Torque and cadence tracking in functional electrical stimulation induced cycling using passivity-based spatial repetitive learning control

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A B S T R A C T

Due to the inherent periodic nature of cycling tasks, iterative and repetitive learning controllers are well motivated for rehabilitative cycling. Motorized functional electrical stimulation induced cycling is a rehabilitation treatment where multiple lower-limb muscle groups are activated jointly with an electric motor to achieve cycling objectives such as speed (cadence) and torque tracking. This paper examines torque tracking accomplished by the stimulation of six lower-limb muscles via a novel spatial repetitive learning control and cadence regulation by an electric motor using a sliding-mode controller. A desired torque trajectory is constructed based on the rider’s kinematic efficiency, which is a function of the crank position. The learning controller takes advantage of the periodicity of the desired torque trajectory to provide a feedforward input to the stimulated muscles. A passivity-based analysis is developed to ensure stability of the torque and cadence closed-loop error systems. The muscle learning and electric motor controllers were implemented in real-time during cycling experiments on five able-bodied individuals and three participants with movement disorders. The combined average cadence tracking error was 0.01 ± 1.20 RPM for a 50 RPM trajectory and the combined average power tracking error was 1.78 ± 1.25 W for a peak power of 10 W.

1. Introduction

Functional Electrical Stimulation (FES) and robotic devices seek to enhance the quality of life of people with neurological conditions (NCs) by restoring mobility. Closed-loop FES control has been implemented in upper-limb tasks (Lew, Alavi, Randhawa, & Menon, 2016; Rouse, Duenas, Cousin, Parikh, & Dixon, 2018), locomotion using neuroprostheses (Alibeji, Kirsch, & Sharma, 2015, 2017; Ha, Murray, & Goldfarb, 2016; Nataraj, Audu, & Triolo, 2017), and lower-limb cycling (Bellman, Cheng, Downey, Hass, & Dixon, 2016; Bellman, Downey, Parikh, & Dixon, 2017). Motorized FES-cycling aims to produce a coordinated movement by artificially activating lower-limb muscles and engaging an electric motor to provide assistance as needed. FES-cycling studies have been found to provide neurological, movement, and sensory gains to people with spinal cord injury (SCI) and post stroke (Ferrante, Pedrocchi, Ferrigno, & Molteni, 2008; Sadowsky et al., 2013).

Cadence and power tracking objectives have been developed for cardiovascular and strength training in FES-cycling. In cadence tracking, a desired speed trajectory is tracked by muscles with or without motorized assistance. In power tracking, a torque trajectory is also tracked along with the speed trajectory. Robust closed-loop controllers leveraging high-gain or high-frequency techniques (e.g., sliding-mode control) have been used for cadence tracking in Bellman et al. (2016, 2017), Farhoud and Erfanian (2014), Hunt et al. (2004) and Kawai, Bellman,
Downey, and Dixon (2019). However, motivation exists to maximize the torque output produced by the lower-limb muscles as a means to build muscle mass (Szecsi, Straube, & Formusek, 2014). Hence, the concurrent objectives of cadence and torque tracking (i.e., power tracking) have been studied for motorized FES-cycling, where each control objective is assigned to the muscles or electric motor. Power tracking in FES-cycling has been investigated using linear feedback control (Hunt et al., 2004), higher-order sliding-mode control (Farhoud & Erfanian, 2014), a Lyapunov-based switched dwell-time analysis (Cousin, Duenas, Rouse, & Dixon, 2017), and a discrete-time analysis where the controller was updated once at the beginning of each crank cycle (Bellman, 2015). None of the aforementioned results exploit the repetitive/periodic nature of cycling to design learning-based controllers while guaranteeing the stability of the human-machine closed-loop system.

Since people undergoing movement therapy often have diminished torque producing capacity, an electric motor is typically used to assist FES-induced cycling. However, the use of an electric motor raises an additional concern for safe interaction between the rider and the motor. Motivated to ensure safe human–robot interaction, passivity theory has been used to design controllers in human applications including exercise machines and exoskeletons (Li & Horowitz, 1997; Zhang & Cheah, 2015). Closed-loop controllers that ensure passivity in the human–robot system are beneficial due to their compliant behavior, which also yield safe performance (Zhang & Cheah, 2015). In this paper, passivity is exploited as a tool to design and analyze switching controllers for cycling.

Learning control techniques, such as iterative learning control (ILC) and repetitive learning control (RLC), have been developed to improve tracking performance for repetitive or periodic systems by using control inputs from previous trials, iterations, cycles, or periods (Arimoto, Kawamura, & Miyazaki, 1984; Bristow, Tharayil, & Alleyne, 2006). ILC and RLC have been extensively applied for tracking of nonlinear systems to enhance performance (Zhang & Cheah, 2015). In this paper, passivity is exploited as a tool to design and analyze switching controllers for cycling.

