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Single Agent Indirect Herding of Multiple Targets: A Switched Adaptive Control Approach

Ryan A. Licitra, Zachary I. Bell, Emily A. Doucette, and Warren E. Dixon

***Abstract*—Indirect herding involves using a set of directly controllable herding agents to indirectly regulate the states of target agents. This letter considers a single herding agent tasked with regulating the states of multiple targets that are repelled from the herder and the goal location. Integral concurrent learning is used to develop an exponentially convergent adaptive controller that is proven to achieve globally uniformly ultimately bounded regulation through a Lyapunov-based analysis. Switched systems methods are used to develop dwell-time conditions that direct the herder to switch between targets. Experimental results with a network of quadcopters are included to illustrate the performance of the developed controller.**

***Index Terms*—Adaptive control, switched control, Lyapunov methods.**

I. Introduction

HERDING is a type of leader-follower network that usually refers to a scenario where the leader directs the other agents (i.e., rather than simply acting as a guidepost for the followers as in general leader-follower networks). Herding generally is used to denote a position consensus objective, and flocking is a similar problem for velocity vector alignment. To draw a distinction with the problems considered in this letter, each of the aforementioned network control problems are labeled as *direct* herding problems since leaders and followers all follow a designed control strategy.

Some applications involve networks of agents where agents do not cooperate and don't share a common objective. The pursuer-evader problem (see [1], [2]) is a common such example. These problems feature evader agents that are essential to the objective but don't cooperate to achieve it. A generalization of the pursuer-evader problem is the herding pursuer-evader problem (see [3]–[7]) where the pursuer is tasked with regulating the evader to a goal location, in addition to the typical objective that ends with capture/interception. These problems are labeled as *indirect* herding problems since only the pursuer agents follow a designed control strategy, while evaders are controlled indirectly by exploiting models of the interaction dynamics (i.e., the evaders can only be controlled through pursuer interactions).

Several solution methods have been developed for *indirect* herding problems. In [3], the *indirect* herding pursuer-evader problem is solved for a single pursuer and single evader with known dynamics by solving an on-line optimal control problem. Approaches such as [4]–[6] solve the indirect herding pursuer-evader problem by using off-line numerical solutions to a differential game where there are equal or more pursuers that chase evaders with known dynamics. The result in [7] also uses an off-line solution to a differential game that can involve more evaders than pursuers, where the herder captures targets at the goal location. The solution in [8] develops a forcing function, based on two or more herders forming an arc, that is used to direct a single target along a desired trajectory. The result in [9] uses a similar arc-based approach to regulate the mean location of a larger herd to a desired goal by considering the entire herd as a single unicycle.

This letter seeks to solve the *indirect* herding problem with more target agents than herders. This added challenge motivates the use of a switched systems analysis to develop dwell-time conditions which dictate how long the herder can chase any given target before it must switch to another target, which is unnecessary when the herder team isn't outnumbered by the targets (see [3]–[6], [8]). In our previous preliminary efforts in [10], the single herder for multiple agents problem was formulated with the same model considered in this letter. However, in [10] a robust sliding mode approach is used to compensate for worse case uncertainties in the target dynamics. The sliding mode controller yielded a strict Lyapunov function that facilitated the switched stability analysis and corresponding dwell time conditions. In comparison, the current result investigates an adaptive control strategy to learn and compensate for the uncertainties in the target dynamics. However, traditional adaptive controllers only yield a negative

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R. A. Licitra, Z. I. Bell, and W. E. Dixon are with the Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611-6250 USA (e-mail: rlicitra@ufl.edu; bellz121@ufl.edu; wdixon@ufl.edu).

E. A. Doucette is with the Munitions Directorate, Air Force Research Laboratory, Eglin AFB, FL 32542 USA (e-mail: emily.doucette@us.af.mil).

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semi-definite Lyapunov derivative unless a very restrictive (and impossible to verify on-line) persistence of excitation (PE) condition is met, which would be problematic for the switched systems analysis and discovery of dwell time conditions. Hence, motivated by the concurrent learning method in [11], the developed adaptive control strategy includes a unique integral concurrent learning (ICL) strategy that does not require measurement of higher order derivatives (see [11]) and yields a finite excitation (FE) condition. Development is provided that shows before sufficient excitation occurs, the targets can be regulated to a neighborhood of the goal positions, and if the FE condition is satisfied the targets are proven to converge to a smaller neighborhood about the goal. The development of the adaptive switching controller is further complicated by the fact that the target agent dynamics don't explicitly contain a control input, so the model of the interaction between the targets and herder is exploited to influence the targets to the goal. A backstepping strategy is used to develop a virtual control input in the target dynamics, and then the actual herder input is used to regulate the backstepping error. Sufficient dwell-time conditions determine the number of targets a single herder can successfully herd for a certain set of parameters and gains. Moreover, the analysis in this letter is agnostic to the specific design of the herding strategy, i.e., a myriad of switching strategies may be employed to meet the sufficient dwell-time conditions. Experimental results employ an example herding strategy that accomplishes the herding task for a set of three quadcopter targets and one herder quadcopter.

