Distributed State Estimation with Deep Neural Networks for Uncertain Nonlinear Systems under Event-Triggered Communication

F. M. Zegers, R. Sun, G. Chowdhary, and W. E. Dixon, "Distributed State Estimation with Deep Neural Networks for Uncertain Nonlinear Systems under Event-Triggered Communication," in IEEE Transactions on Automatic Control. Submitted.

Problem Formulation

A system is modeled as

$$\dot{x}_0 = f(x_0) + d,$$

where

$x_0 \in \mathbb{R}^n$ unmeasurable, system state,

$d \in \mathbb{R}^n$ time-varying disturbance,

$f : \mathbb{R}^n \to \mathbb{R}^n$ uncertain.

Consider a MAS with N agents

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}),$$

$$\mathcal{V} = \{1, 2, ..., N\},$$

$$\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V},$$

$$\hat{x}_i \in \mathbb{R}^n,$$

$$y_i = C_i x_0 \in \mathbb{R}^m.$$

Design an observer capable of reconstructing the system's state, i.e.,

$$\lim_{t\to\infty} \sup \|\hat{x}_i(t) - x_0(t)\| \leq \varepsilon \quad \forall i \in \mathcal{V}$$

where $\varepsilon > 0.$

Moreover,

- Distributed,
- event-triggered ,
- adaptive through DNN.

Assumptions

- The function f is locally Lipschitz.
- The disturbance d is bounded.
- The graph $\mathcal{G}$ is connected for all time.
- The state of the system is bounded, i.e., $x_0 \in \mathcal{D}$.

Since f is Lipschitz continuous and $x_0 \subset \mathcal{D}$,

$$f|_{\mathcal{D}}(x_0) = \underbrace{W_0^\top \sigma(\Phi(x_0))}_{\text{Ideal DNN}} + \epsilon$$

Ideal DNN — Reconstruction error in $\mathbb{R}^n$.

$W_0 \in \mathbb{R}^{L \times n}$ is the ideal outer layer weight matrix

$\sigma : \mathbb{R}^p \to \mathbb{R}^L$ is a user-defined bounded continuous function (sigmoid, Gaussian)

$\Phi : \mathbb{R}^n \to \mathbb{R}^p$ is an ideal continuous function (ideal inner DNN)

$\Phi(x_0) = (W_\ell^\top \phi_\ell \circ W_{\ell-1}^\top \phi_{\ell-1} \circ \ldots \circ W_1^\top \phi_1)(x_0).$

Using the structure of $f|_{\mathcal{D}}$ and Φ ,

$$\hat{f}_i(\hat{x}_i, \widehat{W}_i, \widehat{W}_{1,i}, \ldots, \widehat{W}_{\ell,i}) \triangleq \widehat{W}_i^\top \sigma(\widehat{\Phi}_i(\hat{x}_i)),$$

$\widehat{\Phi}_i(\hat{x}_i)$ is piecewise continuous, $\{T_p^i\}_{p=1}^{\infty}$.

Let $\{t_k^i\}_{k=0}^{\infty}$ be an increasing sequence of sampling and broadcast times, where

$$\tilde{x}_i(t) = \hat{x}_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i).$$

The observe of agent i is designed as

FF Model Approx.

Output feedback

$$\dot{\hat{x}}_i(t) \triangleq \widehat{W}_i^\top(t)\sigma(\widehat{\Phi}_i(\hat{x}_i(t))) + K_1(z_i(t) - C_i^\top(\hat{y}_i(t) - y_i(t))),$$

$$z_i(t) \triangleq \sum_{j \in \mathcal{N}_i} a_{ij}\left(\tilde{x}_j(t) - \tilde{x}_i(t)\right),$$

