UNKNOWN TIME-VARYING INPUT DELAY COMPENSATION FOR UNCERTAIN NONLINEAR SYSTEMS

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2016

To my parents, Vehdet and Saadet Obuz; my wife, Aysel Tekgoz Obuz; and my family for their invaluable support and encouragement

ACKNOWLEDGMENTS

I would like to express sincere gratitude towards Dr. Warren E. Dixon, whose advice and motivation were essential to my academic success. As my academic advisor, he has guided my research with valuable advice and encouraged the creativity regarding research that helped shape this dissertation. As a mentor, he has played a central role in preparing me for my academic career by inspiring me to do independent research.
I would also like to extend my gratitude towards my committee members Dr. Carl Crane, Dr. Prabir Barooah, and Dr. John Shea for their valuable recommendations and insights. I would also like to thank my colleagues at the University of Florida Nonlinear Controls and Robotics laboratory for countless fruitful discussions that have helped shape the ideas in this dissertation. I acknowledge that this dissertation would not have been possible without the support and encouragement provided by my family (Vahdet Obuz, Saadet Obuz, Aysel Tekgoz Obuz, Halil Obuz, Seda Bulut, Mehmet Bulut, Fatih Muhammet Bulut) and my friends (especcially Mustafa Bahar, Irfan Kil, Caglar Doguer, Muharrem Ayar, Eray Caliskan, Enver Tatlicioglu, Ryan Downey, Justin Klotz, Rushikesh Kamalapurkar, Victor Duenas, Indrasis Chakraborty) and without the financial support provided by Turkey Republic National Education Ministry.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

NMES	Neuromuscular Electrical Stimulation

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

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December 2016

Chair: Warren E. Dixon
Major: Mechanical Engineering

Time delay commonly exists in engineering applications such as master-slave robots, haptic systems, networked systems, chemical systems, and biological systems. The system dynamics, communication over a network, and sensing with associated sensor processing (e.g., image-based feedback) can induce time delays that can result in decreased performance and loss of stability. Time delays in physical systems are often time-varying. For example, the input delay in neuromuscular electrical stimulation (NMES) applications often changes with muscle fatigue, communication delays in wireless networks change based on the operating environment and distance between the communicating agents, etc.

NMES is a technique that activates muscle artificially by using electrical impulses to induce a contraction. NMES is a prescribed treatment for various neuromuscular disorders (e.g., providing restored motor function in spinal cord injured individuals, relearning skills after a stroke, and increasing muscle mass). However, feedback control of NMES is difficult since muscles exhibit a delayed response to electrical stimulation. Furthermore, the exact value of the input delay is time-varying. Specifically, muscle groups rapidly fatigue in response to artificial muscle stimulation compared to volitional contractions. Muscle fatigue, in turn, can cause the delay to increase during NMES. Therefore, input delay presents a significant challenge to designing controllers that yield controlled limb movement, especially since the delay is unmeasurable during limb

movement. Motivated by such practical engineering challenges, this dissertation focuses on designing controllers to compensate for time delay disturbances for uncertain nonlinear systems.

Chapter 1 motivates current challenges in the field of input delayed systems. It also provides an overview of pioneering and state of the art control strategies for systems with input delays and discusses the contributions of this dissertation. Chapter 2 details the development of a novel tracking controller for a class of uncertain nonlinear systems subject to unknown time-varying input delay and additive disturbances that provide uniformly ultimate boundedness of the tracking error signals. The techniques used in Chapter 2 are extended in Chapter 3 to investigate compensating for the effects of input delay without delay rate constraints for uncertain Euler-Lagrange systems with unknown time-varying input delay. The techniques in Chapters 2 and 3, compensate for the effects of unknown time-varying input delay by using a constant estimate of the input delay. These techniques are extended in Chapter 4 by using an adaptive based control strategy to estimate an uncertain state-dependent input delay and compensate the effects of input delay despite an uncertain nonlinear system with an unknown time-varying additive disturbances. In Chapter 5, a position tracking control problem is considered for an uncertain time-varying delayed muscle response by using the technique used in Chapter 3 and adding an integral feedback of the error signal to increase tracking performance and robustness of the controller with respect to uncertainties in lower limb dynamics and time-varying input delay. A robust controller is designed in Chapter 6 to ensure reaction torque tracking in isometric NMES for the uncertain, nonlinear dynamics of the lower limb with additive disturbances without knowledge of time-varying input delay. Chapter 7 discusses the main contributions of each chapter, limitations and implementation challenges, and possible future research directions.

CHAPTER 1
INTRODUCTION

1.1 Literature Review

In recent decades, numerous investigations have focused on linear systems that experience a known delay in the control input. Results such as [2–6] assume that the system dynamics are linear and exactly known. However, exact model knowledge of the dynamics may not be available in many engineering systems. Results such as [7–9] develop robust controllers which compensate for known input time delay for systems with uncertain linear plant dynamics.

Efforts in recent years have focused on designing controllers that are subjected to known input delay for nonlinear plant dynamics. Robustness to input delay disturbances is addressed in prominent works such as [10–17] for nonlinear plant dynamics affected by exogenous disturbances and in [18–22] for plant dynamics without exogenous disturbances. However, the controllers in [10–23] require exact knowledge of the time delay duration. In practice, the duration of an input time delay can be challenging to determine for some applications; therefore, it is necessary to develop controllers that do not require exact knowledge of the time delay.

Since uncertainty in the delay can lead to unpredictable closed-loop performance (potentially even instabilities), several recent results have been developed which do not assume the delay is exactly known. Some studies have addressed the problem of compensating for unknown input delay for systems with linear plant dynamics. The unknown input delay problem is investigated in results such as [24–33] for systems with exactly known dynamics and in results such as [34–38] for systems with uncertain dynamics. However, the controllers in [24–38] are developed for linear plant dynamics, where the linearity is specifically exploited in the control design and analysis.

In results such as [19, 39–43] controllers were developed for plants with nonlinear dynamics and an unknown input delay, but require exact model knowledge of the nonlinear dynamics. For example, the controller in [43] is designed to compensate for an arbitrarily large, uncertain, constant input delay with known bounds by using an adaptive prediction based technique for a class of nonlinear systems with exact model knowledge of the dynamics. This results in global asymptotic convergence for the case when the full-state is available for feedback and local regulation for the case of missing measurements of the actuator state by using an adaptive based estimate instead of the actuator state. In [19], a backstepping approach is applied by using Lyapunov-Krasovskii functionals in the stability analysis to prove globally uniformly asymptotic stability for an arbitrarily large but known input delay for known nonlinear plant models; additionally, the authors mentioned that the technique used in [19] can be adapted for the unknown delay case. In [40], direct model reference adaptive control techniques are used to compensate for an unknown, constant, and small input delay for regulation of a nonlinear system. In [41, 43], a predictor-based control design is used to compensate the arbitrarily large input delay under the condition that knowledge of the dynamics is available. However, uncertain dynamics occur in many applications and predictor based delay compensation techniques used in [19, 41, 43] may fail when the system dynamics are not available. Such results specifically exploit exact model knowledge as a means to predict the evolution of the states.

When uncertain nonlinear dynamics are present, the control design is significantly more challenging than when linear or exactly known nonlinear dynamics are present. In general, if the system states evolve according to linear dynamics, the linear behavior can be exploited to predict the system response over the delay interval. Exact knowledge of the dynamics facilitates the ability to predict the state transition for nonlinear systems. The main challenge of designing predictor-based controllers for nonlinear systems is the determination of an implementable controller that uses future values of the state. For uncertain nonlinear systems, the state transition is significantly more difficult to predict, especially if the delay interval is also unknown and/or time-varying.

Given the difficulty in predicting the state transition, the contribution in Chapter 2-4 (and in [11, 12, 15–17, 44–48]) is to treat the input delay and dynamic uncertainty as a disturbance and develop a robust controller that can compensate for these effects.

NMES evokes muscle contractions by applying an external electrical stimulus. NMES is a well-known therapeutic technique that is used to restore motor function [49]; increase muscle size and strength [50]; and assist or elicit functional activities such as cycling [51–55], grasping [56], walking [57, 58], standing [59] and reaching [60]. Although the development of noninvasive closed-loop methods for NMES is necessary, open-loop applications of NMES are still commonly used in physical therapy. However, feedback control of NMES is challenging since $i)$ the mapping from electrical input to generated muscle force is nonlinear [61], $ii)$ the muscle force decays under a constantly applied voltage because of fatigue [62–65], $iii)$ the dynamic model of muscle includes uncertain parameters and unmodeled disturbances [66], and $iv)$ muscle response to electrical stimulation [63, 67–69] is delayed. Further complicating closed-loop control, the input delay is time-varying [70] due to muscle fatigue, which develops more rapidly during NMES than volitional contractions, and it is difficult to measure during feedback control of limb movement. In summary, there is a need for an NMES control design that tracks a desired limb motion while compensating for unknown time-varying input delay effects and uncertain nonlinear muscle dynamics with unmodelled disturbances.

There is a growing interest for NMES control design and its and potential applications. Results such as [71–78] designed NMES controllers for a linear model or where the governing equations are linearized. However, the dynamics of a muscle contraction/limb motion are nonlinear [66]. Some studies have addressed the development of a nonlinear control technique for NMES when considering parametric uncertainty in the dynamics of contraction/limb motion by using sliding mode control [79], adaptive control [80, 81], neural network based control [82], robust control [83], and switching control [84–87]. However, the controllers in [79–88] are developed based on the absence of delayed muscle response, and the performance of NMES control suffers from delayed muscle response [76]. Few results have focused on the delayed response of the muscle when designing a NMES controller [1, 89–93]. The techniques used in [1, 89] ensure uniformly ultimately bounded tracking error signals despite uncertain dynamics with additive disturbances, but it is assumed that the input delay is known and constant. Although the stability analysis of the controller designed in [1] requires exact knowledge of the input delay, the experimental studies demonstrate that the controller is robust uncertainty in the delay during NMES. However, the stability analysis in [1] does not guarantee stability with respect to the maximum mismatch between the estimated delay and the actual delay. Motivated by the difficulty of delay measurement during NMES, an alternative approach is presented in [90, 94] to design a controller that does not require measurement of the delay. The predictor-based controllers in [90, 94] ensure global asymptotic tracking, make the strict assumption of exact model knowledge of the lower limb dynamics and assume that the delay is constant. However, the lower limb dynamics are uncertain [66], and the input delay is generally modeled as time-varying [70]. A need remains for efficient control techniques that can overcome unknown time-varying input delay and uncertain nonlinear dynamics with additive disturbances to ensure tracking of a reference limb motion.

1.2 Outline of the Dissertation

Chapter 2 presents a controller for uncertain nonlinear systems with unknown time-varying input delay and additive disturbances. A novel filtered error signal is designed using the past states in a finite integral over a constant estimated delay interval. A Lyapunov-based stability analysis techniques is provided to prove uniformly ultimate boundedness of the tracking error signals. Simulation are illustrated the performance of the developed robust controller.

Chapter 3 extends the results of Chapter 2 to design a tracking controller for a class of uncertain Euler-Lagrange systems with bounded external disturbances and an uncertain time-varying input delay without delay rate constraints. Lyapunov-Krasovskii functionals are used in the Lyapunov-based stability analysis to provide uniformly ultimately bounded convergence of the tracking error to the origin. Numerical simulation results illustrate the performance of the designed robust controller.

Chapter 4 extends the results of Chapters 2 and 3 to design a tracking controller for a class of uncertain nonlinear systems with bounded external disturbances and an uncertain state-dependent input delay by using adaptive strategies to estimate the input delay rather than using a constant estimate of input delay (cf. Chapters 2 and 3). Lyapunov-Krasovskii functionals are used in the Lyapunov-based stability analysis to provide semi-global uniformly ultimately bounded convergence of the tracking error to the origin. Numerical simulation results illustrate the performance of the designed robust controller.

Chapter 5 is an extended version of the technique used in Chapter 3, where an integral feedback error is included to develop a control method to yield lower limb tracking with NMES, despite an unknown time-varying input delay intrinsic to NMES, uncertain nonlinear dynamics, and additive bounded disturbances. The control development is based on an approach that uses Lyapunov-Krasovskii functionals in a Lyapunov-based stability analysis to prove ultimately bounded tracking error signals. Experimental results are provided to demonstrate the performance of the developed controller.

Chapter 6 includes an implementation of revised controller that is presented in Chapter 2 for isometric NMES. A continuous robust controller is developed to track a reference force. The controller is designed in Chapter 6 compensates for the uncertain time-varying input delay for the uncertain, nonlinear NMES dynamics of the lower limb despite additive bounded disturbances. The purpose of designed controller is force tracking during the isometric NMES. A Lyapunov-based stability analysis is used to prove that the error signals are uniformly ultimately bounded. Experimental results are provided to show the performance of the designed controller.

Chapter 7 includes the conclusion of the dissertation by contributions of each chapter, the limitations and challenges, and the future research directions of this work are discussed.

1.3 Contributions

This section details the contributions of this dissertation over the state-of-the-art.

1.3.1 Robust Control of an Uncertain Nonlinear System with Uncertain Time-varying Input Delay

The approach in Chapter 2 generalizes by using a novel filtered error signal to compensate for an unknown slowly varying input delay for uncertain nonlinear systems affected by additive disturbances. Recently, Fischer et al. presented a robust controller for uncertain nonlinear systems with additive disturbances subject to slowly varying input delay in [12], where it is assumed that the input delay duration is measurable and the absolute value of the second derivative of the delay is bounded by a known constant. In [12], a filtered error signal defined as the finite integral of the control input over the delay interval is used to obtain a delay-free expression for the control input in the closed-loop error system. However, the computation of the finite integral requires exact knowledge of the input delay. In contrast, in Chapter 2, a novel filtered error signal is designed using the past states in a finite integral over a constant estimated delay interval to cope with the lack of delay knowledge, which requires a significantly different stability analysis that takes advantage of Lyapunov-Krasovskii functionals. Techniques used in Chapter 2 provide relaxed requirements of the delay measurement and obviate the need for a bound of the absolute value of the second derivative of the delay. It is assumed that the estimated input delay is selected sufficiently close to the actual time-varying input delay. That is, there are robustness limitations, which can be relaxed with more knowledge about the time-delay. Because it is feasible to obtain lower and upper bounds for the input delay in many applications [95], it is feasible to select a delay estimate in an appropriate range. New sufficient conditions for stability are based on the length of the estimated delay as well as the maximum tolerable error between the actual and estimated input delay. A Lyapunov-based stability analysis is developed in Chapter 2 to prove uniformly ultimate boundedness of the error signals. Numerical simulation results demonstrate the performance of the robust controller.

1.3.2 Robust Control of an Uncertain Euler-lagrange System with Uncertain Time-varying Input Delays without Delay Rate Constraints

Obuz et al. [96] designed a robust controller to compensate for the effects of an unknown time-varying input delay for uncertain nonlinear systems by assuming the first time derivative of input delay is less than 1. The techniques used in Chapter 3 builds upon, but significantly extends, our previous works in [12] and [96]. The contribution of Chapter 3 is that the first and second time derivative of the input delay do not have to be bounded (i.e., delay rate constraints are eliminated) by using the constant estimated delay instead of time-varying known/estimated delay in the novel error signal, and the measurement delay and inertia matrix do not have to be known. Lyapunov-Krasovskii functionals are used in the stability analysis to eliminate requirements for a slowly-varying delay rate. The result is achieved by using a novel filtered error signal that facilitates compensating for the uncertain, time varying input delay in the uncertain Euler-Lagrange system with additive disturbances. In [12], a filtered error signal developed based on the finite integral of the input signals over the known delay interval is used to obtain a delay-free control signal in the closed-loop error system, but the computation of the filtered error signal requires that the delay be known. To cope with the lack of delay knowledge as well as an upper bound on the first and second time derivative of the delay, a novel error signal is designed using the past states in a finite integral over a constant estimated delay interval. Based on the development of this modified filtering error signal, a novel Lyapunov-based stability analysis is developed by using Lyapunov-Krasovskii functionals to prove uniformly ultimate boundedness of the error signals. As opposed to previous results in [12], in Chapter 3, an estimate of the input delay is used in the control, and only bounds on the delay are required to be known rather than the first and second derivative of the delay. Additionally, the maximum tolerable delay error between the estimate of the input delay and actual input delay can be determined based on selection of control gains in the designed controller. This assumption is much easier to satisfy in practice where lower and upper bounds for the input delay can be found [95].

1.3.3 Adaptive Control for an Uncertain Nonlinear System with Uncertain State-Dependent Input Delay

The results in [96, 97] use a constant estimate of the time-varying input delay in the control development. The tracking error signal consists of a finite integral of the input signals over a constant estimate of the input delay interval rather than using exact knowledge of time-varying input delay. In contrast to such results, in Chapter 4, an adaptive estimate is used for the uncertain state-dependent input delay. The time-varying input delay estimate is used to develop a robust controller to compensate for the state-dependent input delay disturbance for uncertain nonlinear dynamics. Gain conditions are derived, which are much more less conservative than the gain conditions in results such as [92, 93, 96, 97]. The stability results in [96, 97] are locally uniformly ultimately bounded of error signals; however, the controller designed Chapter 4 provides semi-global uniformly ultimate boundedness. As opposed to [15, 92, 93, 96–98], the controller compensates for arbitrary long input delays. The gain conditions obtained in this chapter allow arbitrary large delay magnitudes. Furthermore, the maximum tolerable delay error between the actual input delay and the estimated input delay can be determined based on the controller gains. Novel Lyapunov-Krasovskii functionals are used in the Lyapunov-based stability analysis to prove semi-global uniformly ultimate boundedness.

1.3.4 Robust Neuromuscular Electrical Stimulation Control for Unknown Time-varying Input Delayed Muscle Dynamics:Position Tracking Control

Chapter 5 presents a modified version of [92] by adding an integral feedback of the error signal (desired versus actual limb position) to increase tracking performance and robustness of the controller with respect to parametric uncertainty, unmodelled disturbances of the lower limb dynamics, and effects of the unknown time-varying input delay. The controller designed in [92] is a PD-type delay compensating controller for NMES, and Chapter 5 is extended as a PID-type delay compensating controller for NMES and provides an exponential decay rate of the tracking error to its ultimate bound. Due to the different control structure presented in this chapter, the stability analysis of the controller significantly changed. Furthermore, the performance of the developed controller is validated with an expanded experimental section. The key contributions of this chapter are 1) the designed controller compensates for an unknown time-varying input delay, rather than assuming that the delay is constant and known (cf. the control design approach in [1, 89]), and 2) it removes the requirement of exact model knowledge for the lower limb dynamics, (cf. [91, 99]), by using the past states of the controller in a finite integral over an estimated delay interval in the control structure. The designed error signal not only injects a delay-free control signal in the closed-loop dynamics, but also overcomes the requirement of exact input delay measurements. Another contribution in Chapter 5 is that the maximum allowable mismatch between the actual input delay and the estimated input delay is obtained to guarantee stability of dynamics. Since the approximate interval of the time-varying input delay can be experimentally obtained for NMES, the estimated delay can be selected for minimization of the mismatch between the actual input delay and the estimated input. The developed robust controller can compensate for the unknown time-varying input delay despite uncertainty in the lower limb dynamics and external disturbances (e.g., unmodeled dynamics). A Lyapunov-based stability analysis is used to prove uniformly ultimately boundedness of tracking error signals. Experiments were conducted in 10 able-bodied individuals to examine the performance of the developed controller.

1.3.5 Robust Neuromuscular Electrical Stimulation Control for Unknown Time-varying Input Delayed Muscle Dynamics:Force Tracking Control

In Chapter 6, a novel filtered error signal, defined as the finite integral of the time derivative of control input over a constant estimated delay interval, is designed to cope with the lack of delay knowledge. Recently, Merad et al. [100] presented a robust controller that can compensate for known time-varying input delay effects in the closed-loop dynamics by using a filtered error signal. The error signal is designed based on the finite integral of the input signals over the known delay interval to inject a delay-free control signal in the closed-loop error system; however, the exact knowledge of the input delay is required for computation of the error signal. The novel filtered error signal is designed to obtain a delay-free control input expression in the closed-loop error system without requirement of the exact knowledge of the input delay by using a constant estimate of the input delay. It is assumed that the constant estimate of the input delay is selected sufficiently close to the actual time-varying input delay. Since it is feasible to obtain lower and upper bounds for the input delay for NMES [101], it is feasible to select a delay estimate in an appropriate range. The sufficient conditions for stability are based on the maximum tolerable error between the actual and estimated input delay as well as the length of the estimated delay. A Lyapunov-based stability analysis is used to prove uniformly ultimate boundedness of the error signals. Experiments were obtained in 10 able-bodied individuals to demonstrate the performance of the developed controller.

CHAPTER 2
ROBUST CONTROL OF AN UNCERTAIN NONLINEAR SYSTEM WITH UNCERTAIN TIME-VARYING INPUT DELAY

A tracking controller is developed for a class of uncertain nonlinear systems subject to unknown time-varying input delay and additive disturbances. A novel filtered error signal is designed using the past states in a finite integral over a constant estimated delay interval. The maximum tolerable error between unknown time-varying delay and a constant estimate of the delay is determined to establish uniformly ultimately bounded convergence of the tracking error to the origin. The controller development is based on an approach which uses Lyapunov-Krasovskii functionals to analyze the effects of unknown sufficiently slowly time-varying input delays. A stability analysis is provided to prove ultimate boundedness of the tracking error signals. Numerical simulation results illustrate the performance of the developed robust controller.

2.1 Dynamic System

Consider a class of n^{th}-order nonlinear systems

$$\begin{aligned} \dot{x}_i &= x_{i+1},\ i = 1, \ldots, n-1, \\ \dot{x}_n &= f\left(X, t\right) + d + u\left(t - \tau\right), \end{aligned} \tag{2–1}$$

where $x_i \in \mathbb{R}^m,\ i = 1, \ldots, n$ are the measurable system states, $X = \left[x_1^T,\ x_2^T,\ \ldots,\ x_n^T\right]^T \in \mathbb{R}^{mn}$, $u \in \mathbb{R}^m$ is the control input, $f : \mathbb{R}^{mn} \times [t_0, \infty) \to \mathbb{R}^m$ is an uncertain nonlinear function, $d : [t_0, \infty) \to \mathbb{R}^m$ denotes sufficiently smooth unknown additive disturbance (e.g., unmodeled effects), and $\tau : [t_0, \infty) \to \mathbb{R}$ denotes a time-varying unknown positive time delay, where t_0 is the initial time. Throughout the dissertation, delayed functions are denoted as

$$h_\tau \triangleq \begin{cases} h\left(t - \tau\right) & t - \tau \geq t_0 \\ 0 & t - \tau < t_0. \end{cases}$$

The dynamic model of the system in (2–1) can be rewritten as

$$x_1^{(n)} = f\left(X, t\right) + d + u\left(t - \tau\right), \tag{2–2}$$

where the superscript (n) denotes the n^{th} time derivative. Moreover, the dynamic model of the system in (2–1) satisfies the following assumptions.

Assumption 2.1. The function f and its first and second partial derivatives are bounded on each subset of their domain of the form $\Xi \times [t_0, \infty)$, where $\Xi \subset \mathbb{R}^{mn}$ is compact and for any given Ξ, the corresponding bounds are known[1] .

Assumption 2.2. [102] The nonlinear additive disturbance term and its first time derivative (i.e., d, $\dot{d}$) exist and are bounded by known positive constants.

Assumption 2.3. The reference trajectory $x_r \in \mathbb{R}^m$ is designed such that the derivatives $x_r^{(i)}$, $\forall i = 0, 1, ..., (n+2)$ exist and are bounded by known positive constants.

Assumption 2.4. The input delay is bounded such that $\tau\left(t\right) < 1$ for all $t \in \mathbb{R}$, differentiable, and slowly varying such that $|\dot{\tau}| < \varphi < 1$ for all $t \in \mathbb{R}$, where $\varphi \in \mathbb{R}$ is a known positive constant. Additionally, a constant estimate $\hat{\tau} \in \mathbb{R}$ of τ is available and sufficiently accurate such that $\tilde{\tau} \triangleq \tau - \hat{\tau}$, the difference between the input delay and the estimate of the input delay, is bounded by $|\tilde{\tau}| \leq \bar{\tilde{\tau}}$ for all $t \in \mathbb{R}$, where $\bar{\tilde{\tau}} \in \mathbb{R}$ is a known positive constant[2] . Furthermore, it is assumed that the system in (2–1) does not escape to infinity during the time interval $\left[t_0, t_0 + 1\right]$.

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[1] Given a compact set $\Xi \subset \mathbb{R}^{mn}$, the bounds of f, $\frac{\partial f(X,t)}{\partial X}$, $\frac{\partial f(X,t)}{\partial t}$, $\frac{\partial^2 f(X,t)}{\partial^2 X}$, $\frac{\partial^2 f(X,t)}{\partial X \partial t}$, and $\frac{\partial^2 f(X,t)}{\partial^2 t}$ over Ξ are assumed to be known. To satisfy Assumption 1, the aforementioned functions do not need to be bounded for all time. Assumption 1 only requires that provided X is bounded, then the functions are uniformly bounded in t.

[2] Since the maximum tolerable error, $\bar{\tilde{\tau}}$, and the estimate of actual delay, $\hat{\tau}$, are known, the maximum tolerable input delay can be determined. Because the bounds on the input delay are feasible to obtain in many applications [95], Assumption 4 is reasonable.

2.2 Control Development

The objective of the control design is to develop a continuous controller which ensures that the state x_1 of the delayed system in (2–2) tracks a reference trajectory, x_r. To quantify the control objective, a tracking error, denoted by $e_1 \in \mathbb{R}^m$, is defined as

$$e_1 \triangleq x_r - x_1. \tag{2–3}$$

To facilitate the subsequent analysis, auxiliary tracking error signals, denoted by $e_i \in \mathbb{R}^m,\ i = 2, 3, \ldots, n$, are defined as [103]

$$e_2 \triangleq \dot{e}_1 + e_1, \tag{2–4}$$

$$e_3 \triangleq \dot{e}_2 + e_2 + e_1, \tag{2–5}$$

$$\vdots$$

$$e_n \triangleq \dot{e}_{n-1} + e_{n-1} + e_{n-2}. \tag{2–6}$$

A general expression of e_i for $i = 2, 3, \ldots, n$ can be written as

$$e_i = \sum_{j=0}^{i-1} a_{i,j} e_1^{(j)}, \tag{2–7}$$

where $a_{i,j} \in \mathbb{R}$ are defined by Fibanocci numbers [104].[3] To obtain a delay-free control expression for the input in the closed-loop error system, an auxiliary tracking error signal, denoted by $e_u \in \mathbb{R}^m$, is defined as

$$e_u \triangleq -\int_{t-\hat{\tau}}^{t} \dot{u}(\theta)\, d\theta. \tag{2–8}$$

It should be emphasized that the best estimate of τ, denoted by $\hat{\tau}$, is required instead of exact knowledge of τ in the control design. For example, the constant estimate $\hat{\tau}$ may be selected to best approximate the mean of τ. Based on the subsequent stability analysis,

[3] It should be noted that $a_{i,i-1} = 1,\ \forall i = 1, 2, ..., n$.

the following continuous robust controller is designed as

$$u \triangleq k\left(e_n - e_n\left(t_0\right)\right) + \upsilon, \tag{2–9}$$

where $e_n\left(t_0\right) \in \mathbb{R}^m$ is the initial error signal, and $\upsilon \in \mathbb{R}^m$ is the solution to the differential equation

$$\dot{\upsilon} = k\left(\Lambda e_n + \alpha e_u\right), \tag{2–10}$$

where $k \in \mathbb{R}^{m\times m}$ is a constant, diagonal, positive definite gain matrix.

