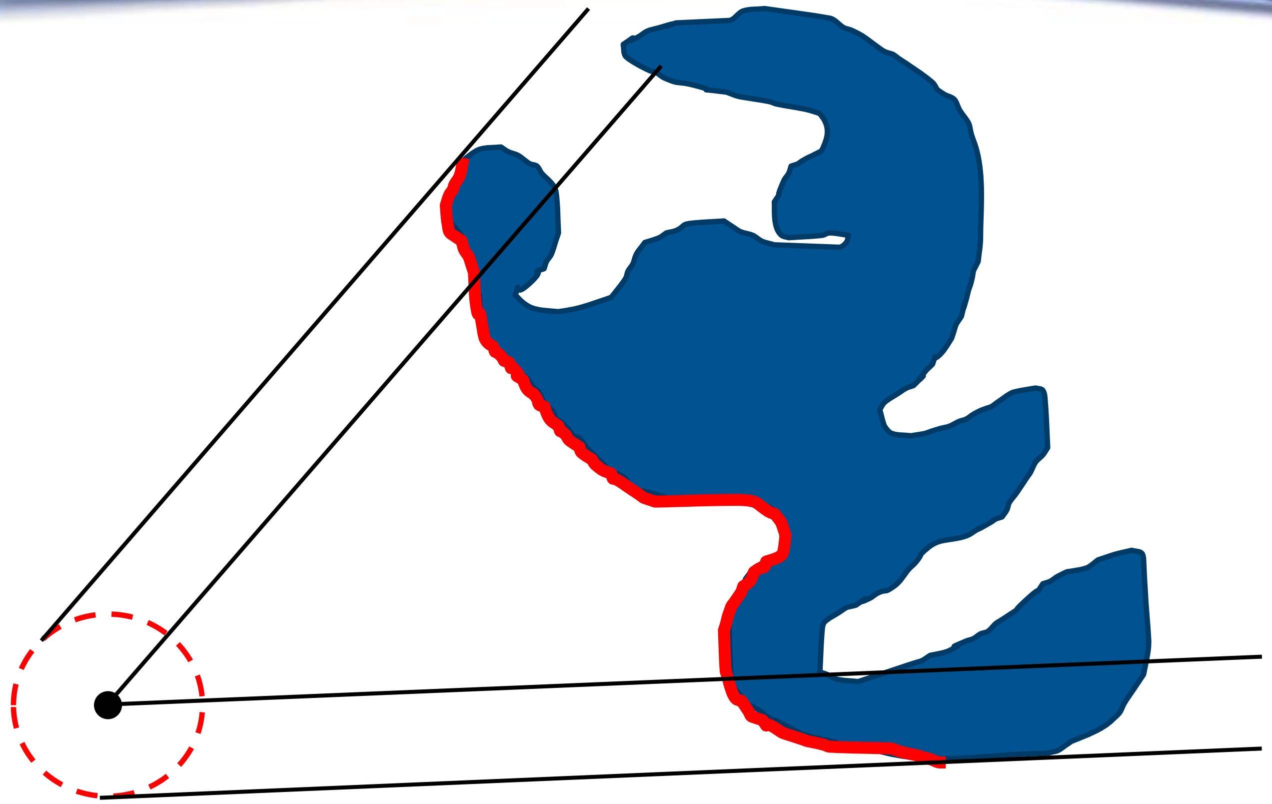
- Current methods use a brute force approach
- New methods aim to reduce computation time from hours to fractions of a second
- Add other animation of old method