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The disturbance stability analysis of et al. (2016) are exploited in the subsequent control design and forward pedaling only. Fig. 1 shows a schematic of the cycle-rider assumed to be continuous from the right and designed to produce forward pedaling only. The crank angle \( q \) and the net torque applied about the crank are positive in the clockwise direction. The knee, hip, and trunk angles are denoted by \( q_{\text{kneq}}, q_{\text{hip}}, \) and \( q_{\text{trunk}} \), respectively. The lengths of the thigh, shank, cycle crank, and horizontal and vertical seat positions are denoted by \( l_{\text{thigh}}, l_{\text{shank}}, l_{\text{crank}} \), and \( l_i \) and \( b_i \), respectively. The regions \( Q_0 \) and \( Q_2 \) denote the crank angles where the muscles and motor are active, respectively.

where \( M: Q \to \mathbb{R}_{>0} \) denotes the combined inertial effects of the rider and cycle defined as \( M \triangleq J_c + M_r \), \( d: \mathbb{R}_{>0} \to \mathbb{R} \) denotes the disturbances applied by the rider and unmodeled effects in the system, and \( B_r \in \mathbb{R}_{>0} \) is the lumped, switched control effectiveness defined as

\[
B_r(q, \dot{q}) \triangleq \sum_{m \in M} B_m(q, \dot{q}) k_m \sigma_m(q).
\]  

The subscript \( \sigma \in \mathcal{P} = \{1, 2, 3, \ldots, n\}, \mathcal{P} \subset \mathbb{N}, n \in \mathbb{N} \) indicates the index of \( B_m \), which switches according to the crank position. The sequence of switching states \( \{\sigma_n\} \) are known and the corresponding sequence of switching times \( \{t_n\} \) are unknown and defined such that each \( t_n \) denotes the instant when \( q \) reaches the corresponding switching state \( \sigma_n \). The switching signal \( \sigma_m \) is assumed to be continuous from the right and designed to produce forward pedaling only. Fig. 1 shows a schematic of the cycle-rider system and illustrates the switching regions for the muscles and motor. The following assumption and properties from Bellman et al. (2016) are exploited in the subsequent control design and stability analysis.

**Assumption 1.** The disturbance \( d \) is bounded as \( |d| \leq \xi_d \) where \( \xi_d \in \mathbb{R}_{>0} \) is a known constant.

**Property 1.** \( c_m \leq M \leq c_M \), where \( c_m, c_M \in \mathbb{R}_{>0} \) are known constants.

**Property 2.** \( |V| \leq c_V |q| \), where \( c_V \in \mathbb{R}_{>0} \) is a known constant.

**Property 3.** \( |G| \leq c_G \), where \( c_G \in \mathbb{R}_{>0} \) is a known constant.

**Property 4.** \( |P| \leq c_{P1} + c_{P2} |\dot{q}| \), where \( c_{P1}, c_{P2} \in \mathbb{R}_{>0} \) are known constants.

**Property 5.** \( \frac{1}{2} \dot{M} - V = 0 \) by skew symmetry.

**Property 6.** The lumped switching control effectiveness is bounded as \( c_b \leq B \leq c_b \), \( \forall r \in \mathcal{P} \), where \( c_b, c_B \in \mathbb{R}_{>0} \) are known constants.

3. Control development

3.1. Cadence control

The first objective is to design a motor controller that tracks a desired cadence trajectory. The measurable angular crank position tracking error \( e: \mathbb{R}_{>0} \to \mathbb{R} \) and auxiliary tracking error \( r: \mathbb{R}_{>0} \to \mathbb{R} \) are defined as

\[
e(t) \triangleq q(t) - q_d(t),
\]

\[
r(t) \triangleq \dot{e}(t) + \alpha e(t),
\]

where \( q_d: \mathbb{R}_{>0} \to \mathbb{R} \) denotes the desired crank position and its first two time derivatives are bounded (i.e., \( |q_d(t)| \leq \xi_1 \) and \( |\dot{q}_d(t)| \leq \xi_2 \), where \( \xi_1, \xi_2 \in \mathbb{R}_{>0} \) are known) and \( \alpha \in \mathbb{R}_{>0} \) is a constant control gain. After taking the time derivative of (8) and premultiplying by \( M \), substituting for (5) and (7) and then performing some algebraic manipulation yield

\[
M \ddot{r} = -V r + \dot{\bar{N}} + \dot{B}_r u_{\text{FES}} + B_r u_e - e,
\]

where the auxiliary signals \( \chi: \mathbb{R}_{>0} \to \mathbb{R} \) and \( \bar{N}: \mathbb{R}_{>0} \to \mathbb{R} \) are defined as