II. Problem Formulation

The herder agent is tasked with regulating $n_t > 1$ targets to a goal location. Each target state is denoted by $x_i \in \mathbb{R}^n$, with a respective constant goal location $x_i^g \in \mathbb{R}^n$, $i \in \{1, 2, \ldots, n_t\}$, and the herder state is $y \in \mathbb{R}^n$, where n is the dimensionality of the system. The target, goal and herder states are assumed to be measurable (available for feedback control). To quantify the control objective, the herding error for each target, denoted by $\bar{x}_i \in \mathbb{R}^n$, is defined as $\bar{x}_i \triangleq x_i - x_i^g$. Unlike leader-follower networks where each agent's interaction is controlled, in the *indirect* herding problem, only the herder's action is controlled and each target's interaction with the herder is inherent to the dynamics of the target. For simplicity,[1] the herder dynamics are given by $\dot{y} = u_y$, where $u_y \in \mathbb{R}^n$ is the subsequently developed herder control input. Each target's dynamic model is assumed to have several qualitative behaviors: targets are repelled by the herder, targets are repelled by the goal location, and otherwise the target remains at rest (i.e., as the norm of the herder distance to a target agent approaches infinity, the target dynamics approach zero).

Remark 1: The qualitative behaviors of the target agents are inspired by practical considerations and behaviors often seen in nature. The assumption that the targets are repelled by the herder is inherent to any herding problem (e.g., prey flee predators, sheep run from a herding dog) and additionally the targets will want to avoid the goal location (e.g., fish do not want to be herded to shallow water by dolphins). Target agents are considered to be content with their current position (e.g., an animal grazing) when the herder is not nearby, and therefore will remain in their current location. The Gaussian potential functions and exponentials used in (1) are commonly used functions (e.g., in path planning literature) to quantify such behaviors. However, other more generalized models are areas of future research.

Given the objective to have one herder regulate the position of n_t targets, a unique challenge is that the herder is required to switch between target agents. The subsequent development assumes that the targets know when they are being chased, and in general, may respond differently when chased. That is, another qualitative behavior is a target may exhibit a more aggressive repulsion from the herder when chased.

To quantify this distinct behavior, let $t_{i,k}^c \in \mathbb{R}$ and $t_{i,k}^u \in \mathbb{R}$ denote the k^{th} instance when the i^{th} target is switched to the *chased* or *unchased* mode, respectively, where $k \in \mathbb{N}$. The contiguous dwell time in the k^{th} activation of the i^{th} target operating in the *chased* or *unchased* mode is denoted by $\Delta t_{i,k}^c \in \mathbb{R}$ and $\Delta t_{i,k}^u \in \mathbb{R}$, and defined as $\Delta t_{i,k}^c \triangleq t_{i,k}^u - t_{i,k}^c$ and $\Delta t_{i,k}^u \triangleq t_{i,k+1}^c - t_{i,k}^u$, respectively. The total amount of time each of these modes is active between switching instances a and b are denoted $T_i^c(a,b) \triangleq \sum_{l=a}^{b} \Delta t_{i,l}^c$ and $T_i^u(a,b) \triangleq \sum_{l=a}^{b} \Delta t_{i,l}^u$, respectively. To quantify the aforementioned qualitative characteristics, the i^{th} target dynamics are modeled as

$$\dot{x}_i = \begin{cases} (\alpha_i(x_i - y) + \beta_i \bar{x}_i)\, \mathrm{e}^{-\chi_i} & t \in \left[t_{i,k}^c, t_{i,k}^u\right) \\ (\gamma \alpha_i(x_i - y) + \beta_i \bar{x}_i)\, \mathrm{e}^{-\chi_i} & t \in \left[t_{i,k}^u, t_{i,k+1}^c\right) \end{cases}, \quad (1)$$

$\forall k \in \mathbb{N}$, where $\gamma \in [0, 1]$ is a known parameter[2] that scales the repulsion effect of the herder on the targets operating in *unchased* mode, $\alpha_i \in \mathbb{R}$ and $\beta_i \in \mathbb{R}$, $i \in \{1, 2, \ldots, n_t\}$ are unknown positive constant parameters with common (without loss of generality) upper and lower bounds denoted by $\bar{(\cdot)}$ and $\underline{(\cdot)}$, respectively, defined as $\bar{\alpha}$, $\underline{\alpha}$, $\bar{\beta}$, and $\underline{\beta}$, and the auxiliary function $\chi_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is defined as $\chi_i \triangleq \frac{1}{2\sigma^2}(x_i - y)^T(x_i - y)$, where $\sigma^2 \in \mathbb{R}$ is a known positive constant that determines the radial size of the Gaussian potential function $\mathrm{e}^{-\chi_i}$. The first term in each equation in (1) is a gradient of the Gaussian potential that models the repulsion interaction between the herder and the i^{th} target, while the second term represents the target's tendency to escape from the goal. To facilitate the subsequent control design and stability analysis, the uncertain parameters in (1) are grouped into the uncertain vector $\theta_i \in \mathbb{R}^2$ as $\theta_i \triangleq [\alpha_i \quad \beta_i]^T$, while $\hat{\theta}_i \in \mathbb{R}^2$ denotes a subsequently designed adaptive estimate of θ_i, and $\tilde{\theta}_i \in \mathbb{R}^2$ denotes the estimation error, defined as $\tilde{\theta}_i \triangleq \theta_i - \hat{\theta}_i$ $\forall i \in \{1, 2, \ldots, n_t\}$.