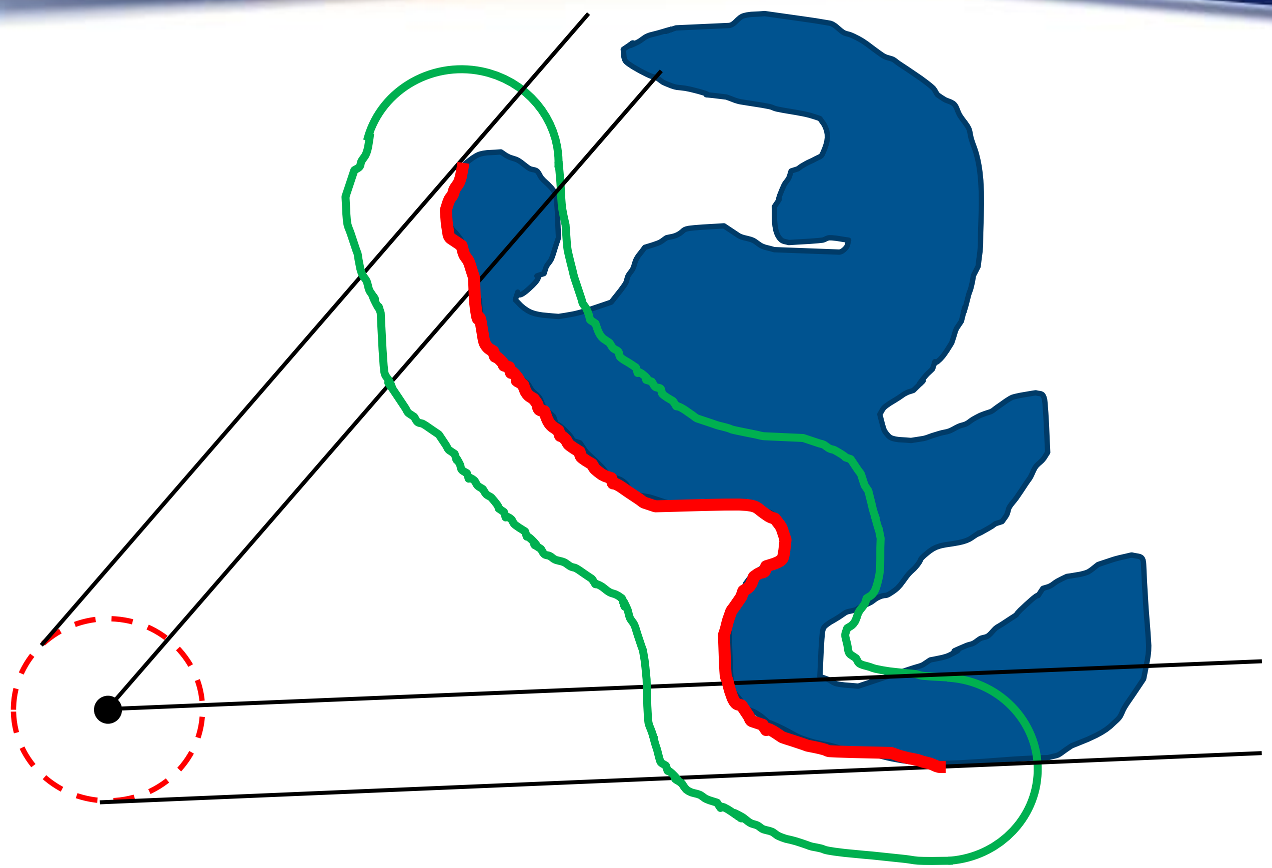
MAUR Computation Methods

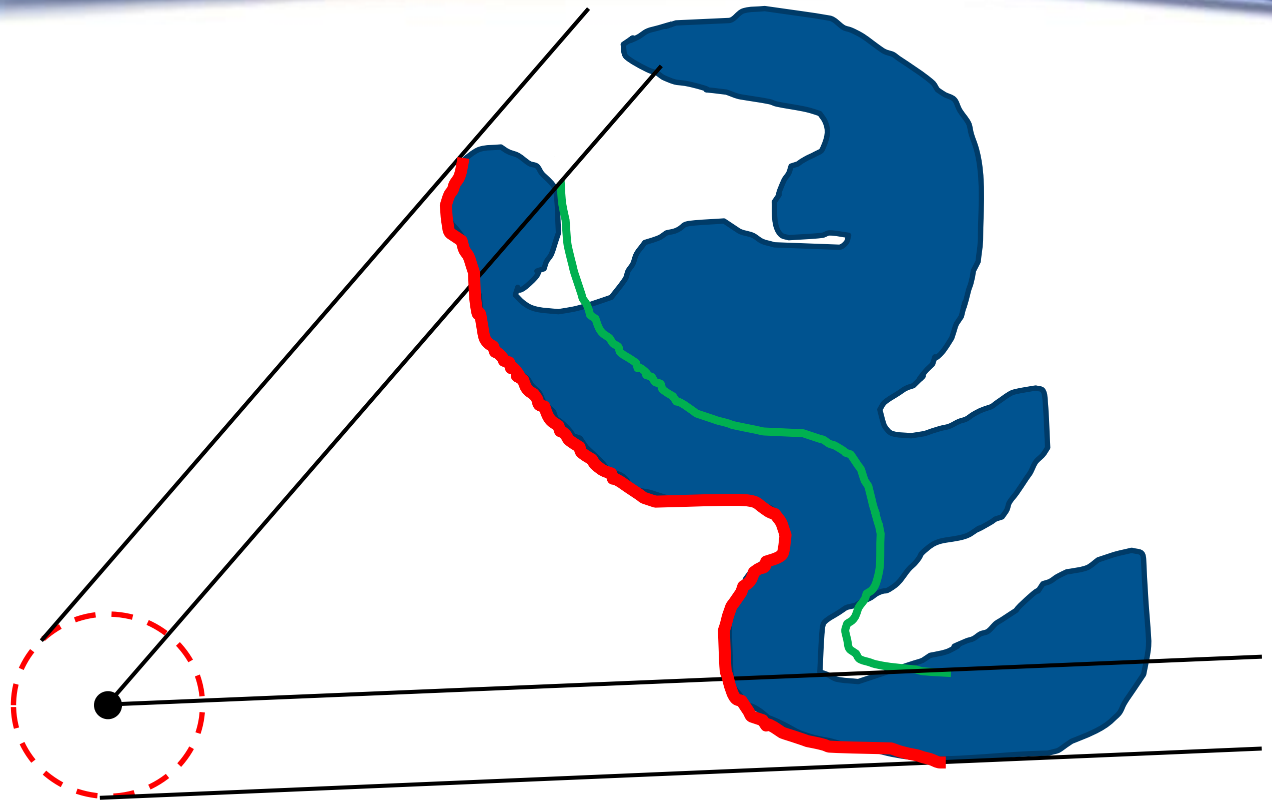
- Current methods use a brute force approach
- New methods aim to reduce computation time from hours to fractions of a second
- Add other animation of old method