2.3 Stability Analysis

To facilitate the stability analysis an auxiliary tracking error signal, denoted by $r \in \mathbb{R}^m$, is defined as[4]

$$r \triangleq \dot{e}_n + \Lambda e_n + \alpha e_u, \tag{2–11}$$

where $\Lambda, \alpha \in \mathbb{R}^{m\times m}$ are constant, diagonal, and positive definite gain matrices. The open loop dynamics for r can be obtained by substituting the first time derivatives of (2–2) and (2–8), the second time derivative of (2–7) with $i = n$, and the $(n+1)^{\text{th}}$ time derivative of (2–3) into (2–11) as

$$\begin{aligned} \dot{r} = &- \dot{f}\left(X, \dot{X}, t\right) - \dot{d} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + x_r^{(n+1)} \\ &- \alpha\dot{u} + \alpha\dot{u}_{\hat{\tau}} - (1-\dot{\tau})\,\dot{u}_\tau + \Lambda\dot{e}_n. \end{aligned} \tag{2–12}$$

Substituting the first time derivative of the controller in (2–9) into (2–12), the closed-loop error system for r can be obtained as

$$\begin{aligned} \dot{r} = &- \dot{f}\left(X, \dot{X}, t\right) - \dot{d} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + x_r^{(n+1)} + \Lambda\dot{e}_n \\ &- \alpha k r + \left(\alpha - I + \dot{\tau} I\right) k r_\tau + \alpha\left(\dot{u}_{\hat{\tau}} - \dot{u}_\tau\right), \end{aligned} \tag{2–13}$$

[4] Since $\dot{e}_n$ is not measurable, r cannot be used in the control design.

where $I \in \mathbb{R}^{m\times m}$ is the identity matrix. The stability analysis can be facilitated by segregating the terms in (2–13) that can be upper bounded by a state-dependent function and terms that can be upper bounded by a constant, such that

$$\dot{r} = -\alpha kr + (\alpha - I + \dot{\tau} I)\, kr_{\tau} + \alpha\left(\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}\right) + \tilde{N} + N_r - e_n. \tag{2–14}$$

The auxiliary functions $\tilde{N} \in \mathbb{R}^m$ and $N_r \in \mathbb{R}^m$ are defined as

$$\tilde{N} \triangleq -\dot{f}\left(X, \dot{X}, t\right) + \dot{f}\left(X_r, \dot{X}_r, t\right) + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + \Lambda \dot{e}_n + e_n, \tag{2–15}$$

$$N_r \triangleq -\dot{f}\left(X_r, \dot{X}_r, t\right) - \dot{d} + x_r^{(n+1)}, \tag{2–16}$$

where $X_r \triangleq \left[x_r^T,\ \dot{x}_r^T,\ \ldots,\ \left(x_r^{(n-1)}\right)^T\right]^T \in \mathbb{R}^{mn}$.

Remark 2.1. Based on Assumptions 2.2 and 2.3, N_r is upper bounded as

$$\sup_{t\in\mathbb{R}} \|N_r\| \le \zeta_{N_r}, \tag{2–17}$$

where $\zeta_{N_r} \in \mathbb{R}$ is a known positive constant.

Remark 2.2. An upper bound can be obtained for (2–15) using Assumption 2.1 and the Lemma 5 in [105] as

$$\left\|\tilde{N}\right\| \le \rho\left(\|z\|\right)\|z\|, \tag{2–18}$$

where ρ is a positive, radially unbounded[5] , and strictly increasing function, and $z \in \mathbb{R}^{(n+2)m}$ is a vector of error signals defined as

$$z \triangleq \left[e_1^T,\ e_2^T, \ldots, e_n^T,\ e_u^T,\ r^T\right]^T. \tag{2–19}$$

Proof. Equation (2–14) can be written as

[5] For some classes of systems, the bounding function ρ can be selected as a constant. For those systems, a global uniformly ultimately bounded result can be obtained as described in Remark 3.

$$\begin{aligned}\tilde{N} =& \nabla_X f\left(X_r, t\right) \dot{X}_r - \nabla_X f\left(X, , t\right) \dot{X} \\ &+ \nabla_t f\left(X_r, t\right) - \nabla_t f\left(X, t\right) + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + \Lambda \dot{e}_n + e_n.\end{aligned}$$

To facilitate the subsequent analysis, $\nabla_X f\left(X_r, t\right) \dot{X}$ is added and subtracted to the right-hand side of the last equation as

$$\begin{aligned}\tilde{N} =& \nabla_X f\left(X_r, t\right) \dot{X}_r - \nabla_X f\left(X_r, t\right) \dot{X} - \nabla_X f\left(X, t\right) \dot{X} + \nabla_X f\left(X_r, t\right) \dot{X} \\ &+ \nabla_t f\left(X_r, t\right) - \nabla_t f\left(X, t\right) + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + \Lambda \dot{e}_n + e_n.\end{aligned}$$

The following equation can be written by using (2–7) and reorganizing the last equation as

$$\begin{aligned}\tilde{N} =& \nabla_X f\left(X_r, t\right) \left(\dot{X}_r - \dot{X}\right) + \left(\nabla_X f\left(X_r, t\right) - \nabla_X f\left(X, t\right)\right) \left(\dot{X} - \dot{X}_r + \dot{X}_r\right) \\ &+ \nabla_t f\left(X_r, t\right) - \nabla_t f\left(X, t\right) + \Lambda \dot{e}_n + e_n \\ &+ \left(a_{n,1} e_1^{(3)} + a_{n,2} e_1^{(4)} + a_{n,3} e_1^{(5)} + ... + a_{n,(n-2)} e_1^{(n)}\right).\end{aligned}$$

By using (2–11), the following equation can be written as

$$\begin{aligned}\left\|\tilde{N}\right\| \leq& \left\|\nabla_X f\left(X_r, t\right)\right\| \left\|\dot{X}_r - \dot{X}\right\| + \left\|\nabla_X f\left(X_r, t\right) - \nabla_X f\left(X, t\right)\right\| \left\|\dot{X}_r - \dot{X}\right\| \\ &+ \left\|\nabla_X f\left(X_r, t\right) - \nabla_X f\left(X, t\right)\right\| \left\|\dot{X}_r\right\| + \left\|\nabla_t f\left(X_r, t\right) - \nabla_t f\left(X, t\right)\right\| \\ &+ \left(a_{n,1} \left\|e_1^{(3)}\right\| + a_{n,2} \left\|e_1^{(4)}\right\| + a_{n,3} \left\|e_1^{(5)}\right\| + ... + a_{n,(n-2)} \left\|e_1^{(n)}\right\|\right) \\ &+ \Lambda \left\|r - \Lambda e_n - \alpha e_u\right\| + \left\|e_n\right\|.\end{aligned}$$

Using Assumption 2.1, 2.3, definition of X and X_r, and Lemma 5 in [105], the following inequality can be obtained[6]

$$\begin{aligned}\left\|\tilde{N}\right\| \leq & c_1\left(\dot{E}\right)+\rho_{01}\left(\|E\|\right)\|E\|\left\|\dot{E}\right\|+\rho_{01}\left(\|E\|\right)\|E\|\left\|\dot{X}_r\right\| \\ & +\rho_{02}\left(\|E\|\right)\|E\|+c_2\|z\|+\left(\Lambda+\Lambda^2+\Lambda\alpha+I_{n\times n}\right)\|z\|,\end{aligned}$$

where $I_{n\times n}$ is identity matrix, $E \triangleq \left[e_1^T,\ \dot{e}_1^T,\ \ldots,\ \left(e_1^{(n-1)}\right)^T\right]^T \in \mathbb{R}^{mn}$, $\rho_{01}, \rho_{02}, \rho_{03} \in \mathbb{R}$ are positive, non-decreasing and globally invertible functions, $c_1, c_2 \in \mathbb{R}$ are known positive numbers based on Assumptions 2.1, 2.3.

$$\begin{aligned}\left\|\tilde{N}\right\| \leq & c_1\|z\|+\rho_{01}\left(\|z\|\right)\|z\|\|z\|+c_3\rho_{01}\left(\|z\|\right)\|z\| \\ & +\rho_{02}\left(\|z\|\right)\|z\|+c_2\|z\|+\left(\Lambda+\Lambda^2+\Lambda\alpha+I_{n\times n}\right)\|z\|.\end{aligned}$$

Since $\rho_{01}\left(\|z\|\right)\|z\|$ can be upper bounded as $\rho_{01}\left(\|z\|\right)\|z\| \leq \rho_1\left(\|z\|\right)$, where ρ_1 is positive, non-decreasing, radially unbounded and globally invertible function, it can be concluded that $\left\|\tilde{N}\right\| \leq \rho\left(\|z\|\right)\|z\|$, where $\rho = \rho_1\left(\|z\|\right)+c_3\rho_{01}\left(\|z\|\right)+\rho_{02}\left(\|z\|\right)+\left(\bar{\Lambda}+\bar{\Lambda}^2+\bar{\Lambda}\bar{\alpha}+1+c_1+c_2\right)$, where $\bar{\Lambda}, \bar{\alpha}$ denote the maximum eigenvalues of Λ and α, respectively. $\square$

To facilitate the subsequent stability analysis, auxiliary bounding constants $\sigma, \delta \in \mathbb{R}$ are defined as

$$\sigma \triangleq \min\left\{1, \left(1-\frac{\epsilon_2}{2}\right), \left(\underline{\Lambda}-\left(\frac{\bar{\alpha}}{2\epsilon_1}+\frac{1}{2\epsilon_2}\right)\right), \frac{\underline{k}\,\underline{\alpha}}{8}, \left(\frac{\omega_2}{4\hat{\tau}}-\bar{\alpha}\epsilon_1\right)\right\} \tag{2–20}$$

[6] Since $e_1^{(n)}$ can be written in term of e_n, $\forall n = 1,2,3,...$ by using (7), the term $\sum_{j=0}^{n-2} a_{n,j}e_1^{(j+2)}$ can be written in term of z.

$$\delta \triangleq \frac{1}{2}\min\Big\{\frac{\sigma}{2}, \frac{\omega_2\underline{k}^3\underline{\alpha}\left(1-\varphi\right)}{4\left(\bar{k}\left(\bar{\alpha}+\varphi-1\right)\right)^2}, \frac{\omega_2\underline{k}^2\bar{\alpha}\epsilon_1}{4\omega_1^2\bar{k}^2}, \frac{1}{4\left(\bar{\bar{\tau}}+\hat{\tau}\right)}\Big\}, \tag{2–21}$$

where $\underline{\Lambda},\ \underline{k},\ \underline{\alpha} \in \mathbb{R}$ denote the minimum eigenvalues of Λ, k, α, respectively, $\bar{k},\ \bar{\alpha} \in \mathbb{R}$ denote the maximum eigenvalues of k and α, respectively, and $\omega_i,\ \epsilon_i \in \mathbb{R}, i = 1, 2$, are known, selectable, positive constants. Let the functions $Q_1,\ Q_2,\ Q_3 \in \mathbb{R}$ be defined as

$$Q_1 \triangleq \frac{\left(\omega_1\bar{k}\right)^2}{\bar{\alpha}\epsilon_1}\int\limits_{t-\hat{\tau}}^{t}\left\|r\left(\theta\right)\right\|^2 d\theta, \tag{2–22}$$

$$Q_2 \triangleq \frac{\left(\bar{k}\left(\bar{\alpha}+\varphi-1\right)\right)^2}{\underline{k}\underline{\alpha}\left(1-\varphi\right)}\int\limits_{t-\tau}^{t}\left\|r\left(\theta\right)\right\|^2 d\theta, \tag{2–23}$$

$$Q_3 \triangleq \omega_2\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\int\limits_{s}^{t}\left\|\dot{u}\left(\theta\right)\right\|^2 d\theta ds, \tag{2–24}$$

and let $y \in \mathbb{R}^{(n+2)m+3}$ be defined as

$$y \triangleq \left[\begin{array}{c} z, \sqrt{Q_1}, \sqrt{Q_2}, \sqrt{Q_3} \end{array}\right]^T. \tag{2–25}$$

For use in the following stability analysis, let

$$\mathscr{D}_1 \triangleq \left\{y \in \mathbb{R}^{(n+2)m+3} |\ \|y\| < \chi_1\right\}, \tag{2–26}$$

where $\chi_1 \triangleq \inf\left\{\rho^{-1}\left([\sqrt{\frac{\sigma\underline{k}\,\underline{\alpha}}{2}}, \infty)\right)\right\}$. Provided $\left\|z\left(\eta\right)\right\| < \gamma,\ \forall\eta \in [t_0, t]$, (2–14) and the fact that $\dot{u} = kr$ can be used to conclude that $\ddot{u} < M$, where γ and M[7] are positive constants. Let $\mathscr{D} \triangleq \mathscr{D}_1 \cap \left(B_\gamma \cap \mathbb{R}^{(n+2)m+3}\right)$ where B_γ denotes a closed ball of radius γ centered at the origin and let

$$\mathcal{S}_{\mathscr{D}} \triangleq \left\{y \in \mathscr{D} |\ \|y\| < \chi_2\right\} \tag{2–27}$$

[7] The subsequent analysis does not assume that the inequality $\ddot{u} < M$ holds for all time. The subsequent analysis only exploits the fact that provided $\left\|z\left(\eta\right)\right\| < \gamma,\ \forall\eta \in [t_0, t]$, then $\ddot{u} < M$.

denote the domain of attraction, where[8] $\chi_2 \triangleq \sqrt{\frac{\min\{\frac{1}{2},\frac{\omega_1}{2}\}}{\max\{1,\frac{\omega_1}{2}\}}}\inf\left\{\rho^{-1}\left(\left[\sqrt{\frac{\sigma \underline{k}\,\underline{\alpha}}{2}},\infty\right)\right)\right\}$.

Theorem 2.1. *Given the dynamics in (1), the controller given in (2–9) and (2–10) ensures uniformly ultimately bounded tracking in the sense that*

$$\limsup_{t\to\infty}\|e_1(t)\| \leq \sqrt{\frac{\max\left\{1,\frac{\omega_1}{2}\right\}\left(2\zeta_{N_r}^2+\underline{\alpha}\,\underline{k}\bar{\alpha}\bar{\bar{\tau}}^2M^2\right)}{\min\left\{\frac{1}{2},\frac{\omega_1}{2}\right\}2\underline{\alpha}\,\underline{k}\delta}}, \tag{2–28}$$

provided that $y(t_0)\in\mathcal{S}_{\mathscr{D}}$ and that the control gains are selected sufficiently large based on the initial conditions of the system such that the following sufficient conditions are satisfied[9] [10]

$$\omega_2 > 4\bar{\alpha}\epsilon_1\hat{\tau}, \qquad \underline{\Lambda} > \frac{\bar{\alpha}}{2\epsilon_1}+\frac{1}{2\epsilon_2}, \qquad 2>\epsilon_2,$$
$$\frac{\frac{\underline{\alpha}\,\underline{k}}{8}-\frac{2\left(\omega_1\bar{k}\right)^2}{\bar{\alpha}\epsilon_1}-\frac{\left(\bar{k}(\bar{\alpha}+\varphi-1)\right)^2}{\underline{\alpha}\,\underline{k}(1-\varphi)}-\omega_2\hat{\tau}\bar{k}^2-\frac{\bar{\alpha}}{2}}{\omega_2\bar{k}^2}\geq\bar{\bar{\tau}}, \quad \chi_2 > \left(\frac{2\zeta_{N_r}^2+\underline{\alpha}\,\underline{k}\bar{\alpha}\bar{\bar{\tau}}^2M^2}{2\underline{\alpha}\,\underline{k}\delta}\right)^{\frac{1}{2}}. \tag{2–29}$$

Proof. Let $V:\mathscr{D}\to\mathbb{R}$ be a continuously differentiable Lyapunov function candidate defined as

$$V \triangleq \frac{1}{2}\sum_{i=1}^{n}e_i^Te_i+\frac{1}{2}r^Tr+\frac{\omega_1}{2}e_u^Te_u+\sum_{i=1}^{3}Q_i. \tag{2–30}$$

In addition, the following upper bound can be provided for Q_3

$$Q_3 \leq \omega_2\left(\bar{\bar{\tau}}+\hat{\tau}\right)\sup_{s\epsilon\left[t-\left(\bar{\bar{\tau}}+\hat{\tau}\right),t\right]}\left[\int_s^t\|\dot{u}(\theta)\|^2ds,\right]\leq\omega_2\left(\bar{\bar{\tau}}+\hat{\tau}\right)\int_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\|\dot{u}(\theta)\|^2d\theta. \tag{2–31}$$

[8] For a set A, the inverse image $\rho^{-1}(A)$ is defined as $\rho^{-1}(A)\triangleq\{a\mid\rho(a)\in A\}$

[9] To achieve a small tracking error for the case of a large value of ζ_{N_r}(i.e., fast dynamics with large disturbances), large gains, small delay, and a better estimate of the delay are required.

[10] By choosing α close to $1-\varphi$, sufficiently small ω_1 and ϵ_1, and a sufficiently large Λ, the gain conditions can be expressed in terms of k, to select ϵ_1 sufficiently small enough, that implies smaller lower bound of ω_2, and $k,\ \hat{\tau},\ \bar{\bar{\tau}},\ \varphi$. The gain k can then be selected provided $\hat{\tau},\ \bar{\bar{\tau}},\ \varphi$ are small.

By applying Leibniz Rule, the time derivatives of (2–22)-(2–24) can be obtained as

$$\dot{Q}_1 = \frac{\left(\omega_1 \bar{k}\right)^2}{\bar{\alpha}\epsilon_1}\left(\|r\|^2 - \|r_{\hat{\tau}}\|^2\right), \tag{2–32}$$

$$\dot{Q}_2 = \frac{\left(\bar{k}\left(\bar{\alpha}+\varphi-1\right)\right)^2}{\underline{k}\,\underline{\alpha}\left(1-\varphi\right)}\left(\|r\|^2 - \left(1-\dot{\tau}\right)\|r_\tau\|^2\right), \tag{2–33}$$

$$\dot{Q}_3 = \omega_2\left(\left(\bar{\bar{\tau}}+\hat{\tau}\right)\bar{k}\,\|r\|^2 - \int_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\|\dot{u}\left(\theta\right)\|^2 d\theta\right). \tag{2–34}$$

Based on (2–30), the following inequalities can be developed:

$$\text{min}\left\{\frac{1}{2}, \frac{\omega_1}{2}\right\}\|y\|^2 \leq V\left(y\right) \leq \text{max}\left\{1, \frac{\omega_1}{2}\right\}\|y\|^2. \tag{2–35}$$

The time derivative of the first term in (2–30) can be obtained by using (2–4)-(2–6), (2–11), and the definition of e_i in (2–7) for $i = n$, as

$$\sum_{i=1}^{n} e_i^T \dot{e}_i = -\sum_{i=1}^{n-1} e_i^T e_i - e_n^T \Lambda e_n + e_{n-1}^T e_n - e_n^T \alpha e_u + e_n^T r. \tag{2–36}$$

By using (2–8), (2–14), (2–32)-(2–34), and (2–36), the time derivative of (2–30) can be determined as

$$\begin{aligned}
\dot{V} = &-\sum_{i=1}^{n-1} e_i^T e_i - e_n^T \Lambda e_n + e_{n-1}^T e_n - e_n^T \alpha e_u + e_n^T r \\
&+ r^T\left(-\alpha k r + \left(\alpha - I + \dot{\tau} I\right) k r_\tau + \alpha\left(\dot{u}_{\hat{\tau}} - \dot{u}_\tau\right) + \tilde{N} + N_r - e_n\right) \\
&+ \omega_1 e_u^T\left(k r_{\hat{\tau}} - k r\right) + \frac{\left(\omega_1 \bar{k}\right)^2}{\bar{\alpha}\epsilon_1}\left(\|r\|^2 - \|r_{\hat{\tau}}\|^2\right) \\
&+ \frac{\left(\bar{k}\left(\bar{\alpha}+\varphi-1\right)\right)^2}{\underline{k}\underline{\alpha}\left(1-\varphi\right)}\left(\|r\|^2 - \left(1-\dot{\tau}\right)\|r_\tau\|^2\right) \\
&+ \omega_2\left(\left(\bar{\bar{\tau}}+\hat{\tau}\right)\bar{k}^2\,\|r\|^2 - \int_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\|\dot{u}\left(\theta\right)\|^2 d\theta\right).
\end{aligned} \tag{2–37}$$

After canceling common terms and using Assumption 2.4, the expression in (2–37) can be upper bounded as

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n-1}\|e_i\|^2 - \underline{\Lambda}\|e_n\|^2 + \left|e_{n-1}^T e_n\right| + \bar{\alpha}\left|e_n^T e_u\right| \\ & + \bar{\alpha}\left|r^T\left(\dot{u}_{\hat{\tau}} - \dot{u}_\tau\right)\right| - \underline{\alpha}\,\underline{k}\|r\|^2 + r^T\tilde{N} + \|r\|\zeta_{N_r} + \bar{k}|\bar{\alpha}+\varphi-1|\left|r^T r_\tau\right| \\ & + \omega_1\bar{k}\left(\|e_u\|\|r_{\hat{\tau}}\| + \|e_u\|\|r\|\right) + \frac{\left(\omega_1\bar{k}\right)^2}{\bar{\alpha}\epsilon_1}\left(\|r\|^2 - \|r_{\hat{\tau}}\|^2\right) \\ & + \frac{\left(\bar{k}\left(\bar{\alpha}+\varphi-1\right)\right)^2}{\underline{k}\,\underline{\alpha}\left(1-\varphi\right)}\left(\|r\|^2 - \left(1-\dot{\tau}\right)\|r_\tau\|^2\right) \\ & + \omega_2\left(\left(\bar{\bar{\tau}}+\hat{\tau}\right)\bar{k}^2\|r\|^2 - \int_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\|\dot{u}(\theta)\|^2 d\theta\right). \end{aligned} \tag{2–38}$$

After using Young's Inequality the following inequalities can be obtained

$$\left|e_n^T e_u\right| \leq \frac{1}{2\epsilon_1}\|e_n\|^2 + \frac{\epsilon_1}{2}\|e_u\|^2, \tag{2–39}$$

$$\left|e_{n-1}^T e_n\right| \leq \frac{\epsilon_2}{2}\|e_{n-1}\|^2 + \frac{1}{2\epsilon_2}\|e_n\|^2, \tag{2–40}$$

$$\left|r^T\left(\dot{u}_{\hat{\tau}} - \dot{u}_\tau\right)\right| \leq \frac{1}{2}\|r\|^2 + \frac{1}{2}\|\dot{u}_{\hat{\tau}} - \dot{u}_\tau\|^2. \tag{2–41}$$

After completing the squares for the cross terms containing r and $r_{\hat{\tau}}$, substituting the time derivative of (2–9) and (2–18), (2–39)-(2–41) into (2–38), and using Assumption 2.4, the following upper bound can be obtained

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n-2}\|e_i\|^2 - \left(1-\frac{\epsilon_2}{2}\right)\|e_{n-1}\|^2 - \left(\underline{\Lambda} - \left(\frac{\bar{\alpha}}{2\epsilon_1} + \frac{1}{2\epsilon_2}\right)\right)\|e_n\|^2 \\ & + \bar{\alpha}\epsilon_1\|e_u\|^2 - \frac{\underline{\alpha}\,\underline{k}}{8}\|r\|^2 - \left(\frac{\underline{\alpha}\,\underline{k}}{8} - \kappa\right)\|r\|^2 + \frac{1}{\underline{\alpha}\,\underline{k}}\rho^2(\|z\|)\|z\|^2 \\ & + \frac{1}{\underline{\alpha}\,\underline{k}}\zeta_{N_r}^2 + \frac{\bar{\alpha}\|\dot{u}_{\hat{\tau}} - \dot{u}_\tau\|^2}{2} - \omega_2\int_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\|\dot{u}(\theta)\|^2 d\theta, \end{aligned} \tag{2–42}$$

where $\kappa \triangleq \frac{2\left(\omega_1 \bar{k}\right)^2}{\bar{\alpha}\epsilon_1} + \frac{\left(\bar{k}(\bar{\alpha}+\varphi-1)\right)^2}{\underline{\alpha}\,\underline{k}(1-\varphi)} + \omega_2\left(\bar{\bar{\tau}} + \hat{\tau}\right)\bar{k}^2 + \frac{\bar{\alpha}}{2}$. The Cauchy-Schwartz inequality is used to develop the following upper bound

$$\left\|e_u\right\|^2 \leq \hat{\tau}\int\limits_{t-\hat{\tau}}^{t}\left\|\dot{u}(\theta)\right\|^2 d\theta. \tag{2–43}$$

Note that using Assumption 2.4, the inequalities $\int\limits_{t-\tau}^{t}\left\|\dot{u}\left(\theta\right)\right\|^2 d\theta \leq \bar{k}^2\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\left\|r\left(\theta\right)\right\|^2 d\theta$ and $\int\limits_{t-\hat{\tau}}^{t}\left\|\dot{u}\left(\theta\right)\right\|^2 d\theta \leq \bar{k}^2\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\left\|r\left(\theta\right)\right\|^2 d\theta$ can be obtained. Moreover, using the expressions in (2–22), (2–23), (2–31) and (2–43), the following inequalities can be obtained

$$-\frac{\omega_2}{4\hat{\tau}}\left\|e_u\right\|^2 \geq -\frac{\omega_2}{4}\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\left\|\dot{u}\left(\theta\right)\right\|^2 d\theta, \tag{2–44}$$

$$-\frac{\omega_2\underline{k}^2\bar{\alpha}\epsilon_1}{4\omega_1^2\bar{k}^2}Q_1 \geq -\frac{\omega_2}{4}\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\left\|\dot{u}\left(\theta\right)\right\|^2 d\theta, \tag{2–45}$$

$$-\frac{\omega_2\underline{k}^3\,\underline{\alpha}\left(1-\varphi\right)Q_2}{4\left(\bar{k}\left(\bar{\alpha}+\varphi-1\right)\right)^2} \geq -\frac{\omega_2}{4}\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\left\|\dot{u}\left(\theta\right)\right\|^2 d\theta, \tag{2–46}$$

$$-\frac{1}{4\left(\bar{\bar{\tau}}+\hat{\tau}\right)}Q_3 \geq -\frac{\omega_2}{4}\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t}\left\|\dot{u}\left(\theta\right)\right\|^2 d\theta. \tag{2–47}$$

By using (2–44)-(2–47), (2–42) can be upper bounded as

$$\begin{aligned}
\dot{V} \leq & -\sum_{i=1}^{n-2}\left\|e_i\right\|^2 - \left(1-\frac{\epsilon_2}{2}\right)\left\|e_{n-1}\right\|^2 - \left(\underline{\Lambda} - \left(\frac{\bar{\alpha}}{2\epsilon_1}+\frac{1}{2\epsilon_2}\right)\right)\left\|e_n\right\|^2 \\
& - \left(\frac{\omega_2}{4\hat{\tau}} - \bar{\alpha}\epsilon_1\right)\left\|e_u\right\|^2 - \frac{\underline{\alpha}\underline{k}}{8}\left\|r\right\|^2 - \left(\frac{\underline{\alpha}\,\underline{k}}{8} - \kappa\right)\left\|r\right\|^2 \\
& + \frac{1}{\underline{\alpha}\,\underline{k}}\rho^2(\left\|z\right\|)\left\|z\right\|^2 + \frac{1}{\underline{\alpha}\,\underline{k}}\zeta_{N_r}^2 + \frac{\bar{\alpha}\left\|\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}\right\|^2}{2} \\
& - \frac{\omega_2\underline{k}^2\bar{\alpha}\epsilon_1}{4\omega_1^2\bar{k}^2}Q_1 - \frac{\omega_2\underline{k}^3\,\underline{\alpha}\left(1-\varphi\right)}{4\left(\bar{k}\left(\bar{\alpha}+\varphi-1\right)\right)^2}Q_2 - \frac{1}{4\left(\bar{\bar{\tau}}+\hat{\tau}\right)}Q_3.
\end{aligned} \tag{2–48}$$

Note that the Mean Value Theorem can be used to obtain the inequality $\|\dot{u}_{\hat{\tau}}-\dot{u}_{\tau}\| \leq \|\ddot{u}(\Theta(t,\hat{\tau}))\| |\tilde{\tau}|$, where $\Theta(t,\hat{\tau})$ is a point in time between $t-\tau$ and $t-\hat{\tau}$. Furthermore, using the gain conditions in (2–29), the definition of σ in (2–20), and the inequality $\|y\| \geq \|z\|$, the following upper bound can be obtained

$$\begin{aligned}\dot{V} \leq & -\left(\frac{\sigma}{2}-\frac{1}{\underline{\alpha}\,\underline{k}}\rho^{2}(\|y\|)\right)\|z\|^{2}-\frac{\sigma}{2}\|z\|^{2}+\frac{1}{\underline{\alpha}\,\underline{k}}\zeta_{N_{r}}^{2}+\frac{\bar{\alpha}\bar{\tilde{\tau}}^{2}\|\ddot{u}(\Theta(t,\hat{\tau}))\|^{2}}{2} \\ & -\frac{\omega_{2}\underline{k}^{2}\bar{\alpha}\epsilon_{1}}{4\omega_{1}^{2}\bar{k}^{2}}Q_{1}-\frac{\omega_{2}\underline{k}^{3}\,\underline{\alpha}(1-\varphi)}{4\left(\bar{k}(\bar{\alpha}+\varphi-1)\right)^{2}}Q_{2}-\frac{1}{4(\bar{\tilde{\tau}}+\hat{\tau})}Q_{3}.\end{aligned} \tag{2–49}$$

Provided $y(\eta) \in \mathscr{D}\ \forall\eta \in [t_{0},t]$, then from the definition of δ in (2–21), the expression in (2–49) reduces to

$$\dot{V} \leq -\delta\|y\|^{2},\ \forall\|y\| \geq \left(\frac{2\zeta_{N_{r}}^{2}+\underline{\alpha}\,\underline{k}\bar{\alpha}\bar{\tilde{\tau}}^{2}M^{2}}{2\underline{\alpha}\,\underline{k}\delta}\right)^{\frac{1}{2}}. \tag{2–50}$$

Using techniques similar to Theorem 4.18 in [106] it can be concluded that y is uniformly ultimately bounded in the sense that $\limsup_{t\to\infty}\|y(t)\| \leq \sqrt{\frac{\max\left\{1,\frac{\omega_{1}}{2}\right\}\left(2\zeta_{N_{r}}^{2}+\underline{\alpha}\,\underline{k}\bar{\alpha}\bar{\tilde{\tau}}^{2}M^{2}\right)}{\min\left\{\frac{1}{2},\frac{\omega_{1}}{2}\right\}2\underline{\alpha}\,\underline{k}\delta}}$ provided $y(t_{0}) \in \mathcal{S}_{\mathscr{D}}$, where uniformity in initial time can be concluded from the independence of δ and the ultimate bound from t_{0}. □

Remark 2.3. If the system dynamics are such that $\tilde{N}$ is linear in z, then the function ρ can be selected to be a constant, i.e., $\rho(\|z\|)=\bar{\rho}$, $\forall z \in \mathbb{R}^{(n+2)m}$ for some known $\bar{\rho}>0$. In this case, the last sufficient condition in (2–29) reduces to

$$\underline{k} \leq \frac{2\bar{\rho}^{2}}{\sigma\underline{\alpha}}, \tag{2–51}$$

and the result is global in the sense that $\mathscr{D}=\mathcal{S}_{\mathscr{D}}=\mathbb{R}^{(n+2)m+3}$.