[1]The control development can be generalized to include herder dynamics (e.g., Euler-Lagrange dynamics) through modifications of the controller and stability results using known methods. Single integrator dynamics are used for simplicity and to focus on the technical challenges uniquely associated with the herding problem without involving additional (more common) challenges associated with herder dynamics.

[2]The scaling parameter γ is required to be known since it is unclear how to learn parameters during the periods in which a target operates in the *unchased* mode.

Remark 2: The following design strategy can be applied to more generalized models, as long as certain properties that are standard in Lyapunov switching stability analysis (e.g., continuous dynamics between switches) or unique to this problem (e.g., error term $x_i - y$ present in the target dynamics to facilitate backstepping) are satisfied. The first term in (1) is derived from taking the negative gradient of a potential function of the square of the norm of the error term $x_i - y$. The gradient of various kernel functions (Gaussian, Epanechnikov, etc.) satisfy this property. Moreover, function approximation strategies could be employed to learn models without a known form.

Given the different dynamics for the *chased* and *unchased* modes in (1), the subsequent development entails the design of a herding controller and switching conditions to ensure the switched system is stable. Since the target agent dynamics in (1) do not explicitly contain a control input, a backstepping strategy is used to inject the desired herder state as a virtual controller, $y_d \in \mathbb{R}^n$, into the dynamics of the target currently operating in the*chased* mode. Therefore, in addition to regulating the target herding error, the subsequent development also entails minimizing the backstepping error $e_y \in \mathbb{R}^n$, defined as

$$e_y(t) \triangleq y_d(t) - y(t). \tag{2}$$

III. Control Development

The following development is based on the strategy that the herder switches between targets to achieve the overall herding objective. Since the herder only chases one target at a time, the herder's controller always uses the i^{th} target as the *chased* target (i.e., unless otherwise stated, the development in this section considers that the i^{th} target is the one currently operating in *chased* mode ($t \in [t_{i,k}^c, t_{i,k}^u), \forall k \in \mathbb{N}$)). To develop the controller, the *chased* target dynamics in (1), as well as the backstepping error in (2), are used to express the open-loop target herding error dynamics as

$$\dot{\bar{x}}_i = \alpha_i \left(x_i + e_y - y_d + \frac{\beta_i}{\alpha_i} \bar{x}_i \right) e^{-\chi_i}. \tag{3}$$

Based on (3) and the subsequent stability analysis, the herder's desired state is designed as

$$y_d \triangleq K_1 \bar{x}_i + x_i^g, \tag{4}$$

where $K_1 = k_1 + k_2$ and k_1, k_2 are positive constant control gains. Using (4), (3) can be rewritten as

$$\dot{\bar{x}}_i = \alpha_i \left(-k_1 \bar{x}_i - \left(k_2 - \left(\frac{\beta_i}{\alpha_i} + 1 \right) \right) \bar{x}_i + e_y \right) e^{-\chi_i}. \tag{5}$$

Given the herder's desired state in (4), the backstepping error dynamics can be determined by taking the time derivative of (2), and using the *chased* dynamics in (1) and the herder dynamics to obtain

$$\begin{aligned} \dot{e}_y &= K_1 (\alpha_i (x_i - y) + \beta_i \bar{x}_i) e^{-\chi_i} - u_y \\ &= Y_i \theta_i - u_y, \end{aligned} \tag{6}$$

where $Y_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times 2}$ is the regression matrix for the currently *chased* target, defined as $Y_i \triangleq [K_1 (x_i - y) e^{-\chi_i} \quad K_1 \bar{x}_i e^{-\chi_i}]$, and θ_i contains the unknown parameters associated with the target currently operating in *chased* mode. Based on (6) and the subsequent stability analysis, the herder control law is designed as

$$u_y \triangleq k_y e_y + \bar{x}_i e^{-\chi_i} + Y_i \hat{\theta}_i, \tag{7}$$

where k_y is a positive constant control gain. Using (7), the closed-loop backstepping dynamics are

$$\dot{e}_y = -k_y e_y - \bar{x}_i e^{-\chi_i} + Y_i \tilde{\theta}_i. \tag{8}$$

The parameter estimate for the *chased* target $\hat{\theta}_i$ in (7) is generated from the ICL-based adaptive update law