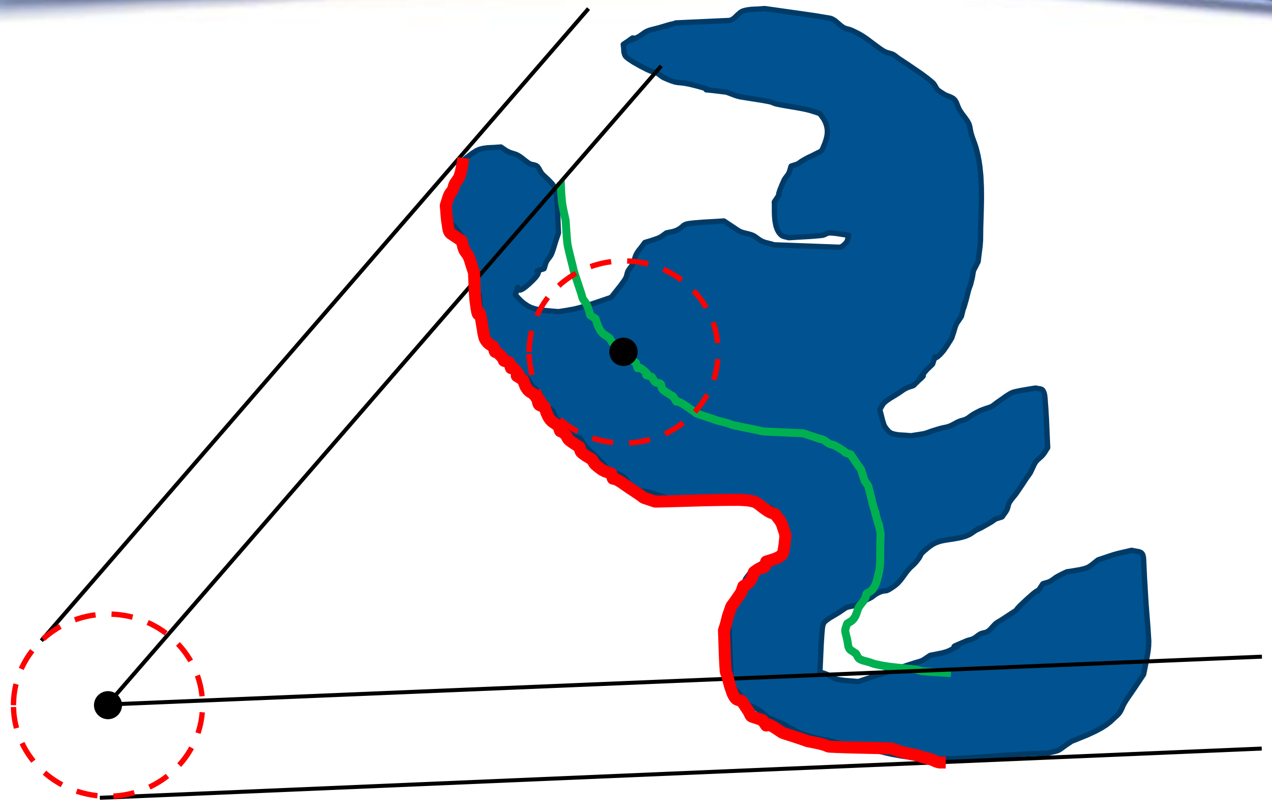












Topological Methods for Guaranteeing Transitions in Switched Systems

Dan P. Guralnik   Sage C. Edwards   Warren E. Dixon

University of Florida/NCR Lab

April 30, 2021

Motivation

Informal Challenge: *Given a controlled hybrid system with uncertain state, how to guarantee transitions into a desired operational mode?*

$\rightsquigarrow$ *crucial for ensuring high-level plan execution*

$\rightsquigarrow$ *complicated geometry of transition boundaries*

$\rightsquigarrow$ *must be addressed with minimal specific geometric insight*

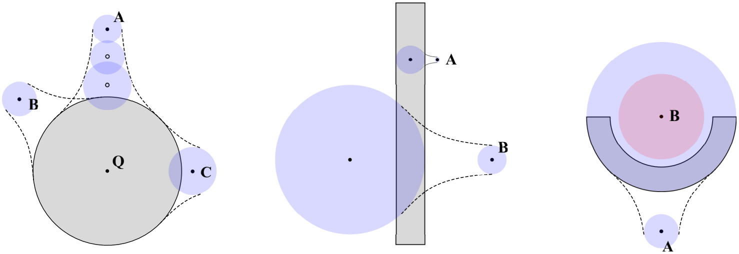
Motivation

Informal Challenge: *Given a controlled hybrid system with uncertain state, how to guarantee transitions into a desired operational mode?*

$\rightsquigarrow$ *crucial for ensuring high-level plan execution*

$\rightsquigarrow$ *complicated geometry of transition boundaries*

$\rightsquigarrow$ *must be addressed with minimal specific geometric insight*



Guaranteed Transitions: Lack of symmetry makes the problem less intuitive. . .

$\rightsquigarrow$ *It's not about thickness of the target domain*

$\rightsquigarrow$ *It's about the transition boundary separating the error cone*

Formal Problem Statement

A setup, successfully generalizing our work in [1]:

- *Incomplete information on state-environment interaction*: agent dynamics over $x \in \mathcal{F}^{\complement}$, $\mathcal{F} \subset \mathbb{R}^n$ closed, given by

$$\|\dot{x} - f(x, u)\| \leq \bar{d}. \tag{1}$$

Formal Problem Statement

A setup, successfully generalizing our work in [1]:

- *Incomplete information on state-environment interaction*: agent dynamics over $x \in \mathcal{F}^{\complement}$, $\mathcal{F} \subset \mathbb{R}^n$ closed, given by

$$\|\dot{x} - f(x,u)\| \leq \bar{d}. \tag{1}$$

- A collection $\mathscr{A}$ of curves $\hat{x} : [0,\infty) \to \mathbb{R}^n$, called *admissible plans*,

$$\mathscr{A}_p \triangleq \{\hat{x} \in \mathscr{A} \colon \hat{x}(0) = p\}, \tag{2}$$