\[
\chi \triangleq W_d - M(q)(\ddot{q}_d - \alpha e) - V(q, \dot{q})(\dot{q}_d - \alpha e) - G(q)
\]

\[
- P(q, \dot{q}) - c_d \ddot{q} + N_d + e,
\]

\[
\bar{N} \triangleq -(W_d + N_d + d),
\]

and the signals \( W_d: \mathbb{R}_{>0} \to \mathbb{R} \) and \( N_d: \mathbb{R}_{>0} \to \mathbb{R} \) are defined as

\[
W_d \triangleq M(q_0)\ddot{q}_d + V(q_0, \dot{q}_0)\dot{q}_d + G(q_0) + c_d \ddot{q}_d \quad \text{and} \quad N_d \triangleq c_{P1} + c_{P2} \ddot{q}_d,
\]

respectively. The auxiliary signal in (11) can be upper bounded as

\[
\bar{N} \leq \vartheta_1,
\]

where \( \vartheta_1 \in \mathbb{R}_{>0} \) is a known positive constant. By using Properties 1–4, (7), (8), and the Mean Value Theorem, an upper bound for (10) can be developed as

\[
\chi \leq \rho(|z|)||z||
\]

where \( z: \mathbb{R}_{>0} \to \mathbb{R}^2 \) is defined as \( z \triangleq [e r]^T \), and \( \rho(\cdot) \in \mathbb{R} \) is a known positive, radially unbounded, nondecreasing function. Given the cadence open-loop error system in (9), the motor control input is designed as

\[
u_e = -k_1 r - (k_2 + k_3 \rho(||z||)) \text{sgn}(r) + v_p,
\]

where \( k_1, k_2, k_3 \in \mathbb{R}_{>0} \) are selectable positive gain constants, \( \text{sgn}(\cdot): \mathbb{R} \to [\text{-}1, 1] \) is the signum function, and \( v_p: \mathbb{R}_{>0} \to \mathbb{R} \) is a subsequently designed control input. The cadence motor control input in (14) includes a feedback term and robust control terms to reject the disturbance in (5) and the state-dependent function in (13). The closed-loop cadence error system is obtained by substituting (14) into (9) as

\[
M \ddot{r} = -V r + \dot{N} + B_r u_{\text{FES}} - e - B_c (k_1 |r| - v_p + (k_2 + k_3 \rho(||z||)) \text{sgn}(r)).
\]

3.2. Spatial learning control for torque tracking

The second objective is to track a desired torque trajectory in the muscle stimulation regions \( q \in Q_m \). The torque tracking error signal is designed based on the difference between the desired torque and an estimate of the active torque produced by the muscle contractions in (3). Torque sensors are commonly included on rehabilitation cycles, which provide a measurement of the net torque contributions about the crank. To obtain direct measurement of muscle force real-time invasive sensing is required, which is not practical. Similar to previous FES experiments (cf. Bellman, 2015; Ha et al., 2016), a baseline measurement of the required torque to drive the cycle-rider system at a desired cadence is obtained without applying FES (i.e., \( r_e = 0 \) such that \( r = r_p \) in (2)) under the assumption that

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2 Functional dependencies are removed henceforth unless they add clarity to the exposition.
the disturbances are sufficiently small. By combining equations in (1) and (2), the dynamics can be expressed as
\[
\tau_a + \tau_r = \tau_e + \tau_p. \quad (16)
\]
A nominal torque measurement \( \tau_n : \mathbb{R} \rightarrow \mathbb{R} \) can be obtained from \( (16) \) as \( \tau_n = \tau_r = \tau_e + \tau_p \) when FES is not applied (i.e., \( \tau_n = 0 \)).

**Assumption 2.** An estimate of the nominal torque measurement \( \hat{\tau}_n : \mathbb{R} \rightarrow \mathbb{R} \) can be obtained using fitting techniques given continuous net torque measurements (Bellman, 2015). The mismatch between the nominal torque and the nominal torque estimate \( \hat{\tau}_n : \mathbb{R} \rightarrow \mathbb{R} \) is defined as \( \hat{\tau}_n \triangleq \tau_n - \tau_n \leq e_n \) where \( e_n \in \mathbb{R}_0 \) is a known upper bound of the estimation error. This assumption is acceptable when FES is not applied (i.e., \( \tau_n = 0 \)) during preliminary testing and if the desired cadence used in this test is the same as during the actual cycling experiment (Cousin et al., 2017).