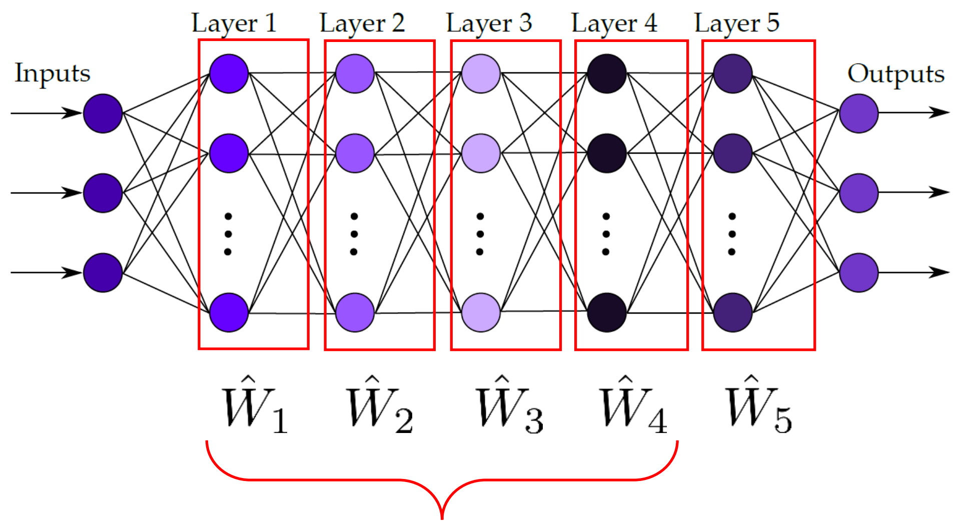
$$\hat{y}_i(t) \triangleq C_i \hat{x}_i(t).$$

Consensus on sampled estimates

$$\dot{\widehat{W}}_i(t) \triangleq \mathrm{proj}(-\Gamma_i \sigma(\widehat{\Phi}_i(\hat{x}_i(t)))(\hat{y}_i(t) - y_i(t))^\top C_i).$$

What about the inner weights and biases?

Inner DNN Training



Levenberg-Marquardt algorithm + Input-Output Data

Levenberg-Marquardt algorithm

- Supervised learning (every input has a corresponding output)
- Nonlinear regression

$$\dot{x}_0 = f(x_0)$$

Output ($\dot{x}_0$), Input (x_0)

$$\dot{x}_0 = f(x_0) + d$$

Unknown Output ($\dot{x}_0$), Unknown Input (x_0), Unknown Disturbance (d)

$$\dot{\hat{x}}_i = \hat{f}_i(\hat{x}_i)$$

Output ($\dot{\hat{x}}_i$), Input ($\hat{x}_i$)

$e_{1,i}(t) \triangleq \hat{x}_i(t) - x_0(t),$ state estimation error.

$e_1(t) \triangleq [e_{1,1}^\top(t), e_{1,2}^\top(t), ..., e_{1,N}^\top(t)]^\top \in \mathbb{R}^{nN}.$

Theorem:
The observer and outer weight update law for each $i \in \mathcal{V}$ ensure the state estimation error is UUB in the sense that

$$\|e_1(t)\|^2 \leq c_1 e^{-\alpha t} + c_2(1 - e^{-\alpha t})$$

provided all assumptions are satisfied, the sufficient parameter conditions (listed in the paper) are satisfied, there exists a matrix K_1 satisfying the bilinear matrix inequality

$$\tfrac{1}{2}(I_N \otimes K_1)C^\top C + \tfrac{1}{2}C^\top C(I_N \otimes K_1) + (L \otimes K_1) \geq k_1 I_{nN},$$

and agent i broadcasts its state estimate as determined by the event-trigger mechanism

$$t_{k+1}^i \triangleq \inf\left\{t > t_k^i : \phi_1\|e_{2,i}(t)\|^2 \geq \phi_2\|z_i(t)\|^2 + \frac{\epsilon}{N}\right\},$$

$$\phi_1 \triangleq \frac{k_2}{2} + \frac{\kappa}{2}\|L \otimes K_1\|^2, \quad \phi_2 \triangleq \frac{k_2}{4\|L \otimes I_n\|^2}.$$

The system model is a 3D Van der Pol oscillator, where

$$f(x_0(t)) = \begin{bmatrix} \mu(x(t) - x^3(t)/3 - y(t)) \\ x(t)/\mu \\ -\mu z(t) \end{bmatrix}, \quad \mu = 0.3.$$