$\rightsquigarrow$ *Usually, expect to have* $\mathscr{A}_p = p + \mathscr{A}_0 = \{p + \hat{x} \colon \hat{x} \in \mathscr{A}_0\}$

$\rightsquigarrow$ *In Sage's problem* $\hat{x}(t) \triangleq p + t\mathbf{v}$*, with* $\|\mathbf{v}\| = v_0$ *a known constant*

Formal Problem Statement

A setup, successfully generalizing our work in [1]:

- *Incomplete information on state-environment interaction*: agent dynamics over $x \in \mathcal{F}^{\complement}$, $\mathcal{F} \subset \mathbb{R}^n$ closed, given by

$$\|\dot{x} - f(x,u)\| \leq \bar{d}. \tag{1}$$

- A collection $\mathscr{A}$ of curves $\hat{x} : [0,\infty) \to \mathbb{R}^n$, called *admissible plans*,

$$\mathscr{A}_p \triangleq \{\hat{x} \in \mathscr{A} \colon \hat{x}(0) = p\}\,, \tag{2}$$

$\rightsquigarrow$ *Usually, expect to have* $\mathscr{A}_p = p + \mathscr{A}_0 = \{p + \hat{x} \colon \hat{x} \in \mathscr{A}_0\}$

$\rightsquigarrow$ *In Sage's problem* $\hat{x}(t) \triangleq p + t\mathbf{v}$*, with* $\|\mathbf{v}\| = v_0$ *a known constant*

- Each plan is *executed* by applying a control $u(t)$, $t \geq 0$, guaranteeing

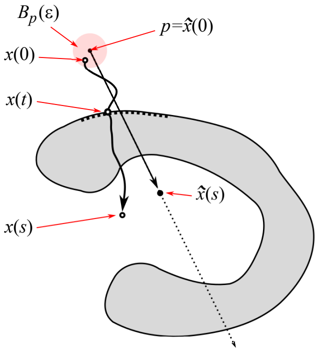
$$\|x(t) - \hat{x}(t)\| \leq \varrho(t),\ t \geq 0 \tag{3}$$

where $\varrho : [0,\infty) \to [0,\infty)$ is a known *error-bounding function* (EBF).

$\rightsquigarrow$ *There can be many EBFs, e.g., depending on* $p = \hat{x}(0)$

Definition. A plan $\hat{x} \in \mathscr{A}_p$ provides *an ε-guaranteed transition into $\mathcal{F}$*, if

$$(\forall_{p^* \in B_p(\varepsilon)})\ (\exists_{t \geq 0})\ (x(0) = p^* \rightarrow x(t) \in \mathcal{F}). \tag{4}$$

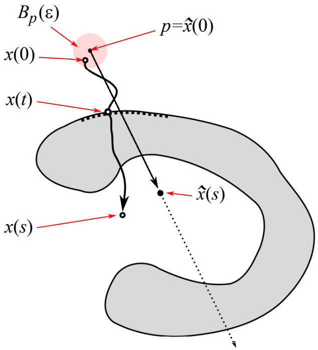


Formal Problem Statement

Definition. A plan $\hat{x} \in \mathscr{A}_p$ provides *an ε-guaranteed transition into $\mathcal{F}$*, if

$$(\forall_{p^* \in B_p(\varepsilon)})\ (\exists_{t \geq 0})\ (x(0) = p^* \rightarrow x(t) \in \mathcal{F}). \qquad (4)$$

- $\mathscr{G}_p(\varepsilon) \subseteq \mathscr{A}_p$ is the collection of *all plans satisfying (4)*.

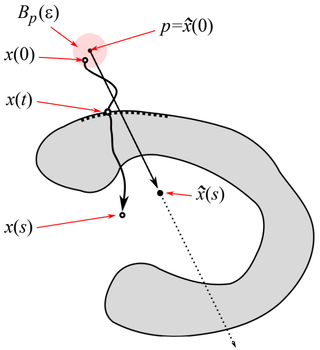


Formal Problem Statement

Definition. A plan $\hat{x} \in \mathscr{A}_p$ provides *an ε-guaranteed transition into $\mathcal{F}$*, if

$$(\forall_{p^* \in B_p(\varepsilon)})\ (\exists_{t \geq 0})\ (x(0) = p^* \to x(t) \in \mathcal{F}). \quad (4)$$

- $\mathscr{G}_p(\varepsilon) \subseteq \mathscr{A}_p$ is the collection of *all plans satisfying* (4).
- $p \in \mathbb{R}^n$ is *ε-feasible*, if $\mathscr{G}_p(\varepsilon) \neq \varnothing$.
- $p \in \mathbb{R}^n$ is *feasible* if it is ε-feasible for some $\varepsilon > 0$.
- WHAT ACTUALLY GUARANTEES A PLAN?