Since $e_{i},\ r,\ e_{u} \in \mathcal{L}_{\infty},\ i=1,2,3,\ldots,n$, from (2–2), $u \in \mathcal{L}_{\infty}$. An analysis of the closed-loop system shows that the remaining signals are bounded.

2.4 Simulation Results

To illustrate performance of the developed controller, numerical simulations were performed using the dynamics of a two-link revolute, direct drive robot[11] with the following dynamics

$$\begin{bmatrix} u_{1_\tau} \\ u_{2_\tau} \end{bmatrix} = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} -p_3s_2\dot{x}_2 & -p_3s_2\left(\dot{x}_1 + \dot{x}_2\right) \\ p_3s_2\dot{x}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \tag{2–52}$$

where $x, \dot{x}, \ddot{x} \in \mathbb{R}^2$. Additive disturbances are applied as $d_1 = 0.2\sin(0.5t)$ and $d_2 = 0.1\sin(0.25t)$. Additionally, $p_1 = 3.473$ kg · m^2, $p_2 = 0.196$ kg · m^2, $p_3 = 0.242$ kg · m^2, $p_4 = 0.238$ kg · m^2, $p_5 = 0.146$ kg · m^2, $f_{d1} = 5.3$ Nm · sec, $f_{d2} = 1, 1$ Nm · sec, and $s_2,\ c_2$ denote $\sin(x_2)$, and $\cos(x_2)$, respectively.

The initial conditions for the system are selected as $x_1, x_2 = 0$. The desired trajectories are selected as

$$x_{d1}(t) = (30\sin(1.5t) + 20)\left(1 - e^{-0.01t^3}\right),$$
$$x_{d2}(t) = -(20\sin(t/2) + 10)\left(1 - e^{-0.01t^3}\right).$$

Several simulation results were obtained using various time-varying delays and different estimated delays, shown in Table 2-1, to demonstrate performance of the developed

[11] Provided the inertia matrix is known, the dynamics in (2–52) can be described using (2–1) [15].

Table 2-1. RMS errors for time-varying time-delay rates and magnitudes.

	$\tau_i(t)$ (ms)	$\hat{\tau}(t)$ (ms)	RMS Error x_1	RMS Error x_2
Case 1	$10\sin(5t)+10$	15	0.1315^o	0.1465^o
Case 2	$10\sin(5t)+10$	100	0.4774^o	0.2212^o
Case 3	$60\sin(t)+60$	75	0.7479^o	0.9928^o

continuous robust controller. Cases 1 and 2 use a high-frequency, low-amplitude oscillating delay for different delay upper-bound estimates. Cases 3 use a low-frequency, high-amplitude oscillating delay for known and unknown delay.

The controller in (2–9) and (2–10) is implemented for each case. The control gains are selected for Cases 1 and 2 as $\alpha=$ diag$\{1,1\}$, $\Lambda=$ diag$\{50,20.5\}$, and$k=$ diag$\{60,6\}$, and for Cases 3 as $\alpha=$ diag$\{1,1\}$, $\Lambda=$ diag$\{23,8\}$, and $k=$ diag$\{140,2.75\}$. The root mean square (RMS) errors obtained for each case are listed in Table 2-1. By comparing the RMS error for Cases 1 and 2, it is clear that selecting a delay estimate closer to the actual upper bound of the unknown delay yields better tracking performance. Case 3 demonstrates that the performance of the developed controller using a constant estimate of the large unknown delay is comparable to the controller in [12] which uses exact knowledge of the time-varying delay[12] . Additionally, the developed controller is reasonably robust even for a constant estimate of the delay when the actual delay is long and time-varying. Results in Figures 2-1 and 2-2 depict the tracking errors, control effort, time-varying delays and estimated delays for Cases 1, and 3, respectively.

[12] The RMS errors for x_1 and x_2 obtained using the controller in [12] for the same delay as Case 3 were 0.7544^o and 0.9722^o, respectively.

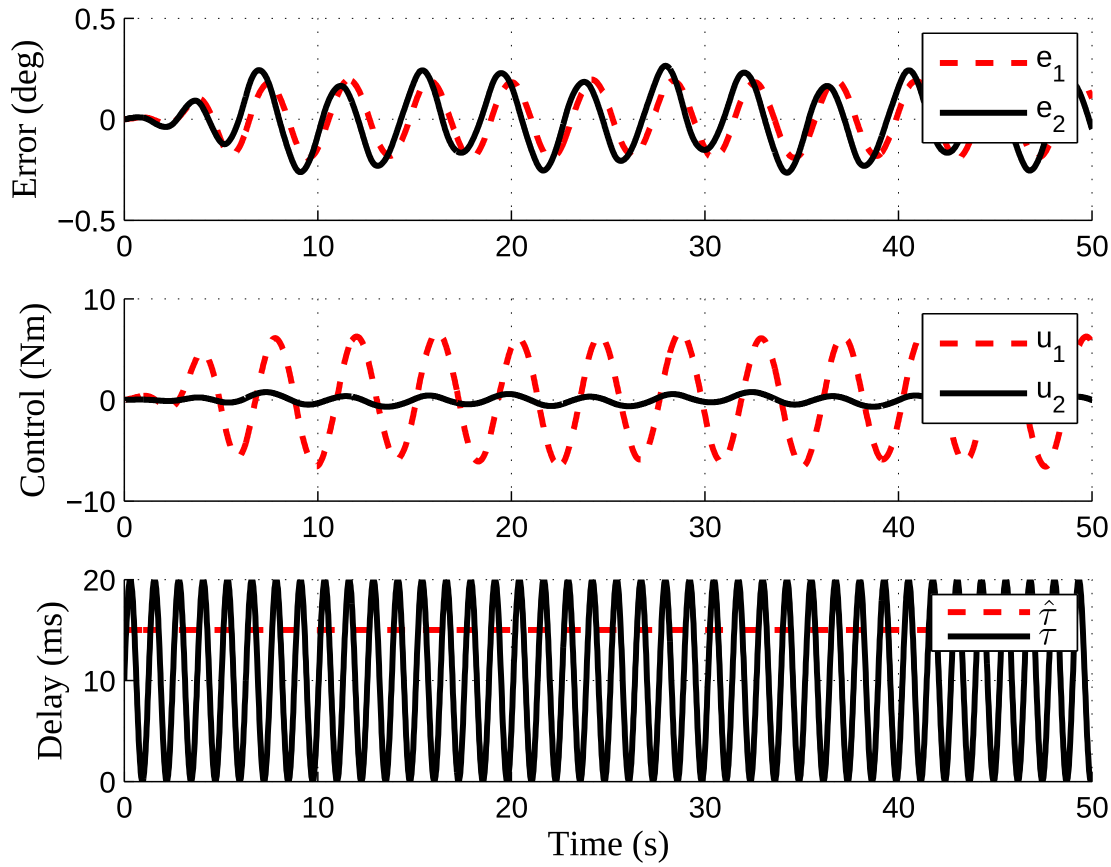

Figure 2-1. Tracking errors, control effort and time-varying delays vs time for Case 1.

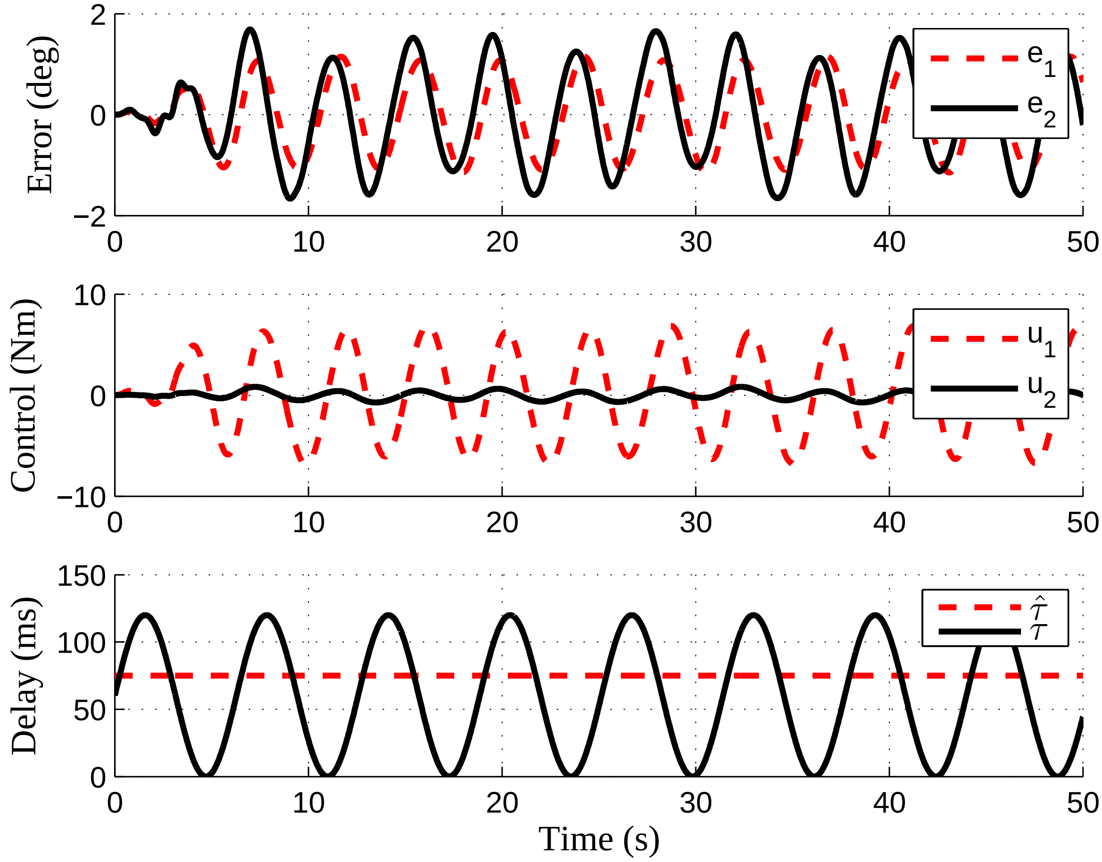

Figure 2-2. Tracking errors, control effort and time-varying delays vs time for Case 3.

2.5 Multiple Delay Case Extension

The results can be extended for the case of multiple delays in the input by redefining the delayed input vector such that $u(t, \tau(t)) \triangleq \begin{bmatrix} u(t-\tau_1), & u(t-\tau_2), ..., u(t-\tau_m) \end{bmatrix}^T$, where $\tau \triangleq [\tau_1, \tau_2, ..., \tau_m]^T$ denotes the input delay, and let $\overline{\tau} \triangleq \max\{\tau_1, \tau_2, ..., \tau_m\}$, where $\overline{\tau} \in \mathbb{R}$ is the maximum value of the input delay, $\tau \in \mathbb{R}$. Additionally, Assumption 2.4 can be modified as

Assumption 2.5. The input delay is bounded such that $\overline{\tau} < 1$ for all $t \in \mathbb{R}$, differentiable, and slowly varying such that $|\bar{\dot{\tau}}| < \varphi < 1$ for all $t \in \mathbb{R}$, where $\dot{\tau} \triangleq$ diag$\{\dot{\tau}_1, \dot{\tau}_2, ..., \dot{\tau}_m\}$, $\bar{\dot{\tau}} \in \mathbb{R}$ is the maximum eigenvalue of $\dot{\tau}$, and $\varphi \in \mathbb{R}$ is a known positive constant. Additionally, a constant estimate $\hat{\tau} \in \mathbb{R}$ of τ is available and sufficiently accurate such that $\tilde{\tau} \triangleq \overline{\tau} - \hat{\tau}$, the difference between the supremum of input delay and the estimate of the input delay is bounded by $|\tilde{\tau}| \leq \bar{\tilde{\tau}}$ for all $t \in \mathbb{R}$, where $\bar{\tilde{\tau}} \in \mathbb{R}$ is a known positive constant.

Moreover, the open-loop dynamics in (2–13) can be modified as[13]

$$\dot{r} = -\dot{f}\left(X, \dot{X}, t\right) - \dot{d} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + x_r^{(n+1)}$$
$$- \alpha\dot{u} + \left(\alpha - I_{m\times m} + \dot{\tau}\right)\dot{u}_{\hat{\tau}} - \left(I_{m\times m} - \dot{\tau}\right)\left(\dot{u}_{\tau} - \dot{u}_{\hat{\tau}}\right) + \Lambda\dot{e}_n.$$

where $I_{m\times m}$ is the identity matrix. Furthermore, the expression (2–14) can be modified as

$$\dot{r} = -\alpha k r + \left(\alpha - I_{m\times m} + \dot{\tau}\right) k r_{\hat{\tau}} + \left(I_{m\times m} - \dot{\tau}\right)\left(\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}\right) + \tilde{N} + N_r - e_n.$$

Uniformly ultimately bounded convergence of the states $e_1, e_2, ..., e_n, e_u, r$ to the origin can then be established using techniques similar to the single delay case.

The performance of the controller is demonstrated in Figure 2-3 for the multiple delay case by using the same dynamics as Section 2.4. The control gains are selected as $\alpha = \text{diag}\{1, 1\},\ \Lambda = \text{diag}\{20, 29\}$, and $k = \text{diag}\{15, 0.9\}$.

[13] The closed-loop dynamics does not include $\dot{u}_{\tau}$. Since the stability analysis does not require exact knowledge of the time delay dynamics, the analysis does not require new Lyapunov Krasovskii functional for every single input delay.

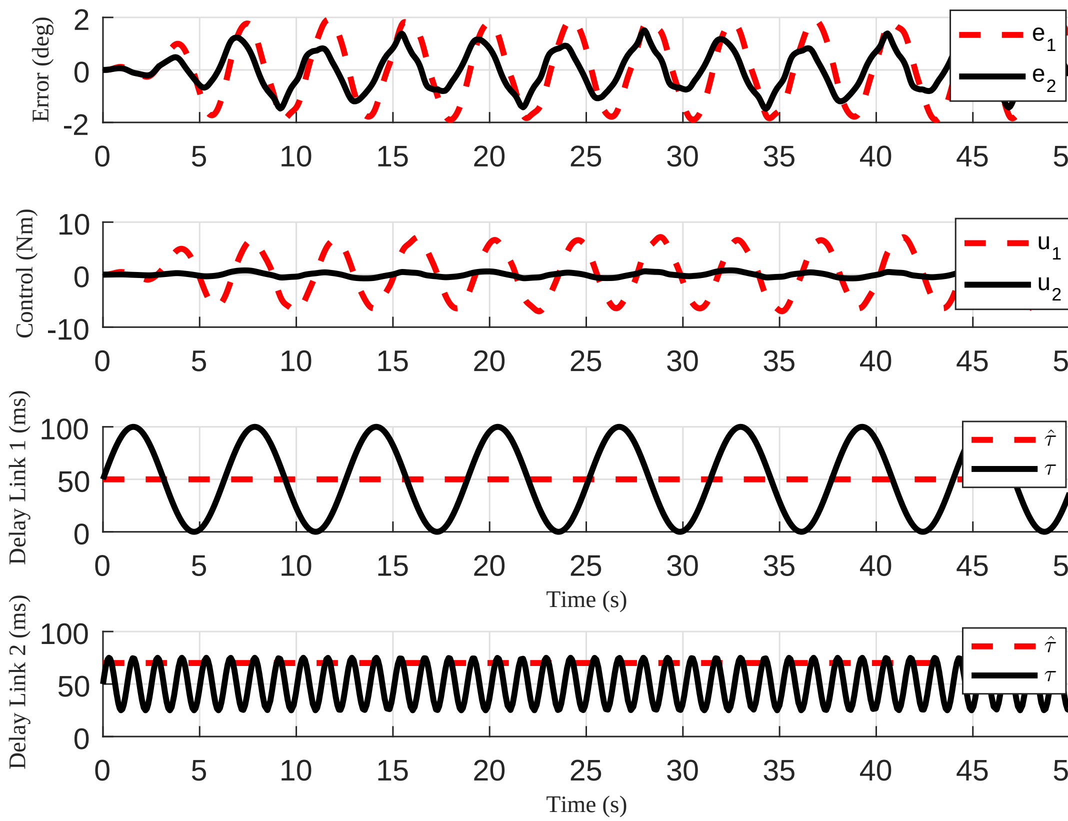

Figure 2-3. Tracking errors, control effort and time-varying delays vs time for multiple delays.

2.6 Conclusion

Novelty of the controller comes from the fact that a continuous robust controller is developed for a class of uncertain nonlinear systems with additive disturbances subject to uncertain time-varying input time delay. A filtered tracking error signal is designed to facilitate the control design and analysis. A Lyapunov-based analysis is used to prove ultimate boundedness of the error signals through the use of Lyapunov-Krasovskii functionals that are uniquely composed of an integral over the estimated delay range rather than the actual delay range. Simulation results indicate the performance of the controller over a range of time varying delays and estimates. Improved performance may be obtained by altering the design to allow for a time-varying estimate of the delay.

CHAPTER 3
ROBUST CONTROL OF AN UNCERTAIN EULER-LAGRANGE SYSTEM WITH UNCERTAIN TIME-VARYING INPUT DELAYS WITHOUT DELAY RATE CONSTRAINTS

In this chapter, a robust controller is designed to track a reference trajectory for unknown non-constant input delayed uncertain Euler-Lagrange systems with bounded additive disturbances. An auxiliary error signal is designed to $i)$ eliminate the delay rate constraints on the input delay since exact knowledge of the input delay is not used in the designed auxiliary error signal, and $ii)$ inject a delay-free control signal in the closed-loop dynamics. The first time-derivative of the designed auxiliary error signal consists of the difference between a delay-free input signal and the delayed control signal using a constant estimated delay. The constant estimated input delay is used instead of a time-varying estimation of the input delay to overcome the delay rate constraints of actual input delay. Novel Lyapunov-Krasovskii functionals are developed without using exact knowledge of the delay to avoid delay rate constraints in the Lyapunov-based stability analysis. Control gain conditions are developed to provide uniformly ultimately bounded convergence of the tracking error to the origin. The control gains are used to determine the maximum allowable error between unknown time-varying delay and a constant estimate of the delay. Numerical simulation results illustrate the performance of the designed robust controller.

3.1 Dynamic Model and Properties

Consider a class of Euler-Lagrange systems defined by

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + d(t) = u(t - \tau(t)), \tag{3–1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ denote the systems states, $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is an uncertain generalized inertia matrix, $V_m : \mathbb{R}^{2n} \to \mathbb{R}^{n\times n}$ is an uncertain generalized centripetal-Coriolis matrix, $G : \mathbb{R}^n \to \mathbb{R}^n$ denotes an uncertain generalized gravity vector, $F : \mathbb{R}^n \to \mathbb{R}^n$ denotes uncertain generalized friction, $d : [t_0, \infty) \to \mathbb{R}^n$ is an uncertain exogenous disturbance (e.g., unmodeled effects), $u(t-\tau(t)) \in \mathbb{R}^n$ represents the generalized delayed input control vector, $\tau : [t_0, \infty) \to \mathbb{R}$ is an uncertain non-negative time-varying delay, and t_0 is the initial time.

The subsequent development is based on the assumption that $q, \dot{q}$ are measurable. In addition, the dynamics of the system in (3–1) satisfies the following assumptions and properties.

Assumption 3.1. The nonlinear exogenous disturbance term is and its first time derivative (i.e., d, $\dot{d}$) exist and are bounded by known positive constants [107–109].

Assumption 3.2. The reference trajectory $q_d \in \mathbb{R}^n$ is designed such that $q_d, \dot{q}_d, \ddot{q}_d$ exist and are bounded by known positive constants.

Assumption 3.3. The input delay is differentiable and bounded as $\tau(t) < 1 \; \forall t \in \mathbb{R}$. Moreover, a positive constant estimate $\hat{\tau} \in \mathbb{R}$ is sufficiently accurate in sense that $|\tilde{\tau}| \leq \bar{\tilde{\tau}}$ where $\tilde{\tau} \triangleq \tau - \hat{\tau}$ and $\bar{\tilde{\tau}} \in \mathbb{R}$ is a known constant[1] . Additionally, the system in (3–1) does not escape to infinity during the time interval $[t_0, t_0 + 1]$ is assumed.

Property 3.1. The inertia matrix M is symmetric positive-definite, and satisfies the following inequality:

$$\underline{m} \|\xi\|^2 \leq \xi^T M \xi \leq \overline{m} \|\xi\|^2 , \quad \forall \xi \in \mathbb{R}^n,$$

where $\underline{m}, \overline{m} \in \mathbb{R}$ are known positive constants.

3.2 Control Objective

The objective is to develop a continuous controller which ensures that the generalized state q of the input-delayed system in (3–1) tracks a reference trajectory, q_d, despite uncertainties and additive disturbances in the dynamics. To quantify the control

[1] Since the maximum tolerable error, $\bar{\tilde{\tau}}$, can be determined based on the selection of control gains, and the estimate of actual delay, $\hat{\tau}$, is known, the maximum tolerable input delay can be determined.

objective, a tracking error, denoted by $e \in \mathbb{R}^n$, is defined as

$$e \triangleq q_d - q. \tag{3–2}$$

To facilitate the subsequent analysis, a measurable auxiliary tracking error, denoted by $r \in \mathbb{R}^n$, is defined as

$$r \triangleq \dot{e} + \alpha e + Be_u, \tag{3–3}$$

where $\alpha, B \in \mathbb{R}^{n\times n}$ are known, diagonal, positive definite constant gains. In (3–3), $e_u \in \mathbb{R}^n$ is an auxiliary signal that is used to obtain a delay-free control signal in the closed-loop systems and is defined as

$$e_u \triangleq -\int_{t-\hat{\tau}}^{t} u(\theta)\, d\theta. \tag{3–4}$$

Since $e, \dot{e}$ are assumed to be measurable, and given that the auxiliary error term e_u is designed based on the estimated delay, r can be computed and used as a feedback control term.

3.3 Control Development

The open-loop error system for r can be obtained by multiplying the time derivative of (3–3) by M and using the expressions in (3–1), (3–2), and (3–4) to yield

$$M\dot{r} = M\ddot{q}_d + V_m\dot{q} + G + F + d + M\alpha\dot{e} + (u_{\hat{\tau}} - u_\tau) + (MB - I_{n\times n})\, u_{\hat{\tau}} - MBu, \tag{3–5}$$

where $I_{n\times n} \in \mathbb{R}^{n\times n}$ denotes the identity matrix. The open-loop error system in (3–5) contains a delay-free control input that is obtained by using the time derivative of (3–4). Based on the subsequent stability analysis, the designed continuous robust control input is designed as

$$u = k_c r, \tag{3–6}$$

where $k_c \in \mathbb{R}^{n\times n}$ is a constant, diagonal, positive definite adjustable control gain matrix. To facilitate the subsequent stability analysis, the terms in (3–5) can be segregated into terms that can be upper bounded by state-dependent functions and terms which can be upper bounded by a known constant such that

$$M\dot{r} = -e - \frac{1}{2}\dot{M}(q,\dot{q})\dot{q}r + \tilde{N} + N_d + (u_{\hat{\tau}} - u_\tau) + (MB - I_{n\times n})u_{\hat{\tau}} - MBu, \tag{3–7}$$

where the terms $\tilde{N}, N_d \in \mathbb{R}^n$ are defined as

$$\begin{aligned}\tilde{N} \triangleq & M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + M(q)\alpha\dot{e} - M(q_d)\ddot{q}_d \\ & - V_m(q_d,\dot{q}_d)\dot{q}_d - G(q_d) - F(\dot{q}_d) + \frac{1}{2}\dot{M}(q,\dot{q})\dot{q}r + e,\end{aligned} \tag{3–8}$$

and

$$N_d \triangleq M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F(\dot{q}_d) + d(t). \tag{3–9}$$

Substituting the expression in (3–6) into (3–7), the closed-loop error system for r can be obtained as

$$M\dot{r} = -e - \frac{1}{2}\dot{M}r + \tilde{N} + N_d + (u_{\hat{\tau}} - u_\tau) + (MB - I_{n\times n})k_c r_{\hat{\tau}} - MBk_c r. \tag{3–10}$$

Remark 3.1. Using the Mean Value Theorem, Property 1, and Assumption 3.2, the expression in (3–8) can be upper bounded as

$$\left\|\tilde{N}\right\| \leq \rho(\|z\|)\|z\|, \tag{3–11}$$

where $\rho : \mathbb{R} \to \mathbb{R}$ is positive definite, non-decreasing, radially unbounded[2] function, and $z \in \mathbb{R}^{3n}$ is a vector of error signals, defined as

$$z \triangleq \begin{bmatrix} e^T & r^T & e_u^T \end{bmatrix}^T. \tag{3–12}$$

Remark 3.2. Using Assumptions 3.1, 3.2 and Property 1, N_d can be upper bounded as

$$\sup_{t\in[0,\infty)} \|N_d\| \leq \varsigma, \tag{3–13}$$

where $\varsigma \in \mathbb{R}$ is a known positive constant.

