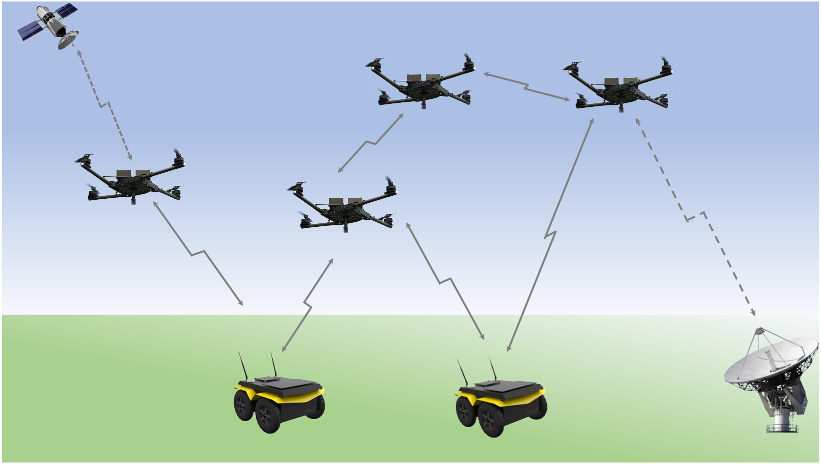
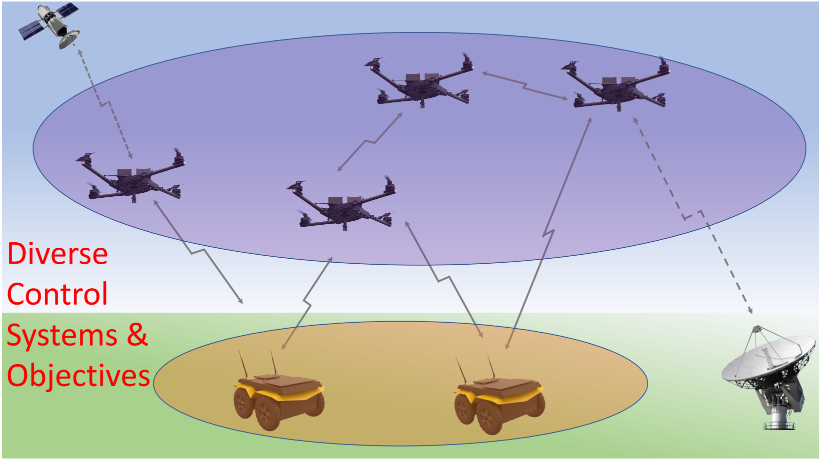
POMDPs and Reinforcement Learning for Cross-Layer Control and Network Optimization

John M. Shea and Caleb M. Bowyer

Big Picture: Joint Optimization of Control and Networks



Challenges



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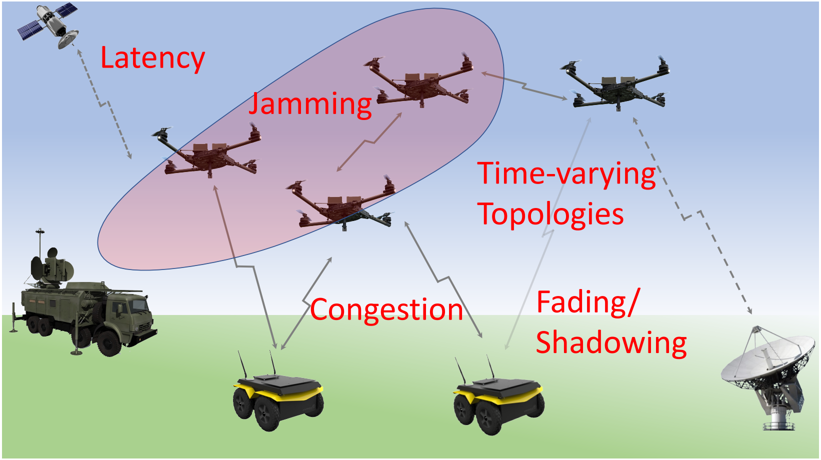
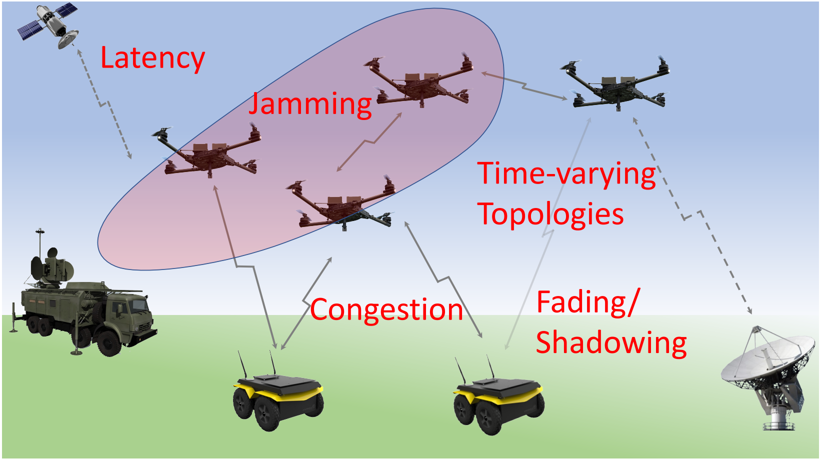