The disturbance acting on the system is

$$d(t) = [0.5\sin(3t),\ 0.75\cos(t),\ \cos(3.75t)]^\top .$$

The MAS observing the system consists of three agents with the following output matrices

$$C_1 = [1\ 0\ 0],\ C_2 = [0\ 1\ 0],\ C_3 = [0\ 0\ 1].$$

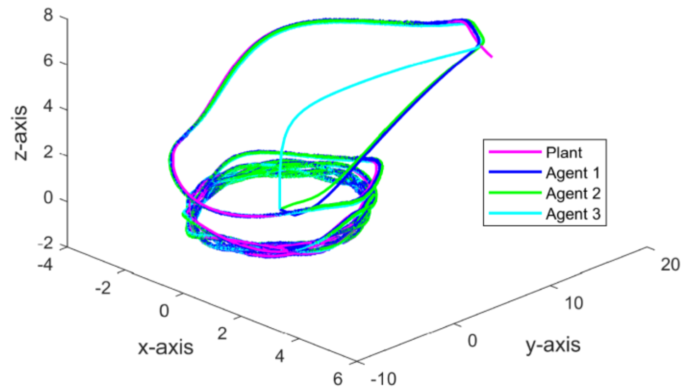
The adjacency matrix encoding the communication topology of the MAS is $\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

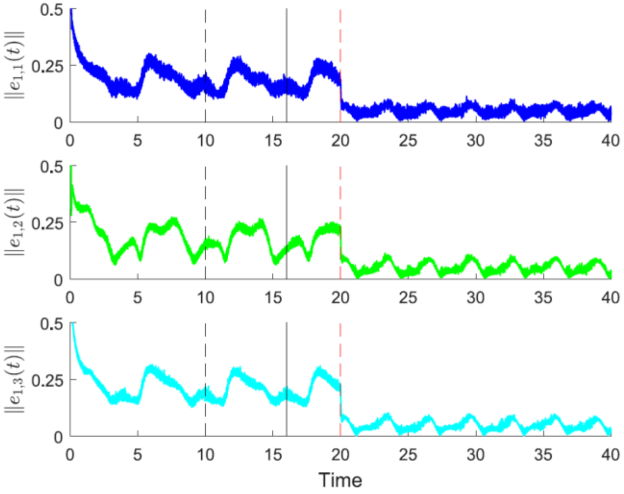
The simulation parameters are

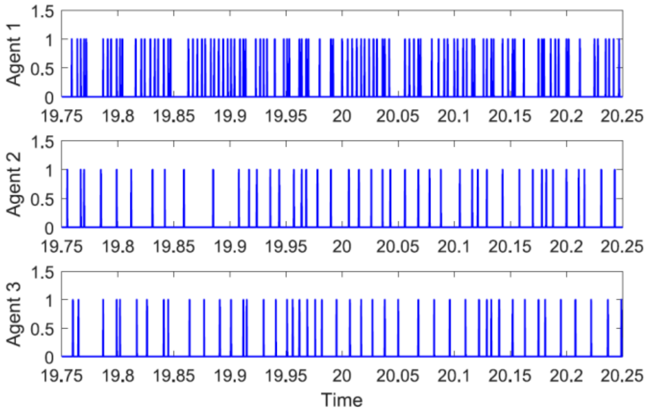
$$N = 3,\ \kappa = 1,\ \epsilon = 3 \times 10^3,\ \Gamma = 3I_3,\ \rho = 2,\ \delta = 0.3,\ k_2 = 10,\ \text{and}\ k_1 = 23.3$$

$$K_1 = \begin{bmatrix} 134.86 & 0 & 0 \\ 0 & 263.23 & 0 \\ 0 & 0 & 263.23 \end{bmatrix}.$$

Simulation Results







- Conclusion
  - Performed distributed state estimation
    - Uncertain nonlinear system,
    - Event-triggered communication,
    - Deep neural networks,
      - Multi-timescale learning (online and offline).
  - Free to adopt several (offline) training strategies
    - Robust to external disturbances (UUB).