3.4 Stability Analysis

To facilitate the subsequent stability analysis, let $y \in \mathbb{R}^{3n+2}$ be defined as

$$y \triangleq \begin{bmatrix} z^T & \sqrt{Q_1} & \sqrt{Q_2} \end{bmatrix}^T, \tag{3–14}$$

where $Q_1, Q_2 \in \mathbb{R}$ denote LK functionals defined as

$$Q_1 \triangleq \left(\frac{\left(2\left(\overline{m}\,\overline{B}-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}} + \frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon} \right) \int_{t-\hat{\tau}}^{t} \|r(\theta)\|^2 d\theta, \tag{3–15}$$

$$Q_2 \triangleq \omega_2 \int_{t-\left(\hat{\tau}+\overline{\tilde{\tau}}\right)}^{t} \int_{s}^{t} \|r(\theta)\|^2\, d\theta ds, \tag{3–16}$$

and $\varepsilon, \omega_1, \omega_2 \in \mathbb{R}$ are known, selectable, positive constants, $\overline{B}$,$\underline{B} \in \mathbb{R}$ are the maximum and minimum eigenvalues of B, respectively, and $\overline{k_c}, \underline{k_c} \in \mathbb{R}$ denote the maximum and minimum eigenvalues of k_c, respectively. Moreover, let the auxiliary bounding constants $\sigma\,, \delta \in \mathbb{R}$ be defined as

$$\sigma \triangleq \min\left\{ \left(\alpha - \frac{\overline{B}^2}{2\varepsilon}\right), \left(\frac{\omega_2}{3\hat{\tau}\overline{k_c}^2} - \varepsilon\right), \frac{\underline{m}\,\underline{B}\,\underline{k_c}}{4} \right\}, \tag{3–17}$$

[2] A global uniformly ultimately bounded stability result can be obtained by providing a constant upper bound for ρ as described in Remark 3.

$$\delta \triangleq \frac{1}{2}\min\left\{ \frac{\sigma}{2}, \frac{\omega_2}{3\left(\frac{\left(2\left(\overline{m}\,\overline{B}-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}} + \frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon}\right)}, \frac{1}{3\left(\hat{\tau}+\bar{\tilde{\tau}}\right)} \right\}. \tag{3–18}$$

Let

$$\mathscr{D}_1 \triangleq \left\{y \in \mathbb{R}^{3n+2} \middle| \, \|y\| < \chi_1 \right\}, \tag{3–19}$$

where $\chi_1 \triangleq$ inf $\left\{\rho^{-1}\left(\left[\sqrt{\frac{\sigma \underline{m}\,\underline{B}\,\underline{k_c}}{8}}, \infty\right)\right)\right\}$. Provided $\|z(\eta)\| < \gamma,\ \forall \eta \in [t_0, t]$, the closed-loop error dynamics in (3–10) and the designed controller in (3–6) can be used to conclude that $\dot{u} < \Upsilon$, where γ and Υ are known positive constants. Let $\mathscr{D} \triangleq \mathscr{D}_1 \cap \left(B_\gamma \cap \mathbb{R}^{3n+2}\right)$ where B_γ denotes a closed ball of radius γ centered at the origin.

Let the region of the attraction $\mathcal{S}_{\mathscr{D}} \subset \mathscr{D}$ be defined as:

$$\mathcal{S}_{\mathscr{D}} \triangleq \left\{y \in \mathscr{D} \middle| \, \|y\| < \chi_2 \right\}, \tag{3–20}$$

where $\chi_2 \triangleq \sqrt{\frac{\phi_1}{\phi_2}}$inf $\left\{\rho^{-1}\left(\left[\sqrt{\frac{\sigma \underline{m}\,\underline{B}\,\underline{k_c}}{8}}, \infty\right)\right)\right\}$, where $\phi_1, \phi_2 \in \mathbb{R}$ are known positive constants that are a lower and an upper bound of the Lyapunov-Krosvskii functionals, respectively, defined as

$$\phi_1 \triangleq \frac{1}{2}\min\{1, \underline{m}, \omega_1\}, \quad \phi_2 \triangleq \max\{\frac{\bar{m}}{2}, \frac{\omega_1}{2}, 1\}. \tag{3–21}$$

Let $V : \mathcal{D} \times [t_0,\ \infty) \to \mathbb{R}$ be a continuously differentiable, positive-definite functional on a domain $\mathcal{D} \subseteq \mathbb{R}^{3n+2}$, defined as

$$V \triangleq \frac{1}{2}e^T e + \frac{1}{2}r^T M r + \frac{\omega_1}{2}e_u^T e_u + Q_1 + Q_2. \tag{3–22}$$

The following inequalities can be obtained for (3–22) :

$$\phi_1 \|y\|^2 \leq V \leq \phi_2 \|y\|^2 .$$

Based on the result of the subsequent stability analysis, the control gains are selected according to the following sufficient conditions

$$3\varepsilon\hat{\tau}\overline{k_c}^2 < \omega_2, \tag{3–23}$$

$$\frac{B^2}{2\varepsilon} < \underline{\alpha}, \tag{3–24}$$

$$\bar{\tilde{\tau}} \leq \frac{\frac{\underline{m}\,\underline{B}\,\underline{k_c}}{2} - \frac{\left(2\left(\overline{m}\,\overline{B}-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}} - \frac{2\left(\omega_1\overline{k_c}\right)^2}{\varepsilon} - \omega_2\hat{\tau}}{\omega_2}, \tag{3–25}$$

$$\sqrt{\frac{2\phi_2\left(\varsigma^2+\Upsilon^2\bar{\tilde{\tau}}^2\right)}{\phi_1\left(\underline{m}\,\underline{B}\,\underline{k_c}\delta\right)}} < \inf\left\{\rho^{-1}\left(\left[\sqrt{\frac{\sigma\underline{m}\,\underline{B}\,\underline{k_c}}{8}},\infty\right)\right)\right\}, \tag{3–26}$$

where $\overline{\alpha}$, $\underline{\alpha} \in \mathbb{R}$ are the maximum and minimum eigenvalues of α, respectively. The gain conditions can be expressed in terms of k_c by selection B close to $\frac{1}{\overline{m}}$, sufficiently small ω_1 and ε, and a sufficiently large α, the smaller lower bound of ω_2 can be provided by selection of sufficiently small ε. The gain k_c then can be selected depend on $\hat{\tau}$, $\bar{\tilde{\tau}}$. It can be concluded that $\bar{\tilde{\tau}}$ can be arranged by selection of control gains and $\hat{\tau}$.

Theorem 3.1. *Given the dynamics in (3–1), the controller in (3–6) ensures uniformly ultimately bounded tracking in the sense that*

$$\limsup_{t\to\infty}\|e(t)\| \leq \sqrt{\frac{\phi_2 4\left(\varsigma^2+\Upsilon^2\bar{\tilde{\tau}}^2\right)}{\phi_1\underline{m}\,\underline{B}\,\underline{k_c}\delta}}, \tag{3–27}$$

provided that $y(t_0) \in \mathcal{S}_{\mathscr{D}}$, the sufficient conditions in (3–23)-(3–26) are satisfied and that the control gains are selected sufficiently large relative to the initial conditions of the system[3] *.*

[3] To achieve a small tracking error for the case of a large value of ς (i.e., fast dynamics with large disturbances), large gains are required. To use a large gains resulted in large Υ, and to compensate the large Υ, a better estimate of the delay and small delay are required to obtain small $\bar{\tilde{\tau}}$. It should be noted that the better estimate can be determined by time-varying delay estimate, however, using time-varying delay estimate requires slowly-varying actual time-delay constraint. The motivation of this chapter is provide uniformly ultimately bounded tracking for the case of the error bound does not depend on delay rate.

Proof. The time derivative of (3–22) can be obtained after applying the Leibniz Rule to obtain the time derivative of (3–15) and (3–16) and utilizing (3–3), (3–4) and (3–10) as

$$\begin{aligned}
\dot{V} &= e^T\left(r-\alpha e-Be_u\right)+r^T\left(-e-\frac{1}{2}\dot{M}\left(q,\dot{q}\right)\dot{q}r+\tilde{N}+N_d+\left(u_{\hat{\tau}}-u_\tau\right)\right)\\
&\quad +r^T\left(\left(MB-I_{n\times n}\right)k_c r_{\hat{\tau}}-MBk_c r\right)+\frac{1}{2}r^T\dot{M}r+\omega_1 e_u^T\left(k_c r_{\hat{\tau}}-k_c r\right)\\
&\quad +\left(\frac{\left(2\left(\overline{m}\,\overline{B}\,-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}}+\frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon}\right)\left(\left\|r\right\|^2-\left\|r_{\hat{\tau}}\right\|^2\right)\\
&\quad +\omega_2\left(\hat{\tau}+\overline{\tilde{\tau}}\right)\left\|r\right\|^2-\omega_2\int_{t-\left(\hat{\tau}+\overline{\tilde{\tau}}\right)}^{t}\left\|r\left(\theta\right)\right\|^2 d\theta.
\end{aligned} \tag{3–28}$$

By using (3–6), (3–9), (3–11) and canceling common terms in (3–28), an upper can be obtained as

$$\begin{aligned}
\dot{V} &\leq -\underline{\alpha}\left\|e\right\|^2+\underline{B}\left\|e\right\|\left\|e_u\right\|+\left\|r\right\|\rho\left(\left\|z\right\|\right)\left\|z\right\|+\varsigma\left\|r\right\|+\left\|r\right\|\left\|u_{\hat{\tau}}-u_\tau\right\|\\
&\quad +\left|\overline{m}\overline{B}-1\right|\overline{k_c}\left\|r\right\|\left\|r_{\hat{\tau}}\right\|-\underline{m}\,\underline{B}\,\underline{k_c}\left\|r\right\|^2+\omega_1\overline{k_c}\left\|e_u\right\|\left\|r_{\hat{\tau}}\right\|+\omega_1\overline{k_c}\left\|e_u\right\|\left\|r\right\|\\
&\quad +\left(\frac{\left(2\left(\overline{m}\,\overline{B}\,-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}}+\frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon}\right)\left(\left\|r\right\|^2-\left\|r_{\hat{\tau}}\right\|^2\right)\\
&\quad +\omega_2\left(\hat{\tau}+\overline{\tilde{\tau}}\right)\left\|r\right\|^2-\omega_2\int_{t-\left(\hat{\tau}+\overline{\tilde{\tau}}\right)}^{t}\left\|r\left(\theta\right)\right\|^2 d\theta.
\end{aligned} \tag{3–29}$$

After the completing the squares for r, $r_{\hat{\tau}}$ and e_u and using Young's inequality, the following upper bound can be obtained

$$\begin{aligned}
\dot{V} &\leq -\underline{\alpha}\left\|e\right\|^2+\frac{\underline{B}^2}{2\varepsilon}\left\|e\right\|^2+\frac{\varepsilon}{2}\left\|e_u\right\|^2\frac{\underline{m}\,\underline{B}\,\underline{k_c}}{16}\left\|r\right\|^2+\frac{4}{\underline{m}\,\underline{B}\,\underline{k_c}}\rho^2\left(\left\|z\right\|\right)\left\|z\right\|^2\\
&\quad +\frac{\underline{m}\,\underline{B}\,\underline{k_c}}{16}\left\|r\right\|^2+\frac{4}{\underline{m}\,\underline{B}\,\underline{k_c}}\varsigma^2+\frac{\underline{m}\,\underline{B}\,\underline{k_c}}{16}\left\|r\right\|^2+\frac{4}{\underline{m}\,\underline{B}\,\underline{k_c}}\left\|u_{\hat{\tau}}-u_\tau\right\|^2\\
&\quad +\frac{\underline{m}\,\underline{B}\,\underline{k_c}}{16}\left\|r\right\|^2+\frac{\left(2\left(\overline{m}\,\overline{B}\,-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}}\left\|r_{\hat{\tau}}\right\|-\underline{m}\,\underline{B}\,\underline{k_c}\left\|r\right\|^2+\frac{\varepsilon}{4}\left\|e_u\right\|^2\\
&\quad +\frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon}\left\|r_{\hat{\tau}}\right\|^2+\frac{\varepsilon}{4}\left\|e_u\right\|^2+\frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon}\left\|r\right\|^2\\
&\quad +\left(\frac{\left(2\left(\overline{m}\,\overline{B}\,-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}}+\frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon}\right)\left(\left\|r\right\|^2-\left\|r_{\hat{\tau}}\right\|^2\right)\\
&\quad +\omega_2\left(\hat{\tau}+\overline{\tilde{\tau}}\right)\left\|r\right\|^2-\omega_2\int_{t-\left(\hat{\tau}+\overline{\tilde{\tau}}\right)}^{t}\left\|r\left(\theta\right)\right\|^2 d\theta.
\end{aligned} \tag{3–30}$$

After canceling common terms in (3–30), an upper bound can be obtained for (3–30) as

$$\begin{aligned}\dot{V} \;\leq\; & -\left(\underline{\alpha}-\frac{\overline{B}^{2}}{2\varepsilon}\right)\|e\|^{2}-\frac{\underline{m}\,\underline{B}\,\underline{k_{c}}}{4}\|r\|^{2}+\varepsilon\|e_{u}\|^{2}+\frac{4}{\underline{m}\,\underline{B}\,\underline{k_{c}}}\rho^{2}(\|z\|)\|z\|^{2}\\ & +\frac{4}{\underline{m}\,\underline{B}\,\underline{k_{c}}}\varsigma^{2}+\frac{4}{\underline{m}\,\underline{B}\,\underline{k_{c}}}\|u_{\hat{\tau}}-u_{\tau}\|^{2}-\left(\frac{\underline{m}\,\underline{B}\,\underline{k_{c}}}{2}-\kappa\right)\|r\|^{2}\\ & -\omega_{2}\int_{t-\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}^{t}\|r(\theta)\|^{2}d\theta,\end{aligned} \tag{3–31}$$

where $\kappa\triangleq\frac{\left(2\left(\overline{m}\,\overline{B}-1\right)\overline{k_{c}}\right)^{2}}{\underline{m}\,\underline{B}\,\underline{k_{c}}}+\frac{2\left(\omega_{1}\overline{k_{c}}\right)^{2}}{\varepsilon}+\omega_{2}\left(\hat{\tau}+\bar{\tilde{\tau}}\right)$. By using the Mean Value Theorem to obtain the inequality such that $\|u_{\hat{\tau}}-u_{\tau}\|\leq\|\dot{u}(\Theta(t,\hat{\tau}))\|\,|\tilde{\tau}|\leq\|\dot{u}(\Theta(t,\hat{\tau}))\|\,\left|\bar{\tilde{\tau}}\right|$, where $\Theta(t,\hat{\tau})$ is a point between $t-\tau$ and $t-\hat{\tau}$. Additionally, using (3–25), the right hand side of (3–31) can be upper bounded as

$$\begin{aligned}\dot{V} \;\leq\; & -\left(\underline{\alpha}-\frac{\overline{B}^{2}}{2\varepsilon}\right)\|e\|^{2}-\frac{\underline{m}\,\underline{B}\,\underline{k_{c}}}{4}\|r\|^{2}+\varepsilon\|e_{u}\|^{2}+\frac{4}{\underline{m}\,\underline{B}\,\underline{k_{c}}}\rho^{2}(\|z\|)\|z\|^{2}\\ & +\frac{4}{\underline{m}\,\underline{B}\,\underline{k_{c}}}\varsigma^{2}+\frac{4}{\underline{m}\,\underline{B}\,\underline{k_{c}}}\|\dot{u}(\Theta(t,\hat{\tau}))\|^{2}\left|\bar{\tilde{\tau}}\right|^{2}-\omega_{2}\int_{t-\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}^{t}\|r(\theta)\|^{2}d\theta.\end{aligned} \tag{3–32}$$

The Cauchy-Schwartz inequality is used to develop an upper bound for $\|e_{u}\|^{2}$ as

$$\|e_{u}\|^{2}\leq\hat{\tau}\int_{t-\hat{\tau}}^{t}\|u(\theta)\|^{2}d\theta\leq\hat{\tau}\overline{k_{c}}^{2}\int_{t-\hat{\tau}}^{t}\|r(\theta)\|^{2}d\theta. \tag{3–33}$$

Additionally, Q_{2} can be upper bounded as

$$Q_{2}\leq\omega_{2}\left(\bar{\tilde{\tau}}+\hat{\tau}\right)\sup_{s\epsilon\left[t-\left(\bar{\tilde{\tau}}+\hat{\tau}\right),t\right]}\left[\int_{s}^{t}\|r(\theta)\|^{2}ds\right]\leq\omega_{2}\left(\bar{\tilde{\tau}}+\hat{\tau}\right)\int_{t-\left(\bar{\tilde{\tau}}+\hat{\tau}\right)}^{t}\|r(\theta)\|^{2}d\theta. \tag{3–34}$$

Using the inequalities $\|y\|\geq\|z\|$, $\int_{t-\hat{\tau}}^{t}\|r(\theta)\|^{2}d\theta\leq\int_{t-\left(\bar{\tilde{\tau}}+\hat{\tau}\right)}^{t}\|r(\theta)\|^{2}d\theta$ and using the definition of σ in (3–17), the gain conditions in (3–23), (3–24), (3–26), and utilizing (3–15), (3–16), (3–33), (3–34), the following upper bound can be obtained

$$\begin{aligned}\dot{V} &\leq -\left(\frac{\sigma}{2}-\frac{4}{\underline{m}\,\underline{B}\,\underline{k_c}}\rho^2(\|y\|)\right)\|z\|^2-\frac{\sigma}{2}\|z\|^2+\frac{4\left(\varsigma^2+\Upsilon^2\bar{\bar{\tau}}^2\right)}{\underline{m}\,\underline{B}\,\underline{k_c}} \\ &\quad -\frac{\omega_2}{3\left(\frac{\left(2\left(\overline{m}\,\overline{B}-1\right)\overline{k_c}\right)^2}{\underline{m}\,\underline{B}\,\underline{k_c}}+\frac{\left(\omega_1\overline{k_c}\right)^2}{\varepsilon}\right)}Q_1-\frac{1}{3\left(\bar{\bar{\tau}}+\hat{\tau}\right)}Q_2. \end{aligned} \tag{3–35}$$

Provided $y(\eta)\in\mathscr{D},\ \forall\eta\in[t_0,t]$ then using the definition δ in (3–18), the expression in (3–35) reduces to

$$\dot{V}\leq-\delta\|y\|^2,\quad\forall\|y\|\geq 2\left(\frac{\varsigma^2+\Upsilon^2\bar{\bar{\tau}}^2}{\underline{m}\,\underline{B}\,\underline{k_c}\delta}\right)^{\frac{1}{2}}. \tag{3–36}$$

It can be concluded that by using techniques similar to Theorem 4.18 in [106], y is uniformly ultimately bounded in the sense that $\limsup_{t\to\infty}\|y(t)\|\leq\sqrt{\frac{\phi_2 4\left(\varsigma^2+\Upsilon^2\bar{\bar{\tau}}^2\right)}{\phi_1\underline{m}\,\underline{B}\,\underline{k_c}\delta}}$, provided $y(t_0)\in\mathcal{S}_{\mathscr{D}}$, where uniformity in initial time can be concluded from the independence of δ and the ultimate bound from t_0. Since $e,\ r,\ e_u\in\mathcal{L}_\infty$, from (3–6), $u\in\mathcal{L}_\infty$. An analysis of the closed-loop system shows that the remaining signals are bounded. □

Remark 3.3. If the system dynamics are such that $\tilde{N}$ is linear in z, then the function ρ can be upper bounded by a known positive constant $(i.e.,\ \rho(\|z\|)=\bar{\rho},\ \forall z\in\mathbb{R}^{3n})$. The sufficient condition in (3–26) can be reduced as

$$k_c\leq\frac{8\bar{\rho}^2}{\underline{m}\,\underline{B}\,\sigma}, \tag{3–37}$$

and the result is global in the sense that $\mathscr{D}=\mathcal{S}_{\mathscr{D}}=\mathbb{R}^{3n+2}$.

3.5 Simulation Results

The performance of the controller defined in (3–6) is demonstrated by using the same dynamics as Section 2.4. Several simulation results were obtained using fast time-varying input delays with different estimated delays, shown in Table 3-1, to illustrate performance of the controller in (3–6). Cases 1, 2, 3 and 4 use a high-frequency, time-varying input delay with different delay upper-bound estimates. The root mean square

Table 3-1. RMS errors for time-varying time-delay rates and magnitudes.

	$\tau_i(t)$ (ms)	$\hat{\tau}(t)$ (ms)	RMS Error x_1	RMS Error x_2
Case 1	$50\sin(100t)+50$	50	0.2492^o	0.3059^o
Case 2	$50\sin(100t)+50$	$50\sin(100t)+50$	0.2477^o	0.2977^o
Case 3	$50\sin(100t)+50$	80	0.3804^o	0.3669^o
Case 4	$50\sin(100t)+50$	120	0.5648^o	0.4434^o

(RMS) errors are determined for each case and are listed in Table 3-1. By comparing the RMS errors for Cases 1, 2, 3 and 4, it is clear that the RMS error is improved as the delay estimation error decreases. Moreover, Cases 1, 3 and 4 demonstrate that using a constant estimate of the unknown delay may yield comparable performance to using the actual input delay. The result in Figures 3-1 presents the tracking errors, actuation effort, time-varying delays and estimated delays for Cases 1.

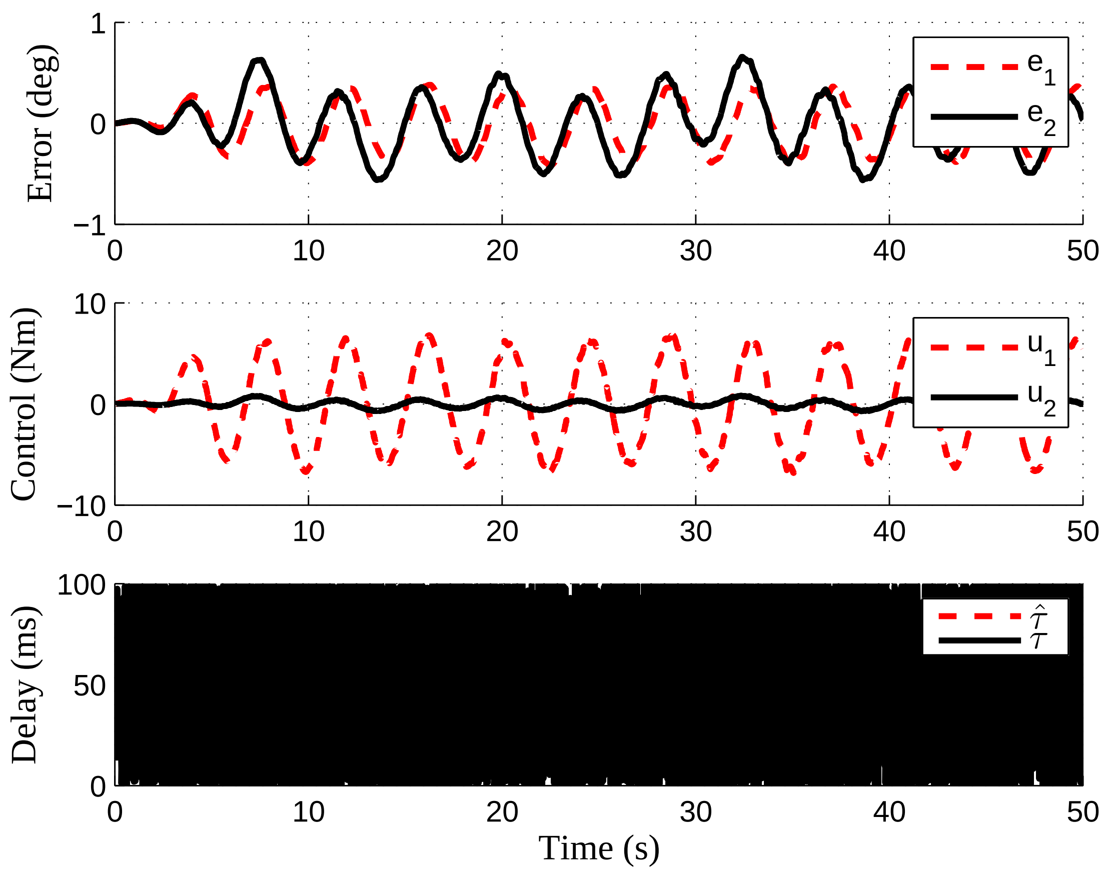

Figure 3-1. Tracking errors, actuation effort and time-varying delays vs time for Case 1.

3.6 Conclusion

A novel continuous robust controller was developed for a class of Euler-Lagrange systems with uncertainties in the plant parameters, additive disturbances and uncertainties in the time-varying input delay. A novel filtered tracking error signal is designed to facilitate the control design and analysis. Techniques used in this chapter to compensate for the time-varying delay results in sufficient conditions that depend on the length of the estimated delay as well as the maximum tolerable error between the estimated and actual delay. Lyapunov-Krasovskii functionals are used in the Lyapunov stability analysis to prove uniformly ultimately bounded tracking.

CHAPTER 4
ADAPTIVE CONTROL FOR AN UNCERTAIN NONLINEAR SYSTEM WITH UNCERTAIN STATE-DEPENDENT INPUT DELAY

In this chapter, a continuous controller is designed to track a reference trajectory despite a state-dependent input delay and uncertain nonlinear dynamics with additive bounded disturbances. A novel tracking error system is developed to inject a delay-free control signal in the closed-loop dynamics. The error signal uses the past derivative of input states in a finite integral over an estimated delay interval to cope with the lack of delay knowledge. The estimated input delay is obtained from an adaptation policy. A novel Lyapunov-based stability analysis is developed by using Lyapunov-Krasovskii functionals to prove semi-global uniformly ultimate boundedness of the error signals.