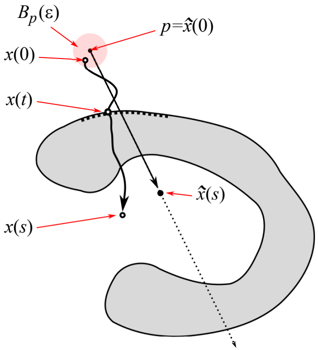


Formal Problem Statement

Definition. A plan $\hat{x} \in \mathscr{A}_p$ provides *an ε-guaranteed transition into $\mathcal{F}$*, if

$$(\forall_{p^* \in B_p(\varepsilon)})\ (\exists_{t \geq 0})\ (x(0) = p^* \to x(t) \in \mathcal{F}). \tag{4}$$

- $\mathscr{G}_p(\varepsilon) \subseteq \mathscr{A}_p$ is the collection of *all plans satisfying (4)*.
- $p \in \mathbb{R}^n$ is *ε-feasible*, if $\mathscr{G}_p(\varepsilon) \neq \varnothing$.
- $p \in \mathbb{R}^n$ is *feasible* if it is ε-feasible for some $\varepsilon > 0$.
- WHAT ACTUALLY GUARANTEES A PLAN?
- **Recall:** Let $U \subset \mathbb{R}^n$ be an open domain and let $p, q \in U$. A closed set $K \subset \mathbb{R}^n$ *separates U between p and q*, if p, q lie in distinct components of $U \setminus K$.
- $\partial\mathcal{F}$ separates $\mathbb{R}^n$ between $p \in \mathcal{F}^{\complement}$ and any $q \in \mathrm{int}(\mathcal{F})$.



Formal Problem Statement

Definition. The *maximum allowed uncertainty radius* (MAUR) at p:

$$\mathbf{M}^*_{\mathcal{F}}(p) \triangleq \begin{cases} \sup\left(\varepsilon > 0 \colon \mathscr{G}_p(\varepsilon) \neq \varnothing\right), & \text{if } p \text{ is feasible,} \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

and denote $\mathcal{M}(\mathcal{F}) \triangleq \left\{f : \mathbb{R}^n \to \mathbb{R}_{\geq 0} : f \leq \mathbf{M}^*_{\mathcal{F}}\right\}$, the *MAUR bounds*.

Problem 1. Find explicit constructions of $f \in \mathcal{M}(\mathcal{F})$.

$\rightsquigarrow$ *The inscribed ball criterion (IBC) leads to one such construction*

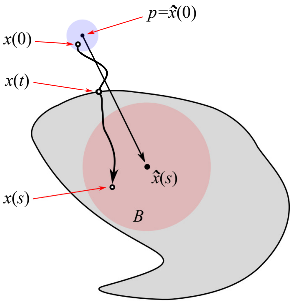
Problem 2. Given $f \in \mathcal{M}(\mathcal{F})$ and $p \in \mathbb{R}^n$, compute $f(p)$ and $\hat{x} \in \mathscr{G}_p(f(p))$.

$\rightsquigarrow$ *Expect: the better f approximates $\mathbf{M}^*_{\mathcal{F}}$, the more complex this problem will be*

Topological Transition Guarantee (TTG)

- An *IBC certificate at p* is an EBF ϱ with
  - $B_{\hat{x}(s)}(\varrho(s)) \subseteq \mathcal{F}$,

  for some $\hat{x} \in \mathscr{A}_p$, $s \geq 0$.

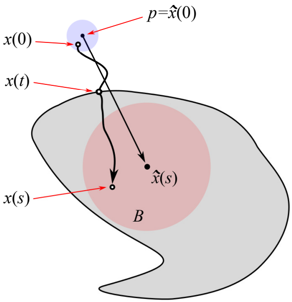


Topological Transition Guarantee (TTG)

- An *IBC certificate at p* is an EBF ϱ with
  - $B_{\hat{x}(s)}(\varrho(s)) \subseteq \mathcal{F}$,

  for some $\hat{x} \in \mathscr{A}_p$, $s \geq 0$.
- Then one defines the *IBC-MAUR*:
  - $$\mathbf{M}^{IBC}_{\mathcal{F}}(p) \triangleq \sup\left(\varrho(0) : \begin{array}{l}\varrho \text{ is an IBC} \\ \text{certificate at } p\end{array}\right)$$

  and $\mathbf{M}^{IBC}_{\mathcal{F}}(p) \triangleq 0$ otherwise.

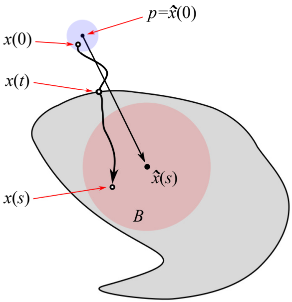


Topological Transition Guarantee (TTG)

- An *IBC certificate at p* is an EBF ϱ with
  - $B_{\hat{x}(s)}(\varrho(s)) \subseteq \mathcal{F}$,

  for some $\hat{x} \in \mathscr{A}_p$, $s \geq 0$.
- Then one defines the *IBC-MAUR*:
  - $\mathbf{M}_{\mathcal{F}}^{IBC}(p) \triangleq \sup\left(\varrho(0) : \begin{array}{l}\varrho \text{ is an IBC} \\ \text{certificate at } p\end{array}\right)$

  and $\mathbf{M}_{\mathcal{F}}^{IBC}(p) \triangleq 0$ otherwise.
- Clearly, $\mathbf{M}_{\mathcal{F}}^{IBC} \leq \mathbf{M}_{\mathcal{F}}^{*}$.



UF UNIVERSITY of FLORIDA · Duke UNIVERSITY · TEXAS The University of Texas at Austin · UC SANTA CRUZ

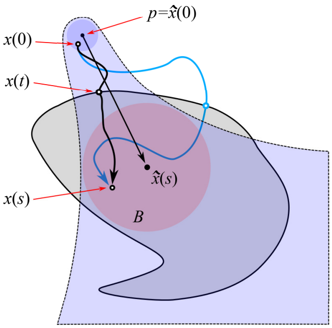
Topological Transition Guarantee (TTG)

- An *IBC certificate at p* is an EBF ϱ with
  - $B_{\hat{x}(s)}(\varrho(s)) \subseteq \mathcal{F}$,

  for some $\hat{x} \in \mathscr{A}_p$, $s \geq 0$.
- Then one defines the *IBC-MAUR*:
  - $\mathbf{M}^{IBC}_{\mathcal{F}}(p) \triangleq \sup\left(\varrho(0): \begin{array}{l}\varrho \text{ is an IBC} \\ \text{certificate at } p\end{array}\right)$

  and $\mathbf{M}^{IBC}_{\mathcal{F}}(p) \triangleq 0$ otherwise.
- Clearly, $\mathbf{M}^{IBC}_{\mathcal{F}} \leq \mathbf{M}^{*}_{\mathcal{F}}$.
- The argument takes no account of the error cone!

