Synchronization of Uncertain Euler–Lagrange Systems With Uncertain Time-Varying Communication Delays

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***Abstract*—A decentralized controller is designed for leader-based synchronization of communication-delayed networked agents. The agents have heterogeneous dynamics modeled by uncertain, nonlinear Euler–Lagrange equations of motion affected by heterogeneous, unknown, exogenous disturbances. The developed controller requires only one-hop (delayed) communication from network neighbors and the communication delays are assumed to be heterogeneous, uncertain, and time-varying. Each agent uses an estimate of communication delay to provide feedback of estimated recent tracking error. Simulation results are provided to demonstrate the improved performance of the developed controller over other popular control designs.**

***Index Terms*—Communication delay, decentralized control, nonlinear systems, synchronization.**

I. Introduction

SYNCHRONIZATION describes a type of cooperative control in which networked autonomous agents act independently to accomplish a network-wide objective, and generally refers to matching the states of all dynamical systems connected in the network (see [1]–[6]). Applications include collective satellite interferometry, social network influence, neural networks, surveillance by a formation of autonomous ground or air vehicles, etc. (see [3], [7]–[11]). Synchronizing control policies are typically developed with a distributed interaction framework, in which agents sense or communicate with neighbors to inform the control policy. Network leaders can be used to influence a subset of the networked agents so that the networked "follower" agents track a useful state trajectory instead of regulating to a constant consensus value dependent on the networked agents' initial conditions, where the network leader may be a preset time-varying trajectory, often called a "virtual leader," or may represent a physical system with which the connected follower agents interact. The framework of cooperative control is particularly suited for applications where only following the leader could yield disruptive effects (e.g., graph segregation and collisions) due to the differences in the dynamic response or initial conditions of neighboring agents, or when only a subset of the follower agents receive information from the network leader. Limiting interaction with the network leader to a strict subset of the followers provides a more realistic framework for practical scenarios.

Communication delay, also known as broadcast, coupling or transmission delay, is a latency in interagent interaction. Even a small communication delay, such as that caused by information processing or a communication protocol, can cause networked autonomous systems to become unstable (see [7]), and hence, analysis is motivated to ensure stability. Control policies designed in [12]–[31] provide stability for communication-delayed autonomous synchronizing agents without a network leader. As demonstrated in [12], despite the effects of communication delay, asymptotic convergence to a stationary consensus point is achievable for scenarios without a network leader. The framework for communication-delayed synchronization is generalized in [3], [25], and [32]–[35] to include a reference trajectory or network leader, wherein each follower maintains knowledge of the leader's state. As shown in [3], despite the effects of communication delay, asymptotic convergence of the networked agents to the leader trajectory is possible when each follower agent is directly connected to the leader.

Synchronization to a time-varying leader trajectory wherein not every follower agent is connected to the leader constitutes a more challenging control problem: if an agent is not directly connected to the leader, it must rely on the delayed state of neighboring follower agent(s) between itself and the leader, i.e., the effect of a change in the leader's state may not be evident to a follower agent until information has been passed through multiple communication delays. The control policies designed in [25] and [36]–[39] address this more challenging interaction framework for communication-delayed leader–follower synchronization. The work in [36] is designed for follower agents with single integrator dynamics, state communication without delay, and the additional communication of control effort, which is uniformly delayed. The control policy in [37] is developed for follower agents

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with single integrator dynamics and uniformly delayed state communication, and the controller in [25] is developed for single or double integrator dynamics and uniformly delayed state communication and control inputs. However, an analysis which considers single or double integrator dynamics does not account for the potentially destabilizing state drift that can be caused by drift dynamics, which are present in many engineering systems, during the period of communication delay. Synchronization with delayed state communication is considered in [38] and [39] for follower agents with more general nonlinear dynamics; however, the approaches assume that the follower agents' dynamics are globally Lipschitz, which is restrictive and excludes many physical and electrical systems. Because globally Lipschitz nonlinear dynamics can be uniformly upper-bounded by a linear expression, the results in [38] and [39] develop a stability analysis which does not account for general nonlinearities. Hence, the developments in [25] and [36]–[38] do not directly apply to networks with agents which have general nonlinear dynamics. A new strategy is required for demonstrating stability in synchronization of a network of agents with general uncertain nonlinear dynamics, delayed communication, and restrictive connectivity to a time-varying leader trajectory.