$$\dot{\hat{\theta}}_i \triangleq \text{proj}\left\{ \Gamma Y_i^T e_y + k_{cl} \Gamma S_{cl} \right\} \tag{9}$$

where proj$\{\cdot\}$ is a smooth projection operator, $S_{cl} \triangleq \sum_{j=1}^{N_i} (\mathcal{Y}_{ij}^T (K_1 (\bar{x}_i(t_j) - \bar{x}_i(t_j - \Delta t)) - \mathcal{Y}_{ij} \hat{\theta}_i))$, $\Gamma \in \mathbb{R}^{2\times 2}$ and $k_{cl} \in \mathbb{R}$ are constant, positive definite control gains, $N_i \in \mathbb{Z}$ is a constant that represents the number of saved data points for the data stack of the i^{th} target, $t_j \in [0, t]$ are time points between the initial time and the current time, $\Delta t \in \mathbb{R}$ is a positive constant denoting the size of the window of integration, $\mathcal{Y}_{ij} \triangleq \mathcal{Y}_i(t_j) \in \mathbb{R}^{n\times 2}$ is the integrated regression matrix at $t = t_j$,

$$\mathcal{Y}_i(t) \triangleq \begin{cases} 0_{n\times 2} & t \in [0, \Delta t] \\ \int_{t-\Delta t}^{t} Y_i(x_i(\varsigma), y(\varsigma)) d\varsigma & t > \Delta t \end{cases}, \tag{10}$$

and $0_{n\times 2}$ denotes an $n \times 2$ matrix of zeros.

Remark 3: The first term in (9) is a traditional gradient-based adaptive control term, which uses the system error signal as feedback to estimate the unknown parameter vector θ_i and is used to cancel coupled terms in the stability analysis. A smooth projection operator is used to bound the adaptive update law to facilitate the analysis prior to parameter identification. See [12, Remark 3.6 or Sec. 4.4] for details of the projection operator. The summation of terms are unique to integral concurrent learning and involve the use of recorded (concurrent to the controller execution) input/output data for parameter identification with finite excitation. In particular, $\mathcal{Y}_{ij}$ refers to a single data point of the integral of the regression matrix Y_i, relaxing the need for measurement of higher order derivative terms (in this case velocity). Since integral concurrent learning is based on collecting input/output data, it can be employed independently of the dynamics of the model provided the uncertainty satisfies the linear-in-the-uncertain-parameters assumption. Moreover, as long as the system dimension is 2 or greater, the 2 unknown parameters could be computed using a least squares formulation (which requires the FE condition in this letter) or by simply measuring the velocity at any instance that the target, herder, and goal are non-collinear, however it is not assumed that velocity measurements are available in this letter (and the use of an ICL-based update law eliminates this requirement).

The data points that are saved are selected to maximize the minimum eigenvalue of $\sum_{j=1}^{N_i} (\mathcal{Y}_{ij}^T \mathcal{Y}_{ij})$ (See [13] for methods of selecting data for concurrent learning). Integrating the definition of $Y_i \theta_i$, applying the Fundamental Theorem of Calculus, and substituting in (10) yields $K_1 (\bar{x}_i(t) - \bar{x}_i(t - \Delta t)) = \mathcal{Y}_i(t) \theta_i \ \forall t > \Delta t$, which can be used to rewrite the adaptive update law (9) in the following equivalent but non-implementable[3] form:

$$\dot{\hat{\theta}}_i = \text{proj}\left\{ \Gamma Y_i^T e_y + k_{cl}\Gamma \sum_{j=1}^{N_i} \left(\mathcal{Y}_{ij}^T \mathcal{Y}_{ij}\right)\tilde{\theta}_i \right\}. \tag{11}$$

Additionally, since it is infeasible to learn the parameters of targets operating in the *unchased* mode, the adaptive update law will be turned off during these periods, i.e., $\dot{\hat{\theta}}_i = 0, \quad \forall t \in [t_{i,k}^u, t_{i,k+1}^c), \ \forall k \in \mathbb{N}$.

IV. Stability Analysis

The subsequent stability analysis considers the behavior of the i^{th} target when it is in the *chased* and *unchased* modes. Two time phases must be also considered: an initial phase before sufficient data has been collected to satisfy the FE condition, and a second phase after sufficient excitation has occurred. Specifically, ICL assumes that the following FE[4] condition is satisfied

$$\exists \underline{\lambda}, \tau_i > 0 : \forall t \geq \tau_i, \ \lambda_{\min}\left\{ \sum_{j=1}^{N_i} \mathcal{Y}_{ij}^T \mathcal{Y}_{ij} \right\} \geq \underline{\lambda}, \tag{12}$$