4.1 Dynamic Model and Properties

Consider a class of $(n+1)^{th}$-order nonlinear systems

$$\begin{aligned} \dot{x}_i &= x_{i+1},\ i=1,\ldots,n, \\ \dot{x}_{n+1} &= f(X)+d(t)+u(t-g(X)), \end{aligned} \tag{4–1}$$

where $x_i(t)\in\mathbb{R}^m,\ i=1,\ldots,n$ are the system states, $X(t)=\left[x_1^T\,x_2^T\,\ldots\,x_n^T\right]^T\in\mathbb{R}^{m\times n}$, $u(t)\in\mathbb{R}^m$ is the control input, $f(X):\mathbb{R}^{m\times n}\times[t_0,\infty)\to\mathbb{R}^m$ is an uncertain nonlinear function, uniformly bounded in t, where t_0 is the initial time. $d:[t_0,\infty)\to\mathbb{R}^m$ is a sufficiently smooth unknown additive disturbance (e.g., unmodeled effects). $g(X):\mathbb{R}^{m\times n}\times[t_0,\infty)\to\mathbb{R}$ is an uncertain nonlinear function that presents a nonnegative input delay. The subsequent development is based on the assumption that $x_i,\ i=1,\ldots,n+1$ are measurable outputs. The dynamic model of the system in (4–1) can be rewritten as

$$x_1^{(n+1)}=f(X)+d(t)+u(t-\tau). \tag{4–2}$$

The system dynamics in (4–1) satisfies the following assumptions.

Assumption 4.1. The function f and its first partial derivatives are bounded on each subset of their domain of the form $\Xi \times [t_0, \infty)$, where $\Xi \in \mathbb{R}^{m \times n}$ is compact and for any given Ξ, the corresponding bounds are known such that the bounds of f, $\frac{\partial f(X,t)}{\partial X}$, $\frac{\partial f(X,t)}{\partial t}$ over Ξ are assumed to be known.

Assumption 4.2. The nonlinear additive disturbance term and its first time derivatives (i.e., d, $\dot{d}$) exist and are bounded by known constants.

Assumption 4.3. The reference trajectory $x_d \in \mathbb{R}^m$ is designed such that $x_d^{(i)} \in \mathbb{R}^m$, $\forall i = 0, 1, ..., (n+2)$ exist and are bounded by known positive constants, where the superscript (i) denotes the i^{th} time derivative.

Assumption 4.4. The dynamics of input delay $g(X)$ are assumed to be bounded by known positive constants such that $\underline{\tau} < g(X) < \bar{\tau}$, where $\underline{\tau},\ \bar{\tau} \in \mathbb{R}$. Furthermore, $g(X)$ can be linearly parametrized such that $g(X) \triangleq Y(X)\theta$, where $Y(X) \in \mathbb{R}^{1 \times h}$ denotes a regression matrix that is assumed to be known, $\theta \in \mathbb{R}^h$ contains the unknown constants of input delay dynamics, $g(X)$, where $h \in \mathbb{R}$. Furthermore, it is assumed that the system in (4–1) does not escape to infinity during the time interval $[t_0, t_0 + \bar{\tau}]$.

4.2 Control Objective

The control objective for the state x_1 of the input delayed system in (4–2) to track a reference trajectory. To quantify the control objective, a tracking error, denoted by $e_1 \in \mathbb{R}^m$, is defined as

$$e_1 \triangleq x - x_d. \tag{4–3}$$

To facilitate the subsequent analysis, auxiliary tracking error signals, denoted by $e_i(t) \in \mathbb{R}^m,\ i = 2, 3, \ldots, n$, can be defined the same as (2–4)-(2–7). Another auxiliary tracking error signal, denoted by $r(t) \in \mathbb{R}^m$, is defined as

$$r \triangleq \dot{e}_n + \alpha e_n + e^{\eta_1 \bar{\tau}} e_{u1} + e_{u2}, \tag{4–4}$$

where $\alpha \in \mathbb{R}^{m\times m}$ is a constant, diagonal, positive definite gain matrix, and $\eta_1 \in \mathbb{R}$ is a constant, positive control gain. To facilitate the stability analysis, an auxiliary tracking error signal $e_{u1} \in \mathbb{R}^m$ is defined as

$$e_{u1} \triangleq \int_{t-\bar{\tau}}^{t-Y_d\hat{\theta}} e^{\eta_1(\xi-t)} u\left(\xi\right) d\xi. \tag{4–5}$$

The desired regression matrix is defined by $Y_d \triangleq Y\left(X_d\right) \in \mathbb{R}^{1\times h}$, where $X_d \triangleq \left[x_d^T,\ \dot{x}_d^T,\ \ldots,\ \left(x_d^{(n-1)}\right)^T\right]^T \in \mathbb{R}^{mn}$, and $\hat{\theta} \in \mathbb{R}^h$ denotes an adaptive estimate. Furthermore, to inject the first time derivative of the delay-free control input into the closed-loop error system, an auxiliary tracking error signal, denoted by $e_{u2} \in \mathbb{R}^m$, is defined as

$$e_{u2} \triangleq \int_{t-Y_d\hat{\theta}}^{t} e^{\eta_2(\xi-t)} \dot{u}\left(\xi\right) d\xi, \tag{4–6}$$

where $\eta_2 \in \mathbb{R}$ is a constant, positive control gain and an auxiliary term, denoted by $\Omega \in \mathbb{R}^m$, is defined as

$$\Omega \triangleq \frac{\Gamma^{-1} Y_d^T}{\left\|Y_d\right\|^2}\left(\kappa + \dot{Y}_d \Gamma \hat{\theta}\right), \tag{4–7}$$

where $\Gamma \in \mathbb{R}^{m\times m}$ is a constant, diagonal, positive definite gain matrix, and $\kappa \in \mathbb{R}$ is defined as

$$\begin{aligned}\kappa \triangleq & e^{\eta_1\left(\bar{\tau}-Y_d\hat{\theta}\right)}\left(r^T + \omega_1 e_{u1}^T\right) u\left(t - Y_d\hat{\theta}\right) - e^{-\eta_2 Y_d\hat{\theta}}\left(r^T + \omega_2 e_{u2}^T\right)\dot{u}\left(t - Y_d\hat{\theta}\right) \\ & + \frac{\varepsilon_1\omega_1}{2} e^{\eta_3\left(\bar{\tau}-Y_d\hat{\theta}\right)}\left\|u\left(t - Y_d\hat{\theta}\right)\right\|^2,\end{aligned} \tag{4–8}$$

where $\omega_1, \omega_2, \varepsilon_1 \in \mathbb{R}$ are selectable positive constants. The adaptive estimate of θ is generated from the following differential equation as

$$\dot{\hat{\theta}}_i \triangleq \text{proj}\left\{\Omega_i\right\} = \begin{cases} 0, & \text{if } \Omega_i > \frac{\bar{\tau}Y_d^T}{\|Y_d\|^2} \\ 0, & \text{if } \Omega_i < \frac{\underline{\tau}Y_d^T}{\|Y_d\|^2} \\ \Omega_i, & \text{otherwise}, \end{cases} \tag{4–9}$$

where $\dot{\hat{\theta}}_i, \Omega_i$ denote the i^{th} component of $\dot{\hat{\theta}}$ and Ω, respectively. The projection operator ensures that $\underline{\tau} < Y_d\hat{\theta} < \bar{\tau}$. The time derivative of (4–4) can be obtained by substituting (4–2), time derivatives of (2–4)-(2–7), (4–5) and (4–6) into (4–4), as

$$\begin{aligned} \dot{r} &= f\left(X\right) + d\left(t\right) + \alpha\dot{e}_n + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} - x_d^{(n+1)} - \eta_1 e^{\eta_1\bar{\tau}} e_{u1} - \eta_2 e_{u2} \\ &\quad + \left(1 - \dot{Y}_d\hat{\theta} - Y_d\dot{\hat{\theta}}\right)\left(e^{-\eta_1\left(Y_d\hat{\theta}-\bar{\tau}\right)} u\left(t - Y_d\hat{\theta}\right) - e^{-\eta_2 Y_d\hat{\theta}}\dot{u}\left(t - Y_d\hat{\theta}\right)\right) \\ &\quad + \dot{u} + u\left(t - Y\theta\right) - u\left(t - \bar{\tau}\right). \end{aligned} \tag{4–10}$$

The equation in (4–10) facilitates the stability analysis by segregating terms that can be upper bounded by a state-dependent term and terms that can be upper bounded by constants, such that

$$\begin{aligned} \dot{r} &= N_d + \tilde{N} - e_n + \dot{u} + u\left(t - Y\theta\right) - u\left(t - \bar{\tau}\right) \\ &\quad + \left(1 - \dot{Y}_d\hat{\theta} - Y_d\dot{\hat{\theta}}\right)\left(e^{-\eta_1\left(Y_d\hat{\theta}-\bar{\tau}\right)} u\left(t - Y_d\hat{\theta}\right) - e^{-\eta_2 Y_d\hat{\theta}}\dot{u}\left(t - Y_d\hat{\theta}\right)\right), \end{aligned} \tag{4–11}$$

where the auxiliary function $\tilde{N} \in \mathbb{R}^m$ and $N_r \in \mathbb{R}^m$ are defined as

$$\tilde{N} \triangleq f\left(X\right) - f\left(X_d\right) + \alpha\dot{e}_n + e_n - \eta_1 e^{\eta_1\bar{\tau}} e_{u1} - \eta_2 e_{u2} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)}, \tag{4–12}$$

$$N_d \triangleq f\left(X_r\right) + d\left(t\right) - x_r^{(n+1)}. \tag{4–13}$$

Based on (4–11) and the subsequent stability analysis,

$$u(t) = -k\left(e_n(t) - e_n(t_0)\right) - \upsilon, \tag{4–14}$$

where $\upsilon \in \mathbb{R}^m$ is the solution of the following differential equation

$$\dot{\upsilon}(t) = k\left(\alpha e_n(t) + e^{\eta_1 \bar{\tau}} e_{u1} + e_{u2}\right), \tag{4–15}$$

where $k \in \mathbb{R}^{m \times m}$ is a positive constant control gain matrix. Substituting (4–14) and (4–15) into (4–11), the following expression can be obtained

$$\begin{aligned}\dot{r} &= N_d + \tilde{N} - e_n - kr + u(t - Y\theta) - u(t - \bar{\tau}) \\ &\quad + \left(1 - \dot{Y}_d\hat{\theta} - Y_d\dot{\hat{\theta}}\right)\left(e^{-\eta_1\left(Y_d\hat{\theta} - \bar{\tau}\right)} u\left(t - Y_d\hat{\theta}\right) - e^{-\eta_2 Y_d\hat{\theta}}\dot{u}\left(t - Y_d\hat{\theta}\right)\right).\end{aligned} \tag{4–16}$$

Remark 4.1. Based on Assumptions 4.2 and 4.3, N_d is bounded as

$$\sup_{t \in \mathbb{R}} \|N_d\| \leq \Phi, \tag{4–17}$$

where $\Phi \in \mathbb{R}$ is a known positive constant.

Remark 4.2. The Mean Value Theorem and Lemma 5 in [105] can be utilized to obtain an upper bound for (4–12) as

$$\left\|\tilde{N}\right\| \leq \rho(\|z\|)\|z\|, \tag{4–18}$$

where ρ is a positive, non-decreasing, radially unbounded function, and $z \in \mathbb{R}^{(n+3)m}$ denotes the vector

$$z \triangleq \left[e_1^T,\ e_2^T, \ldots, e_n^T,\ e_{u1},\ e_{u1},\ r^T\right]^T. \tag{4–19}$$

To facilitate the subsequent stability analysis, auxiliary bounding constants $\sigma, \delta \in \mathbb{R}$ are defined as

$$\sigma \triangleq \min\Bigg\{\frac{1}{2}, \left(\alpha_{min} - \frac{1+\varepsilon_1\omega_1 e^{2\eta_1\bar{\tau}}}{2} - \frac{\varepsilon_2\omega_2}{2k_{min}}\right),$$
$$\frac{k_{min}}{2} - k_{max}\left(\frac{\varepsilon_2\omega_2}{2} - (\bar{\tau} - \underline{\tau})\left(e^{\eta_4\bar{\tau}} + \frac{1}{4}\right)\right),$$
$$\omega_1\left(\eta_1 e^{\eta_1\bar{\tau}} - \frac{1}{\varepsilon_1}\right), \omega_2\left(\eta_2 - \frac{\omega_2 k_{max}}{\varepsilon_2}\right)\Bigg\}, \tag{4–20}$$
$$\delta \triangleq \min\left\{\frac{\sigma}{2},\ \eta_3 e^{\eta_3\bar{\tau}}\varepsilon_1\omega_1, \eta_4 e^{\eta_4\underline{\tau}} k_{max}(\bar{\tau} - \underline{\tau}),\ \frac{\gamma}{\beta}\right\}, \tag{4–21}$$

where $\eta_4, \beta, \gamma, \varepsilon_2 \in \mathbb{R}$ are known selectable positive constants and $\alpha_{max}, \alpha_{min}, k_{max}, k_{min} \in \mathbb{R}$ are the maximum and minimum eigenvalues of α and k, repectively. Let the functions $Q_1,\ Q_2,\ Q_3 \in \mathbb{R}$ be defined as

$$Q_1 \triangleq\ e^{\eta_3\bar{\tau}}\frac{\varepsilon_1\omega_1}{2}\int_{t-\bar{\tau}}^{t-Y_d\hat{\theta}} e^{\eta_3(\xi-t)}\left|\left|u(\xi)\right|\right|^2 d\xi, \tag{4–22}$$
$$Q_2 \triangleq\ e^{\eta_4\Theta}k_{max}(\bar{\tau} - \underline{\tau})\int_{t-\Theta}^{t} e^{\eta_4(\xi-t)}\left|\left|r(\xi)\right|\right|^2 d\xi, \tag{4–23}$$
$$Q_3 \triangleq\ \frac{\beta}{2}\left(Y_d\tilde{\theta}\right)^2, \tag{4–24}$$

where $\Theta \in \mathbb{R}$ is a point in time between $t - \bar{\tau}$ and $t - Y\theta$. Let $y \in \mathbb{R}^{(n+3)m+3}$ be defined as

$$y \triangleq \left[\ z^T, \sqrt{Q_1}, \sqrt{Q_2}, \sqrt{Q_3}\ \right]^T. \tag{4–25}$$

For use in the following stability analysis, let

$$\mathscr{D} \triangleq \left\{y \in \mathbb{R}^{(n+3)m+3} \middle|\ \|y\| < \chi_1\right\}, \tag{4–26}$$

where $\chi_1 \triangleq \text{inf}\left\{\rho^{-1}\left([\sqrt{\frac{\sigma k_{min}}{2}}, \infty)\right)\right\}$. The domain of attraction of the dynamics can be defined as

$$\mathcal{S}_{\mathscr{D}} \triangleq \left\{y \in \mathscr{D}|\ \|y\| < \chi_2\right\}, \tag{4–27}$$

where $\chi_2 \triangleq \sqrt{\frac{\lambda_1}{\lambda_2}}\text{inf}\left\{\rho^{-1}\left([\sqrt{\frac{\sigma k_{min}}{2}}, \infty)\right)\right\}$ and $\lambda_1, \lambda_2 \in \mathbb{R}$ are positive constants, defined as

$$\lambda_1 \triangleq \min\{\frac{1}{2}, \frac{\omega_1 e^{\eta_1 \bar{\tau}}}{2}, \frac{\omega_2}{2}\}, \qquad \lambda_2 \triangleq \max\{\frac{1}{2}, \frac{\omega_1 e^{\eta_1 \bar{\tau}}}{2}, \frac{\omega_2}{2}\}.$$

4.3 Stability Analysis

Theorem 4.1. *Given the dynamics in (4–2), the controller given in (4–14) and (4–15) ensures semi-global uniformly ultimately bounded tracking in the sense that*

$$\left\| e_1^{(i)}(t) \right\| \leq \epsilon_0 \exp\left(-\epsilon_1 (t - t_0)\right) + \epsilon_2, \;\; i = 0, \ldots, n \tag{4–28}$$

where $\epsilon_0 \triangleq \sqrt{2\left(V(t_0) - \frac{\lambda_2 \acute{\Phi}}{\delta}\right)}$, $\epsilon_1 \triangleq \frac{2\delta}{\lambda_2}$ *and* $\epsilon_2 \triangleq \sqrt{\frac{\lambda_2 \acute{\Phi}}{\delta}}$, $\acute{\Phi} \triangleq \frac{\Phi^2}{k_{min}} + \frac{\gamma}{2}(\bar{\tau} - \underline{\tau})^2 + \frac{(\Gamma_{max} + \beta(\bar{\tau} - \underline{\tau}))^2}{4\Gamma_{min}} + \beta(\bar{\tau} - \underline{\tau})\varphi$, *where* $\varphi, \gamma \in \mathbb{R}$ *are known positive constants,*[1] *and* $\Gamma_{max}, \Gamma_{min} \in \mathbb{R}$ *are the maximum and minimum eigenvalue of* Γ*, respectively. The expression in (4–28) can be satisfied provided* $y(t_0) \in \mathcal{S}_{\mathscr{D}}$ *and the control gains are selected sufficiently large relative to the initial conditions of the system such that the following sufficient conditions are satisfied*

$$\alpha_{min} > \frac{1 + \varepsilon_1 \omega_1 e^{2\eta_1 \bar{\tau}}}{2} + \frac{\varepsilon_2 \omega_2}{2k_{min}}, \;\; \eta_1 e^{\eta_1 \bar{\tau}} > \frac{1}{\varepsilon_1}, \;\; \eta_2 > \frac{\omega_2 k_{max}}{\varepsilon_2} \tag{4–29}$$

$$\frac{\frac{k_{min}}{k_{max}} - \varepsilon_2 \omega_2}{2e^{\eta_4 \Theta} + \frac{1}{2}} > (\bar{\tau} - \underline{\tau}). \tag{4–30}$$

Proof. Let $V : \mathscr{D} \to \mathbb{R}$ be a continuously differentiable Lyapunov function, and is defined as

$$V \triangleq \frac{1}{2}\sum_{i=1}^{n} e_i^T e_i + \frac{1}{2} r^T r + \frac{\omega_1 e^{\eta_1 \bar{\tau}}}{2} e_{u1}^T e_{u1} + \frac{\omega_2}{2} e_{u2}^T e_{u2} + Q, \tag{4–31}$$

[1] Using Assumptions 4.3, 4.4, $\dot{Y}_d \theta$ is bounded by a known constant such that $\varphi \geq \dot{Y}_d \theta$.

where $Q = Q_1 + Q_2 + Q_3$. The Lyapunov function in (4–31) satisfies the following inequalities:

$$\lambda_1 \|y\|^2 \leq V(y) \leq \lambda_2 \|y\|^2 . \tag{4–32}$$

The time derivative of $\frac{1}{2}\sum_{i=1}^{n} e_i^T e_i$ can be obtained by using (2–4)-(2–7) for $i = n$, and (4–4) as

$$\sum_{i=1}^{n} e_i^T \dot{e}_i = -\sum_{i=1}^{n-2} e_i^T e_i - \|e_{n-1}\|^2 - e_n^T \alpha e_n + e_{n-1}^T e_n + e_n^T \left(r - e^{\eta_1 \bar{\tau}} e_{u1} - e_{u2}\right). \tag{4–33}$$

By using (4–5), (4–6), (4–11), (4–33), and using the time derivatives of (4–22)-(4–24), the time derivative of (4–31) can be determined as

$$\begin{aligned}
\dot{V} = & -\sum_{i=1}^{n-2} e_i^T e_i - \|e_{n-1}\|^2 - e_n^T \alpha e_n + e_{n-1}^T e_n + e_n^T \left(r - e^{\eta_1 \bar{\tau}} e_{u1} - e_{u2}\right) \\
& + r^T \left(N_d + \tilde{N} + e_n\right) - r^T k r + r^T \left(u\left(t - Y\theta\right) - u\left(t - \bar{\tau}\right)\right) \\
& - \omega_1 e_{u1}^T u\left(t - \bar{\tau}\right) - \omega_2 e_{u2}^T k r - \eta_1 \omega_1 e^{\eta_1 \bar{\tau}} e_{u1}^T e_{u1} - \eta_2 \omega_2 e_{u2}^T e_{u2} \\
& - \eta_3 e^{\eta_3 \bar{\tau}} \varepsilon_1 \omega_1 Q_1 - \frac{\varepsilon_1 \omega_1}{2} \left\|u\left(t - \bar{\tau}\right)\right\|^2 - \eta_4 e^{\eta_4 \Theta} k \left(\bar{\tau} - \underline{\tau}\right) Q_2 \\
& + e^{\eta_4 \Theta} k \left(\bar{\tau} - \underline{\tau}\right) \|r\|^2 - k \left(\bar{\tau} - \underline{\tau}\right) \left\|r\left(t - \Theta\right)\right\|^2 + \beta Y_d \tilde{\theta} \frac{d}{dt}\left(Y_d \tilde{\theta}\right) \\
& + \left(1 - \dot{Y}_d \hat{\theta} - Y_d \dot{\hat{\theta}}\right) \kappa.
\end{aligned} \tag{4–34}$$

After canceling common terms, and using (4–7)-(4–9), (4–34) can be upper bounded as

$$\begin{aligned}
\dot{V} \leq & -\sum_{i=1}^{n-2} e_i^T e_i - \|e_{n-1}\|^2 - \alpha_{min} \|e_n\|^2 + \left|e_{n-1}^T e_n\right| + e^{\eta_1 \bar{\tau}} \left|e_n^T e_{u1}\right| + \left|e_n^T e_{u2}\right| \\
& + \|r\| \left\|\tilde{N}\right\| + \|r\| \|N_d\| + \|r\| \left\|u\left(t - Y\theta\right) - u\left(t - \bar{\tau}\right)\right\| - k_{min} \|r\|^2 \\
& + \omega_1 \left|e_{u1}^T u\left(t - \bar{\tau}\right)\right| + \omega_2 k_{max} \left|e_{u2}^T r\right| - \eta_1 \omega_1 e^{\eta_1 \bar{\tau}} \|e_{u1}\|^2 - \eta_2 \omega_2 \|e_{u2}\|^2 \\
& - \eta_3 e^{\eta_3 \bar{\tau}} \varepsilon_1 \omega_1 Q_1 - \eta_4 e^{\eta_4 \Theta} k_{max} \left(\bar{\tau} - \underline{\tau}\right) Q_2 - \frac{\varepsilon_1 \omega_1}{2} \left\|u\left(t - \bar{\tau}\right)\right\|^2 \\
& + e^{\eta_4 \Theta} k_{max} \left(\bar{\tau} - \underline{\tau}\right) \|r\|^2 + \frac{d}{dt}\left(Y_d \Gamma \hat{\theta}\right) - \frac{d}{dt}\left(Y_d \hat{\theta}\right) \frac{d}{dt}\left(Y_d \Gamma \hat{\theta}\right)
\end{aligned}$$

$$- k_{max} \left(\bar{\tau} - \underline{\tau} \right) \left\| r \left(t - \Theta \right) \right\|^2 + \beta Y_d \tilde{\theta} \frac{d}{dt} \left(Y_d \tilde{\theta} \right). \tag{4–35}$$

After using Young's Inequality, (4–9), Assumption 4.4 and canceling common terms in (4–35), an upper bound can be obtained for (4–35) as

$$\begin{aligned}
\dot{V} \leq & -\sum_{i=1}^{n-2} e_i^T e_i - \frac{1}{2} \left\| e_{n-1} \right\|^2 - \omega_1 \left(\eta_1 e^{\eta_1 \bar{\tau}} - \frac{1}{\varepsilon_1} \right) \left\| e_{u1} \right\|^2 - \omega_2 \left(\eta_2 - \frac{\omega_2 k_{max}}{\varepsilon_2} \right) \left\| e_{u2} \right\|^2 \\
& - \left(\alpha_{min} - \frac{1 + \varepsilon_1 \omega_1 e^{2\eta_1 \bar{\tau}}}{2} - \frac{\varepsilon_2 \omega_2}{2 k_{min}} \right) \left\| e_n \right\|^2 + \frac{k_{max} \varepsilon_2 \omega_2}{2} \left\| r \right\|^2 \\
& - \eta_3 e^{\eta_3 \bar{\tau}} \varepsilon_1 \omega_1 Q_1 - \eta_4 e^{\eta_4 \Theta} k_{max} \left(\bar{\tau} - \underline{\tau} \right) Q_2 - k_{max} \left(\bar{\tau} - \underline{\tau} \right) \left\| r \left(t - \Theta \right) \right\|^2 \\
& + e^{\eta_4 \Theta} k_{max} \left(\bar{\tau} - \underline{\tau} \right) \left\| r \right\|^2 + \left\| r \right\| \left\| u \left(t - Y\theta \right) - u \left(t - \bar{\tau} \right) \right\| \\
& + \left\| r \right\| \left\| \tilde{N} \right\| + \left\| r \right\| \left\| N_d \right\| - k_{min} \left\| r \right\|^2 + \left(\Gamma_{max} + \beta \left(\bar{\tau} - \underline{\tau} \right) \right) \left| \frac{d}{dt} \left(Y_d \hat{\theta} \right) \right| \\
& + \beta \left(\left(\bar{\tau} - \underline{\tau} \right) \right) \left| \frac{d}{dt} \left(Y_d \theta \right) \right| - \Gamma_{min} \left(\frac{d}{dt} \left(Y_d \hat{\theta} \right) \right)^2 - \frac{\gamma}{2} \left(\bar{\tau} - \underline{\tau} \right)^2 + \frac{\gamma}{2} \left(\bar{\tau} - \underline{\tau} \right)^2.
\end{aligned} \tag{4–36}$$

The Mean Value Theorem can be used to obtain the inequality $\left\| u \left(t - Y\theta \right) - u \left(t - \bar{\tau} \right) \right\| \leq \left\| \dot{u} \left(\mathit{\Theta} \left(t, \hat{\tau} \right) \right) \right\| \left| \bar{\tau} - Y\theta \right|$, where $\mathit{\Theta} \left(t, \hat{\tau} \right)$ is a point in time between $t - \bar{\tau}$ and $t - Y\theta$. After completing the squares for the cross terms containing r and $\frac{d}{dt} \left(Y_d \hat{\theta} \right)$, using Assumption 4.4, expressions defined in (4–9), (4–17)-(4–18), the following upper bound can be obtained for (4–36) as

$$\begin{aligned}
\dot{V} \leq & -\sum_{i=1}^{n-2} e_i^T e_i - \frac{1}{2} \left\| e_{n-1} \right\|^2 - \left(\alpha_{min} - \frac{1 + \varepsilon_1 \omega_1 e^{2\eta_1 \bar{\tau}}}{2} - \frac{\varepsilon_2 \omega_2}{2 k_{min}} \right) \left\| e_n \right\|^2 \\
& - \left(\frac{k_{min}}{2} - k_{max} \left(\frac{\varepsilon_2 \omega_2}{2} - \left(\bar{\tau} - \underline{\tau} \right) \left(e^{\eta_4 \Theta} + \frac{1}{4} \right) \right) \right) \left\| r \right\|^2 + \frac{\rho^2 \left(\left\| z \right\| \right) \left\| z \right\|^2}{k_{min}} \\
& - \omega_1 \left(\eta_1 e^{\eta_1 \bar{\tau}} - \frac{1}{\varepsilon_1} \right) \left\| e_{u1} \right\|^2 - \omega_2 \left(\eta_2 - \frac{\omega_2 k_{max}}{\varepsilon_2} \right) \left\| e_{u2} \right\|^2 + \frac{\Phi^2}{k_{min}} \\
& - \eta_3 e^{\eta_3 \bar{\tau}} \varepsilon_1 \omega_1 Q_1 - \eta_4 e^{\eta_4 \Theta} k_{max} \left(\bar{\tau} - \underline{\tau} \right) Q_2 + \frac{\gamma}{2} \left(\bar{\tau} - \underline{\tau} \right)^2 \\
& + \frac{\left(\Gamma_{max} + \beta \left(\bar{\tau} - \underline{\tau} \right) \right)^2}{4 \Gamma_{min}} + \beta \left(\left(\bar{\tau} - \underline{\tau} \right) \right) \left| \frac{d}{dt} \left(Y_d \theta \right) \right| - \frac{\gamma}{2} \left(Y_d \tilde{\theta} \right)^2.
\end{aligned} \tag{4–37}$$

Using Assumptions 4.3, 4.4, the definition of Q_3 in (4–24), the definition σ in (4–20), the gain conditions in (4–29)-(4–30), and the inequality $\|y\| \geq \|z\|$, the following upper bound can be obtained

$$\dot{V} \leq -\left(\frac{\sigma}{2} - \frac{1}{k_{min}}\rho^2(\|y\|)\right)\|z\|^2 - \frac{\sigma}{2}\|z\|^2 - \eta_3 e^{\eta_3\bar{\tau}}\varepsilon_1\omega_1 Q_1$$
$$- \eta_4 e^{\eta_4\Theta}k_{max}\left(\bar{\tau} - \underline{\tau}\right)Q_2 - \frac{\gamma}{\beta}Q_3 + \acute{\Phi}. \tag{4–38}$$

Provided $y \in \mathscr{D}$, then by using the definition of δ in (4–21), (4–38) can be upper bounded as

$$\dot{V} \leq -\delta\|y\|^2 + \acute{\Phi}. \tag{4–39}$$

Using (4–32) and (4–39), an upper bound can be obtained for (4–39) as

$$\dot{V} \leq -\frac{\delta}{\lambda_2}V + \acute{\Phi}. \tag{4–40}$$

The solution of the differential inequality in (4–40) can be obtained as

$$V(t) \leq \Psi, \tag{4–41}$$

where

$$\Psi \triangleq \left(V\left(t_0\right) - \frac{\lambda_2\acute{\Phi}}{\delta}\right)\exp\left(-\frac{\delta}{\lambda_2}\left(t - t_0\right)\right) + \frac{\lambda_2\acute{\Phi}}{\delta}.$$

Using (4–31) and (4–41), the following upper bounds can be obtained for $e_i, \;\; i = 1, 2, 3, \ldots, n$, and r, e_{u1}, e_{u2} as

$$\|e_i\| \leq \sqrt{2\Psi}, \; \|r\| \leq \sqrt{2\Psi}, \; \|e_{u1}\| \leq \sqrt{\frac{2\Psi}{\omega_1 e^{\eta_1\bar{\tau}}}}, \; \|e_{u2}\| \leq \sqrt{\frac{2\Psi}{\omega_2}}.$$

□

4.4 Conclusion

A robust tracking controller is designed to compensate for the disturbance of a state-dependent input delay for uncertain nonlinear systems with additive disturbances. To overcome of requirement of exact knowledge of input delay dynamics, an estimation of the input delay is obtained by using an adaptive estimation strategy. A novel tracking error signal is designed to obtain a delay-free control signal in the closed-loop dynamics. A Lyapunov-based stability analysis is used to prove semi-global uniformly ultimately boundedness of error signals by using novel Lyapunov-Krasovskii functionals.