This paper, and the preliminary work in [40], consider the problem of synchronization of a leader–follower network of agents with heterogeneous dynamics described by nonlinear second-order Euler–Lagrange equations of motion affected by an unknown, time-varying, exogenous input disturbance. Euler–Lagrange dynamics are used due to their ability to capture nonlinearities intrinsic to many engineering systems, such as robotic systems, power generators, etc. (see [41]), and are still the subject of active research efforts (see [32], [42], [43]). Moreover, these nonlinear systems typically interact over a communication protocol which can impose delay, thus motivating the study of communication-delayed networked nonlinear systems (see [32]). For example, robotic manipulators which are working collaboratively via data communication to articulate an object may suffer from communication delays that degrade overall performance. The leader agent has a time-varying trajectory and is assumed to interact with at least one follower agent. The follower agents are delayed in communicating state information and do not communicate control effort information. The communication delay is assumed to be uncertain, heterogeneous, time-varying and bounded. Motivated by recent results (see [18], [44]) which demonstrate that approximate knowledge of delay can be incorporated into a controller for improved performance, an estimate of the communication delay is used to provide feedback of an estimated recent tracking error in a delay-affected proportional-derivative-based decentralized controller.

The most relevant literature includes [25], [36]–[38], and [45]. Compared to [36], this paper considers delayed state communication with nonlinear dynamics. Compared to [25] and [37], which address constant, heterogeneous and known communication delays and linear dynamics, this paper considers time-varying, heterogeneous, and uncertain communication delays and nonlinear dynamics. Compared to [38], this paper considers heterogeneous communication delays, including between the leader and followers, and more general uncertain nonlinear dynamics. Compared to [45], this paper allows for arbitrary selection of the leader trajectory instead of requiring the network leader's trajectory to evolve according to dynamics which are identical to that of each follower. Additionally, compared to all literature on cooperative control with communication delay, a novel approach is taken here in that a controller is developed which takes advantage of both feedback with self-delay and feedback without self-delay, which are independent types of feedback defined in Sections III and [16]. These two types of feedback are crafted into a neighborhood-based distributed controller, requiring only one-hop communication, which is shown to outperform competing types of communication-delayed controllers in the case studied in the simulation section. A Lyapunov-based stability analysis using Lyapunov–Krasovskii (LK) functionals is provided to develop sufficient conditions for uniformly ultimately bounded (UUB) convergence to the leader state for each follower agent. Simulation results are provided to demonstrate the comparatively improved performance of the developed controller.

II. Problem Formulation

A. Graph Theory Preliminaries

Consider a network with one leader and a finite number $\mathcal{F} \in \mathbb{Z}_{>0}$ of follower agents. The interaction among the follower agents is described by a fixed undirected graph $\mathcal{G}_\mathcal{F} = \{\mathcal{V}_\mathcal{F}, \mathcal{E}_\mathcal{F}\}$, where $\mathcal{V}_\mathcal{F} \triangleq \{1, \ldots, \mathcal{F}\}$ is the node set representing the follower agents and $\mathcal{E}_\mathcal{F} \subseteq \mathcal{V}_\mathcal{F} \times \mathcal{V}_\mathcal{F}$ is an edge set representing the communication links among the follower agents. An edge, represented by the pair (j, i), belongs to $\mathcal{E}_\mathcal{F}$ if agent $j \in \mathcal{V}_\mathcal{F}$ communicates information to agent $i \in \mathcal{V}_\mathcal{F}$. The neighbor set in $\mathcal{G}_\mathcal{F}$ for agent $i \in \mathcal{V}_\mathcal{F}$ is defined as $\mathcal{N}_{\mathcal{F}i} \triangleq \{j \in \mathcal{V}_\mathcal{F} \mid (j, i) \in \mathcal{E}_\mathcal{F}\}$. Connections in $\mathcal{G}_\mathcal{F}$ are described by the adjacency matrix $A \triangleq [a_{ij}] \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$, where $a_{ij} > 0$ if $(j, i) \in \mathcal{E}_\mathcal{F}$ and $a_{ij} = 0$, otherwise. The Laplacian matrix $\mathcal{L}_\mathcal{F} \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$ of graph $\mathcal{G}_\mathcal{F}$ is defined as $\mathcal{L}_\mathcal{F} \triangleq D - A$, where $D \triangleq \mathrm{diag}\{d_1, \ldots, d_\mathcal{F}\} \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$ is the degree matrix and $d_i \triangleq \sum_{j\in\mathcal{N}_{\mathcal{F}i}} a_{ij}$. A leader-included directed supergraph of $\mathcal{G}_\mathcal{F}$ can be constructed as $\mathcal{G} = \{\mathcal{V}_\mathcal{F} \cup L, \mathcal{E}_\mathcal{F} \cup \mathcal{E}_L\}$, where the node L represents the leader agent and the ordered pair $(L, i) \in \mathcal{E}_L$ if and only if agent $i \in \mathcal{V}_\mathcal{F}$ receives information from the leader. The leader-included neighbor set is defined as $\bar{\mathcal{N}}_{\mathcal{F}i} \triangleq \{j \in \mathcal{V}_\mathcal{F} \cup \{L\} \mid (j, i) \in \mathcal{E}_\mathcal{F} \cup \mathcal{E}_L\}$. The diagonal leader-connectivity (pinning) matrix $B \triangleq \mathrm{diag}\{b_1, \ldots, b_\mathcal{F}\} \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$ is defined such that $b_i > 0$ if $(L, i) \in \mathcal{E}_L$ and $b_i = 0$ otherwise. The following assumption specifies the class of communication networks considered in this paper.