where $\lambda_{\min}\{\cdot\}$ refers to the minimum eigenvalue of $\{\cdot\}$. In Section IV-A, Lemma 1 shows that during periods when a target is *chased*, the system states associated with the i^{th} target are asymptotically stable prior to sufficient excitation ($t \in [0, \tau_i)$) and exponentially stable after sufficient excitation ($t \in [\tau_i, \infty)$). Lemma 2 in Section IV-B shows that when the i^{th} target is *unchased*, the target states remain bounded for all bounded t. Once these convergence analyses have been completed, a combined analysis will be carried out to discover how the overall system evolves when subject to a discrete switching signal. Specifically, in Section IV-C, Theorems 1 and 2 provide an ultimate bound for the system states associated with the i^{th} target during the two time phases, respectively, provided that the developed dwell time conditions are met. The ultimate bound in Theorem 2 is proven to be smaller than that in Theorem 1 based on the fact that the system states converge exponentially during periods that the target operates in chased mode once (12) is satisfied (up until the point that (12) is satisfied there are additional terms that prevent pure exponential convergence).

To facilitate the following analysis, let $V_i : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^2 \to \mathbb{R}$ be a positive definite, continuously differentiable candidate Lyapunov function, defined as

$$V_i(z_i(t)) \triangleq \frac{1}{2\alpha_i}\bar{x}_i^T \bar{x}_i + \frac{1}{2}e_y^T e_y + \frac{1}{2}\tilde{\theta}_i^T \Gamma^{-1}\tilde{\theta}_i, \tag{13}$$

which can be bounded by

$$c_1\|z_i(t)\|^2 \leq V_i(z_i(t)) \leq c_2\|z_i(t)\|^2, \tag{14}$$

where $z_i \triangleq [\,\bar{x}_i^T \quad e_y^T \quad \tilde{\theta}_i^T\,]^T$ and $c_1, c_2 \in \mathbb{R}$ are known positive bounding constants. Moreover, since the use of the projection algorithm in (11) ensures that $\tilde{\theta}_i, \hat{\theta}_i \in \mathcal{L}_\infty$, then the Lyapunov function candidate can also be upper bounded as

$$V_i(z_i(t)) \leq c_3\left\|\left[\,\bar{x}_i^T \ e_y^T\,\right]\right\|^2 + c_4, \tag{15}$$

where $c_3, c_4 \in \mathbb{R}$ are known positive bounding constants.

A. Target Operating in the Chased Mode

Lemma 1: The controller given in (4), (7), and the adaptive update law in (9) ensure that all system signals associated with the i^{th} target are bounded under closed-loop operation and that $\forall t \in [t_{i,k}^c, t_{i,k}^u), \forall k \in \mathbb{N}$,

$$\|z_i(t)\|^2 \leq \frac{c_2}{c_1}\left\|z_i\left(t_{i,k}^c\right)\right\|^2 \mathrm{e}^{-\lambda_1\left(t-t_{i,k}^c\right)} + \frac{c_4}{c_1} \tag{16}$$

provided that the gains are selected according to the sufficient condition

$$k_2 \geq \frac{\bar{\beta}}{\underline{\alpha}} + 1. \tag{17}$$

Moreover, provided the inequality in (12) is satisfied (i.e., the trajectories are sufficiently exciting), then $\forall t \in [t_{i,k}^c, t_{i,k}^u) \cap [\tau_i, \infty), \forall k \in \mathbb{N}$,

$$\|z_i(t)\|^2 \leq \frac{c_2}{c_1}\left\|z_i\left(t_{i,k}^c\right)\right\|^2 \mathrm{e}^{-\lambda_2\left(t-t_{i,k}^c\right)}. \tag{18}$$

Proof: Using (5), (8), and (11), and provided that the gain condition (17) is satisfied, the time derivative of (13) during $t \in [t_{i,k}^c, t_{i,k}^u), \forall k \in \mathbb{N}$ can be upper bounded as

$$\dot{V}_i(z_i(t)) \leq -k_1\|\bar{x}_i(t)\|^2 \mathrm{e}^{-\chi_i} - k_y\left\|e_y(t)\right\|^2. \tag{19}$$

Since $V_i \geq 0$ and $\dot{V}_i \leq 0$, $V_i \in \mathcal{L}_\infty$; therefore, $\bar{x}_i, e_y, \tilde{\theta}_i \in \mathcal{L}_\infty$. Since $\bar{x}_i \in \mathcal{L}_\infty$ and the goal position $x_i^g \in \mathcal{L}_\infty$ by assumption then (4) and the target herding error can be used to prove that $x_i, y_d \in \mathcal{L}_\infty$. Since $e_y, y_d \in \mathcal{L}_\infty$, (2) indicates that $y \in \mathcal{L}_\infty$. Since $x_i, y \in \mathcal{L}_\infty$, then $\chi_i \in \mathcal{L}_\infty$. Since $\chi_i \in \mathcal{L}_\infty$, then $\exists \ \bar{\chi}_i > 0 : \chi_i(t) \leq \bar{\chi}_i \quad \forall t \in [t_{i,k}^c, t_{i,k}^u), \ \forall k \in \mathbb{N}$. The facts that $x_i, \bar{x}_i, y, \chi_i \in \mathcal{L}_\infty$ can be used to show that the regression matrix $Y_i \in \mathcal{L}_\infty$, and hence, $u_y \in \mathcal{L}_\infty$ from (7).