Latency
Jamming
Time-varying
Topologies
Congestion
Fading/
Shadowing

Challenges



Multi-objective, distributed, partially observable
⇒ Partially observable stochastic game

Constrained Problem: Distributed Sensing

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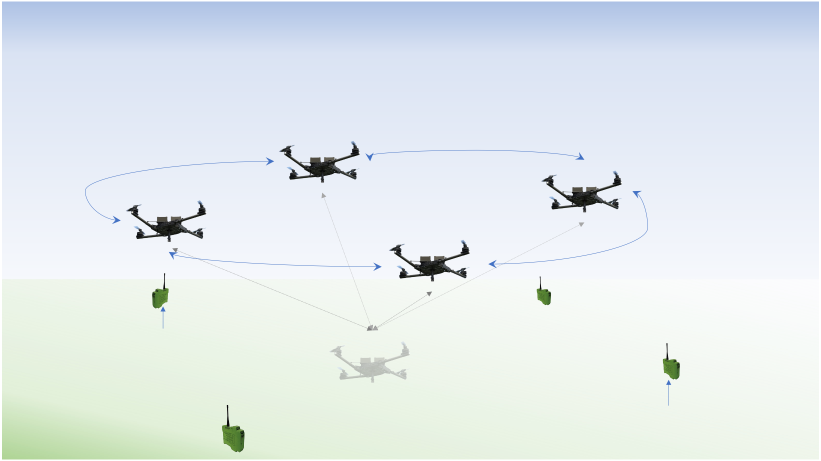
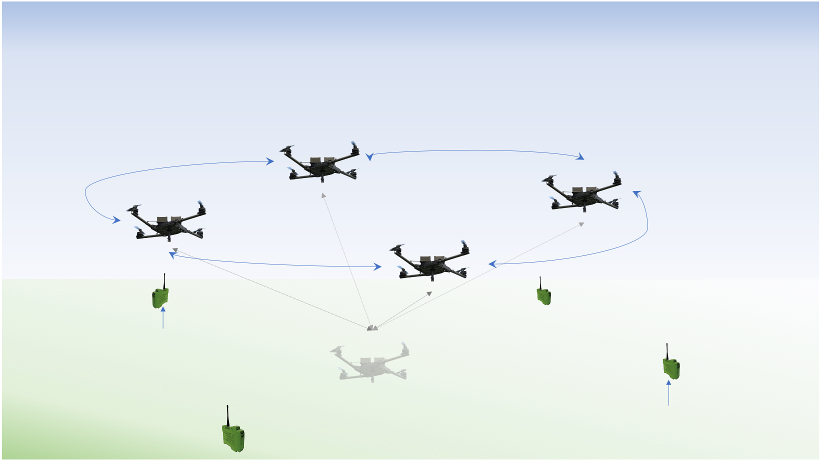
Distributed Sensing and Coordination: Who senses and transmits? $\Rightarrow$ Partially observable, multi-agent MDP

Constrained Problem: Distributed RF Localization



Constrained Problem: Distributed RF Localization



Coordinated sensing and localization: When to sense? When to synchronize? $\Rightarrow$ Partially Observable MDP

Application: Distributed Localization in GPS-Denied Environments

- Desire low cost, low complexity, robust, high-performance solutions to tracking/RADAR in GPS-denied environments

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- Desire low cost, low complexity, robust, high-performance solutions to tracking/RADAR in GPS-denied environments
  - **Low cost, low complexity:** sensors have unreliable clocks and noisy RF
  - **Robust:** no single point of failure $\Rightarrow$ distributed sensors with robustness to failure of individual sensors
  - **High-performance:** need to generate reliable localization estimates using noisy ToF measurements from noisy clocks

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Need for Synchronization

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- With unreliable clocks, timing drifts between synchronization times reduces localization accuracy
- Clock synchronization requires communication among sensors and localization may not be possible during the synchronization times

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Need to optimize between localization and synchronization

System Model

- Network of m stationary sensing agents
- Single asset to be tracked:
  - Asset transmits beacon signal at known times to agents to facilitate tracking in GPS-denied environment
  - Asset moves according to known Markov model

System Model – cont.