CHAPTER 5
ROBUST NEUROMUSCULAR ELECTRICAL STIMULATION CONTROL FOR UNKNOWN TIME-VARYING INPUT DELAYED MUSCLE DYNAMICS: POSITION TRACKING CONTROL

In this chapter, a control method is developed to yield lower limb tracking with NMES, despite an unknown time-varying input delay intrinsic to NMES, uncertain nonlinear dynamics, and additive bounded disturbances. The designed error signal not only injects a delay-free control signal in the closed-loop dynamics, but also overcomes the requirement of exact input delay measurements. In this chapter the maximum allowable mismatch between the actual input delay and the estimated input delay is obtained to guarantee stability of the dynamics. Since the approximate interval of the time-varying input delay can be experimentally obtained for NMES, the estimated delay can be selected for minimization of the mismatch between the actual input delay and the estimated input delay during NMES. A Lyapunov-based stability analysis is used to prove uniformly ultimately boundedness of tracking error signals. Experiments were conducted in 10 able-bodied individuals to examine the performance of the developed controller.

5.1 Knee Joint Dynamics

The knee-joint dynamics are modeled in [83] as

$$M_I\left(\ddot{q}\right)+M_g\left(q\right)+M_e\left(q\right)+M_v\left(\dot{q}\right)+\bar{d}=\mu, \tag{5–1}$$

where $M_I:\mathbb{R}\to\mathbb{R}$ denotes the inertial effects of the shank-foot complex about the knee-joint, $M_g:\mathbb{R}\to\mathbb{R}$ denotes the gravitational component, $M_e:\mathbb{R}\to\mathbb{R}$ denotes the elastic effects depending on joint stiffness, $M_v:\mathbb{R}\to\mathbb{R}$, denotes the viscous effects due to the damping in the musculotendon complex, $\bar{d}:[t_0,\infty)\to\mathbb{R}$ is a sufficiently smooth unknown time-varying exogenous disturbance (e.g., unmodeled dynamics) where $t_0\in\mathbb{R}$ is the initial time, and $\mu:\mathbb{R}\to\mathbb{R}$ symbolizes the torque that is produced at the knee-joint as a result of electrical stimulation. The inertial and gravitational effects are modeled for the knee-joint dynamics in (5–1) as

$$M_I(\ddot{q}) \triangleq J\ddot{q}, \quad M_g(q) \triangleq mgl\sin(q), \tag{5–2}$$

where $q,\ \dot{q},\ \ddot{q} \in \mathbb{R}$ symbolize the angular position, velocity, and acceleration of the shank about the knee-joint, respectively. The inertia of shank and foot symbolizes by J, m is the combined mass of the shank and foot, g is the gravitational acceleration, and l is the distance between the knee-joint and the lumped center of mass of the shank and foot, where $J,\ m,\ g,\ l \in \mathbb{R}$ are uncertain positive constants. The elastic effect is modeled for the knee-joint dynamics as

$$M_e(q) \triangleq k_1\exp(-k_2 q)(q - k_3), \tag{5–3}$$

where $k_1,\ k_2,\ k_3 \in \mathbb{R}$ are uncertain positive constants. The viscous effect is modeled for the knee-joint dynamics as

$$M_v(\dot{q}) \triangleq -B_1\tanh(-B_2\dot{q}) + B_3\dot{q}, \tag{5–4}$$

where $B_1,\ B_2,\ B_3\ \in\ \mathbb{R}$ are uncertain positive constants. The subsequent analysis assumes q and $\dot{q}$ are measurable outputs.

Electrical stimulation of the quadriceps produces the sum of the muscle forces generated by contractile, elastic and viscous elements [83]. The produced muscle forces yield a torque about the knee. The generated torque can be controlled through muscle forces to track a desired position of the limb. The resulting torque about the knee is delayed since the force development in the muscle is delayed, due to the finite propagation time of chemical ions and action potential along the T-tubule system and the stretching of the elastic components in the muscle [69, 101, 110]. The muscle force produced by the delayed implemented input signal, $F \in \mathbb{R}$ is defined as

$$F \triangleq \Omega\xi(q, \dot{q})u_\tau, \tag{5–5}$$

where $\Omega : [t_0, \infty) \to \mathbb{R}$ symbolizes an unknown positive time-varying function exhibiting fatigue and potentiation, $\xi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a sufficiently smooth, unknown nonlinear function that depends on the knee-joint angle and angular velocity [83], and $u_\tau \in \mathbb{R}$ is the delayed voltage potential across the quadriceps muscle, where $\tau : [t_0, \infty) \to \mathbb{R}$ denotes the electromechanical delay which is the delay between the application of voltage and the onset of force production at the knee-joint [79, 111] during electrical stimulation.[1] The knee torque is defined as

$$\mu \triangleq \zeta(q)F, \tag{5–6}$$

where $\zeta : \mathbb{R} \to \mathbb{R}$ is a positive moment arm that changes with the extension and flexion of the leg [112, 113]. Figure 5-1 depicts the dynamics of the knee-joint and the generation of muscle force due to the voltage potential across the quadriceps muscle group.

The subsequent control development is based on the following assumptions.

Assumption 5.1. The moment arm ζ and the functions ξ, Ω are assumed to be positive and bounded along their first and second time derivatives [112–115].

Assumption 5.2. The nonlinear additive disturbance and its first and second time derivatives exist and are bounded by known positive constants [98].

Assumption 5.3. The reference trajectory $q_r \in \mathbb{R}$ is designed such that $q_r,\ \dot{q}_r,\ \ddot{q}_r$ exist and are bounded by known positive constants.

Assumption 5.4. The mismatch between the actual input delay $\tau(t)$ and the constant estimated input delay $\hat{\tau} \in \mathbb{R}$ is bounded by a known constant $\bar{\tilde{\tau}} \in \mathbb{R}$ such that $\sup_{t \in \mathbb{R}} |\tau - \hat{\tau}| \leq \bar{\tilde{\tau}}$.

[1] Measurement of electromechanical delay is challenging due to the many factors that cause it, such as fatigue, different types of muscle contractions, the finite propagation time of chemical ions, etc.; due to the measurement difficulty, the delay is assumed to be unknown and time-varying.

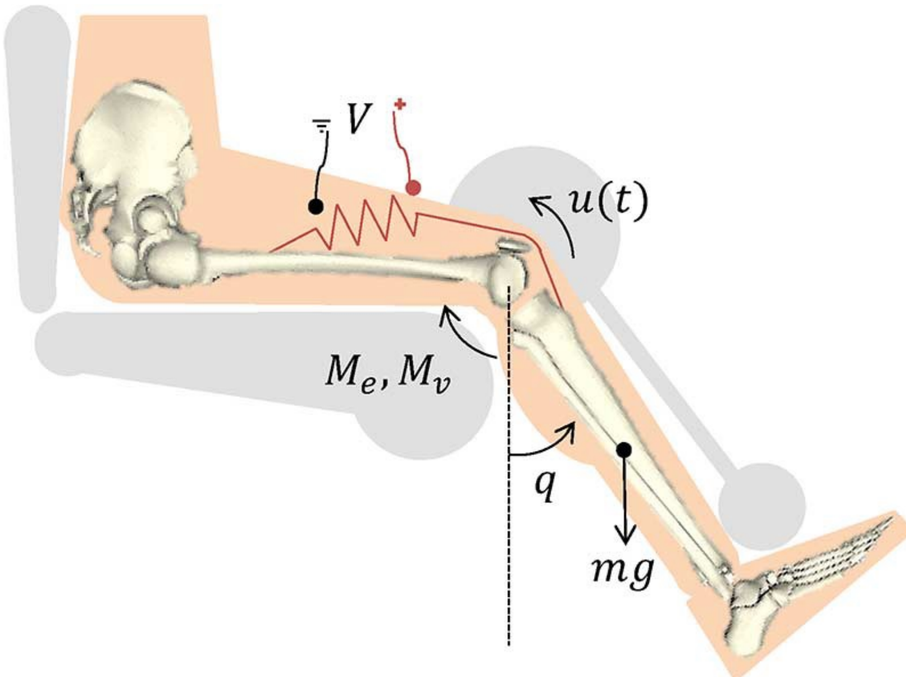

Figure 5-1. Schematic [1] of the knee-joint dynamics and the torque production about the knee caused by the voltage potential V applied to the of the quadriceps muscle group.

The dynamics of knee-joint can be simplified by using the expressions in (5–1)-(5–6) as

$$M(q,\dot{q})\,\ddot{q}+f(q,\dot{q})+d=u_{\tau}, \tag{5–7}$$

where $M:\mathbb{R}\times\mathbb{R}\to\mathbb{R}\triangleq\frac{J}{\Omega\xi\zeta}$, $f:\mathbb{R}\times\mathbb{R}\to\mathbb{R}\triangleq\frac{M_e+M_g+M_v}{\Omega\xi\zeta}$, and $d\triangleq\frac{\bar{d}}{\Omega\xi\zeta}$. Using Assumption 5.1, M can be bounded as

$$\underline{m}\leq|M(x_1,x_2)|\leq\overline{m} \tag{5–8}$$

for all $x_1,x_2\in\mathbb{R}$, where $\underline{m},\overline{m}\in\mathbb{R}$ are known positive constants.

5.2 Control Development

The goal is to enable the knee-joint angle $q(t)$ to track a given reference trajectory $q_r(t)$ despite an unknown time-varying input delay and uncertainties in the dynamic model subjected to additive bounded disturbances. To facilitate the subsequent analysis, a measurable auxiliary tracking error, denoted by $e_1 \in \mathbb{R}$, is defined as[2]

$$e_1 \triangleq \int_{t_0}^{t} \left(q_r\left(\theta\right) - q\left(\theta\right)\right) d\theta. \tag{5–9}$$

To facilitate the subsequent analysis, an auxiliary tracking error, denoted by $e_2 \in \mathbb{R}$, is defined as

$$e_2 \triangleq \dot{e}_1 + \alpha e_1, \tag{5–10}$$

where $\alpha \in \mathbb{R}$ is a selectable, positive, constant control gain. To further facilitate the subsequent analysis, a measurable auxiliary tracking error, denoted by $r \in \mathbb{R}$, is defined as

$$r \triangleq \dot{e}_2 + \beta e_2 + \eta e_u, \tag{5–11}$$

where $\beta, \eta \in \mathbb{R}$ are known, selectable, positive, constant control gains. To integrate a delay-free input term in the closed-loop error system, the same tracking error signal e_u, defined in (3–4), is used. By multiplying the time derivative of (5–11) by M and using (5–7), (5–9), (5–10), and (3–4), the open-loop dynamics for r can be obtained as

$$\begin{aligned} M\left(q,\dot{q}\right)\dot{r} =& M\left(q,\dot{q}\right)\ddot{q}_r + f\left(q,\dot{q}\right) + d + M\left(q,\dot{q}\right)\left(\alpha+\beta\right)\dot{e}_2 - M\left(q,\dot{q}\right)\alpha^2\dot{e}_1 \\ &+ u_{\hat{\tau}} - u_{\tau} + \left(M\left(q,\dot{q}\right)\eta - 1\right)u_{\hat{\tau}} - M\left(q,\dot{q}\right)\eta u. \end{aligned} \tag{5–12}$$

Based on subsequent stability analysis, the following continuous robust controller is designed as

$$u = k_c r, \tag{5–13}$$

[2] The control objective can be quantified in terms of the first time derivative of e_1.

where $k_c \in \mathbb{R}$ is a positive, constant, selectable control gain. Substituting (5–13) into (5–12), the closed-loop error system for r can be obtained as

$$M(q,\dot{q})\dot{r} = M(q,\dot{q})\ddot{q}_r + f(q,\dot{q}) + d - M(q,\dot{q})\alpha^2\dot{e}_1 + M(q,\dot{q})(\alpha+\beta)\dot{e}_2$$
$$+ u_{\hat{\tau}} - u_\tau + (M(q,\dot{q})\eta - 1)k_c r_{\hat{\tau}} - M(q,\dot{q})\eta k_c r. \tag{5–14}$$

The closed-loop error system for r in (5–14) can be segregated by terms that can be upper bounded by a constant and terms that can be upper bounded by a state-dependent function such that

$$M(q,\dot{q})\dot{r} = \tilde{N} + N_r - \frac{1}{2}\dot{M}(q,\dot{q},\ddot{q})r - e_2 + u_{\hat{\tau}} - u_\tau$$
$$+ (M(q,\dot{q})\eta - 1)k_c r_{\hat{\tau}} - M(q,\dot{q})\eta k_c r, \tag{5–15}$$

where the auxiliary terms $\tilde{N}, N_r \in \mathbb{R}$ are defined as

$$\tilde{N} \triangleq (M(q,\dot{q}) - M(q_r,\dot{q}_r))\ddot{q}_r + f(q,\dot{q}) - f(q_r,\dot{q}_r) + M(q,\dot{q})(\alpha+\beta)\dot{e}_2$$
$$- M(q,\dot{q})\alpha^2\dot{e}_1 + \frac{1}{2}\dot{M}(q,\dot{q},\ddot{q})r + e_2, \tag{5–16}$$

and

$$N_r \triangleq d + M(q_r,\dot{q}_r)\ddot{q}_r + f(q_r,\dot{q}_r). \tag{5–17}$$

Remark 5.1. An upper bound can be obtained for N_r based on Assumptions 5.1, 5.2, and 5.3 as

$$\sup_{t\in\mathbb{R}} |N_r| \leq \Phi, \tag{5–18}$$

where $\Phi \in \mathbb{R}$ is a known positive constant.

Remark 5.2. $\tilde{N}$ is upper bounded by a state-dependent function. Using Lemma 5 in [105] and Assumption 5.1, an upper bound for (5–16) can be obtained as

$$\left|\tilde{N}\right| \leq \rho\left(\|z\|\right)\|z\|, \tag{5–19}$$

where ρ is a positive, radially unbounded, and strictly increasing function and $z \in \mathbb{R}^4$ is a vector of error signals defined as

$$z \triangleq \begin{bmatrix} e_1 & e_2 & r & e_u \end{bmatrix}^T. \tag{5–20}$$

To facilitate the subsequent stability analysis, auxiliary bounding positive constants[3] $\sigma\,, \delta \in \mathbb{R}$ are defined as

$$\sigma \triangleq \min\left\{\left(\alpha - \frac{1}{2\varepsilon_1}\right), \left(\beta - \frac{\varepsilon_1}{2} - \frac{\varepsilon_2\eta^2}{2}\right), \frac{1}{3}\underline{m}\eta k_c, \left(\frac{\omega_2}{3\hat{\tau}k_c^2} - \frac{1}{\varepsilon_2}\right)\right\} \tag{5–21}$$

$$\delta \triangleq \min\left\{\frac{\sigma}{2}, \frac{\omega_2}{3\left(\varepsilon_2\left(\omega_1 k_c\right)^2 + \frac{\left(3k_c\left|\bar{m}\eta - 1\right|\right)^2}{4\underline{m}\eta k_c}\right)}, \frac{1}{3\left(\bar{\tilde{\tau}} + \hat{\tau}\right)}\right\}, \tag{5–22}$$

where $\omega_1, \omega_2, \varepsilon_1, \varepsilon_2, \varepsilon_3 \in \mathbb{R}$ are known, selectable, positive constants. Let the functions $Q_1,\ Q_2 : \mathbb{R} \to \mathbb{R}$ be defined as

$$Q_1 \triangleq \left(\varepsilon_2\left(\omega_1 k_c\right)^2 + \frac{\left(3k_c\left|\bar{m}\eta - 1\right|\right)^2}{4\underline{m}\eta k_c}\right)\int_{t-\hat{\tau}}^{t}\left|r\left(\theta\right)\right|^2 d\theta, \tag{5–23}$$

$$Q_2 \triangleq \omega_2 \int_{t-\left(\bar{\tilde{\tau}}+\hat{\tau}\right)}^{t}\int_{s}^{t}\left|r\left(\theta\right)\right|^2 d\theta ds, \tag{5–24}$$

and let $y \in \mathbb{R}^6$ be defined as

[3] σ and δ will be used as bounding constants (convergence decay rate and definition of the domain of attraction) in Section 5.3.

$$y \triangleq \left[\begin{array}{c} z^T, \sqrt{Q_1}, \sqrt{Q_2} \end{array}\right]^T. \tag{5–25}$$

For use in the following stability analysis, let

$$\mathscr{D}_1 \triangleq \left\{ y \in \mathbb{R}^6 | \, \|y\| < \chi \right\}, \tag{5–26}$$

where $\chi \triangleq \inf \left\{ \rho^{-1} \left((\sqrt{\frac{2\sigma \underline{m} \eta k_c}{9}}, \infty) \right) \right\}$. Provided $\|z(\Sigma)\| < \gamma, \; \forall \Sigma \in [t_0, t]$, (5–15) and the fact that $u = k_c r$ can be used to conclude that $\dot{u} < \Upsilon$, where γ and Υ are positive constants. Let $\mathscr{D} \triangleq \mathscr{D}_1 \cap (B_\gamma \cap \mathbb{R}^6)$ where B_γ denotes a closed ball of radius γ centered at the origin and let

$$\mathcal{S}_{\mathscr{D}} \triangleq \left\{ y \in \mathscr{D} | \, \|y\| < \sqrt{\frac{\lambda_1}{\lambda_2}} \chi \right\}, \tag{5–27}$$

denote the set of stabilizing initial conditions, where $\lambda_1 \triangleq \min\{\frac{1}{2}, \frac{\underline{m}}{2}, \frac{\omega_1}{2}\}$ and $\lambda_2 \triangleq \max\{1, \frac{\bar{m}}{2}, \frac{\omega_1}{2}\}$.

5.3 Stability Analysis

Theorem 5.1. *Given the dynamics in (1), the controller given in (5–13) ensures uniformly ultimately bounded tracking in the sense that*

$$|q_r - q| \leq \epsilon_0 \exp\left(-\epsilon_1 (t - t_0)\right) + \epsilon_2, \tag{5–28}$$

where $\epsilon_0 \triangleq (1+\alpha) \sqrt{\left(2V(t_0) - \frac{2\lambda_2 \upsilon}{\delta}\right)}$, $\epsilon_1 \triangleq -\frac{\delta}{2\lambda_2}$ *and* $\epsilon_2 \triangleq (1+\alpha)\sqrt{\frac{2\lambda_2 \upsilon}{\delta}}$ *with* $\upsilon \triangleq \frac{18\varepsilon_3 \Phi^2 + 4\underline{m}\eta k_c \bar{\bar{\tau}}^2 \Upsilon^2}{8\underline{m}\eta k_c \varepsilon_3}$, *provided* $y(t_0) \in \mathcal{S}_{\mathscr{D}}$ *and the control gains are selected sufficiently large relative to the initial conditions of the system such that the following sufficient conditions are satisfied*[4]

[4] The sufficient gain conditions can be met provided $\hat{\tau}$,$\bar{\bar{\tau}}$ are small enough.

$$\alpha > \frac{1}{2\varepsilon_1}, \quad \beta > \frac{\varepsilon_{1+}\varepsilon_2\eta^2}{2}, \quad \omega_2 > \frac{3\hat{\tau}k_c^2}{\varepsilon_2},$$
$$\frac{\frac{1}{3}\underline{m}\eta k_c - \frac{\varepsilon_3}{2} - 2\varepsilon_2\left(\omega_1 k_c\right)^2 - \frac{\left(3k_c\left|\bar{m}\eta-1\right|\right)^2}{4\underline{m}\eta k_c} - \omega_2\hat{\tau}}{\omega_2} \geq \bar{\bar{\tau}}. \tag{5–29}$$

Proof. Let $V : \mathscr{D} \to \mathbb{R}$ be a Lyapunov function candidate defined as

$$V \triangleq \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}Mr^2 + \frac{\omega_1}{2}e_u^2 + Q_1 + Q_2. \tag{5–30}$$

The Lyapunov function candidate (5–30) can be bounded as

$$\lambda_1 \left\|y\right\|^2 \leq V\left(y\right) \leq \lambda_2 \left\|y\right\|^2. \tag{5–31}$$

Using (3–4), (5–9)-(5–11), (5–15), and by applying the Leibniz Rule for (5–23)-(5–24), the time derivative of (5–30) can be determined

$$\begin{aligned}\dot{V} &= e_1\left(e_2 - \alpha e_1\right) + e_2\left(r - \beta e_2 - \eta e_u\right) + r\left(\tilde{N} + N_r - \frac{1}{2}\dot{M}r - e_2 - M\eta k_c r\right)\\
&+ \frac{1}{2}\dot{M}r^2 + r\left(\left(M\eta - 1\right)k_c r_{\hat{\tau}} + \left(u_{\hat{\tau}} - u_{\tau}\right)\right) + \omega_1 e_u\left(k_c r_{\hat{\tau}} - k_c r\right)\\
&+ \left(\varepsilon_2\left(\omega_1 k_c\right)^2 + \frac{\left(3k_c\left|\bar{m}\eta - 1\right|\right)^2}{4\underline{m}\eta k_c}\right)\left(r^2 - r_{\hat{\tau}}^2\right)\\
&+ \omega_2\left(\left(\bar{\bar{\tau}} + \hat{\tau}\right)r^2 - \int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t} r\left(\theta\right)^2 d\theta\right).\end{aligned} \tag{5–32}$$

By using (5–18) and (5–19) and canceling common terms in (5–32), an upper bound for (5–32) can be obtained as

$$\begin{aligned}\dot{V} &\leq \left|e_1 e_2\right| - \alpha e_1^2 - \beta e_2^2 + \eta\left|e_2 e_u\right| + \left|r\right|\rho\left(\left\|z\right\|\right)\left\|z\right\| + \left|r\right|\Phi\\
&- M\eta k_c r^2 + k_c\left(M\eta - 1\right)r r_{\hat{\tau}} + \left|r\left(u_{\hat{\tau}} - u_{\tau}\right)\right| + \omega_1 k_c\left(\left|e_u r_{\hat{\tau}}\right| + \left|e_u r\right|\right)\end{aligned}$$

$$+\left(\varepsilon_2\left(\omega_1 k_c\right)^2+\frac{\left(3k_c\left|\bar{m}\eta-1\right|\right)^2}{4\underline{m}\eta k_c}\right)\left(r^2-r_{\hat{\tau}}^2\right)$$

$$+\,\omega_2\left(\left(\bar{\bar{\tau}}+\hat{\tau}\right)r^2-\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t} r\left(\theta\right)^2 d\theta\right). \tag{5–33}$$

Using Young's Inequality, the following inequalities can be obtained

$$\left|e_1 e_2\right| \leq \frac{1}{2\varepsilon_1}e_1^2+\frac{\varepsilon_1}{2}e_2^2, \tag{5–34}$$

$$\left|e_2 e_u\right| \leq \frac{1}{2\varepsilon_2\eta}e_u^2+\frac{\varepsilon_2\eta}{2}e_2^2, \tag{5–35}$$

$$\left|r\left(u_{\hat{\tau}}-u_{\tau}\right)\right| \leq \frac{\varepsilon_3}{2}r^2+\frac{1}{2\varepsilon_3}\left|u_{\hat{\tau}}-u_{\tau}\right|^2. \tag{5–36}$$