Subtracting the nominal torque estimate \( \hat{\tau}_n \) from both sides of \( (16) \) yields
\[
\tau_a = \hat{\tau}_n + \tau_n - \tau_r. \quad (17)
\]
Combining \( (17) \) with the estimate of the net active muscle torque \( \hat{\tau}_a \) defined as \( \hat{\tau}_a \triangleq \hat{\tau}_n - \tau_e \) yields
\[
\tau_a = \tau_a - \hat{\tau}_a. \quad (18)
\]
To quantify the torque control objective, an integral torque tracking error-like term \( e_r : \mathbb{R} \rightarrow \mathbb{R} \) is defined as
\[
e_r = \int_{t_0}^{t} \left( \tau_a(p) - \hat{\tau}_a(p) \right) dp, \quad (19)
\]
where \( \tau_a : \mathbb{R} \rightarrow \mathbb{R} \) denotes a bounded periodic desired torque trajectory such that \( |\tau_a| \leq \beta_d \).

**Remark 1.** In \( (19) \), the torque trajectory \( \tau_a \) is a function of time. However in the experiments in Section 5, the desired torque trajectory \( \tau_a \) is a bounded periodic function of the crank angle \( q \in [0, 2\pi] \). Hence, a mapping between time and space is needed. This mapping is feasible since there exists a relationship between time and crank position. The angular speed of the system is defined as \( \dot{q} \triangleq dq/dt \), which can be integrated to yield \( q = \int_{0}^{t} \dot{q}(\psi) d\psi \triangleq f(t) \). In cycling only forward pedaling is allowed (no change of direction) and the desired cadence \( \dot{q_d} \) is positive. Moreover, the cadence controller in \( (14) \) is designed and proven to achieve \( \dot{q} > 0 \) (i.e., the actual cadence is nonzero) based on the stability proof in Section 4. Hence, \( q \) is a strictly increasing function of \( t \), (i.e., the relationship between \( t \) and \( q \) is bijective Xu & Huang, 2008). Thus the function \( q = f(t) \) is analytic and the inverse function \( t = f^{-1}(q) \) exists globally. Therefore, any function of \( t \) can be expressed as a spatial function of \( q \), e.g., \( \tau_a(t) \) can be expressed as \( \tau_a(f^{-1}(q)) \).

The open-loop error system is obtained by taking the time derivative of \( (19) \) and using \( (18), (3), (4), \) and \( (6) \) as
\[
\dot{e}_r = \tau_d - B_\tau u_{FES} + \tilde{\tau}_n. \quad (20)
\]
Given the open-loop error system \( (20) \), the muscle control input is designed as
\[
u_{FES} = \bar{W}_d + k_\xi e_r + k_\chi \tilde{W}_d, \quad (21)
\]
where \( k_\xi, k_\chi \in \mathbb{R}_0 \) are positive constant control gains, and \( \bar{W}_d : \mathbb{R} \rightarrow \mathbb{R} \) is the subsequently designed RLC update law.

**Remark 2.** The RLC is typically designed based on the knowledge of the time period \( T \) of a periodic process (Dixon et al., 2002; Sun et al., 2006). In this paper, the RLC is designed based on the system periodicity (crank position) of the desired torque trajectory \( \tau_d \). Based on the mapping described in Remark 1, a spatial RLC denoted as \( W_d(t) = W_d(f^{-1}(q)) \) can be designed leveraging the fact that \( q - 2\pi \equiv f(t - T) \) and the existence of the map \( t - T = f^{-1}(q - 2\pi) \). Knowledge of the period \( T \) (i.e., the time to complete a revolution) is not necessary for the implementation of \( W_d \), but it can be computed as \( T = \int_{t-q}^{t-q} dt = \frac{1}{d} \int_{0}^{d} dq \). The period \( T \) varies across crank cycles because it depends on the achieved cadence tracking performance.

The RLC update law in \( (21) \) is defined as
\[
\dot{\bar{W}}_d = \Gamma \text{sat}_{\beta_1}(\bar{W}_d(t - T)) + k_\xi e_r, \quad (22)
\]
where \( \Gamma \) is a strictly increasing function of \( e_r \) satisfying \( k_\xi < \beta_1 \). The closed-loop error system \( (21) \) is obtained by substituting \( (21) \) into \( (20) \) as
\[
\dot{e}_r = \bar{W}_d + \bar{W}_d + \tilde{\tau}_n - B_\tau (\bar{W}_d + k_\xi e_r + k_\chi \tilde{W}_d), \quad (24)
\]
where \( \bar{W}_d : \mathbb{R} \rightarrow \mathbb{R} \) is the learning estimation error defined as \( \bar{W}_d \triangleq \tau_a - \tilde{\tau}_n \). Based on the periodicity and boundedness of \( \tilde{\tau}_n \), \( \bar{W}_d(t) = \text{sat}_{\beta_1}(\bar{W}_d(t)) = \text{sat}_{\beta_1}(\bar{W}_d(t - T)) \). Hence, by exploiting \( (22) \), the following expression can be developed
\[
\dot{\bar{W}}_d = \text{sat}_{\beta_1}(\bar{W}_d(t - T)) - \Gamma \text{sat}_{\beta_1}(\bar{W}_d(t - T)) - k_\xi e_r. \quad (25)
\]