Based on (15), the inequality in (19) can be upper bounded as

$$\dot{V}_i(z_i(t)) \leq -\lambda_1(V_i(z_i(t)) - c_4) \tag{20}$$

where $\lambda_1 \triangleq \frac{1}{c_3}\min\{k_1 \min_i \mathrm{e}^{-\bar{\chi}_i}, k_y\}$. Applying the Comparison Lemma [14, Lemma 3.4] to (20) yields

$$V_i(z_i(t)) \leq V_i\left(z_i\left(t_{i,k}^c\right)\right) \mathrm{e}^{-\lambda_1\left(t-t_{i,k}^c\right)} + c_4, \tag{21}$$

$\forall t \in [t_{i,k}^c, t_{i,k}^u), \forall k \in \mathbb{N}$, which can be used with (14) to yield (16).

Once sufficient data has been collected (i.e., $t \in [t_{i,k}^c, t_{i,k}^u) \cap [\tau_i, \infty)$), it can be shown using (14) that

$$\dot{V}_i(z_i(t)) \leq -\lambda_2 V_i(z_i(t)), \tag{22}$$

$\forall t \in [t_{i,k}^c, t_{i,k}^u) \cap [\tau_i, \infty), \forall k \in \mathbb{N}$, where $\lambda_2 \triangleq \frac{1}{c_2}\min\{k_1 \min_i \mathrm{e}^{-\bar{\chi}_i}, k_y, k_{cl}\underline{\lambda}\}$. Applying the Comparison Lemma [14, Lemma 3.4] to (22) yields

$$V_i(z_i(t)) \leq V_i\left(z_i\left(t_{i,k}^c\right)\right) \mathrm{e}^{-\lambda_2\left(t-t_{i,k}^c\right)} \tag{23}$$

[3]The expression in (11) contains $\tilde{\theta}_i$ which is unknown.

[4]The condition in (12) requires that the system be sufficiently excited, which is a milder (can be satisfied in finite time τ_i) condition than the typical PE condition.

$\forall t \in [t^c_{i,k}, t^u_{i,k}) \cap [\tau_i, \infty)$, $\forall k \in \mathbb{N}$, which can be used with (14) to yield (18). ■

B. Target Operating in the Unchased Mode

Lemma 2: During $t \in [t^u_{i,k}, t^c_{i,k+1})$, $\forall k \in \mathbb{N}$, the system states associated with the i^{th} target remain bounded for all bounded t.

Proof: Using (1), (8), and $\dot{\hat{\theta}}_i = 0$, the time derivative of (13) during $t \in [t^u_{i,k}, t^c_{i,k+1})$, $\forall k \in \mathbb{N}$ can be upper bounded by

$$\dot{V}_i(z_i(t)) \leq \kappa_1 \|z_i(t)\|^2 + \kappa_2 \|\bar{x}_c(t)\|^2 + \kappa_3, \quad (24)$$

where $\kappa_1, \kappa_2, \kappa_3 \in \mathbb{R}$ are positive constants and the term with the subscript c refers to the target currently operating in *chased* mode. Using the fact that the currently *chased* target error trajectory is bounded based on the analysis in Section IV-A, (24) can be upper bounded as

$$\dot{V}_i(z_i(t)) \leq \kappa_1 \|z_i(t)\|^2 + \kappa_4, \quad (25)$$

where $\kappa_4 \in \mathbb{R}$ is a known positive bounding constant. Using (14), (25) can be upper bounded as

$$\dot{V}_i(z_i(t)) \leq \frac{\kappa_1}{c_1} V_i(z_i(t)) + \kappa_4. \quad (26)$$

Applying the Comparison Lemma [14, Lemma 3.4] to (26), and upper bounding, yields

$$V_i(z_i(t)) \leq \left(V_i\left(z_i\left(t^u_{i,k}\right)\right) + \frac{\kappa_4 c_1}{\kappa_1} \right) e^{\frac{\kappa_1}{c_1}\left(t - t^u_{i,k}\right)} - \frac{\kappa_4 c_1}{\kappa_1}. \quad (27)$$

■

C. Combined Analysis

Consider the analysis of the i^{th} target in Sections IV-A and IV-B, and recall the definitions of T^c_i and T^u_i from Section III. The following switched systems analysis shows that the i^{th} target's error trajectory converges to an ultimate bound. To facilitate the analysis, let ν_1, ν_2 denote positive constants, where $\nu_1 \triangleq e^{\frac{\kappa_1}{c_1} T^u_i(km,(k+1)m-1) - \lambda_1 T^c_i(km,(k+1)m-1)}$, where $m \in \mathbb{N}$, c_1 is introduced in (14), λ_1 is introduced in (20), and κ_1 is introduced in (24).