After completing the squares for r, e_u and $r_{\hat{\tau}}$ and substituting (5–13), (5–34)-(5–36) into (5–33), the following upper bound can be obtained

$$\dot{V} \leq -\left(\alpha-\frac{1}{2\varepsilon_1}\right)e_1^2-\left(\beta-\frac{\varepsilon_1}{2}-\frac{\varepsilon_2\eta^2}{2}\right)e_2^2-\frac{1}{3}\underline{m}\eta k_c r^2+\frac{1}{\varepsilon_2}e_u^2+\frac{9}{4\underline{m}\eta k_c}\rho^2\left(\|z\|\right)\|z\|^2$$

$$+\frac{9}{4\underline{m}\eta k_c}\Phi^2-\left(\frac{1}{3}\underline{m}\eta k_c-\kappa\right)r^2+\frac{1}{2\varepsilon_3}\left|u_{\hat{\tau}}-u_{\tau}\right|^2-\omega_2\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t} r\left(\theta\right)^2 d\theta, \tag{5–37}$$

where $\kappa \triangleq \frac{\varepsilon_3}{2}+2\varepsilon_2\left(\omega_1 k_c\right)^2+\omega_2\left(\bar{\bar{\tau}}+\hat{\tau}\right)+\frac{\left(3k_c\left|\bar{m}\eta-1\right|\right)^2}{4\underline{m}\eta k_c}$. The Cauchy-Schwarz inequality is used to develop an upper bound for e_u^2 such that $e_u^2 \leq \hat{\tau}\int\limits_{t-\hat{\tau}}^{t} u(\theta)^2 d\theta$. Additionally, an upper bound for Q_2 can be obtained as $Q_2 \leq \omega_2\left(\bar{\bar{\tau}}+\hat{\tau}\right)\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t} r\left(\theta\right)^2 d\theta$. Furthermore, by using Assumption 5.4, $\int\limits_{t-\hat{\tau}}^{t} r\left(\theta\right)^2 d\theta \leq \int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t} r\left(\theta\right)^2 d\theta$ can be obtained. By using these developed upper bounds for $e_u^2,\ Q_1,\ Q_2$, the following upper bound can be developed

$$\dot{V} \leq -\left(\alpha-\frac{1}{2\varepsilon_1}\right)e_1^2-\left(\beta-\frac{\varepsilon_1}{2}-\frac{\varepsilon_2\eta^2}{2}\right)e_2^2-\frac{1}{3}\underline{m}\eta k_c r^2-\left(\frac{\omega_2}{3\hat{\tau}k_c^2}-\frac{1}{\varepsilon_2}\right)e_u^2$$

$$+\frac{9}{4\underline{m}\eta k_c}\rho^2\left(\|z\|\right)\|z\|^2+\frac{9}{4\underline{m}\eta k_c}\Phi^2+\frac{1}{2\varepsilon_3}\left|u_{\hat{\tau}}-u_\tau\right|^2-\left(\frac{1}{3}\underline{m}\eta k_c-\kappa\right)r^2$$
$$-\frac{\omega_2}{3\left(\varepsilon_2\left(\omega_1 k_c\right)^2+\frac{\left(3k_c\left|\bar{m}\eta-1\right|\right)^2}{4\underline{m}\eta k_c}\right)}Q_1-\frac{1}{3\left(\bar{\bar{\tau}}+\hat{\tau}\right)}Q_2. \tag{5–38}$$

Using the definition σ in (5–21), the gain conditions in (5–29), the inequality $\|y\|\geq\|z\|$, and the Mean Value Theorem $|u_{\hat{\tau}}-u_\tau|\leq|\dot{u}\left(\Theta\left(t,\hat{\tau}\right)\right)||\tilde{\tau}|$, where $\Theta\left(t,\hat{\tau}\right)$ is a point between $t-\tau$ and $t-\hat{\tau}$, the following upper bound can be obtained

$$\dot{V}\leq-\left(\frac{\sigma}{2}-\frac{9}{4\underline{m}\eta k_c}\rho^2(\|y\|)\right)\|z\|^2-\frac{\sigma}{2}\|z\|^2+\frac{9}{4\underline{m}\eta k_c}\Phi^2+\frac{\bar{\tilde{\tau}}^2\left|\dot{u}\left(\Theta\left(t,\hat{\tau}\right)\right)\right|^2}{2\varepsilon_3}$$
$$-\frac{\omega_2}{3\left(\varepsilon_2\left(\omega_1 k_c\right)^2+\frac{\left(3k_c\left|\bar{m}\eta-1\right|\right)^2}{4\underline{m}\eta k_c}\right)}Q_1-\frac{1}{3\left(\bar{\bar{\tau}}+\hat{\tau}\right)}Q_2. \tag{5–39}$$

Provided $y\in\mathscr{D}$, then by using the definition of δ in (5–22), the expression in (5–39) reduces to

$$\dot{V}\leq-\delta\|y\|^2+\upsilon. \tag{5–40}$$

Using (5–31) and (5–40), an upper bound can be obtained for (5–40) as

$$\dot{V}\leq-\frac{\delta}{\lambda_2}V+\upsilon, \tag{5–41}$$

and hence,

$$|e_1|\leq\sqrt{2\left(V\left(t_0\right)-\frac{\lambda_2\upsilon}{\delta}\right)\exp\left(-\frac{\delta}{\lambda_2}\left(t-t_0\right)\right)+\frac{\lambda_2\upsilon}{\delta}},$$
$$|e_2|\leq\sqrt{2\left(V\left(t_0\right)-\frac{\lambda_2\upsilon}{\delta}\right)\exp\left(-\frac{\delta}{\lambda_2}\left(t-t_0\right)\right)+\frac{\lambda_2\upsilon}{\delta}},$$
$$|r|\leq\sqrt{2\left(\frac{V\left(t_0\right)}{\overline{m}}-\frac{\lambda_2\upsilon}{\delta\overline{m}}\right)\exp\left(-\frac{\delta}{\lambda_2}\left(t-t_0\right)\right)+\frac{2\lambda_2\upsilon}{\delta\overline{m}}},$$

$$|e_u| \leq \sqrt{2\left(\frac{V(t_0)}{\omega_1} - \frac{\lambda_2 \upsilon}{\delta\omega_1}\right)\exp\left(-\frac{\delta}{\lambda_2}(t-t_0)\right) + \frac{2\lambda_2\upsilon}{\delta\omega_1}}.$$

An upper bound can be obtained for $|q_r - q|$ by using the time derivative of (5–9), (5–10) and the inequality $\sqrt{\left(V(t_0) - \frac{2\lambda\upsilon}{\delta}\right)\exp\left(-\frac{\delta}{\lambda_2}(t-t_0)\right) + \frac{\lambda_2\upsilon}{\delta}} \leq \sqrt{\left(V(t_0) - \frac{\lambda_2\upsilon}{\delta}\right)\exp\left(-\frac{\delta}{\lambda_2}(t-t_0)\right)} + \sqrt{\frac{\lambda_2\upsilon}{\delta}}$ as

$$|q_r - q| \leq (1+\alpha)\left(\sqrt{2V(t_0) - \frac{2\lambda_2\upsilon}{\delta}}\exp\left(-\frac{\delta}{2\lambda_2}(t-t_0)\right) + \sqrt{\frac{2\lambda_2\upsilon}{\delta}}\right).$$

It can be concluded that y is uniformly ultimately bounded in the sense that $|e(t)| \leq \epsilon_0 \exp(-\epsilon_1(t-t_0)) + \epsilon_2$, provided $y(t_0) \in \mathcal{S}_{\mathscr{D}}$, where uniformity in initial time can be concluded from the independence of δ and the ultimate bound from t_0.Since $e_1, e_2, r, e_u \in \mathcal{L}_\infty$, from (5–13), $u \in \mathcal{L}_\infty$ and the remaining signals are bounded. □

5.4 Experiments

The performance of the developed controller in (5–13) was examined through a series of experiments. Surface electrical stimulation was applied to the quadriceps muscle group to elicit contractions during knee-joint angle tracking trials. For all trials, the control algorithm in (5–13) was used to vary the current amplitude in real time, while the pulsewidth (PW) and stimulation frequency were set to constant values.

5.4.1 Subjects

Ten able-bodied individuals (9 male, 1 female, aged 21-31) participated in the experiments. Prior to participation, written informed consent was obtained from all participants, as approved by the institutional review board at the University of Florida.

5.4.2 Apparatus

The experimental test bed, shown in Figure 5-2, consisted of the following: 1) a modified leg extension machine equipped with orthotic boots to fix the ankle and securely fasten the shank and the foot, 2) optical encoders (BEI technologies) to measure the leg angle (i.e., the angle between the vertical and the shank), 3) a current-controlled 8-channel stimulator (RehaStim, Hasomed GmbH), 4) a data acquisition board (Quanser QPIDe), 5) a personal computer running Matlab/Simulink, and 6) a pair of 3" by 5" Valutrode® surface electrodes placed proximally and distally over the quadriceps muscle group.[5]

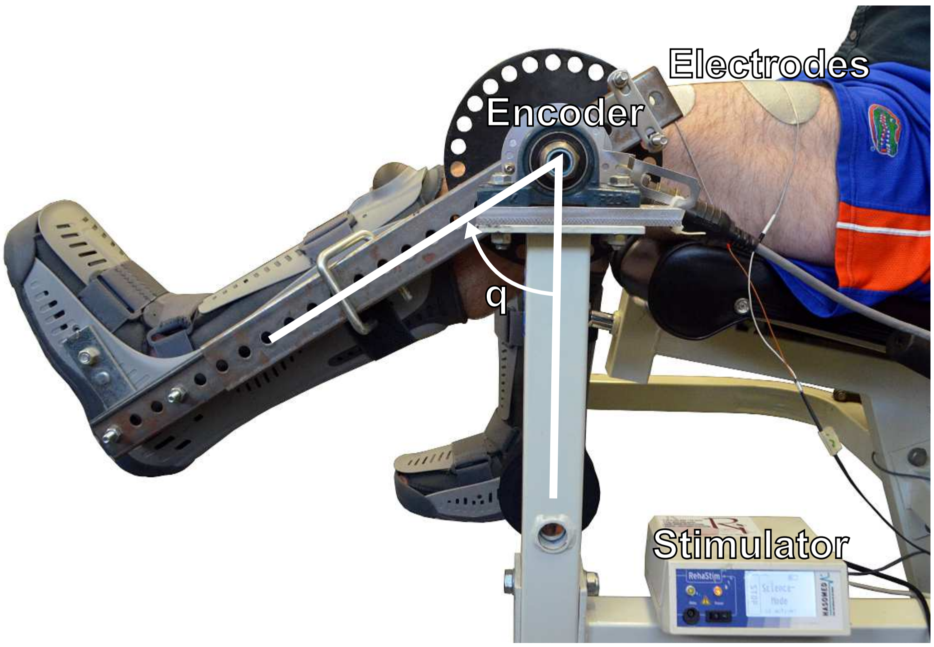

Figure 5-2. A modified leg extension machine was fitted with optical encoders to measure the knee-joint angle and provide feedback to the developed control algorithm running on a personal computer.

5.4.3 Dynamic Trials

During the experiments, electrical pulses were delivered at a constant stimulation frequency of 35 Hz and the pulsewidth was fixed to a constant value that depended on the individual.[6] The control gains were adjusted during pretrial tests to achieve

[5] Surface electrodes for the study were provided compliments of Axelgaard Manufacturing Co., Ltd.

[6] Different responses to stimulation were obtained when testing across participants (i.e., greater or weaker muscle force was produced for a nominal stimulation

trajectory tracking where the desired angular trajectory of the knee joint was selected as a sinusoid with a range of 5° to 50° and a period of 2 seconds. The constant estimate of the time varying delay was selected as $\hat{\tau} \in [0.085, 0.1]$ seconds based on the results reported in previous NMES studies [89]. After determining suitable gains in the pretrial tests, the tracking trial was run for one of the lower limbs for a testing duration of 45 seconds. The control performance was evaluated by calculating the root-mean-square (RMS) tracking error over the entire trial. The experiment duration was reduced if before reaching the 45 seconds limit the RMS error increased by 3°after the steady state baseline RMS error was established.[7] Then the process was repeated for the other lower limb starting with the pretrial tests. Table 5-1 presents the mean RMS error over the entire experiment duration and the pulsewidth used in all the tracking trials. Table 5-2 presents the gains from Table I in an equivalent percentage form. The pulsewidth was selected for each leg during the pretrial tests due to the difference in muscle response to stimulation depending on asymmetries between lower limbs due to muscle mass, pain threshold, and past history of musculoskeletal injuries or medical interventions. An illustrative example of a complete dynamic tracking trial is shown in Figure 5-3.

5.5 Conclusion

A robust PID-type delay-compensating controller for NMES was designed to provide limb tracking for uncertain lower limb dynamics subject to bounded unknown additive

intensity). Although the main gain k_c can be decreased/increased to compensate for stronger/weaker responses, the stimulator has finite resolution. Therefore, the constant value of pulsewidth was either reduced or increased so that the resulting control input would be within an acceptable range. Additionally, other factors were considered for modifying the pulsewidth, such as tracking accuracy and pain threshold of the participants.

[7] The start of the steady state baseline RMS error is defined as the point at which the error no longer decreases from the initial overshoot error during the first 10 seconds of the dynamic trial.

Table 5-1. RMS Error (Degrees), controller gains, estimate of delay, and selected pulsewidth (PW)

Subject	Leg	RMS Error (deg.)	$\mathbf{k_c}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\eta}$	$\hat{\tau}$(ms)	PW(μs)
S1	R	3.31	26	0.32	8.68	0.5	85	200
	L	4.38	23	0.40	7.60	0.5	85	200
S2	R	3.37	30	0.32	8.80	0.5	95	400
	L	3.54	30	0.32	8.80	0.5	95	400
S3	R	3.69	25	0.34	8.16	0.5	100	400
	L	3.24	18	0.35	7.95	0.5	100	300
S4	R	4.15	13	0.41	7.09	0.5	100	200
	L	4.35	11	0.41	7.09	0.5	100	200
S5	R	3.46	22	0.39	7.11	0.5	95	200
	L	4.10	25	0.39	7.11	0.5	95	200
S6	R	4.58	35	0.32	8.88	0.5	100	500
	L	4.37	30	0.32	8.88	0.5	100	500
S7	R	3.52	18	0.36	8.14	0.5	95	200
	L	3.84	15	0.42	6.88	0.5	95	200
S8	R	3.22	18	0.36	8.14	0.5	95	200
	L	4.08	15	0.42	6.88	0.5	95	200
S9	R	4.16	16	0.46	6.04	0.5	95	200
	L	3.94	18	0.46	6.04	0.5	95	200
S10	R	4.15	20	0.41	7.09	0.5	100	300
	L	3.50	26	0.46	6.04	0.5	100	300
Mean		3.85	-	-	-		96	-
SD		0.43	-	-	-		5	-

Table 5-2. Percentage of controller gains

Subject	Leg	Proportional (Percentage)	Integration (Percentage)	Derivative (Percentage)	Compensator (Percentage)
S1	R	67.67	21.05	7.52	3.76
	L	64.00	24.00	8.00	4.00
S2	R	68.15	20.74	7.40	3.70
	L	68.15	20.74	7.40	3.70
S3	R	66.41	21.87	7.80	3.91
	L	65.87	22.22	7.94	3.96
S4	R	63.03	24.37	8.40	4.20
	L	63.03	24.37	8.40	4.20
S5	R	63.56	23.73	8.47	4.24
	L	63.56	23.73	8.47	4.24
S6	R	68.15	20.74	7.40	3.70
	L	68.15	20.74	7.40	3.70
S7	R	65.90	22.48	7.75	3.88
	L	62.40	24.79	8.55	4.27
S8	R	65.90	22.48	7.75	3.88
	L	62.40	24.79	8.55	4.27
S9	R	60.19	25.93	9.26	4.63
	L	60.19	25.93	9.26	4.63
S10	R	63.03	24.37	8.4	4.2
	L	60.19	25.93	9.26	4.63

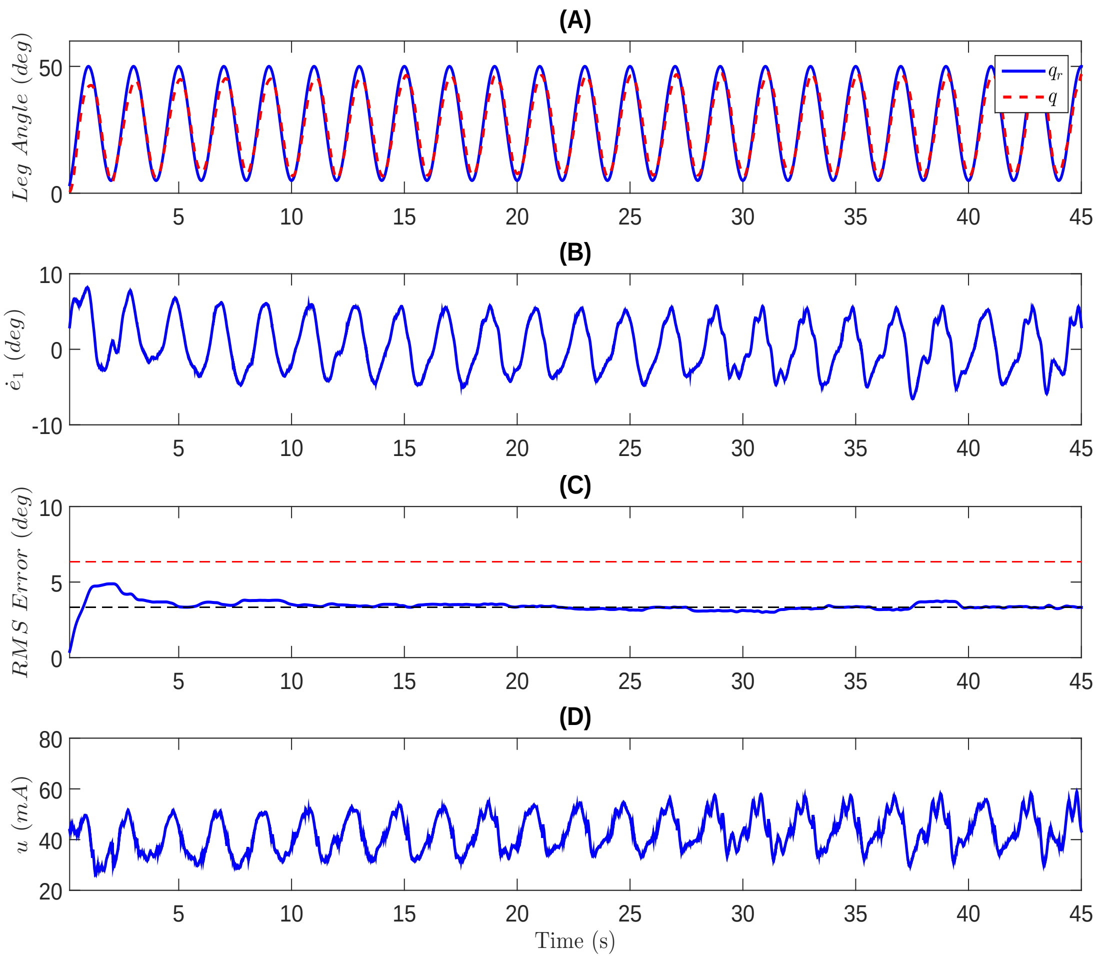

Figure 5-3. Tracking performance example taken from the right leg of subject 1 (S1-Right). Plot A includes the desired trajectory (blue solid line) and the actual leg angle (red dashed line). Plot B illustrates the angle tracking error. Plot C depicts the RMS tracking error calculated online. The black dashed lines in Plot C indicate the baseline of the RMS error and the red dashed lines indicate the limit at which the trial would terminate if reached. Plot D depicts the control input (current amplitude in mA).

disturbances with an unknown time-varying input delay. An auxiliary tracking error signal was designed to inject a delay-free control signal in the closed-loop dynamics without measuring time-varying input delay. Lyapunov-Krasovskii functionals are used in the Lyapunov-based stability analysis to provide uniformly ultimate boundedness of error for tracking reference limb movement. Additionally, the exponential decay rate of the tracking error is simplified in this chapter. Experiments were conducted with 10 able-bodied individuals to examine the performance of the designed controller. Further work is planned to extend this research by using an estimated time-varying delay in the control design instead of using a constant estimated delay to achieve more precise tracking of reference limb movement.

CHAPTER 6
ROBUST NEUROMUSCULAR ELECTRICAL STIMULATION CONTROL FOR UNKNOWN TIME-VARYING INPUT DELAYED MUSCLE DYNAMICS: FORCE TRACKING CONTROL

Recent results indicate that muscles have a delayed response to neuromuscular electrical stimulation (NMES). Muscle groups are known to rapidly fatigue in response to external muscle stimulation when compared to volitional contractions, and recent results indicate that this input delay increases as the muscle fatigues. Since the exact value of the time-varying input delay is difficult to measure during feedback control, the uncertain input delay presents a significant challenge to designing controllers for force tracking during isometric NMES. In this chapter, a continuous robust controller is developed that compensates for the uncertain time-varying input delay for the uncertain, nonlinear NMES dynamics of the lower limb despite additive bounded disturbances. In this chapter, an auxiliary error signal is designed to obtain the first time derivative of delay-free control signal in the closed-loop dynamics. The first time derivative of the designed auxiliary error signal is defined as the difference between the first time derivatives of delay-free control signal and the delayed control signal using a constant estimated delay. The maximum error between the actual input delay and estimated input delay is determined based on selection of the control gains to guarantee stability of the system. A Lyapunov-based stability analysis is used to prove that the error signals are uniformly ultimately bounded. Experimental results are obtained in 10 able-bodied individuals to show the performance of the designed controller.

6.1 Control Design

The nonlinear input delayed model for the isometric knee-joint dynamics [111] is defined as

$$R(t) \triangleq f(q) + d(t) + \Omega(t)u\left(t - \tau(t)\right), \tag{6–1}$$

where $R : [t_0, \infty) \to \mathbb{R}$ denotes the reaction torque, $f : \mathbb{R} \to \mathbb{R}$ is a continuous function that represents the gravity and the elastic components, $q \in \mathbb{R}$ denotes the constant, finite knee-joint angle, $d : [t_0, \infty) \to \mathbb{R}$ is an unknown time-varying exogenous disturbance, $\Omega : [t_0, \infty) \to \mathbb{R}$ is an unknown nonzero time-varying function relating the input voltage to the produced torque, $u : [t_0, \infty) \to \mathbb{R}$ is the input voltage, and $\tau : [t_0, \infty) \to \mathbb{R}$ denotes an uncertain time-varying input delay, where t_0 is the initial time. The subsequent control development is based on the following assumptions.

Assumption 6.1. The desired reaction torque is designed such that $R_d,\ \dot{R}_d \in \mathcal{L}_\infty$ are bounded by known positive constants.

Assumption 6.2. The nonlinear additive disturbance term and its first time derivative exist and are bounded by known positive constants

Assumption 6.3. The positive nonzero unknown function Ω is bounded[1] such that $\underline{\Omega} \leq \Omega \leq \overline{\Omega}$ for all t, where $\underline{\Omega}$ and $\overline{\Omega}$ are positive known constants. The first time-derivative of Ω exists and is bounded above and below by known constants.

Assumption 6.4. The input delay is bounded such that $\tau(t) < \Sigma$ for all $t \in \mathbb{R}$, differentiable, and slowly varying[2] such that $|\dot{\tau}| < \bar{\dot{\tau}} < 1$ for all $t \in \mathbb{R}$, where $\bar{\dot{\tau}}, \Sigma \in \mathbb{R}$ are known positive constants. Additionally, a constant estimate $\hat{\tau} \in \mathbb{R}$ of τ is available and sufficiently accurate such that $\tilde{\tau} \triangleq \tau - \hat{\tau}$ is bounded by $|\tilde{\tau}| \leq \bar{\tilde{\tau}}$ for all $t \in \mathbb{R}$, where $\bar{\tilde{\tau}} \in \mathbb{R}$ is a known positive constant. Additionally, the isometric knee joint dynamics in (6–1) do not escape to infinity during the time interval $[t_0, t_0 + \Sigma]$.

6.2 Control Development

The control objective is to ensure the reaction torque R of the input delayed system in (6–1) tracks a desired reaction torque R_d and accounts for the effects of an uncertain time-varying input delay, uncertainties, and exogenous bounded disturbances in the

[1] The bounds of Ω can be determined experimentally by using minimum and maximum applied torque that can be produced for a given input.

[2] NMES input delay is a slowly varying signal and preliminary experiments validate the assumption [89].

dynamic model. To quantify the control objective, the reaction torque tracking error, denoted by $e \in \mathbb{R}$, is defined as

$$e \triangleq R_d - R. \tag{6–2}$$

To facilitate the subsequent analysis, a measurable auxiliary tracking error, denoted by $r \in \mathbb{R}$, is defined as

$$r \triangleq \alpha e + \beta e_u, \tag{6–3}$$

where $\alpha, \beta \in \mathbb{R}$ are positive constant control gains. The same tracking error signal e_u, defined in (2–8), is used to inject a delay-free input term in the closed-loop error system. The open-loop dynamics for r can be obtained by multiplying (6–3) by Ω^{-1}, taking the time-derivative, and using the expressions in (6–1), (6–2), and (2–8) to yield

$$\begin{aligned} \frac{d}{dt}\left(\Omega^{-1}r\right) =& \frac{d}{dt}\left(\Omega^{-1}\alpha(R_d - f - d)\right) + \frac{d}{dt}\left(\Omega^{-1}\right)\beta e_u + \alpha(1-\dot{\tau})\left(\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}\right) \\ & - \Omega^{-1}\beta\dot{u} + \left(\Omega^{-1}\beta - \alpha\left(1-\dot{\tau}\right)\right)\dot{u}_{\hat{\tau}}. \end{aligned} \tag{6–4}$$

Based on the subsequent stability analysis, the following continuous robust controller is designed:

$$\dot{u} = k_c r, \tag{6–5}$$

where $k_c \in \mathbb{R}$ is a constant, positive selectable control gain. Substituting (6–5) into (6–4), the closed-loop error system for r can be obtained as

$$\begin{aligned} \frac{d}{dt}\left(\Omega^{-1}r\right) =& \frac{d}{dt}\left(\Omega^{-1}\alpha(R_d - f - d)\right) + \frac{d}{dt}\left(\Omega^{-1}\right)\beta e_u + \alpha(1-\dot{\tau})\left(\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}\right) \\ & - \Omega^{-1}\beta k_c r + \left(\Omega^{-1}\beta - \alpha\left(1-\dot{\tau}\right)\right)k_c r_{\hat{\tau}}. \end{aligned} \tag{6–6}$$

To facilitate the stability analysis, (6–6) is segregated into terms which can be upper bounded linearly in error-dependent functions and terms that can be upper bounded by a constant, such that

$$\Omega^{-1}\dot{r} = \tilde{N} + N_r - \frac{1}{2}\frac{d}{dt}\left(\Omega^{-1}\right)r + \alpha(1-\dot{\tau})\left(\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}\right)$$

$$- \Omega^{-1}\beta k_c r + \left(\Omega^{-1}\beta - \alpha\left(1-\dot{\tau}\right)\right) k_c r_{\hat{\tau}}, \tag{6–7}$$

where the auxiliary terms $\tilde{N}, N_r \in \mathbb{R}$ are defined as

$$\tilde{N} \triangleq -\frac{1}{2}\frac{d}{dt}\left(\Omega^{-1}\right) r + \frac{d}{dt}\left(\Omega^{-1}\right)\beta e_u, \tag{6–8}$$

$$N_r \triangleq \frac{d}{dt}\left(\Omega^{-1}\alpha(R_d - f - d)\right). \tag{6–9}$$

Remark 6.1. Assumption 6.3 can be used to obtain an upper bound for (6–8) as

$$\left|\tilde{N}\right| \leq \Lambda \left\|z\right\|, \tag{6–10}$$

where $\Lambda \in \mathbb{R}$ is a known positive constant, and $z \in \mathbb{R}^2$ is a vector of collected error signals defined as

$$z \triangleq \begin{bmatrix} e_u & r \end{bmatrix}^T. \tag{6–11}$$

Remark 6.2. Based on Assumptions 6.1, 6.2 and 6.3, N_r is upper bounded as

$$\sup_{t\in\mathbb{R}} \left|N_r\right| \leq \Phi, \tag{6–12}$$

where $\Phi \in \mathbb{R}$ is a known positive constant.