Assumption 1: The follower graph $\mathcal{G}_\mathcal{F}$ is connected and at least one follower agent receives information from the leader.

B. Dynamic Model and Properties

Let the dynamics of follower agent $i \in \mathcal{V}_\mathcal{F}$ be represented by Euler–Lagrange equations of motion of the form

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + F_i(\dot{q}_i) + G_i(q_i) = u_i + d_i(t) \qquad (1)$$

where $q_i \in \mathbb{R}^m$ is the generalized configuration coordinate, $M_i : \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is the inertia matrix, $C_i : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is the Coriolis/centrifugal matrix such that $C_i\dot{q}_i$ captures the apparent forces due to Coriolis and centrifugal effects (see [41, Ch. 2.2]), $F_i : \mathbb{R}^m \to \mathbb{R}^m$ represents the effects of viscous friction, $G_i : \mathbb{R}^m \to \mathbb{R}^m$ represents gravitational effects, $u_i \in \mathbb{R}^m$ is the vector of control inputs, and $d_i : \mathbb{R} \to \mathbb{R}^m$ is the time-varying, unknown, exogenous disturbance that captures the effects of input disturbances and unmodeled effects. The time-varying state of the leader is denoted by $q_L : \mathbb{R} \to \mathbb{R}^m$. To simplify analysis, the following assumptions concerning the Euler–Lagrange dynamics, external disturbances, and leader trajectory are made.

Assumption 2 [41, Ch. 2.3]: For each follower agent $i \in \mathcal{V}_\mathcal{F}$, the inertia matrix is positive definite and symmetric, and there exist positive constants $\underline{m}, \bar{m} \in \mathbb{R}$ such that the inertia matrix satisfies the inequalities $\underline{m}\|\xi\|^2 \le \xi^T M_i(\psi)\xi \le \bar{m}\|\xi\|^2$ for all $\xi, \psi \in \mathbb{R}^m$ and $i \in \mathcal{V}_\mathcal{F}$.

Assumption 3: For each follower agent $i \in \mathcal{V}_\mathcal{F}$, the dynamics are sufficiently smooth such that the functions M_i, C_i, F_i, and G_i are first-order differentiable, i.e., the first-order derivative is bounded if $q_i, \dot{q}_i, \ddot{q}_i \in \mathcal{L}_\infty$.

Assumption 4: For each follower agent $i \in \mathcal{V}_\mathcal{F}$, the vector of time-varying input disturbances is continuous and bounded such that $\sup_{t\in\mathbb{R}} \|d_i(t)\| \le \bar{d}$ for some known positive constant $\bar{d} \in \mathbb{R}$.

Assumption 5: The leader state is bounded and sufficiently smooth such that $q_L, \dot{q}_L, \ddot{q}_L \in \mathcal{L}_\infty$.

The communication delay between agents is modeled such that, at time t, agent $i \in \mathcal{V}_\mathcal{F}$ is unaware of the set of recent states $\{q_j(\sigma) \mid t - \tau_{ji}(t) < \sigma \le t\}$ of a neighbor $j \in \bar{\mathcal{N}}_{\mathcal{F}i}$ $(i \ne j)$, where $\tau_{ji} : \mathbb{R} \to \mathbb{R}_{\ge 0}$ is the positive, time-varying, uncertain communication delay. The communication delays in the network need not be homogeneous, i.e., the communication delays may be different for each interaction link. The communication delay may even differ between an interacting pair of agents, i.e., it may be that $\tau_{ij}(t) \ne \tau_{ji}(t)$ for $i, j \in \mathcal{V}_\mathcal{F}$. The following assumption specifies the class of delays considered in this paper.

Assumption 6 (see [37], [38]): The uncertain, time-varying delay τ_{ji} is bounded above by a known constant $\bar{\tau} \in \mathbb{R}_{>0}$ such that $\sup_{t\in\mathbb{R}} \tau_{ji}(t) < \bar{\tau}$, τ_{ji} is differentiable, and τ_{ji} changes sufficiently slowly such that $\sup_{t\in\mathbb{R}} |\dot{\tau}_{ji}(t)| < 1$, for each $(j, i) \in \mathcal{E}_\mathcal{F} \cup \mathcal{E}_L$. There is no delay in agent $i \in \mathcal{V}_\mathcal{F}$ knowing its own state, q_i.