- Future Work
  - Under what conditions does $\dot{\hat{x}}_i \to \dot{x}_0$? Are there any offline learning strategies capable ensuring $\hat{f}_i \to f$?
  - Develop a training strategy that runs consensus on the neural network weights.
  - Develop an observer that does not require all states of the system to be measurable by the MAS.
  - Consider sensor/measurement noise (unmatched uncertainty).

Event/Self-Triggered Multi-Agent System Rendezvous with Graph Maintenance

F. M. Zegers, D. Guralnik, and W. E. Dixon, "Event/Self-Triggered Multi-Agent System Rendezvous with Graph Maintenance," in IEEE Control Systems Letters/IEEE Conference on Decision and Control. Submitted.

Problem Formulation

Consider a multi-agent system of N agents.

- Dynamics of agent p

$$\dot{x}_p = u_p$$

$x_p \in \mathbb{R}^n$, position of agent p

$u_p \in \mathbb{R}^n$, control of agent p

- Undirected Network Topology

$$\mathcal{G}_R(\mathbf{x}(0)) = (\mathcal{V}, \mathcal{E}(\mathbf{x}(0)), W(\mathbf{x})),$$
$$\mathbf{x} \triangleq (x_p)_{p\in\mathcal{V}} \in \texttt{Conf}(\mathcal{V}) \triangleq (\mathbb{R}^n)^{\mathcal{V}},$$
$$\mathcal{V} = \{1, 2, ..., N\},$$
$$\mathcal{E}_R(\mathbf{x}(0)) = \{(p,q) \in \mathcal{V} \times \mathcal{V} : \|x_p(0) - x_q(0)\| \leq R\},$$
$$W(\mathbf{x}) = [w_{pq}] \in \mathbb{R}^{N\times N}.$$

Design an event/self-triggered controller capable of

- achieving $\nu-$approximate rendezvous, and
- maintaining the edges of the initial $\mathcal{G}_R(\mathbf{x}(0))$.

Key Assumption

- The initial graph $\mathcal{G}_R(\mathbf{x}(0))$ is connected, and every edge $(p,q) \in \mathcal{E}(\mathbf{x}(0))$ satisfies $\|x_p(0) - x_q(0)\| < R$.

Let $r : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be a non-decreasing continuous function such that $r(0) > 0$. Also, let

$$P(\rho) \triangleq \int_0^\rho r(s) s ds, \quad \rho \in \mathbb{R}_{\geq 0}.$$

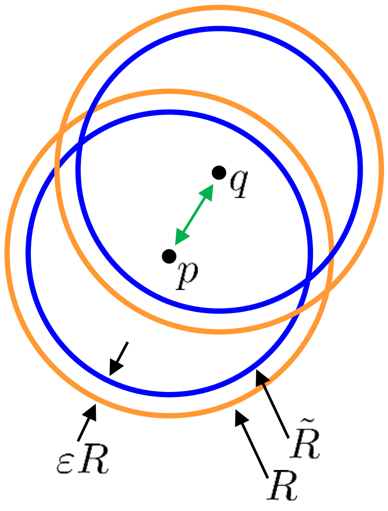
The potential and edge weight assigned to edge $\{p, q\} \in \mathcal{E}$ are

$$V_{pq}(\mathbf{x}) \triangleq P\left(\|x_p - x_q\|\right),$$

$$w_{pq} \triangleq r\left(\|x_p - x_q\|\right).$$

Next, define $\varepsilon > 0$ and $\tilde{R} \triangleq R(1-\varepsilon)$.

$$r(s) \triangleq \mu \cdot \begin{cases} 1, & s \in [0, \tilde{R}] \\ 1 + \omega(s^2 - \tilde{R}^2), & s \in [\tilde{R}, R] \\ 1 + \omega(R^2 - \tilde{R}^2), & s \geq R. \end{cases}$$



Hybrid Controller and Closed-Loop Dynamics

The controller of agent p is $u_p \triangleq \eta_p$.