**4. Stability analysis**

The stability of the RLC muscle and sliding-mode motor controllers can be examined independently through the following two theorems. **Theorem 1** shows that the closed-loop torque error system is output strictly passive (OSP) and ensures asymptotic tracking or uniformly ultimately bounded (UUB) tracking provided two conditions, which hold for all time, are satisfied, respectively. **Theorem 2** shows that the closed-loop cadence error system is OSP and exponential tracking is achieved for \( q \neq \Omega_d \) ensuring passivity with respect to the muscle input. **Lemma 1** is used to prove that the time derivative of the torque tracking error in \( (19) \) is uniformly bounded.

**Theorem 1.** The closed-loop error system in \( (24) \) is OSP from input \( \nu_1 \) to output \( e_r \) if \( q \neq \Omega_d \). The controller designed in \( (21) \) and RLC in \( (22) \) ensures asymptotic tracking if \( \lim_{t \to \infty} e_r(t) = 0 \) and UUB tracking if \( \lim_{t \to \infty} e_r(t) < \lambda_2 \) in the sense that
\[
|e_r| \leq \sqrt{e_r(t_0)\epsilon_1 - \frac{\lambda_2}{\lambda_3} (1 - e^{-\lambda_3})} + \sqrt{\frac{\lambda_2}{\lambda_3} (1 - e^{-\lambda_3})}, \quad (26)
\]
where \( \lambda_1 \triangleq \min \left( \frac{1}{2}, \frac{\lambda_2}{\lambda_3} \right), \lambda_2 \triangleq \max \left( \frac{1}{2}, \frac{\lambda_2}{\lambda_3} \right), \lambda_3 \triangleq \lambda_5 + \frac{k_\xi e_r + k_\chi \tilde{W}_d}{\lambda_3}, \) and \( \lambda_5 \triangleq \frac{k_\xi e_r}{\lambda_3}. \)

For \( q \neq \Omega_d \), the torque controller in \( (21) \) and desired torque trajectory \( \tau_d \) are zero.
\begin{proof}
Let \( V_1 : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R} \) be a nonnegative, continuously differentiable, storage function defined as
\[
V_1 \leq \frac{1}{2} \dot{e}_1^2 + \frac{1}{2k_e} \int_{t-T}^{t} \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right)^2 d\psi. \quad (27)
\]

The storage function in (27) satisfies the following inequalities:
\[
\lambda_1 \|w\|^2 \leq V_1 (w, t) \leq \lambda_2 \|w\|^2, \quad (28)
\]
\[
V_1 (w, t) \leq \lambda_3 \|e_r\|^2 + \lambda_4 \leq, \quad (29)
\]
where \( w \triangleq \{e_r, \sqrt{Q_1} \}, \; Q_1 \triangleq \int_{-T}^{t} \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right)^2 d\psi \), and \( \lambda_3, \lambda_4 \) are known positive bounding constants. Let \( w(t) \) be a Filippov solution to the differential inclusion \( \dot{w} \in K[h(w)] \), where \( K \cdot \) is defined as \( \text{Filippov (1964)} \) and \( h \) is defined using (24) as \( h \triangleq [h_1 \; h_2] \), where \( h_1 \triangleq \dot{W}_d + \dot{\tau}_d - B_r (\dot{W}_d + k_4 e_r + k_5 \dot{W}_d), \; h_2 \triangleq \frac{1}{2k_e} \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right)^2 - \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right)^2 \right). \)

The control input in (4) has the discontinuous lumped control effectiveness \( B_r \); hence, the time derivative of (27) exists almost everywhere (a.e.), i.e., for almost all \( t \). Based on Fischer, Kamalapurkar, and Dixon (2013, Lemma 1), \( V_1 (w(t), t) \triangleq \frac{1}{T} V_1 (w(t), t) \), where \( V_1 \) is the generalized time derivative of (27) along the Filippov trajectories of \( w = h(w) \) and is defined as \( V_1 \triangleq \int_{t_0}^{t} \frac{1}{T} \dot{e}_1 \right)^2 \). Then, since \( V_1 (w, t) \) is continuously differentiable in \( w, \partial V_1 = [V V_1] \), thus \( V_1 \leq \frac{1}{2k_e} \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right)^2 \), and after substituting for (24), the generalized time derivative of (27) can be expressed as
\[
\dot{V}_1 \leq \frac{1}{2k_e} \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right)^2. \quad (30)
\]