Each agent maintains an estimate of the duration of communication delay for all incoming communication, i.e., agent $i \in \mathcal{V}_\mathcal{F}$ estimates τ_{ji} with $\hat{\tau}_{ji} : \mathbb{R} \to \mathbb{R}_{>0}$ for every neighbor $j \in \bar{\mathcal{N}}_{\mathcal{F}i}$, where $\hat{\tau}_{ji}$ is upper-bounded by the known constant $\bar{\bar{\tau}} \in \mathbb{R}_{>0}$ for each communication channel $(j, i) \in \mathcal{E}_\mathcal{F} \cup \mathcal{E}_L$. The following assumption is similar in effect to the condition in [16] which stipulates that both delay and its estimate are bounded by a known constant.

Assumption 7: The difference between the communication delay τ_{ji} and delay estimate $\hat{\tau}_{ji}$ is upper-bounded by a known constant $\bar{\bar{\tau}} \in \mathbb{R}_{>0}$ such that $\sup_{t\in\mathbb{R}} |\tau_{ji}(t) - \hat{\tau}_{ji}(t)| < \bar{\bar{\tau}}$, $\hat{\tau}_{ji}$ is differentiable, and $\hat{\tau}_{ji}$ changes sufficiently slowly such that $\sup_{t\in\mathbb{R}} |\dot{\hat{\tau}}_{ji}(t)| < 1$, for each $(j, i) \in \mathcal{E}_\mathcal{F} \cup \mathcal{E}_L$.

There are multiple ways to obtain an estimate of communication delay, and the specific application may dictate the methodology used (see [18], [34], [46]–[49]). In this paper, a specific method of estimating communication delay is not considered, but it may be approximated analytically (e.g., inspection of the network's communication protocols and hardware components) or experimentally (e.g., inspection of message timestamps).

For implementation purposes, it is also assumed that for every agent $i \in \mathcal{V}_\mathcal{F}$, the delayed state $q_j(t - \tau_{ji}(t))$ has been communicated to agent i from every neighbor $j \in \bar{\mathcal{N}}_{\mathcal{F}i}$ for at least $\bar{\tau} + \bar{\bar{\tau}}$ seconds before control implementation. Note that this approach does not omit the case in which some communication channels may have no delay.

C. Control Objective

Similar to traditional synchronization approaches (see [1]–[4]), the objective is to design a decentralized controller u_i so that the states of the networked agents are driven toward the state of the network leader such that $\limsup_{t\to\infty} \|q_i(t) - q_L(t)\| \le \varepsilon$ for each $i \in \mathcal{V}_\mathcal{F}$ and some $\varepsilon \in \mathbb{R}_{>0}$. The bound ε is desired to be as small as possible.

III. Controller Development

Feedback control policies for network synchronization typically use error signals with the form $e_i \triangleq \sum_{j\in\mathcal{N}_{\mathcal{F}i}} a_{ij}(q_j(t) - q_i(t)) + b_i(q_L(t) - q_i(t))$ (see [1]–[4]). However, because the network considered in this scenario is affected by communication delay, the error signal e_i is not implementable. Alternatively, a new feedback signal $e_{\tau i} \in \mathbb{R}^m$ is developed to implement the delay estimates $\hat{\tau}_{ji}$ and encourage state agreement among the leader and follower agents as

$$\begin{aligned} e_{\tau i} \triangleq\ & \frac{\kappa_1}{|\mathcal{N}_{\mathcal{F}i}|}\sum_{j\in\mathcal{N}_{\mathcal{F}i}} a_{ij}\big(q_j\big(t - \tau_{ji}(t)\big) - q_i(t)\big) \\ & + \frac{\kappa_2}{|\mathcal{N}_{\mathcal{F}i}|}\sum_{j\in\mathcal{N}_{\mathcal{F}i}} a_{ij}\big(q_j\big(t - \tau_{ji}(t)\big) - q_i\big(t - \hat{\tau}_{ji}(t)\big)\big) \\ & + \kappa_1 b_i(q_L(t - \tau_{Li}(t)) - q_i(t)) \\ & + \kappa_2 b_i\big(q_L(t - \tau_{Li}(t)) - q_i\big(t - \hat{\tau}_{Li}(t)\big)\big) \end{aligned} \tag{2}$$

and an auxiliary delayed error signal $r_{\tau i} \in \mathbb{R}^m$ is analogously defined as

$$\begin{aligned} r_{\tau i} \triangleq\ & \frac{\kappa_1}{|\mathcal{N}_{\mathcal{F}i}|}\sum_{j\in\mathcal{N}_{\mathcal{F}i}} a_{ij}\big(\dot{q}_j\big(t - \tau_{ji}(t)\big) - \dot{q}_i(t)\big) \\ & + \frac{\kappa_2}{|\mathcal{N}_{\mathcal{F}i}|}\sum_{j\in\mathcal{N}_{\mathcal{F}i}} a_{ij}\big(\dot{q}_j\big(t - \tau_{ji}(t)\big) - \dot{q}_i\big(t - \hat{\tau}_{ji}(t)\big)\big) \\ & + \kappa_1 b_i(\dot{q}_L(t - \tau_{Li}(t)) - \dot{q}_i(t)) \\ & + \kappa_2 b_i\big(\dot{q}_L(t - \tau_{Li}(t)) - \dot{q}_i\big(t - \hat{\tau}_{Li}(t)\big)\big) + \lambda e_{\tau i} \end{aligned} \tag{3}$$