- Sensors measure time-of-flights (ToFs) of beacon signal and reference leader agent localizes (LOC) asset once measurements are fused
- Each agent's clock drifts independently and variance of clock signals increase with time
- Agents can synchronize (SYNCH) clocks at expense of not being able to measure ToFs during that time

Model-Free Localization for general $3D$ space

- Let coordinates of asset and sensor i in interval k be $(x_{k,a}, y_{k,a}, z_{k,a})$ and (x_i, y_i, z_i)

Model-Free Localization for general $3D$ space

- Let coordinates of asset and sensor i in interval k be $(x_{k,a}, y_{k,a}, z_{k,a})$ and (x_i, y_i, z_i)
- Using sensor $m-1$ as a reference, form linear equations $\mathbf{A} \cdot \mathbf{v}_k = \boldsymbol{\beta}_k$
- Here $\mathbf{v}_k = [x_{k,a}, y_{k,a}, z_{k,a}]^T$, $\mathbf{A}$ is a matrix with row i given by

$$\mathbf{A}_i = [2(x_i - x_{m-1}),\ 2(y_i - y_{m-1}),\ 2(z_i - z_{m-1})], \\ i \in \{0, 1, \ldots, m-2\},$$

and $\boldsymbol{\beta}_k$ is a column vector with component

$$\beta_i = c^2\left(\widehat{\tau}_{k,i}^2 - \widehat{\tau}_{k,m-1}^2\right) - \left(x_i^2 - x_{m-1}^2\right) - \left(y_i^2 - y_{m-1}^2\right) \\ - \left(z_i^2 - z_{m-1}^2\right), \qquad i \in \{0, 1, \ldots, m-2\}$$

Localization Solution

- The linear least squares solution is $[\widehat{x}_{k,a}, \widehat{y}_{k,a}, \widehat{z}_{k,a}]^T = \mathbf{A}^\dagger \boldsymbol{\beta}_k$ where $\mathbf{A}^\dagger$ is the Moore-Penrose pseduo-inverse of $\mathbf{A}$
- Not always solvable, depending on sensor topology – can also solve constrained least squares problem

Optimal Coordination of Localization and Synchronization

- Pure localization generally not good enough because of noisy clocks
- Does not inform system of when SYNC is needed

- Resolve both problems by treating tracking problem as HMM and treating choice of SYNC/LOC as control problem

Optimal Coordination of Localization and Synchronization

- Pure localization generally not good enough because of noisy clocks
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- Resolve both problems by treating tracking problem as HMM and treating choice of SYNC/LOC as control problem
  - Since true state of asset not directly observable, this results in a **Partially Observable Markov Decision Process (POMDP)**

POMDP

1. State space $\mathcal{M} \times \mathcal{T}$, tuples of movement state $\mathcal{M}$ and time since last sync $\mathcal{T} = \mathbb{Z}^+$
2. Two controls $\mathcal{U} = \{u_l, u_s\}$, corresponding to *localize* or *synchronize*
3. Continuous set of noisy observations from ToF measurements, $\mathcal{O}$
4. State-to-state transition function: $p_{ij}(u) = \Pr(M_{k+1} = j | M_k = i, U_k = u) \; \forall \; i, j \in \mathcal{M}$ based on Markov movement model
   (Note time since last sync is deterministic given previous state and control

POMDP – cont.

5. State-to-observation density function: $q_{jo}(u) = f(O_{k+1} = o | M_{k+1} = j, U_k = u) \; \forall \; j \in \mathcal{M}, \; \forall \; o \in \mathcal{O}$ from ToF noise
6. Cost function: MMSE of position estimate

Belief States, Observation Sequences and Control Sequences

- Given:
  - $\mathbf{o}_k$: vector of observations up to interval k
  - $\mathbf{u}_{k-1}$: vector of controls leading up to interval $k-1$