  $\rightsquigarrow$ *Information about $x(t)$ is thrown away*

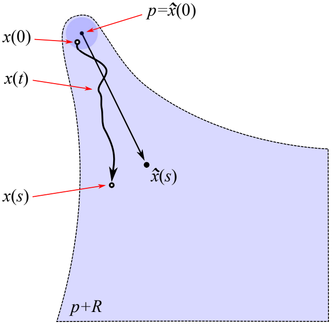


Topological Transition Guarantee (TTG)

- Let $\mathcal{U}$ be a collection of open domains such that
  - $\mathbb{R}^n \in \mathcal{U}$,
  - $0 \in R$ for all $R \in \mathcal{U}$.

  ⇝ *In [1], $\mathcal{U}$ consists of all strips centered at $0 \in \mathbb{R}^2$*

⇝ *$\mathcal{U}$ expresses a bound on the growth of error cones*



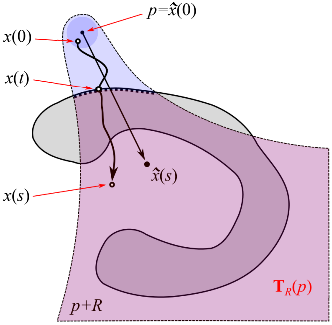
Topological Transition Guarantee (TTG)

- Let $\mathcal{U}$ be a collection of open domains such that
  - $\mathbb{R}^n \in \mathcal{U}$,
  - $0 \in R$ for all $R \in \mathcal{U}$.

  ⇝ *In [1], $\mathcal{U}$ consists of all strips centered at $0 \in \mathbb{R}^2$*

  ⇝ *$\mathcal{U}$ expresses a bound on the growth of error cones*

- Given $R \in \mathcal{U}$, $p \in \mathbb{R}^n$ the *target at p*, $\mathbf{T}_R(p)$, is defined as the set of $q \in \mathbb{R}^n$ separated from p by $\partial\mathcal{F}$ in $p + R$.



UF UNIVERSITY of FLORIDA · Duke UNIVERSITY · TEXAS The University of Texas at Austin · UC SANTA CRUZ

Topological Transition Guarantee (TTG)

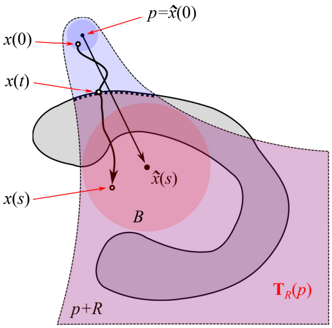
- Let $\mathcal{U}$ be a collection of open domains such that
  - $\mathbb{R}^n \in \mathcal{U}$,
  - $0 \in R$ for all $R \in \mathcal{U}$.

  ⇝ *In [1], $\mathcal{U}$ consists of all strips centered at $0 \in \mathbb{R}^2$*

  ⇝ *$\mathcal{U}$ expresses a bound on the growth of error cones*

- Given $R \in \mathcal{U}$, $p \in \mathbb{R}^n$ the *target at p*, $\mathbf{T}_R(p)$, is defined as the set of $q \in \mathbb{R}^n$ separated from p by $\partial\mathcal{F}$ in $p + R$.
- A *$\mathcal{U}$-certificate at p* is an EBF ϱ, with:
  - $B \triangleq B_{\hat{x}(s)}(\varrho(s)) \subseteq \mathbf{T}_R(p)$, and
  - $\hat{x}(t) \in p + R$ for all $0 \leq t \leq s$.

  for some $\hat{x} \in \mathscr{A}_p$, $s \geq 0$ and $R \in \mathcal{U}$.

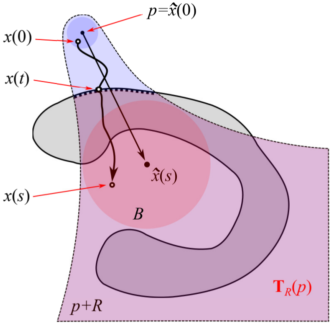


UF UNIVERSITY of FLORIDA · Duke UNIVERSITY · TEXAS The University of Texas at Austin · UC SANTA CRUZ

Topological Transition Guarantee (TTG)

- We can then define the *$\mathcal{U}$-MAUR*:
  - $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p) \triangleq \sup \left(\varrho(0) : \begin{array}{l} \varrho \text{ is a } \mathcal{U}\text{-} \\ \text{certificate at } p \end{array} \right)$

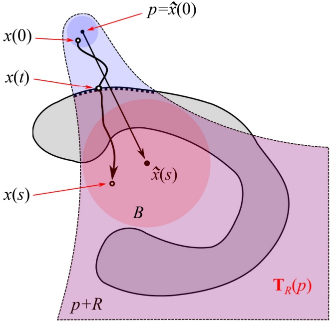
  and $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p) \triangleq 0$ otherwise.