Theorem 1: The controllers in (4) and (7), and the adaptive update law in (9) ensure that all signals associated with the i^{th} target remain bounded for all time $t \in [0, \tau_i)$ and

$$\limsup_t \|z_i(t)\|^2 \leq \frac{\nu_2}{c_1(1-\nu_1)} e^{\frac{\kappa_1}{c_1} T^u_{i,\max}}, \quad (28)$$

where $T^u_{i,\max} \triangleq \sup_k T^u_i(km, (k+1)m-1)$, provided there exists an $m < \infty$ and sequences $\{\Delta t^c_{i,k}\}^\infty_{k=0}$ and $\{\Delta t^u_{i,k}\}^\infty_{k=0}$ such that $\forall k \in \mathbb{N}$

$$T^u_i(km, (k+1)m-1) < \frac{\lambda_1 c_1}{\kappa_1} T^c_i(km, (k+1)m-1). \quad (29)$$

Remark 4: The inequality in (28) states that the square of the norm of the system states are worst-case bounded by the expression on the right hand side, which contains constants over which the user has some influence (based on the selection of gains and parameters).

Proof: Consider a single cycle of the i^{th} target switching to *chased*, *unchased*,and back to *chased* mode, i.e., $t \in [t^c_{i,k}, t^c_{i,k+1})$. Using (21) and (27), the evolution of V_i over m cycles can be written as $V_i(z_i(t^c_{i,(k+1)m})) \leq \nu_1 V_i(z_i(t^c_{i,km})) + \nu_2$, where $\nu_1 < 1$ provided (29) is satisfied. Let $\{s_{i,k}\}^\infty_{k=0}$ be a sequence defined by the recurrence relation $s_{i,k+1} = M_1(s_{i,k})$, with initial condition $s_{i,0} = V_i(z_i(t^c_{i,0}))$, where $M_1 : \mathbb{R} \to \mathbb{R}$ is defined as $M_1(s) \triangleq \nu_1 s + \nu_2$. Since $\nu_1 < 1$, M_1 is a contraction [15, Definition 9.22], and thus all initial conditions, $s_{i,0}$, approach the fixed point $s = \frac{\nu_2}{1-\nu_1}$ [15, Th. 9.23]. Since the sequence $\{s_{i,k}\}$ upper bounds V_i, in the sense that $V_i(z_i(t^c_{i,km})) \leq s_{i,k}$, V_i is ultimately bounded. However, since the dwell time condition (29) is specified over m cycles rather than a single cycle, V_i may grow within $[t^c_{i,km}, t^c_{i,(k+1)m}]$. Thus, the ultimate bound of z_i is given by (28). ■

Theorem 1 indicates that during the initial phase (i.e., $t \in [0, \tau_i)$), the closed-loop system is ultimately bounded. The following theorem establishes that when sufficient excitation occurs (i.e., $t \in [\tau_i, \infty)$), then the resulting bound can be decreased further. To facilitate this further analysis, let ν_3, ν_4 denote positive constants, where $\nu_3 \triangleq e^{\frac{\kappa_1}{c_1} T^u_i(km,(k+1)m-1) - \lambda_2 T^c_i(km,(k+1)m-1)}$, where $m \in \mathbb{N}$, c_1 is introduced in (14), λ_2 is introduced in (22), and κ_1 is introduced in (24).

Theorem 2: The controllers in (4) and (7), and the adaptive update law in (9) ensure that all signals associated with the i^{th} target remain bounded for all time $t \in [\tau_i, \infty)$ and

$$\limsup_t \|z_i(t)\|^2 \leq \frac{\nu_4}{c_1(1-\nu_3)} e^{\frac{\kappa_1}{c_1} T^u_{i,\max}}, \quad (30)$$

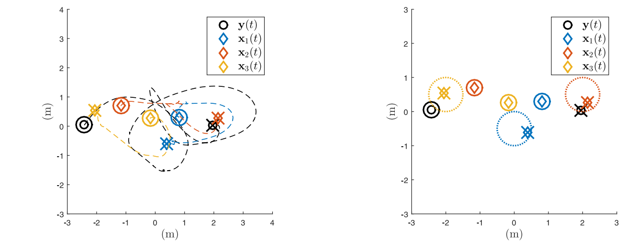
provided there exists an $m < \infty$ and sequences $\{\Delta t^c_{i,k}\}^\infty_{k=0}$ and $\{\Delta t^u_{i,k}\}^\infty_{k=0}$ such that $\forall k \in \mathbb{N}$

$$T^u_i(km, (k+1)m-1) < \frac{\lambda_2 c_1}{\kappa_1} T^c_i(km, (k+1)m-1). \quad (31)$$

Remark 5: The ultimate bound in (30) is smaller than that in (28) based on the fact that the $+c_4$ term in (21), used in Theorem 1, does not appear in (23), used in Theorem 2.