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- Given:
  - $\mathbf{o}_k$: vector of observations up to interval k
  - $\mathbf{u}_{k-1}$: vector of controls leading up to interval $k-1$
- Belief state at interval k is $\mathbf{b}_k$:

$$b_k(m) = \Pr\left(M_k = m \,|\mathbf{o}_k, \mathbf{u}_{k-1}\right)$$

- the maximum a posteriori (MAP) estimate of the asset state is

$$\widehat{M}_k = \arg\max_{m \in \mathcal{M}} \ \mathbf{b}_k\left(m\right).$$

Belief Update

- Continuous observation space (localization results) – most papers consider only a finite observation space

$$b_{k+1}(m_{k+1}) = \frac{f\left(\mathbf{o}_{k+1}, m_{k+1} \left| \mathbf{u}_k \right.\right)}{f\left(\mathbf{o}_{k+1} \left| \mathbf{u}_k \right.\right)} \tag{1}$$

where

$$f(\mathbf{o}_{k+1}, m_{k+1} \left| \mathbf{u}_k \right.) = f(o_{k+1} \left| m_{k+1} \right.) \sum_{m_k \in \mathcal{M}} \Pr(m_{k+1} \left| m_k, u_k \right.) \cdot f(\mathbf{o}_k, m_k \left| \mathbf{u}_{k-1} \right.)$$

Localization Density in Belief Update

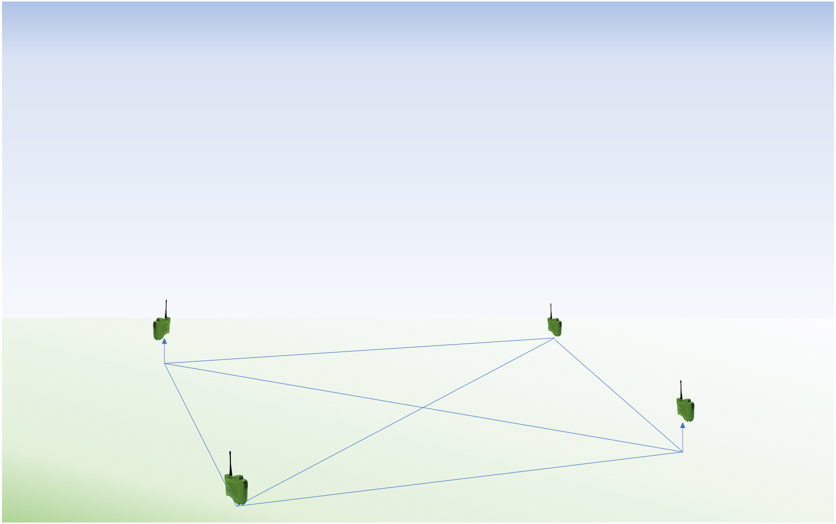
- the conditional distribution of o_k given m_k and $T_k^{(s)}$ is modeled as multi-variate Gaussian with mean determined by M_k and covariances determined by ToF variance, σ_N
- Relation between σ_N and covariances is determined empirically:
  - position estimate variances are proportional to σ_N^2 (input ToF Variance)
  - different coordinates are approximately uncorrelated and are thus treated as independent

Beleif Updates During Sync

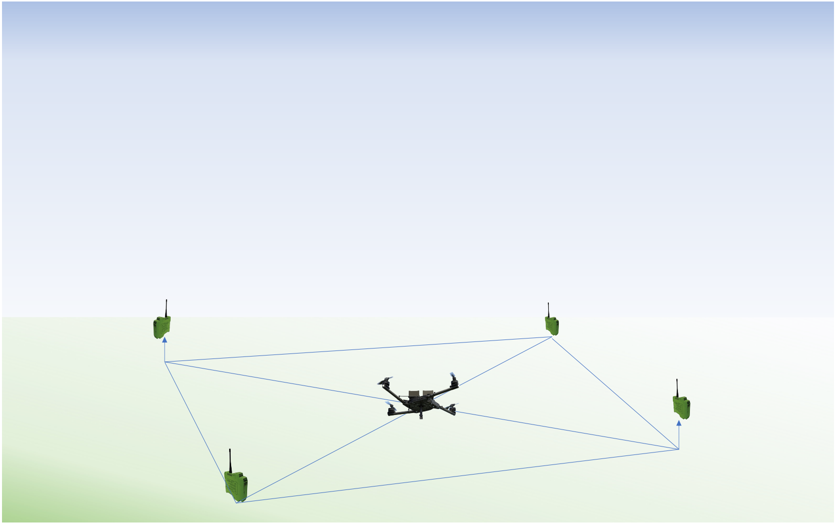
- If the control is **sync**, then no measurement o_{k+1} is available from localization;
- Update the belief by applying the Markov model transitions probabilities