By employing the following property
\[
\left( \frac{1}{2} \dot{\tau}_d (\tau_d (\psi) - T) \right)^2 \leq \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right)^2, \quad (31)
\]
proven similarly as in Dixon et al. (2002, Appendix I) using \( B_r < B_e \), using \textbf{Property 6} to lower bound \( K[B_e] \), substituting for (25), and canceling terms, an upper bound for (30) can be developed as
\[
\dot{V}_1 \leq -\delta_1 e_r^2 + v_1 e_r, \quad (31)
\]
where \( v_1 = (1 + c_k - 5c_k \dot{W}_d + e_n, \; \delta_1 \triangleq c_k k_4 + \frac{1}{2} \). Integrating (31) yields
\[
\int_{t_0}^{t} v_1 \frac{d}{d\psi} \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right) d\psi \geq \int_{t_0}^{t} \left( \text{sat}_{k_e} (\tau_d (\psi)) - \Gamma \text{sat}_{k_e} (\dot{W}_d (\psi)) \right) d\psi. \quad (32)
\]
Hence, the closed-loop system in (24) is OSP from input \( v_1 \) to output \( e_r \). To ensure stability of the closed loop error system in (24), additional analysis is needed. The upperbound in (31) can be rewritten as
\[
\dot{V}_1 \leq -k_4 c_k e_r^2 + e_r \dot{W}_d (1 + c_k - 5c_k \dot{W}_d) + e_r e_n - \frac{1}{2} k_1 e_r^2. \quad (32)
\]
Selecting \( \delta_3 \triangleq \frac{k_1}{c_k} \) and \( k_1 \triangleq \frac{1}{2} k_1 e_r^2 \) and substituting them into (32) yields
\[
\dot{V}_1 \leq -k_4 c_k e_r^2 + e_r e_r e_n \left( 1 - \frac{1}{2} k_1 \right), \quad (33)
\]
where asymptotic tracking is achieved if \( e_r e_r e_n > \varepsilon_2 \) and by invoking (Fischer et al., 2013, Corollary 2) and since \( \dot{V}_1 (w, t) \)
\[
\dot{V}_1 \leq -k_4 c_k e_r^2 + e_r e_r e_n \left( 1 - \frac{1}{2} k_1 \right), \quad (33)
\]
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where asymptotic tracking is achieved if \( e_r e_r e_n > \varepsilon_2 \) and by invoking (Fischer et al., 2013, Corollary 2) and since \( \dot{V}_1 (w, t) \)
\[
\dot{V}_1 \leq -k_4 c_k e_r^2 + e_r e_r e_n \left( 1 - \frac{1}{2} k_1 \right), \quad (33)
\]
where asymptotic tracking is achieved if \( e_r e_r e_n > \varepsilon_2 \) and by invoking (Fischer et al., 2013, Corollary 2) and since \( \dot{V}_1 (w, t) \)
Integrating (41) yields \( \int_0^t v_2(\phi) r(\phi) d\phi \geq a \frac{c}{2} \left( V_2(t) - V_2(t_0) + \int_{t_0}^t \delta_2 |z(\phi)|^2 d\phi \right) \), where \( \delta_2 = \min \{ \alpha, k_x c \} \), and \( v_2 = B_s u_{\text{TES}} + c_r v_p \), which can be used to prove that the closed-loop system in (15) is OSP from input \( v_2 \) to output \( r \), provided the gain conditions in (39) are satisfied. Since the integral term in the right-hand side of passivity inequality is positive definite. Moreover, by setting \( v_p \triangleq -k_p r \), \( k_p \in \mathbb{R}_{>0} \) in (41), \( V_2 \frac{a c}{2} \triangleq -\delta_2 V_2 \), \( \delta_3 = \frac{\min \{ k_x, k_y \} }{\gamma q} \), during \( q \notin \mathcal{Q}_M \) since \( \alpha = 0 \implies B_s = 0 \). Therefore, the state-periodic desired torque trajectory is nonzero during the stimulation regions \( q \in \mathcal{Q}_M \) and is defined as 

\[
\phi(d(q)) = \begin{cases} 
A_0 \sin \left( \frac{q - q_1}{2 (q_2 - q_1)} \pi \right) & q_1 < q \leq \frac{1}{2} q_2 \\
A_0 \cos \left( \frac{q - q_2}{2 (q_3 - q_2)} \pi \right) + \frac{A_0}{2} & q_2 < q \leq q_3 \\
A_0 \sin \left( \frac{q - q_3}{2 (q_4 - q_3)} \pi \right) & q_3 < q \leq \frac{q_4 + q_5}{2} \\
A_0 \cos \left( \frac{q - \frac{q_4 + q_5}{2}}{2 (q_4 - q_3)} \pi \right) + \frac{A_0}{2} & \frac{q_4 + q_5}{2} < q \leq q_4 \\
0 & q_4 < q \leq q_5,
\end{cases}
\]