The hybrid system for agent p is

$$\mathcal{H}: \begin{cases} \dot{x}_p = \eta_p, & \\ \dot{\tau}_p = 1, \quad \dot{\eta}_p = 0_n, & \mathbf{T}_p(\boldsymbol{\xi}) > 0 \\ \tau_p^+ = 0, \quad \eta_p^+ = \sum_{q \in \mathcal{N}_p} w_{pq}\left(x_q - x_p\right), & \mathbf{T}_p(\boldsymbol{\xi}) = 0, \end{cases}$$

where

1. $\boldsymbol{\eta} \triangleq (\eta_p)_{p\in\mathcal{V}} \in \mathtt{Conf}(\mathcal{V})$, $\boldsymbol{\tau} \triangleq (\tau_p)_{p\in\mathcal{V}} \in [0,\infty)^{\mathcal{V}}$,
   $\boldsymbol{\xi} \triangleq [\mathbf{x}^\top, \boldsymbol{\eta}^\top, \boldsymbol{\tau}^\top]^\top \in \mathcal{X} \triangleq \mathtt{Conf}(\mathcal{V}) \times \mathtt{Conf}(\mathcal{V}) \times [0,\infty)$,

2. For each $p \in \mathcal{V}$, the trigger of agent p, i.e., $\mathbf{T}_p : \mathcal{X} \to \mathbb{R}$, is a continuous function that satisfies
   $$\mathbf{T}_p(\boldsymbol{\xi}) = 0 \Longrightarrow \mathbf{T}_p(\boldsymbol{\xi}^+) > 0.$$

$$C \triangleq \bigcap_{p\in\mathcal{V}} [\mathbf{T}_p > 0], \ D \triangleq \overline{C} \cap \bigcup_{p\in\mathcal{V}} [\mathbf{T}_p = 0]$$

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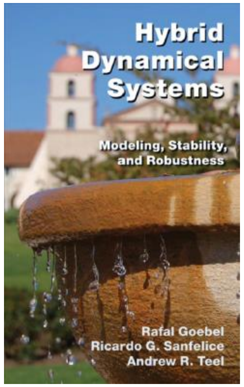
Supporting Items

Lemma 1. Every initial condition $\phi(0,0) \in \overline{C}$ determines one and only one maximal solution of the hybrid system $\mathcal{H}$ as defined in (6). Moreover, every maximal solution of $\mathcal{H}$ is either $t-$complete or Zeno (complete).

Sketch of Proof: Use Proposition 2.10 & properties of our $\mathcal{H}$.



Additional Restrictions on $\mathbf{T}_p$:

$\mathbf{T}_p$ is an *admissible trigger,* if $\mathbf{T}_p$ is continuously differential, $\mathbf{T}_p \leq f_p$ throughout $\mathcal{X}$ and there exist $\mathfrak{m}, \mathfrak{h} > 0$ such that

$$(a)\ \ \mathbf{T}_p(\boldsymbol{\xi}^+) \geq \mathfrak{h} \text{ at each jump of } \mathcal{H},$$

$$(b)\ \ \frac{d}{dt}\mathbf{T}_p \geq -\mathfrak{m} \text{ holds along any solution of } \mathcal{H}.$$

Theorem 1. Given $\tilde{R} = R(1-\varepsilon)$ satisfying (3) and a connected graph $\mathcal{G}_R(\mathbf{x}(0))$, if $\{\mathbf{T}_p\}_{p\in\mathcal{V}}$ is a collection of admissible triggers, then every solution of $\mathcal{H}$ satisfying (10) and initiating from $\mathcal{C}_{\tilde{R}}(\mathcal{G})$ remains $\mathcal{C}_R(\mathcal{G})$, is $t-$complete, and the controllers u_p are bounded for all time.