$$\mathbf{b}_{k+1} = \mathbf{P} \cdot \mathbf{b}_k$$

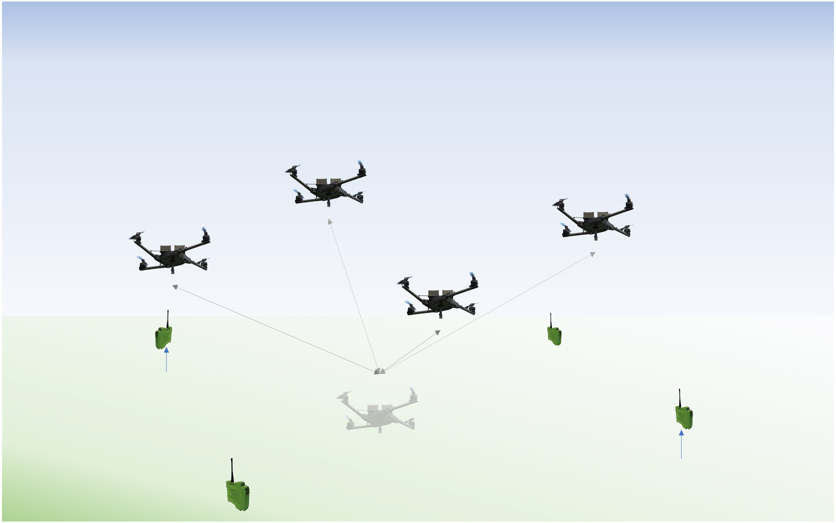
Simulation Model: 3D Sensing Model



3D Sensing Model and Asset Ground Station



3D Sensing and Asset Movement Model



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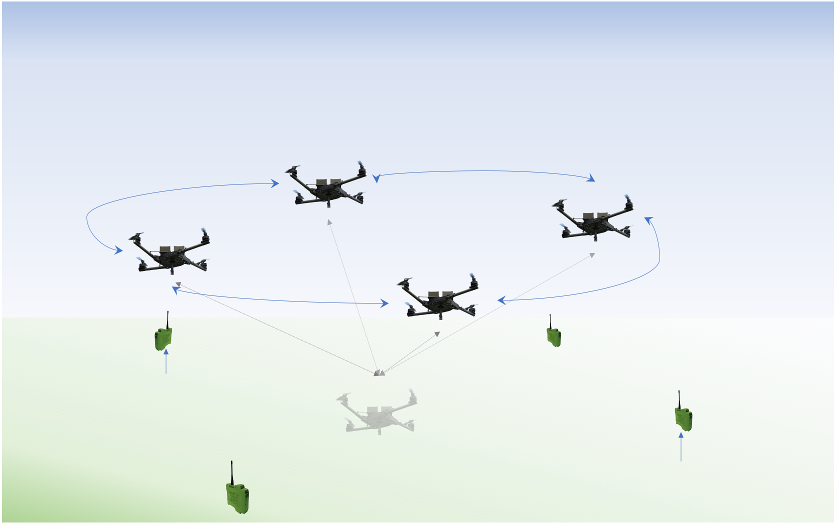

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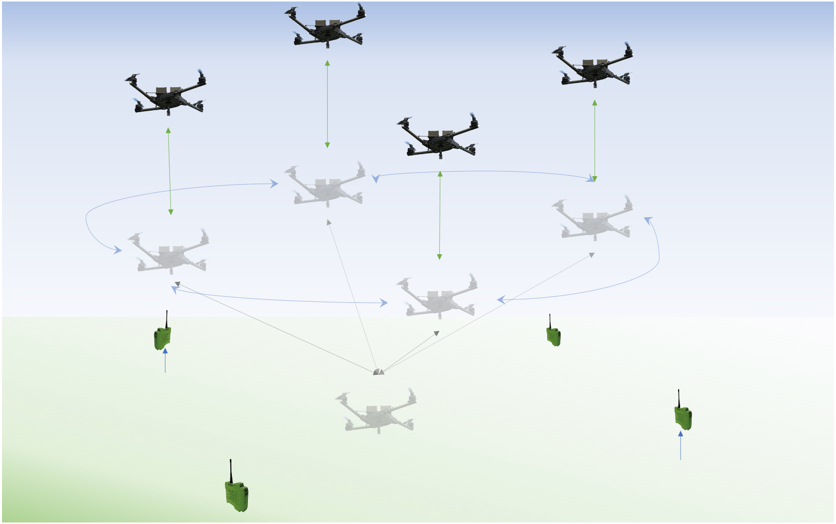