Topological Transition Guarantee (TTG)

- We can then define the *$\mathcal{U}$-MAUR*:
  - $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p) \triangleq \sup \left(\varrho(0) : \begin{array}{l} \varrho \text{ is a } \mathcal{U}\text{-} \\ \text{certificate at } p \end{array} \right)$

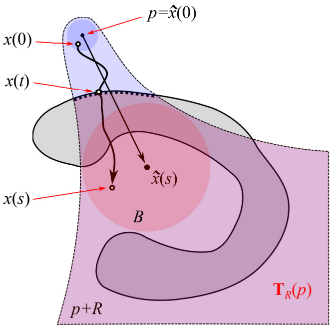
  and $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p) \triangleq 0$ otherwise.
- Since $\mathbf{T}_{\mathbb{R}^n}(p) = \mathcal{F}$, we have
  - $\mathbf{M}^{IBC}_{\mathcal{F}} \leq \mathbf{M}^{\mathcal{U}}_{\mathcal{F}} \leq \mathbf{M}^{*}_{\mathcal{F}}$.



Topological Transition Guarantee (TTG)

- We can then define the *$\mathcal{U}$-MAUR*:
  - $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p) \triangleq \sup \left(\varrho(0) : \begin{array}{l} \varrho \text{ is a } \mathcal{U}\text{-} \\ \text{certificate at } p \end{array} \right)$

  and $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p) \triangleq 0$ otherwise.
- Since $\mathbf{T}_{\mathbb{R}^n}(p) = \mathcal{F}$, we have
  - $\mathbf{M}^{IBC}_{\mathcal{F}} \leq \mathbf{M}^{\mathcal{U}}_{\mathcal{F}} \leq \mathbf{M}^{*}_{\mathcal{F}}$.
- HENCE A FOCUS ON COMPUTABILITY OF LOWER BOUNDS $f \leq \mathbf{M}^{\mathcal{U}}_{\mathcal{F}}$, OBTAINED BY
  - restircting $\mathcal{U}$: strips, cylinders, cones
  - restricting ϱ: specific EBFs
  - restricting $\partial\mathcal{F}$: collared spheres



Towards Computable TTG MAURs

- *Jordan Curves [2]* provide a "separation standard" in the plane.
  - $\partial\mathcal{F}$ can be any continuous simple closed curve (SCC)
  - $\partial\mathcal{F}$ can be lower-approximated by polygonal SCCs

Towards Computable TTG MAURs

- *Jordan Curves [2]* provide a "separation standard" in the plane.
  - $\partial\mathcal{F}$ can be any continuous simple closed curve (SCC)
  - $\partial\mathcal{F}$ can be lower-approximated by polygonal SCCs
- For simple parametric $\mathcal{U}$ (strips, cones), lower bounds on $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p)$ are. . .
  - Piecewise regular for regular increasing ϱ
  - Solutions to a constrained optimization problem

    $\rightsquigarrow$ *Combinatorial/Topological aspects to be resolved*
  - Computations are parallelizable.

    $\rightsquigarrow$ *Target balls picked via edge-by-edge optimization*

Towards Computable TTG MAURs

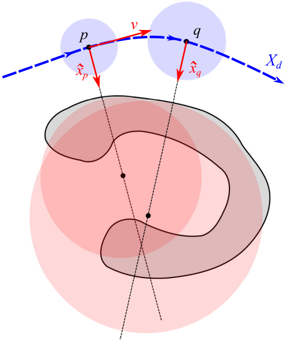
- *Jordan Curves [2]* provide a "separation standard" in the plane.
  - $\partial\mathcal{F}$ can be any continuous simple closed curve (SCC)
  - $\partial\mathcal{F}$ can be lower-approximated by polygonal SCCs
- For simple parametric $\mathcal{U}$ (strips, cones), lower bounds on $\mathbf{M}^{\mathcal{U}}_{\mathcal{F}}(p)$ are...
  - Piecewise regular for regular increasing ϱ
  - Solutions to a constrained optimization problem

    $\rightsquigarrow$ *Combinatorial/Topological aspects to be resolved*
  - Computations are parallelizable.

    $\rightsquigarrow$ *Target balls picked via edge-by-edge optimization*
- Higher-dimensional analog via *collared spheres [3]*:
  - An embedding $\gamma : \mathbb{S}^{n-1} \times \{0\} \hookrightarrow \mathbb{R}^n$ is *collared*, if it extends to an embedding of $\mathbb{S}^{n-1} \times [-1,1] \hookrightarrow \mathbb{R}^n$.
  - *Generalized Schoenflies [4] implies:* If $\gamma : \mathbb{S}^{n-1} \hookrightarrow \mathbb{S}^n$ is collared, then $\mathbb{S}^n \smallsetminus \gamma(\mathbb{S}^{n-1})$ is the disjoint union of two open balls.

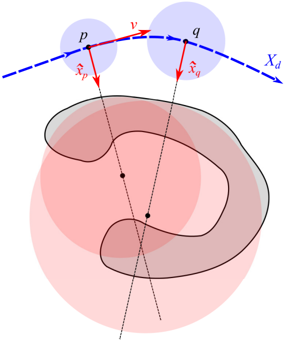
Future Directions: Reactive TTGs

- **Returning to Single-Agent Relay Tracking,**
  - *Available plans:* For all p, $\mathscr{A}_p \equiv \mathbb{S}^{n-1}$;
  - *Desired TTG:* $f \in \mathcal{M}(\mathcal{F})$ with $f \leq \mathbf{M}^{\mathcal{U}}_{\mathcal{F}}$;
  - *Task:* X_d, parametric or subdivided.