To facilitate the subsequent stability analysis, auxiliary bounding constants $\sigma\,, \delta \in \mathbb{R}$ are defined as

$$\sigma = \min\left\{\frac{\beta k_c}{8\overline{\Omega}}, \frac{\omega_2}{3\hat{\tau}k_c^2} - \frac{1}{2}\right\}, \tag{6–13}$$

$$\delta = \frac{1}{2}\min\left\{\frac{\sigma}{2}, \frac{\omega_2}{3\left(\frac{\left(\frac{\beta}{\underline{\Omega}} - \alpha\left(1-\bar{\dot{\tau}}\right)k_c\right)^2}{\frac{1}{\overline{\Omega}}\beta k_c} + \left(\omega_1 k_c\right)^2\right)}, \frac{1}{3\left(\hat{\tau} + \bar{\tilde{\tau}}\right)}\right\}, \tag{6–14}$$

where $\omega_1,\ \omega_2\ \in \mathbb{R}$, are positive control gains. Let the Lyapunov Krasovskii functions $Q_1,\ Q_2 \in \mathbb{R}$ be defined as

$$Q_1 \triangleq \left(\frac{\left(\frac{\beta}{\underline{\Omega}} - \alpha\left(1+\bar{\dot{\tau}}\right)k_c\right)^2}{\frac{1}{\overline{\Omega}}\beta k_c} + \left(\omega_1 k_c\right)^2\right)\int_{t-\hat{\tau}}^{t} r^2(\theta)d\theta, \tag{6–15}$$

$$Q_2 \triangleq \omega_2 \int\limits_{t-\left(\hat{\tau}+\bar{\bar{\tau}}\right)}^{t} \int\limits_{s}^{t} r^2(\theta)d\theta ds, \tag{6–16}$$

and let $y \in \mathbb{R}^4$ be defined as

$$y \triangleq \left[\begin{array}{c} e_u, r, \sqrt{Q_1}, \sqrt{Q_2} \end{array} \right]^T. \tag{6–17}$$

Provided $\|z(\eta)\| < \gamma,\ \forall \eta \in [t-\Sigma, t]$, (6–5) and (6–7) can be used to conclude that $\ddot{u}(\eta) < M,\ \forall \eta \in [t-\Sigma, t]$, where γ and M are positive constants. Let $\mathscr{D} \triangleq B_\gamma$ where $\mathscr{D}$ denotes the set of stabilizing initial conditions and B_γ denotes a closed ball of radius γ centered at the origin that is subset of $\mathbb{R}^4$.

6.3 Stability Analysis

Theorem 6.1. *Given the dynamics in (1), the controller given in (6–5) ensures uniformly ultimately bounded tracking*[3] *in the sense that*

$$\limsup_{t\to\infty} |e(t)| \leq \frac{(1+\beta)}{\alpha} \sqrt{\frac{\max\left\{\frac{1}{2\Omega}, \frac{\omega_1}{2}, 1\right\} \left(2\left(\alpha(1+\bar{\tau})M\bar{\bar{\tau}}\right)^2 + \Phi^2\right)}{\min\left\{\frac{1}{2\Omega}, \frac{\omega_1}{2}, 1\right\} \frac{1}{\Omega}\beta k_c \delta}},\ \forall t \geq t_0 + \Sigma \tag{6–18}$$

provided that $y(\eta) \in \mathscr{D},\ \forall \eta \in [t_0, t_0+\Sigma]$ *and that the control gains are selected sufficiently large relative to the initial conditions of the system such that the following sufficient conditions are satisfied*

$$k_c \geq \frac{4\Lambda^2}{\sigma\bar{\Omega}\beta}, \tag{6–19}$$

$$\omega_2 > \frac{3\hat{\tau}k_c^2}{2}, \tag{6–20}$$

$$\frac{\frac{\beta k_c}{8\Omega} - 2\left(\omega_1 k_c\right)^2 - \frac{\frac{\beta}{\Omega} - \alpha\left(1+\bar{\tau}\right)k_c}{\frac{1}{\Omega}\beta k_c} - \omega_2\hat{\tau}}{\omega_2} \geq \bar{\bar{\tau}}, \tag{6–21}$$

[3] To achieve a small tracking error for the case of a large value of Φ (i.e., large disturbances), large gains are required. Large gains result in a large M, and therefore, a sufficiently small $\bar{\bar{\tau}}$ is required.

$$\gamma > \sqrt{\frac{\max\left\{\frac{1}{2\Omega}, \frac{\omega_1}{2}, 1\right\}\left(2\left(\alpha(1+\bar{\dot{\tau}})M\bar{\bar{\tau}}\right)^2 + \Phi^2\right)}{\min\left\{\frac{1}{2\bar{\Omega}}, \frac{\omega_1}{2}, 1\right\}\frac{1}{\Omega}\beta k_c \delta}}. \tag{6–22}$$

For sufficiently small $\hat{\tau}$ and $\bar{\bar{\tau}}$, the gain condition in (6–21) can be satisfied by selecting ω_1, ω_2, α to be sufficiently small and β to be close to $\underline{\Omega}\alpha\left(1+\bar{\dot{\tau}}\right)k_c$, where k_c is selected to satisfy the gain condition in (6–19) and (6–20).

Proof. Let $V : \mathscr{D} \to \mathbb{R}$ be a continuously differentiable Lyapunov function candidate defined as

$$V \triangleq \frac{1}{2}\Omega^{-1}r^2 + \frac{\omega_1}{2}e_u^2 + Q_1 + Q_2. \tag{6–23}$$

The time derivatives of (6–15) and (6–16) can be obtained by using the Leibniz Rule as

$$\dot{Q}_1 = \left(\frac{\left(\frac{\beta}{\underline{\Omega}} - \alpha\left(1+\bar{\dot{\tau}}\right)k_c\right)^2}{\frac{1}{\bar{\bar{\Omega}}}\beta k_c} + \left(\omega_1 k_c\right)^2\right)\left(r^2 - r_{\hat{\tau}}^2\right), \tag{6–24}$$

$$\dot{Q}_2 = \omega_2\left(\left(\hat{\tau}+\bar{\bar{\tau}}\right)r^2 - \int\limits_{t-\left(\hat{\tau}+\bar{\bar{\tau}}\right)}^{t} r^2(\theta)d\theta\right). \tag{6–25}$$

The following inequalities can be obtained for (6–23):

$$\min\left\{\frac{1}{2\bar{\Omega}}, \frac{\omega_1}{2}, 1\right\}\|y\|^2 \leq V(y) \leq \max\left\{\frac{1}{2\underline{\Omega}}, \frac{\omega_1}{2}, 1\right\}\|y\|^2. \tag{6–26}$$

The time derivative of (6–23) can be determined by using (2–8), (6–7), (6–24), and (6–25) as

$$\begin{aligned}\dot{V} =& r\left(N_r + \tilde{N} + \alpha(1-\dot{\tau})\left(\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}\right) - \frac{1}{2}\frac{d}{dt}\left(\Omega^{-1}\right)r\right) + \omega_1 e_u\left(\dot{u}_{\hat{\tau}} - \dot{u}\right)\\ &+ r\left(\left(\Omega^{-1}\beta - \alpha\left(1-\dot{\tau}\right)\right)k_c r_{\hat{\tau}} - \Omega^{-1}\beta k_c r\right) + \frac{1}{2}\frac{d}{dt}\left(\Omega^{-1}\right)r^2\\ &+ \left(\frac{\left(\frac{\beta}{\underline{\Omega}} - \alpha\left(1+\bar{\dot{\tau}}\right)k_c\right)^2}{\frac{1}{\bar{\bar{\Omega}}}\beta k_c} + \left(\omega_1 k_c\right)^2\right)\left(r^2 - r_{\hat{\tau}}^2\right)\end{aligned}$$

$$+\omega_2\left(\left(\hat{\tau}+\bar{\bar{\tau}}\right)r^2-\int\limits_{t-\left(\hat{\tau}+\bar{\bar{\tau}}\right)}^{t} r^2(\theta)d\theta\right). \tag{6–27}$$

By canceling common terms in (6–27), using Assumption 6.4, and (6–5), an upper bound can be obtained for (6–27) as

$$\begin{aligned}\dot{V} \leq & rN_r + r\tilde{N} + \alpha(1-\dot{\tau})\left|\left(\dot{u}_{\hat{\tau}}-\dot{u}_{\tau}\right)r\right| - \Omega^{-1}\beta k_c r^2 \\ & + \left|\Omega^{-1}\beta-\alpha\left(1-\dot{\tau}\right)\right| k_c\left|r_{\hat{\tau}}r\right| + \omega_1 k_c\left|e_u r_{\hat{\tau}}\right| + \omega_1 k_c\left|e_u r\right| \\ & + \left(\frac{\left(\frac{\beta}{\Omega}-\alpha\left(1+\dot{\bar{\tau}}\right)k_c\right)^2}{\frac{1}{\Omega}\beta k_c} + \left(\omega_1 k_c\right)^2\right)\left(r^2-r_{\hat{\tau}}^2\right) \\ & + \omega_2\left(\left(\bar{\bar{\tau}}+\hat{\tau}\right)r^2-\int\limits_{t-\left(\bar{\bar{\tau}}+\hat{\tau}\right)}^{t} r^2(\theta)d\theta\right).\end{aligned} \tag{6–28}$$

After completing the squares for r, e_u, and $r_{\hat{\tau}}$ and substituting (6–5), (6–10), (6–12) into (6–28), the following upper bound can be obtained

$$\begin{aligned}\dot{V} \leq & \frac{1}{\Omega^{-1}\beta k_c}\Phi^2 + \frac{2\Lambda^2}{\Omega^{-1}\beta k_c}\left\|z\right\|^2 + \frac{2\left(\alpha(1+\dot{\bar{\tau}})\right)^2}{\Omega^{-1}\beta k_c}\left(\dot{u}_{\hat{\tau}}-\dot{u}_{\tau}\right)^2 \\ & -\left(\frac{\Omega^{-1}\beta k_c}{8}-\kappa\right)r^2 + \frac{e_u^2}{2} - \frac{\Omega^{-1}\beta k_c}{8}r^2 - \omega_2\int\limits_{t-\left(\hat{\tau}+\bar{\bar{\tau}}\right)}^{t}\left|r\left(\theta\right)\right|^2 d\theta,\end{aligned} \tag{6–29}$$

where $\kappa \triangleq 2\left(\omega_1 k_c\right)^2 + \frac{\left(\frac{\beta}{\Omega}-\alpha\left(1-\dot{\bar{\tau}}\right)k_c\right)^2}{\frac{1}{\Omega}\beta k_c} + \omega_2\left(\hat{\tau}+\bar{\bar{\tau}}\right)$. The Cauchy-Schwartz inequality is used to develop an upper bound for e_u^2 as

$$e_u^2 \leq \hat{\tau}k_c^2\int\limits_{t-\hat{\tau}}^{t} r^2(\theta)d\theta. \tag{6–30}$$

Additionally, an upper bound can be provided for Q_2 as

$$Q_2 \leq \omega_2\left(\hat{\tau}+\bar{\bar{\tau}}\right)\sup_{s\epsilon\left[t-\left(\hat{\tau}+\bar{\bar{\tau}}\right),t\right]}\left[\int\limits_{s}^{t} r^2(\theta)d\theta\right] \leq \omega_2\left(\hat{\tau}+\bar{\bar{\tau}}\right)\int\limits_{t-\left(\hat{\tau}+\bar{\bar{\tau}}\right)}^{t} r^2(\theta)d\theta. \tag{6–31}$$

By using Assumption 6.4, the inequality $\int\limits_{t-\hat{\tau}}^{t} r^2(\theta)d\theta \leq \int\limits_{t-\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}^{t} r^2(\theta)d\theta$ can be obtained. In addition, using (6–15), (6–16), (6–30), and (6–31), the following inequalities can be obtained

$$\frac{\omega_2}{3\hat{\tau}k_c^2}e_u^2 \leq \frac{\omega_2}{3}\int\limits_{t-\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}^{t} r^2(\theta)d\theta, \tag{6–32}$$

$$\frac{Q_1}{\left(\frac{\left(\frac{\beta}{\Omega}-\alpha\left(1+\bar{\tau}\right)k_c\right)^2}{\frac{1}{\Omega}\beta k_c}+\left(\omega_1 k_c\right)^2\right)\frac{3}{\omega_2}} \leq \frac{\omega_2}{3}\int\limits_{t-\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}^{t} r^2(\theta)d\theta, \tag{6–33}$$

$$\frac{Q_2}{3\left(\hat{\tau}+\bar{\tilde{\tau}}\right)} \leq \frac{\omega_2}{3}\int\limits_{t-\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}^{t} r^2(\theta)d\theta. \tag{6–34}$$

Using (6–32)-(6–34), the right-hand-side of (6–29) can be upper bounded as

$$\begin{aligned}\dot{V} \leq & \frac{1}{\Omega^{-1}\beta k_c}\Phi^2+\frac{2\Lambda^2}{\Omega^{-1}\beta k_c}\left\|z\right\|^2+\frac{2\left(\alpha(1+\bar{\tau})\right)^2}{\Omega^{-1}\beta k_c}\left(\dot{u}_{\hat{\tau}}-\dot{u}_{\tau}\right)^2 \\ & -\left(\frac{\Omega^{-1}\beta k_c}{8}-\kappa\right)r^2-\frac{\beta k_c}{8\overline{\Omega}}r^2-\left(\frac{\omega_2}{3\hat{\tau}k_c^2}-\frac{1}{2}\right)e_u^2 \\ & -\frac{\omega_2}{3\left(\frac{\left(\frac{\beta}{\Omega}-\alpha\left(1-\bar{\tau}\right)k_c\right)^2}{\frac{1}{\Omega}\beta k_c}+\left(\omega_1 k_c\right)^2\right)}Q_1-\frac{1}{3\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}Q_2.\end{aligned} \tag{6–35}$$

By using the Mean Value Theorem, the following inequality can be obtained $\left\|\dot{u}_{\hat{\tau}}-\dot{u}_{\tau}\right\| \leq \left\|\ddot{u}\left(\Theta\left(t,\hat{\tau}\right)\right)\right\|\left|\tilde{\tau}\right|$, where $\Theta\left(t,\hat{\tau}\right)$ is a point between $t-\tau$ and $t-\hat{\tau}$. Furthermore, using the gain conditions in (6–19)-(6–21), the definition of σ in (6–13) and the definition of the domain B_γ, the following upper bound can be obtained

$$\begin{aligned}\dot{V} \leq & -\frac{\sigma}{2}\left\|z\right\|^2+\left(\frac{2\left(\alpha(1+\bar{\tau})M\bar{\tilde{\tau}}\right)^2+\Phi^2}{\Omega^{-1}\beta k_c}\right) \\ & -\frac{\omega_2}{3\left(\frac{\left(\frac{\beta}{\Omega}-\alpha\left(1-\bar{\tau}\right)k_c\right)^2}{\frac{1}{\Omega}\beta k_c}+\left(\omega_1 k_c\right)^2\right)}Q_1-\frac{1}{3\left(\hat{\tau}+\bar{\tilde{\tau}}\right)}Q_2.\end{aligned} \tag{6–36}$$

Using the definition δ in (6–14), the expression in (6–36) reduces to

$$\dot{V} \leq -\delta \left\| y \right\|^2, \; \forall \left\| y \right\| \geq \left(\frac{2\overline{\Omega}\left(\alpha(1+\bar{\tau})M\bar{\bar{\tau}}\right)^2 + \Phi^2}{\beta k_c \delta} \right)^{\frac{1}{2}}. \tag{6–37}$$

It can be concluded that by using the gain condition in (6–22) and an extension to Theorem 4.18 in [106], y is uniformly ultimately bounded in the sense that $\limsup_{t\to\infty} \left\| y\left(t\right) \right\| \leq \sqrt{\frac{\max\left\{\frac{1}{2\underline{\Omega}}, \frac{\omega_1}{2}, 1\right\}\left(2\left(\alpha(1+\bar{\tau})M\bar{\bar{\tau}}\right)^2 + \Phi^2\right)}{\min\left\{\frac{1}{2\overline{\Omega}}, \frac{\omega_1}{2}, 1\right\}\frac{1}{\overline{\Omega}}\beta k_c \delta}}$, $\forall t \geq t_0 + \Sigma$, provided $y\left(\eta\right) \in \mathscr{D}, \; \forall \eta \in \left[t_0, t_0 + \Sigma\right]$, where uniformity in initial time can be concluded from the independence of δ and the ultimate bound from t_0. Since $r, \; e_u \in \mathcal{L}_\infty$, from (6–3), $e \in \mathcal{L}_\infty$. In addition, using Assumptions 6.1, 6.2, 6.3 and the dynamics (6–1), $u \in \mathcal{L}_\infty$. An analysis of the closed-loop system shows that the remaining signals are bounded. □

6.4 Experiments

The performance of the developed controller in (6–5) was tested during experiments of the knee-shank segment. Surface electrical stimulation was applied to the quadriceps muscle group to trigger contractions during isometric torque tracking trials. The control algorithm in (6–5) was used to modify the pulsewidth (PW) in real time, while the stimulation amplitude and stimulation frequency were set to a constant value. The same subjects and apparatuses defined in Section 5.4 are used for testing.

For all experiments, the stimulation frequency and the current amplitude were fixed at 35 Hz and 90 mA, respectively. [4] Biphasic rectangular pulses were used

[4] Different responses to stimulation were obtained when testing across participants (i.e., greater or weaker muscle force was produced for a nominal stimulation intensity). While the main gain k_c can be decreased/increased to compensate for stronger/weaker responses, the stimulator has finite resolution. Therefore, the constant value of pulsewidth was either reduced or increased so that the resulting control input lies within an acceptable range. Other factors were considered for modifying the pulsewidth, such as tracking accuracy and pain threshold of the participants.

during all the trials. The desired torque trajectory was designed to be a periodic smooth trapezoidal profile with upper and lower limits set to 25 N·m and 5 N·m, respectively with a period of 4 seconds. Following Assumption 6.4 described in section 6.1, the constant estimate of the slowly time-varying delay was selected as $\hat{\tau} \in [0.085, 0.1]$ seconds based on the experimental results reported in recent NMES studies [89, 100][5] . The control gains were adjusted during pretrial tests to reach the target isometric torque pattern. Afterwards, the tracking trial was run for one of the participant's knee-shank complex (selected randomly) for a testing duration of 60 seconds. The control performance was evaluated by calculating the root-mean-square (RMS) tracking error over the entire trial. Then the process was repeated for the other lower limb starting with the pretrial tests. Table 6-1 presents the mean RMS error over the entire trial, the selected gains, and the constant delay estimate $\hat{\tau}$ across all the tracking trials. An illustrative example of a complete torque tracking trial is shown in Figure 6-1.

6.5 Conclusion

A continuous robust tracking controller was developed to yield reaction torque tracking for the uncertain, nonlinear dynamics of the lower limb with additive disturbances without knowledge of time-varying input delay. A filtered tracking error signal was designed to facilitate the control design and analysis. A novel Lyapunov-based stability analysis was developed to prove ultimately bounded tracking error signals. Experiments were obtained by 10 able-bodied individuals to demonstrate the performance of the designed controller.

[5] The constant estimate $\hat{\tau}$ varied for each participant. This estimate magnitude was determined during pretrial tests and is dependent on skin resistance, electrode placement, sensitivity to stimulation intensity, among other physiological conditions such as chemical reactions and calcium dynamics during skeletal contractions.

Table 6-1. RMS Error, controller gains, estimate of delay, and selected pulsewidth (PW)

Subject-Leg	RMS Error (N·m)	$\mathbf{k_c}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\hat{\boldsymbol{\tau}}$(s)
S1-Right	0.9721	30	5	2.5	0.095
S1-Left	1.1939	35	5	2.5	0.095
S2-Right	1.0044	30	4	5	0.085
S2-Left	0.9684	35	5	1.8	0.1
S3-Right	0.9335	27	6	2.5	0.09
S3-Left	0.9313	25	6	2.3	0.09
S4-Right	0.9832	38	8	3.8	0.09
S4-Left	1.5218	52	8	2.5	0.1
S5-Right	0.8478	32	6	2.1	0.09
S5-Left	0.6758	32	6	2.1	0.09
S6-Right	0.7759	33	6	3	0.095
S6-Left	0.7286	32	7	2	0.095
S7-Right	0.8718	32	6	2.1	0.085
S7-Left	1.3899	60	6	2.1	0.085
S8-Right	0.8415	28	6	3	0.09
S8-Left	1.0642	35	6	2.8	0.09
S9-Right	0.8163	24	5	2.5	0.095
S9-Left	1.1150	32	7	2.1	0.085
S10-Right	1.1438	45	6.5	1.8	0.095
S10-Left	0.8969	45	6.5	1.8	0.095
Mean	0.9838	-	-	-	0.092
SD	0.2108	-	-	-	0.005

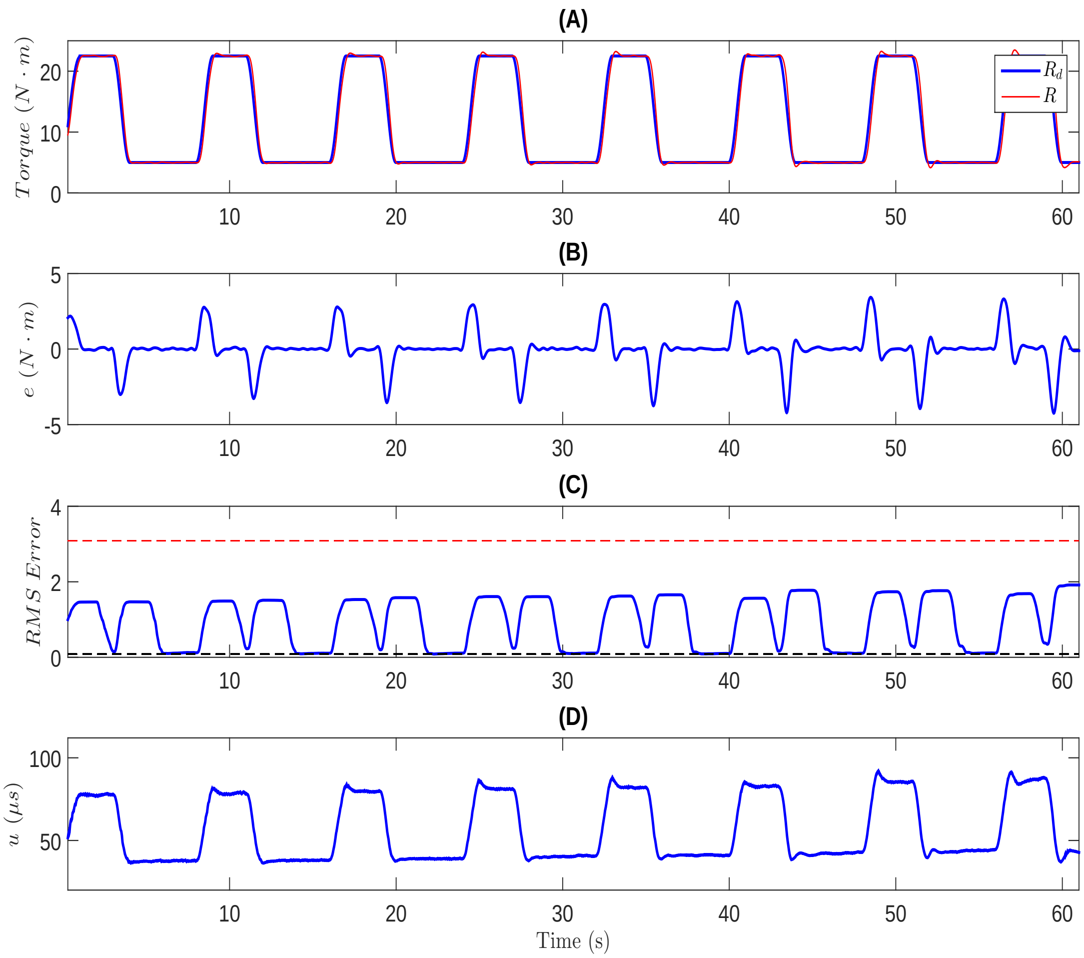

Figure 6-1. Tracking performance example taken from the left leg of subject 2 (S2-Left). Plot A includes the desired isometric torque pattern (blue solid line) and the actual isometric torque produced by the quadriceps muscle group (solid red line). Plot B illustrates the instantaneous torque tracking error. Plot C depicts the RMS error calculated online. The black dashed lines in Plot C indicate the baseline of the RMS error and the red dashed lines indicate the limit at which the trial would terminate if reached. Plot D depicts the control input (pulsewidth in μs).

CHAPTER 7
CONCLUSION

The main contributions of each chapter, limitations and implementation challenges and possible future research directions of the dissertation are discussed in this chapter.

The focus of this dissertation is to develop a control methods for uncertain nonlinear systems subjected to unknown time-varying and state-dependent input delay with additive disturbances. Motivated by real engineering applications that are affected by nonlinearity and uncertainty in the dynamics and time delays in the input, it is necessary to develop a control methods that can be used to compensate input time delay disturbances on uncertain nonlinear systems. This dissertation focuses on various applications of unknown non-constant input delay for uncertain nonlinear systems such as force tracking for isometric NMES, position tracking for NMES, and position tracking for uncertain Euler-Lagrange systems. Chapter 1 focuses on introducing the relevant literature for input time-delay systems, analysis, and design techniques of NMES. In Chapter 2, a continuous controller is developed for a class of uncertain nonlinear systems that includes unknown time-varying additive bounded disturbances and unknown time-varying input delay. The continuous controller achieves uniformly ultimately bounded error tracking. The controller in Chapter 3 is motivated by the need to compensate effects of unknown input time-delay which can be fast varying for uncertain Euler-Lagrange dynamics with unknown time-varying additive disturbances. The controller guarantees uniformly ultimately bounded tracking error regulation. Simulations on a two-link robot manipulator are performed to show the performance of the controller for various delay rates and rate changes. A robust controller is designed in Chapter 4, an adaptive based control strategy is used to estimate an uncertain state-dependent input delay while compensating for input delay disturbances for an uncertain nonlinear system with unknown additive disturbances. The controller in Chapter 4 guarantees semi-globaly uniformly ultimately bounded tracking error regulation. Chapter

5 focuses on unknown input delay compensation for position tracking of lower limb dynamics. The designed controller achieves uniformly ultimately bounded tracking error regulation despite uncertainty and nonlinearity in the lower limb dynamics without input delay measurements. Experiments illustrate the performance of the controller. The development of the controller in Chapter 6 is motivated by the need to design a continuous controller that provides force tracking for isometric NMES. The designed controller achieves uniformly ultimately bounded tracking error regulation during NMES. Experiments illustrate the effectiveness of the designed method.

This dissertation presented control design techniques that are successfully implemented to a class of uncertain nonlinear systems such as NMES and Euler-Lagrange dynamics. Although the developed methods can be applied to a wide range of systems, there are several limitations and open theoretical problems. In this chapter, the remaining open problems, limitations and future research directions are discussed.

In Chapter3, the input delay is assumed that the delay can be fast ($|\bar{\dot{\tau}}|$<1) and continuous. Practically, there are open questions for input time-delayed systems that are not continuous in the delay. In Chapter 4, the dynamics of uncertain state-dependent input delay are assumed to be linearly parameterizable. However, the dynamics of input delay might not satisfy this assumption. In Chapters 2-6, the developed control strategies provide uniformly ultimately bounded tracking. No result is available for uncertain nonlinear systems with a time-varying input delay that can provide asymptotic tracking. Achieving an asymptotic result for an unknown time-varying input delay case for uncertain nonlinear systems is still a remaining open problem. In Chapters 2-6, the developed control strategies are based on the assumption of full-state feedback is available. A remaining open problem is the development of an output feedback solution. The development in Chapters 2-6 does not consider controller saturation. The inclusion of control saturation remains an open problem. In Chapters 5 and 6, the controllers are only considers the case when an input delay exists when the muscle contracts; however, an input delay likely also exist when the muscle relaxes. To the best of author's knowledge, there is no result in literature exists for the case of compensating for a negative input delay in the dynamics. To reduce the tracking error, the control gains should be delay dependent. A remaining open problem is to develop delay dependent control gains through a scheduling strategy. Another possible extension of this dissertation would be to consider multi-agent systems where unknown delays may be considered as communication and input disturbances.

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BIOGRAPHICAL SKETCH

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