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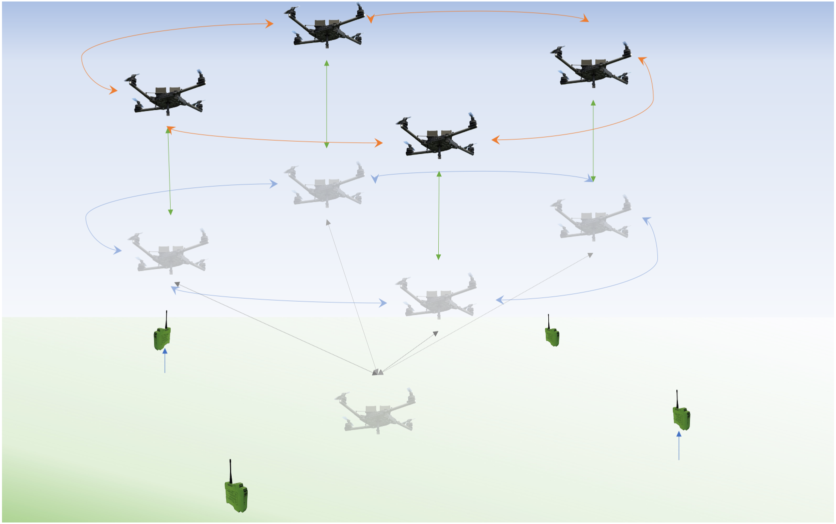
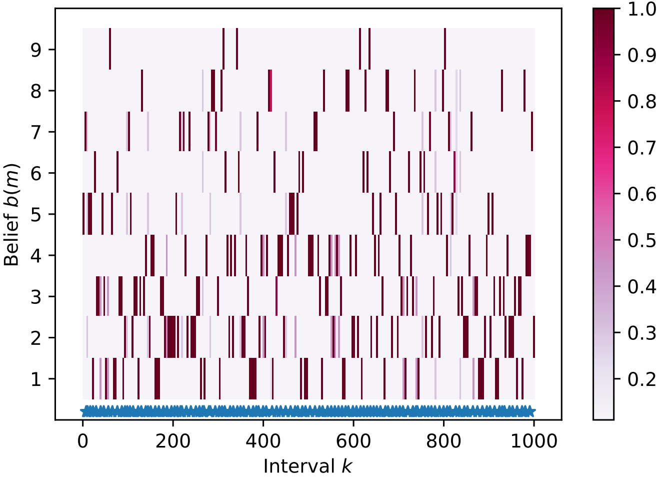
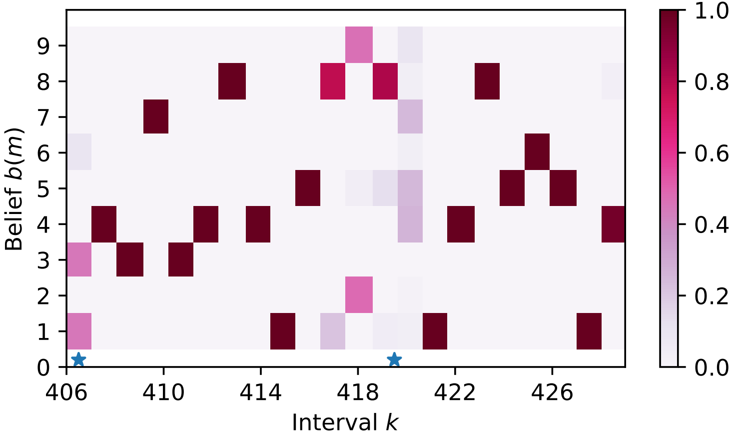


Belief State Evolution



Belief State Evolution Detailed View



Belief State

- Belief state is a sufficient statistic for deciding the control u_k at stage k
- However: state space has $|\mathcal{M}|$ continuous dimensions
- To apply Q-learning, need to do some form of approximation:

Approximate Q-Learning

Two fundamental approaches

- **Q-function Approximation:**
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- **Q-function Approximation:**
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  - Nonlinear: Deep Q-learning
- **Belief State Approximation:**
  - Approximate belief distribution by parameterized distribution (e.g., Gaussian) and quantize parameters: our triple Q (TQ)-learning
  - Belief uncertainty can be represented by universal measures (entropy) or application-specific measures (EMSE)

Belief State Compression

- Compress beliefs and syncing information into triple of **discrete** values $\underline{\mathrm{m}}_k = [\hat{m}_k, T_k^{(s)}, \nu_k]$:

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- Compress beliefs and syncing information into triple of **discrete** values $\underline{\mathrm{m}}_k = [\hat{m}_k, T_k^{(s)}, \nu_k]$:
  - $\hat{m}_k$: ML estimate for movement state
  - $T_k^{(s)}$: is the number of time since last **sync**
  - ν_k: Quantized EMSE
- Called: *Triple Q-Learning* (TQ-Learning)