where $|\cdot|$ denotes set cardinality, $\lambda \in \mathbb{R}_{>0}$ is a constant control gain, and $\kappa_1 \in \mathbb{R}_{\ge 0}$, $\kappa_2 \in \mathbb{R}_{\ge 0}$ are constant weighting parameters selected such that $\kappa_1 + \kappa_2 = 1$. Thus, neighbors' delayed state and state derivative are to be used for control purposes with the implementable error signals $e_{\tau i}$ and $r_{\tau i}$.[1] There are two types of feedback in $e_{\tau i}$. Terms multiplied by κ_1 in (2) provide the difference between a neighbor's delayed state and an agent's own current state and is normalized by the number of neighbors. This term can help promote overall stability (see the following simulation section) of the networked systems and will be referred to as *feedback without self-delay*, as in [16]. Terms multiplied by κ_2 in (2) provide the normalized difference between a neighbor's delayed state and an agent's own state manually delayed by an estimate of the delay corresponding to that communication channel. This term can improve performance in synchronization by correcting for estimated tracking errors in the recent history of the agents' trajectories. This type of feedback will be referred to as *feedback with inexact self-delay* if $\tau_{ji} \neq \hat{\tau}_{ji}$ for some $t \in \mathbb{R}$ and *feedback with exact self-delay* if $\tau_{ji} \equiv \hat{\tau}_{ji}$. The weighting parameters κ_1 and κ_2 are used to relatively weight the contribution of the terms associated with feedback without self-delay and feedback with self-delay, where the following simulation section demonstrates the benefit of using a weighted combination of these two types of feedback in leader tracking performance. Division by the neighbor set cardinality $|\mathcal{N}_{\mathcal{F}i}|$ is used to improve the network scaling properties of the subsequently defined controller by dividing the summation of neighbor feedback by the total number of neighbors. The tuning parameters a_{ij} and b_i may be adjusted to emphasize either leader tracking or (follower) neighbor tracking in closed-loop performance.

A communication-delayed proportional-derivative-based controller, based on one-hop neighbor feedback, is designed for agent $i \in \mathcal{V}_{\mathcal{F}}$ as

$$u_i = kr_{\tau i} \tag{4}$$

where $k \in \mathbb{R}_{>0}$ is a constant control gain. Note that, as opposed to the controller in [37], it is not assumed that the communication delay duration is exactly known.

IV. Convergence Analysis

Stability of the communication-delayed decentralized controller in (4) is analyzed in this section. The subsequent Theorem 1 demonstrates that the controller in (4) yields UUB leader–follower convergence for a network of agents with uncertain nonlinear dynamics given by (1), provided the given sufficient conditions are satisfied, where the agents' dynamics are affected by heterogeneous uncertain time-varying delays and input disturbances. Some of the details are omitted for brevity, but can be found in [50, Ch. 4].

Theorem 1: For sufficiently small communication delays and delay estimate errors, there exists a selection for the gain k such that the communication-delayed controller in (4) provides UUB synchronization for a network of agents with dynamics given by (1) in the sense that $\limsup_{t\to\infty} \|q_i(t) - q_L(t)\| < \varepsilon$ for some $\varepsilon \in \mathbb{R}_{>0}$ and every follower agent $i \in \mathcal{V}_{\mathcal{F}}$, provided the initial conditions of the agents lie within a gain-dependent ball centered at the origin (described in the proof by the set $\mathcal{S}_{\mathfrak{D}}$), Assumptions 1–7 are satisfied, the gain λ satisfies $\lambda > (1/2)$, and the parameters κ_1, κ_2, and a_{ij} are selected within the aforementioned conditions.

Proof: See the Appendix for the proof, including an explicit definition of ε and sufficient conditions that illustrate the dependencies between the gain k, delays, and dynamics. ■

As previously mentioned, leader–follower synchronization in a framework with a time-varying leader trajectory and limited leader connectivity restricts follower agents which are multiple hops away from the leader from timely access to the leader's state. As such, no results which consider a time-varying leader trajectory and limited leader connectivity have successfully achieved asymptotic regulation of the synchronization errors; instead, ultimately bounded convergence is achieved. Similarly, the result in this paper achieves UUB synchronization of the leader and followers' states, where the bound on the synchronization error is diminished by lower communication delays, disturbances, leader trajectory acceleration content, etc. [see (24)]. The synchronization error bound, ε, is nontrivial to compute for complicated nonlinear dynamics, such as those described in the following simulation section; however, ε can simply be computed for linear or less complicated nonlinear agent dynamics with a priori knowledge of bounds of terms such as the communication delay, communication delay estimate inaccuracy, the maximum disturbance magnitude, etc. (see the proof in the Appendix).