where \( q_1, q_3 \in \mathbb{R}_{<0} \) and \( q_2, q_4 \in \mathbb{R}_{>0} \) are the known and constant predefined crank angles that determine the start and the end of the stimulation regions, respectively. The peak torque amplitude \( A_0 \in \mathbb{R}_{>0} \) is defined as \( A_0 \triangleq \frac{1}{2} A_0 \), where \( B_s \triangleq 10 \) W, is the maximum desired power demand (unless stated otherwise), and \( q_d \) is the desired cadence. Since the electric motor tracks cadence throughout the entire crank cycle (i.e., the motor advances the crank position in and out of the torque tracking regions), the spatial RLC, which is initialized to zero, can be implemented without enforcing the typical resetting condition before the start of a new crank cycle or period. The control gains included in (4), (14), (21), and (22) were selected as follows: \( k_m \in [4.5, 5] \), \( \alpha \triangleq 2.5 \), \( k_i \triangleq 9 \), \( k_2 \triangleq 0.1 \), \( k_3 \triangleq 0.01 \), \( k_4 \in [60, 250] \), \( k_5 \triangleq 0.5 \), \( k_p \triangleq 0.001 \), \( \Gamma \in [0.95, 1] \), and \( k_l \in [25, 45] \).

5.3. Results

Table 2 summarizes the average of the cadence error (i.e., the time derivative of (7)), the average of the torque tracking error in (19), the average of the time derivative of (19), and the average power tracking error for \( t \in [t_e, t_u] \) sec. The power tracking error can be computed as the difference between the active power output \( P_a = \tau \dot{\theta} \) and the desired power output \( P_d = \tau \dot{\theta}_d \). For data analysis and to account for the time-delayed nature of muscle activation, the actual torque error \( \tau - \tau_{\text{desired}} \) was computed by averaging the active torque output \( \tau \) within a time window of 100 ms after the stimulation inputs were applied and turned off. Fig. 2 illustrates the switched muscle control inputs, the desired and actual torque output, and the RLC input for participant S3 (as a common example) after 2 min of cycling. Fig. 3 shows the root-mean-squared error (RMS) of the cadence tracking error calculated over a moving time interval window of 12 s, and

### Table 1

Demographics of participants with a neurological condition.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Age</th>
<th>Sex</th>
<th>Injury</th>
<th>Months since injury</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25</td>
<td>M</td>
<td>SB LS-S1</td>
<td>Since birth</td>
</tr>
<tr>
<td>B</td>
<td>32</td>
<td>M</td>
<td>SCI CS-C7, T12</td>
<td>76</td>
</tr>
<tr>
<td>C</td>
<td>28</td>
<td>F</td>
<td>MS</td>
<td>96</td>
</tr>
</tbody>
</table>

The experiments included a warm up, a passive torque estimation trial and the main cycling trial. Warm up trials at different speeds with and without open-loop stimulation pulse trains were conducted for the participants with NCs to acclimate them to the cycle. An estimate of the nominal torque \( \tau_n \) was obtained in a separate trial where the muscles were not stimulated and the electric motor was used to passively rotate the participant’s legs. The FES-cycling trial had a duration of \( t = 180 \) s. The electric motor tracked a time-varying cadence trajectory that reached a steady state value of 50 RPM after \( t_1 = 16 \) s. When the experiment duration reached \( t_2 = 21 \) s, the torque controller in (21) with RLC in (22) was activated and hence torque tracking started.

The desired torque trajectory was designed as a modified function of the knee joint torque transfer ratio, which can be computed as a function of the crank position. Therefore, the state-periodic desired torque trajectory is nonzero during the stimulation regions \( q \in \mathcal{Q}_M \) and is defined as

\[
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themusclestimulationintensities $u_m$ (top), active torque output $\tau_a$ and desired torque $\tau_d$ (middle), and RLC input $\dot{W}_d$ (bottom) during a single crank cycle for participant S3 after 2 min of cycling. The vertical solid bars correspond to the time where the torque output rises above zero, which illustrates the fact that muscle activation is affected by the muscle electromechanical delay (EMD) (Downey, Merad, Gonzalez, & Dixon, 2017).

The average cadence tracking errors were comparable between able-bodied participants (0.02 ± 1.17 RPM) and individuals with NCs (0.01 ± 1.23 RPM). The average power tracking errors varied between healthy (2.18 ± 1.52 W) and impaired (1.11 ± 0.58 W) individuals, primarily due to the differences in muscle strength and desired peak torques. The muscle torque RLC controller improved the power tracking with increasing cycles as depicted in Fig. 5. The muscle and motor controllers were able to compensate for different peak torques and cadences, respectively, as reported in Table 3.