TQ-Learning Update

- Use tabular Q-learning with usual update rule:

$$Q(\underline{\mathrm{m}}, u) \mathrel{+}= \alpha \left[c + \gamma \min_{u'} Q(g(\underline{\mathrm{m}}, u), u') - Q(\underline{\mathrm{m}}, u) \right].$$

TQ-Learning Update

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$$Q(\underline{\mathrm{m}}, u) += \alpha \left[c + \gamma \min_{u'} Q(g(\underline{\mathrm{m}}, u), u') - Q(\underline{\mathrm{m}}, u) \right].$$

- Here, c is the cost of performing u from whatever true state the asset actually is in, g is a generic state update function, and u' is the control that minimizes the cost in the next interval.

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  - γ: discount factor

Replicated Q-learning (RQ-learning)

- Replicated Q-learning uses one vector $\mathbf{q}_u$ for each control $u \in \mathcal{U}$ and approximates the value of the Q-function for belief state $\mathbf{b}$ as $Q_{\mathbf{b}}(u) = \mathbf{q}_u \cdot \mathbf{b}$.
- Because our POMDP state contains both motion state M_k and time since last sync, $T_k^{(s)}$,
- The $\mathbf{q}_u$ vector's elements are updated by

$$q_u(x) = q_u(x) + \alpha b(x) \left[c + \gamma \min_{u'} Q_{\mathbf{b}'}(u') - q_u(x) \right] \forall x \in \mathcal{X}.$$

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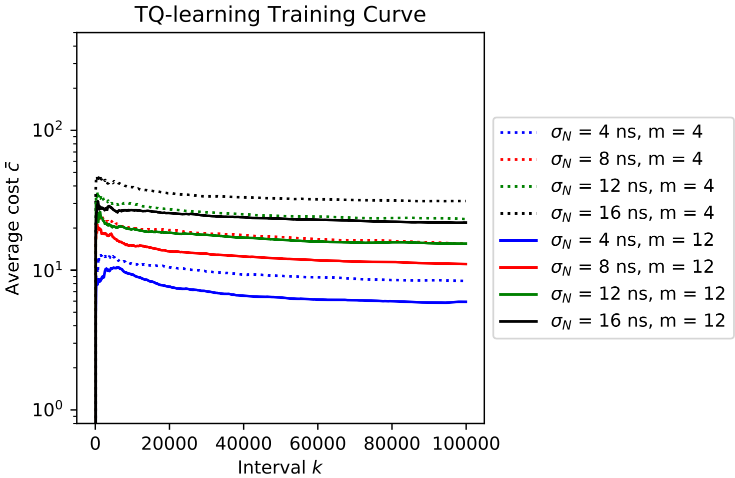
- Exploit deterministic part of state to represent as $2T_{max}$ vectors of dimensions $|\mathcal{M}|$

Fixed-Rate Deterministic Policies (Model-Free)

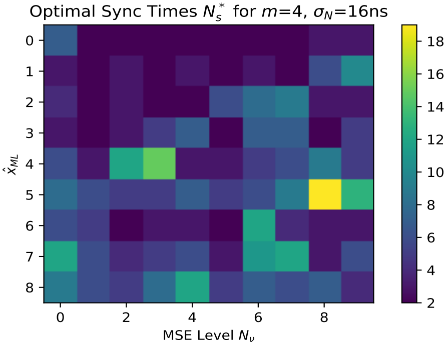
- Fixed-rate deterministic (FRD) is the standard approach used in most of the sensing literature that addresses timing synchronization
- FRD syncs every k intervals, where the value of k is optimzied to minimize the MSE

TQ-learning Training Curve:



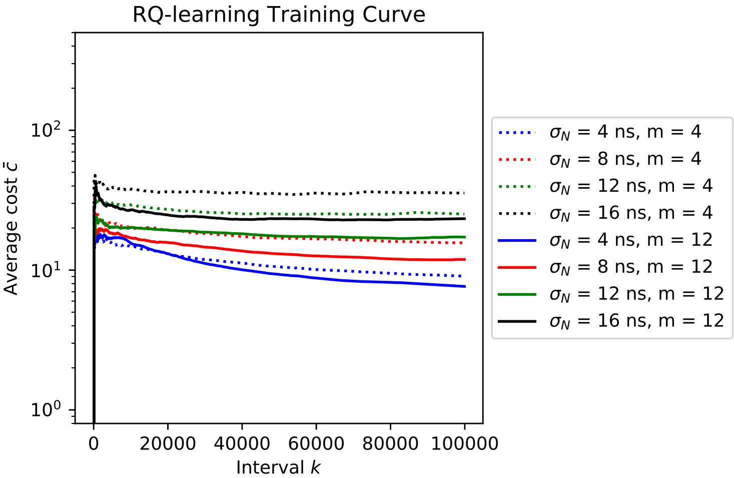
TQ-learning Optimal Sync Times

High Input ToF Variance



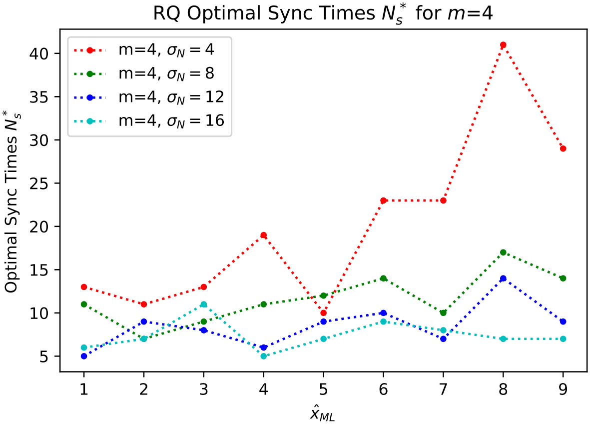
- Note that for the highest input ToF variance, the higher levels of the quantized MSE are reached.
- Also, note that generally the network can optimally wait longer periods of time to resync at the lowest MSE level.

RQ-learning Training Curve:

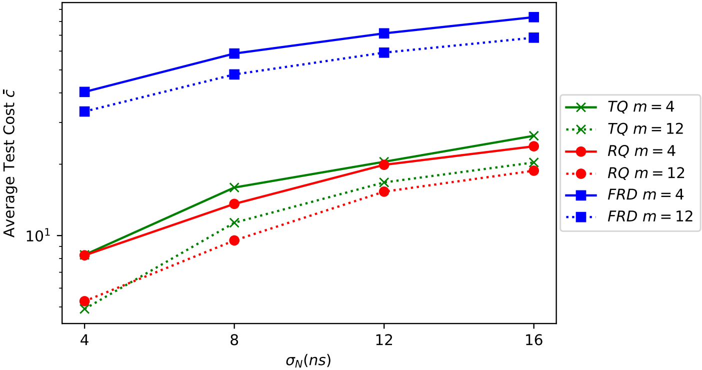


RQ-learning Approximate Sync Times (ML state)

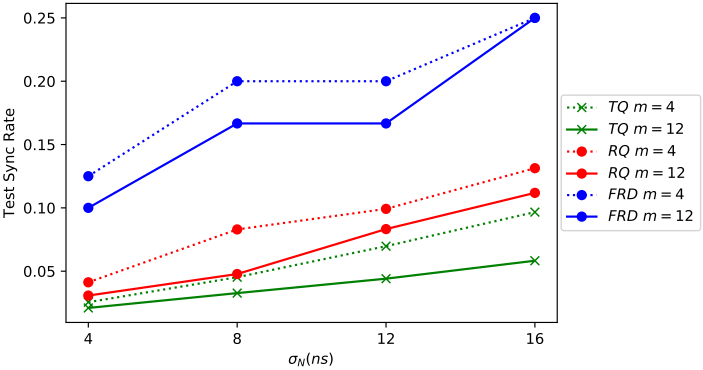
- RQ-learning uses the entire belief vector, so no simple visualization of optimal sync times
- Below we show the sync times if all the belief was concentrated on the ML state:



Performance: Localization MSE



Performance: Average Sync Rates



Conclusion

- Formulated problem of optimizing synchronization times for system of distributed sensors tracking a mobile asset as a POMDP
- Applied dimensionality reduction techniques to perform Q-learning on that POMDP:
  - TQ-learning replaces belief distribution with ML estimate for motion state, time since last sync, and (quantized) expected MSE
  - "old" RQ-learning uses one q vector for each control of size $|\mathcal{M}|$.
  - "new" RQ-learning uses one q vector for each control and stage number of size $|\mathcal{M}|$.
- Q-learning approaches outperform best FRD policies
- Deep Q-learning techniques are a good match for POMDP problems because they can accept the continuous belief state info $\rightarrow$ currently investigating

Conclusion

- Develop approaches for distributed decision making in sensor networks (partially observable MAMDPs)
- Want to deploy and test these ideas using our AFOSR DURIP-funded testbed
- Work towards general framework for joint optimization of stochastic controls and networks