V. Simulation

Simulation results are provided to demonstrate the capability of the proposed controller in (4) over other similar control methods in obtaining approximate convergence in leader–follower synchronization, despite the effects of uncertain, time-varying, heterogeneous communication delays. The leader–follower network is modeled with 11 follower agents, where three agents interact with the leader, as depicted in Fig. 1. Similar to [1], [3], [4], and [51]–[53], each follower agent has nonlinear Euler–Lagrange dynamics that represent a two-link revolute-joint robotic manipulator (see [44]) modeled as

$$\begin{aligned} u_i = &\begin{bmatrix} p_{1,i} + 2p_{3,i}c_{2,i} & p_{2,i} + p_{3,i}c_{2,i} \\ p_{2,i} + p_{3,i}c_{2,i} & p_{2,i} \end{bmatrix}\ddot{q}_i \\ &+ \begin{bmatrix} -p_{3,i}s_{2,i}\dot{q}_{i,2} & -p_{3,i}s_{2,i}(\dot{q}_{i,1} + \dot{q}_{i,2}) \\ p_{3,i}s_{2,i}\dot{q}_{i,1} & 0 \end{bmatrix}\dot{q}_i \\ &+ \begin{bmatrix} f_{d1,i} & 0 \\ 0 & f_{d2,i} \end{bmatrix}\dot{q}_i + d_i \end{aligned}$$

where $p_{1,i}, p_{2,i}, p_{3,i}, f_{d1,i}, f_{d2,i} \in \mathbb{R}_{>0}$ are heterogeneous constant parameters described in [54] that depend on the mass distribution and viscous friction properties of the manipulator, $q_i \in \mathbb{R}^2$ describes the joint angle in radians, $q_{i,1}, q_{i,2}$, respectively denote the first and second entry of the vector q_i, $c_{2,i} \triangleq \cos(q_{i,2})$, $s_{2,i} \triangleq \sin(q_{i,2})$, and the disturbance $d_i \in \mathbb{R}^2$ is modeled as $d_i = \begin{bmatrix} \text{rand}(-1,1) \\ \text{rand}(-1,1) \end{bmatrix}$ N · m, where rand$(-1, 1)$ samples randomly in $(-1, 1)$ with a uniform distribution.

[1] It is assumed that a neighbor's delayed state derivative is communicated, not computed; i.e., agent j obtains and then communicates $q_j(t)$ and $\dot{q}_j(t)$ to a neighbor i with a communication delay $\tau_{ji}(t)$. In other words, this approach does not solve the communication-delayed output feedback problem: numerical computation of the delayed state derivative may be skewed by effects of the time-varying delay.

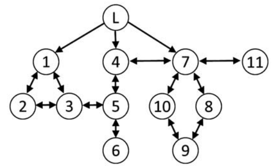

Fig. 1. Communication network topology.

TABLE I
HETEROGENEOUS COMMUNICATION DELAY ESTIMATE FOR EACH COMMUNICATION LINK

Channel	$\hat{\tau}_{ji}$ (s)	Channel	$\hat{\tau}_{ji}$ (s)
$(L,1)$	0.011	$(5,6)$	0.027
$(L,4)$	0.011	$(6,5)$	0.033
$(L,7)$	0.011	$(7,4)$	0.022
$(1,2)$	0.041	$(7,8)$	0.029
$(1,3)$	0.028	$(7,10)$	0.030
$(2,1)$	0.018	$(7,11)$	0.027
$(2,3)$	0.032	$(8,7)$	0.025
$(3,1)$	0.032	$(8,9)$	0.033
$(3,2)$	0.012	$(9,8)$	0.026
$(3,5)$	0.026	$(9,10)$	0.020
$(4,5)$	0.025	$(10,7)$	0.021
$(4,7)$	0.015	$(10,9)$	0.036
$(5,3)$	0.034	$(11,7)$	0.037
$(5,4)$	0.017		

The robotic manipulators' actions may represent, for example, cooperative interaction with a work-piece in a manufacturing setting. The nonzero adjacency gains are selected as $a_{ij} = 1$ $\forall (j,i) \in \mathcal{E}_{\mathcal{F}}$ and the nonzero pinning gains are selected as $b_i = 1$ $\forall i : (L,i) \in \mathcal{E}_L$. The leader state is assigned the trajectory $q_L = \begin{bmatrix} \sin(t) \\ 0.5\cos(t) \end{bmatrix}$ rad. The uncertain communication delay for each interagent interaction lies between 8 and 44 ms and is modeled as $\tau_{ji} = \hat{\tau}_{ji} + 0.003\,\mathrm{rand}(-1,1)$, where the constant delay approximations $\hat{\tau}_{ji}$ are given in Table I. While these communication delays are larger than what may be seen in application, they are used to stress the control environment and demonstrate the performance of the developed control policy. Similarly, while the communication topology shown in Fig. 1 is likely more complicated than would be implemented, the added complexity is used for demonstration purposes. To maintain consistency between simulation trials, the random number generator is started with the same seed for each simulation.