$$\Delta\mathbf{x} \triangleq (x_p - x_q)_{pq\in\mathcal{E}}$$

Theorem 2. Let $\nu > 0$, $\tilde{R} = R(1-\varepsilon)$ satisfy (3), and $\mathcal{G}_R(\mathbf{x}(0))$ be a connected graph. Suppose $0 < \beta < \sigma K \tilde{R}^2$ and $\{\mathbf{T}_p\}_{p\in\mathcal{V}}$ is a collection of admissible triggers such that, over solutions of $\mathcal{H}$ satisfying (10) and initiating from $\mathcal{C}_{\tilde{R}}(\mathcal{G})$,

$$\mathbf{T}_p + \sigma K \tilde{R}^2 \leq f_p + \beta$$

holds for all $p \in \mathcal{V}$. Then, any such solution satisfies

$$\|\Delta\mathbf{x}(t)\|_\infty^2 \leq \frac{r(R)|\mathcal{E}|}{r(0)}\left(\|\Delta\mathbf{x}(0)\|_\infty^2 \, \mathrm{e}^{-\frac{NKt}{r(R)|\mathcal{E}|}} + \sigma\tilde{R}^2\right).$$

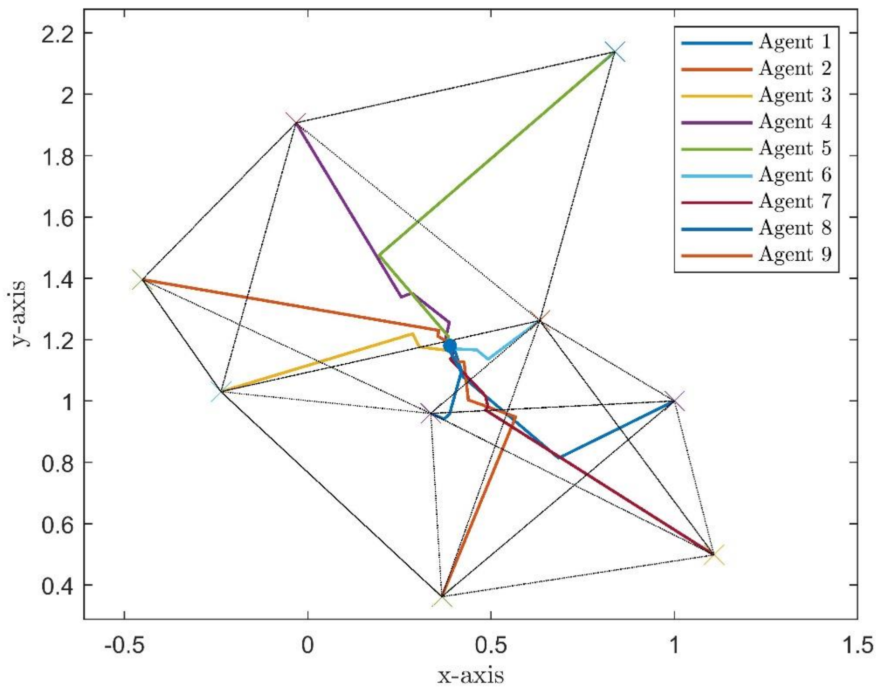
In this sense, $\mathcal{R}_\nu$ is exponentially stable for

$$\sigma = \frac{\nu^2}{|\mathcal{E}|\tilde{R}^2}\cdot\frac{r(0)}{r(R)}.$$

In particular, $\nu'-$ approximate rendezvous is achieved for every such maximal solution, for any $\nu' > \nu$.

Simulation parameters: $\mathbf{T}_{p,1}(\boldsymbol{\xi}) = \|\eta_p\|^2 - \|\zeta_p\|^2 + \alpha(\|\eta_p\|)$

$$\alpha(s) = \begin{cases} \beta - \frac{\beta}{\gamma}s, & s \in [0, \gamma] \\ 0, & s \geq \gamma \end{cases}$$



Closing

- Conclusion
  - Event/self-triggered approximate rendezvous framed in the hybrid systems setting.
  - Allows the consideration of multiple event/self trigger mechanisms.
  - Edge potentials
    - bounded,
    - arbitrarily small buffer, and
    - adjustable given any initial configuration.

- Future Work
  - Develop robustness to perturbations in the state and trigger
    - Hybrid Basic Conditions.
  - Consider more complex dynamics.
  - Develop self-triggers that adhere to prescribed periods of radio silence.
  - Extend development to other types of mobile network control problems.

Thank You