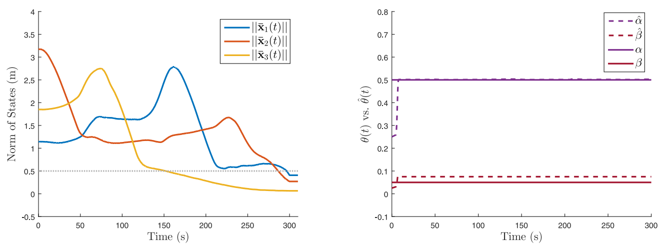
Proof: This proof follows the same strategy as that of Theorem 1 for $t \in [t^c_{i,k}, t^c_{i,k+1}) \cap [\tau_i, \infty)$. Provided (31) is satisfied $\nu_3 < 1$. By establishing $\{s_{i,k}\}^\infty_{k=0}$ as a sequence defined by the recurrence relation $s_{i,k+1} = M_2(s_{i,k})$ with initial condition $s_{i,0} = V_i(z_i(t^c_{i,q_i}))$, where $q_i \triangleq \underset{k}{\operatorname{argmin}}\{t^c_{i,k} > \tau_i\}$ and $M_2 : \mathbb{R} \to \mathbb{R}$ is defined as $M_2(s) \triangleq \nu_3 s + \nu_4$, then following the same arguments in Theorem 1, the result in (30) can be concluded. ■

Remark 6: Let $\bar{T}^c_{tot} \in \mathbb{R}$ and $\bar{T}^u_{tot} \in \mathbb{R}$ denote the average total time target agents spend operating in the *chased* and *unchased* modes, respectively. Using (29) and (31), an average dwell-time condition for all target agents over all time can be written as $\bar{T}^u_{tot} < \frac{\lambda_c c_1}{\kappa_1} \bar{T}^c_{tot}$, where $\lambda_c = \min\{\lambda_1, \lambda_2\}$. Since only one target will operate in the *chased* mode at any given time, for n_t targets the average total time targets spend operating in the *chased* mode is $\bar{T}^c_{tot} = \frac{1}{n_t}(\bar{T}^c_{tot} + \bar{T}^u_{tot})$. Thus, the maximum number of target agents that a single herding agent can successfully herd must satisfy $n_t < \frac{\lambda_c c_1}{\kappa_1} + 1$.



(a) (b)

Fig. 1. (a) Herder and targets trajectories. Each agent's starting location is marked by a circle, and the ending locations are marked by an X. (b) Starting and ending positions of all agents. A 0.5m radius ball around each target's goal location is marked by a dotted line circle.



(a) (b)

Fig. 2. (a) Norms of the target states are shown. Each agent is driven to within 0.5 m of their goal location (gray dotted line). (b) The actual vs. estimated parameters are shown.

V. Experiments

Experimental results[5] were obtained using Parrot Bebop 2 quadcopter platforms that served as a herding agent and three (i.e., $n_t = 3$) homogeneous target agents. A NaturalPoint, Inc. OptiTrack motion capture system was used to record the position of each agent at all times for feedback control. The switching strategy employed was as follows: the herder selects the target furthest from its goal initially, regulates that target to a ball that is 50% (a design parameter) of the target's previous distance, then switches to the next furthest target, and repeats until the target error is within some tolerance (0.5m was set as a reasonable stopping condition for the experiments). Since the agents were homogeneous, a single parameter estimate vector (and thus data history stack) was shared between all agents. The goal locations for each target were $x_1^g = \begin{bmatrix} 0.0 & -0.5 \end{bmatrix}^T$, $x_2^g = \begin{bmatrix} 2.0 & 0.5 \end{bmatrix}^T$, $x_3^g = \begin{bmatrix} -2.0 & 0.5 \end{bmatrix}^T$. The constant parameters for the target dynamics are $\sigma^2 = 1$, $\alpha = 0.5$, $\beta = 0.05$, $\gamma = 0.1$, $\underline{\alpha} = 0.25$, $\bar{\alpha} = 0.75$, $\bar{\beta} = 0.075$, $\hat{\theta}(0) = \begin{bmatrix} 0.25 & 0.025 \end{bmatrix}^T$, and the integration window was $\Delta t = 0.1\ s$. Figure 1 shows the overall paths of the agents and their starting and ending positions, demonstrating that the herding agent successfully regulated the target agents within the 0.5m radius of the goal locations. Figure 2 illustrates the norms of the target errors versus time and the ultimately bounded convergence of the adaptive estimate errors.

[5]A video of a typical run of this experiment is available at [16].

VI. Conclusion

An adaptive switching controller was developed using Lyapunov-based stability analysis for a single herding agent to ensure global uniform ultimate boundedness of n_t target agents to unique goal locations, despite their tendency to flee and lack of explicit control input. Each target's error trajectory was analyzed when operating in both *chased* and *unchased* modes, and dwell-time conditions were developed to ensure overall convergence to an ultimate bound. Experimental results demonstrate the validity of the results shown in this letter for an example herder strategy. Future efforts include generalizing the target and herder dynamics, investigating limits on the herder's control authority as a function of the dynamics of the target agents, and extending the results to multiple herders.

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