Recent results have developed distributed controllers for consensus and synchronization applications affected by communication delay by employing control terms that contain self-delayed feedback or feedback without self-delay (see [16], [25], [36]–[38]). The comparison in this simulation section demonstrates the impact of this paper's contribution by showing that a combination of these two types of feedback, as depicted in (2)–(4), can provide improved leader-tracking performance. The contributions of self-delayed feedback and feedback without self-delay, both alone and mixed, are compared by simulating the closed-loop system with various values for κ_1 and κ_2. Specifically, gain tuning was performed for three different implementations of the control policy in (2)–(4).

1) Only feedback without self-delay ($\kappa_1 = 1$, $\kappa_2 = 0$).
2) Only feedback with self-delay ($\kappa_1 = 0$, $\kappa_2 = 1$).

TABLE II
TUNED GAINS AND ASSOCIATED COSTS FOR (A) ONLY FEEDBACK WITHOUT SELF-DELAY, (B) ONLY FEEDBACK WITH INEXACT SELF-DELAY, AND (C) A MIXTURE OF FEEDBACK WITH SELF-DELAY AND WITHOUT SELF-DELAY, SHOWING THAT (C) PROVIDES 9% IMPROVED LEADER-TRACKING PERFORMANCE OVER (A) AND 73% IMPROVED LEADER-TRACKING PERFORMANCE OVER (B)

	k	λ	κ_1	κ_2	err
(A)	21.3	30	1	0	80.3
(B)	1.2	30	0	1	275
(C)	22.3	30	0.84	0.16	73.1

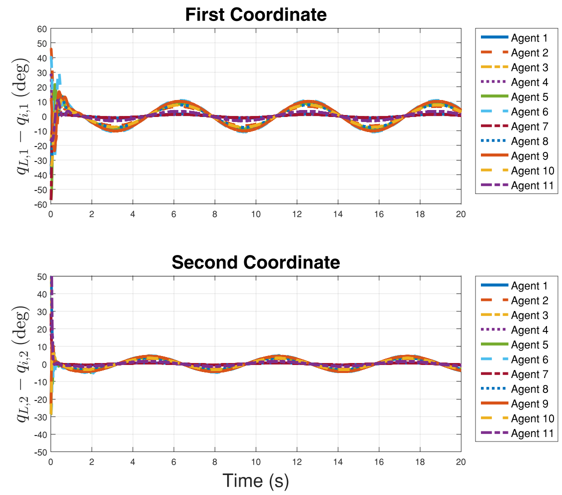

Fig. 2. Using feedback without self-delay alone ($\kappa_1 = 1$, $\kappa_2 = 0$) provides stability with a relatively high gain ($k = 21.3$), but using a mixture of feedback without self-delay and feedback with inexact self-delay can improve leader-tracking performance by 9%.

3) A mixture of feedback with self-delay and without self-delay ($\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_1 + \kappa_2 = 1$).

The gains k and λ were tuned by selecting values such that every combination of $k \in (0.1, 0.2, \ldots, 30)$ and $\lambda \in (0.1, 0.2, \ldots, 30)$ is used in simulation. For implementation 3, every combination of the aforementioned values for k and λ was used in conjunction with each value of κ_1 and κ_2 such that $\kappa_1 \in (0.01, 0.02, \ldots, 0.99)$, $\kappa_2 = 1 - \kappa_1$. To better simulate a real-world scenario, inexact estimates for the communication delays are used for feedback with self-delay. The estimates of the communication delays are constant and are shown in Table I. Ten simulation trials are run for each gain combination, where each trial has unique joint angle initial conditions picked such that $q_{i,1}, q_{i,2} \in [-1, 1]$ rad and $\dot{q}_i(0) = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$ rad/s for $i = 1, \ldots, 4$, where $q_{i,j}$ denotes the jth entry of the vector q_i. The simulation results using these different gain combinations are vetted using the leader tracking-based cost function

$$\mathrm{err} \triangleq \sum_{i=1}^{11} \sum_{j=1}^{2} \mathrm{rms}\big(q_{L,j} - q_{i,j}\big) \tag{5}$$

where $\mathrm{rms}(\cdot)$ denotes the root-mean-square (RMS) of the argument's sampled trajectory, converted from radians to degrees, between 0 and 20 s. The gain combinations which produced the lowest cost for the three different control implementations and the associated costs are shown in Table II, and the according simulation results are shown in Figs. 2–4.