5.4. Discussion

The average cadence tracking errors were comparable between able-bodied participants (0.02 ± 1.17 RPM) and individuals with NCs (0.01 ± 1.23 RPM). The average power tracking errors varied between healthy (2.18 ± 1.52 W) and impaired (1.11 ± 0.58 W) individuals, primarily due to the differences in muscle strength and desired peak torques. The muscle torque RLC controller improved the power tracking with increasing cycles as depicted in Fig. 5. The muscle and motor controllers were able to compensate for different peak torques and cadences, respectively, as reported in Table 3.

The results presented in this study align qualitatively with previously reported power tracking experiments. In Bellman (2015), three experimental results with healthy individuals were reported to track discrete power, where the power measurement was averaged over a full crank cycle and the controller was updated only at the beginning of the crank cycle. In Cousin et al.
controllers were designed for muscles to track cadence and the electric motor to track a resistive torque. A closed-loop controller with motor assistance was reported in Hunt et al. (2004), where the power output was monitored for a paraplegic. In Farhoud and Erfanian (2014), a FES-cycling power control objective with varying amplitudes between 5 and 10 W was implemented with three paraplegics via sliding mode control. Despite the existing power tracking results, the lack of homogeneity in reporting the power tracking performance makes difficult the cross comparisons among the published results.

The implementation of the experiments presented several challenges. The active torque elicited by the participants was obtained by subtracting the nominal torque estimate from the net torque measurement. However, the noise in the nominal torque estimate of the rider’s passive dynamics affected the quality of active torque signal. Future efforts should focus on improving the estimation of the rider’s passive dynamics. Muscle fatigue and electromechanical delay (EMD) are two factors that degrade power tracking performance. Muscle fatigue is a well known issue in FES research and asynchronous stimulation patterns have been developed in Downey, Bellman, Kawai, Gregory, and Dixon (2015) to alleviate the effects of fatigue. However, muscle response to EMD depends on the muscle activation dynamics, which is affected by EMD. As depicted in Fig. 2, there exists a muscle contraction delay illustrated by the time difference between the onset of the stimulation and the point where the participant’s active torque rises above the zero torque baseline. Recently in Downey et al. (2017), it was concluded for the quadriceps that the EMD increases as the number of muscle contraction increases under isometric conditions. Hence, muscle fatigue and delay are important factors to consider for the development of rehabilitative treatments using FES.

The experiments with participants with NCs presented additional challenges. Participant B experienced muscle weakness, thus high stimulation intensities were needed to evoke muscle contractions, and suffered intermittent spasms that acted as disturbances. Participants A and C also exhibited reduced muscle strength, which lead to reduce the peak power from 10 W to 5 W. Moreover, participants A and C elicited asymmetric torque profiles between right and left legs. To compensate for torque asymmetries, a split-crank bicycle is an ideal testbed for further experimentation. Despite these challenges, the torque controller adjusted the muscle stimulation intensities to successfully complete the experiments. Clinical trials with populations with other NCs such as individuals with traumatic brain injury, Parkinson’s disease, etc., are required to investigate the long-term benefits of the developed control methods.

6. Conclusion

A motor cadence controller and a muscle torque controller were implemented in this paper to achieve power tracking in FES-cycling experiments. The switched muscle torque controller included a spatial dimension of input to cope with the state periodicity of the desired torque trajectory. A passivity-based analysis was developed to ensure stability of the torque and cadence closed-loop systems. The average cadence tracking error was 0.01±1.20 RPM and the average power tracking error was 1.78±1.25 W including all the participants. Muscle fatigue and the EMD are important factors that degrade the efficacy of the control methodology. As in results such as Downey et al. (2017), Merad, Downey, Obuz, and Dixon (2016) and Sharma, Gregory, and Dixon (2011), the muscle exhibits a delayed response to external electrical stimulation, and this response varies in time with muscle fatigue. Future efforts are required to develop controllers and analysis methods that can compensate for such delays in switched systems. Longitudinal studies are required to test the long-term benefits of learning control methods for power tracking.

References


Table 3

<table>
<thead>
<tr>
<th>Cadence (RPM)</th>
<th>Power (W)</th>
<th>Power error (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Peak</td>
<td>0.08 ± 0.11</td>
</tr>
<tr>
<td>10</td>
<td>Peak</td>
<td>0.17 ± 0.25</td>
</tr>
<tr>
<td>10</td>
<td>Peak</td>
<td>1.67 ± 0.95</td>
</tr>
<tr>
<td>10</td>
<td>Peak</td>
<td>2.15 ± 1.55</td>
</tr>
</tbody>
</table>

Mean: 0.01 ± 0.08