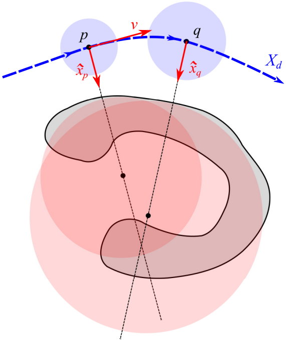
Future Directions: Reactive TTGs

- **Returning to Single-Agent Relay Tracking,**
  - *Available plans:* For all p, $\mathscr{A}_p \equiv \mathbb{S}^{n-1}$;
  - *Desired TTG:* $f \in \mathcal{M}(\mathcal{F})$ with $f \leq \mathbf{M}^{\mathcal{U}}_{\mathcal{F}}$;
  - *Task:* X_d, parametric or subdivided.
- **Observe:** TTG direction $\hat{x}_p$ is well-defined for all $p \in \mathcal{F}^{\complement}$ except for a null set.
  - f increases in the direction $\hat{x}_p$;
  - f may vary along X_d (see figure).



Future Directions: Reactive TTGs

- **Returning to Single-Agent Relay Tracking,**
  - *Available plans:* For all p, $\mathscr{A}_p \equiv \mathbb{S}^{n-1}$;
  - *Desired TTG:* $f \in \mathcal{M}(\mathcal{F})$ with $f \leq \mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$;
  - *Task:* X_d, parametric or subdivided.
- **Observe:** TTG direction $\hat{x}_p$ is well-defined for all $p \in \mathcal{F}^{\complement}$ except for a null set.
  - f increases in the direction $\hat{x}_p$;
  - f may vary along X_d (see figure).
- **Problem 3:** Compute approximations $f \leq \mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$ in closed form.
- **Problem 4:** Determine the relationship between ∇f and TTG plans.
  - Characterizing TTG plans *locally*?
  - Value tradeoffs *à-la* [5, 6] between tracking X_d and detours into $\mathcal{F}$?



Future directions: Learning TTGs

In the absence of complete knowledge of $\mathcal{F}$...

- **Instead of learning $\mathcal{F}$, learn $\mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$ (or possibly $\nabla\mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$), as a model.**

Future directions: Learning TTGs

In the absence of complete knowledge of $\mathcal{F}$...

- **Instead of learning $\mathcal{F}$, learn $\mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$ (or possibly $\nabla \mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$), as a model.**
- With f_t approximating $\mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$ at time t,
  - f_t corresponding to a polygonal approximation $\mathcal{F}_t$ of $\mathcal{F}$;
  - Monotonicity, $t \leq s \implies f_t \leq f_s$, ensure a valid TTG for all time.
  - **Challenge:** Maintain a connected model of $\mathcal{F}$, or—
  - **Challenge:** Extend methods over disconnected $\mathcal{F}$, and—
  - **Challenge:** Directed *exploration* of $\partial \mathcal{F}$ may be required.

Future directions: Learning TTGs

In the absence of complete knowledge of $\mathcal{F}$...

- **Instead of learning $\mathcal{F}$, learn $\mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$ (or possibly $\nabla\mathrm{M}_{\mathcal{F}}^{\mathcal{U}}$), as a model.**
- With f_t approximating $\mathbf{M}_{\mathcal{F}}^{\mathcal{U}}$ at time t,
  - f_t corresponding to a polygonal approximation $\mathcal{F}_t$ of $\mathcal{F}$;
  - Monotonicity, $t \leq s \implies f_t \leq f_s$, ensure a valid TTG for all time.
  - **Challenge:** Maintain a connected model of $\mathcal{F}$, or—
  - **Challenge:** Extend methods over disconnected $\mathcal{F}$, and—
  - **Challenge:** Directed *exploration* of $\partial\mathcal{F}$ may be required.
- Closed parametric form $\mathbf{M}_{\mathcal{F}}^{\mathcal{U}} = H(c^*)$, $c^* \in \mathbb{R}^N$ means...
  - f_t takes the form of $H(c_t)$, $c_t \in \mathbb{R}^N$;
  - Think of N as a bound on the complexity of $\mathcal{F}_t$;
  - We could attempt learning c^*;
  - A natural loss function is, e.g., $\mathcal{L}(c) \triangleq \left\|\mathbf{M}_{\mathcal{F}}^{\mathcal{U}} - H(c)\right\|_2^2$;
  - **Question:** Does GD over $\mathcal{L}$ respond well to the above challenges?
  - **Question:** Could exploration be *guided* so that GD responds adequately to these challenges?

Thank You for Your Attention!

References

[1] S. C. Edwards, D. M. Le, D. P. Guralnik, and W. E. Dixon, "A Topologically Inspired Path-Following Method with Intermittent State Feedback," *IEEE Robotics Automation Letters*, 2021.

[2] J. Gallier and D. Xu, *A Guide to the Classification Theorem for Compact Surfaces*, ch. Appendix E, pp. 157–163. Springer, 2012.

[3] A. Hatcher, *Algebraic Topology*. Cambridge University Press, Cambridge, 2002, 2002.

[4] M. Brown, "A proof of the generalized Schoenflies theorem," *Bulletin of the American Mathematical Society*, vol. 66, no. 2, pp. 74–76, 1960.

[5] P. Reverdy and D. E. Koditschek, "A dynamical system for prioritizing and coordinating motivations," *SIAM Journal on Applied Dynamical Systems*, vol. 17, no. 2, pp. 1683–1715, 2018.

[6] P. Reverdy, "A route to limit cycles via unfolding the pitchfork with feedback," in *2019 American Control Conference (ACC)*, pp. 3057–3062, 2019.