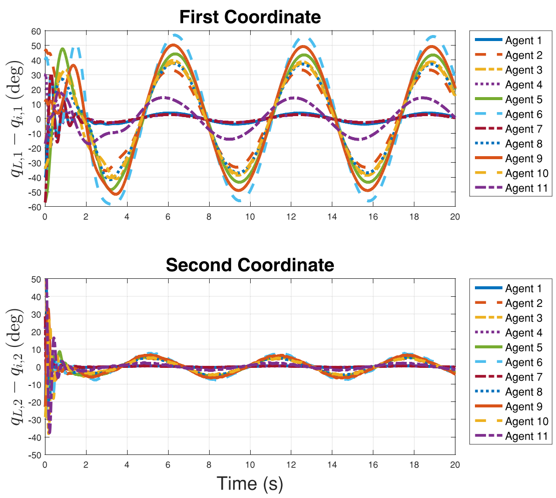

Fig. 3. Using feedback with inexact self-delay alone ($\kappa_1 = 0$, $\kappa_2 = 1$) can only provide stability with a relatively low gain ($k = 1.2$), and using a mixture of feedback without self-delay and feedback with inexact self-delay can improve leader-tracking performance by 73%.

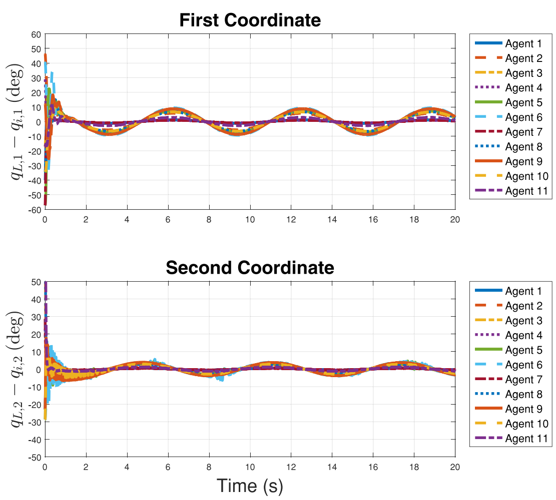

Fig. 4. Using a mixture of feedback without self-delay and feedback with inexact self-delay ($\kappa_1 = 0.84$, $\kappa_2 = 0.16$) with delay estimates shown in Table I provides stability with a relatively high gain ($k = 22.3$), resulting in leader-tracking performance that is improved over feedback without self-delay or feedback with inexact self-delay alone by 9% and 73%, respectively.

As seen in Table II, for the given simulation setting, the developed control implementation 3 (mixture of feedback without self-delay and feedback with self-delay) outperforms conventional communication-delayed cooperative control approaches, giving a 9% performance increase over implementation 1 (no self-delayed feedback) and a 73% performance increase over implementation 2 (only self-delayed feedback), in terms of the RMS error between a follower agent's state component and the corresponding value of the leader's state component, summed across each component of the state and each follower agent, as depicted in (5). Whereas implementations 1 and 3 remain stable for higher selections of the gain k, implementation 2 produces degraded performance upon gain selection of $k \geq 1.3$ and $\lambda = 30$, which demonstrates the sensitivity of only using feedback with self-delay. Implementation 3 produces the best leader-tracking performance by combining feedback without self-delay, which is helpful in stabilization, and feedback with self-delay, which can provide better tracking performance by comparing signals closer in time.

Note that the conditions in the simulation section violate Assumption 6: the more challenging case of a discrete random delay is considered, rather than a delay that changes sufficiently slowly. These conditions are selected to illustrate the robustness of the controller. Specifically, since the delay can change infinitely fast in the simulation example, the sufficient gain conditions cannot be satisfied; yet, for the selected gains, the controller is still able to yield approximate leader–follower synchronization.

VI. Conclusion

A decentralized, proportional-derivative-based controller is presented for UUB leader-synchronization of a network wherein all agents may be affected by communication delay. The follower agents are modeled with heterogeneous, uncertain Euler–Lagrange equations of motion affected by time-varying, uncertain exogenous disturbances. The communication delay between any two agents is considered to be heterogeneous, time-varying, and uncertain. An estimate of the communication delay is used in the controller to estimate recent tracking errors. The benefit of using a mixture of feedback without self-delay and feedback with inexact self-delay is demonstrated in simulation. Some prominent assumptions are that the delays and delay estimation errors are sufficiently small and the leader trajectory is bounded and sufficiently smooth.

Compared to the relevant literature, the more general nonlinear dynamics considered in this paper challenge the convergence analysis because the uncertain volatility of the dynamics can disturb and possibly destabilize the dynamic response of the closed-loop networked systems; specifically, much technical development was performed in bounding the effects of the uncertain nonlinear dynamics in the derivative of the Lyapunov function and establishment of a set of stabilizing initial conditions.

The approach in this paper may lead to other exciting methods to improve the performance of decentralized control in networks affected by communication delay. For example, the controller in (2)–(4) may be improved by the development of a distributed algorithm which changes edge weights or the neighbor set based on local network structure and estimates of neighbors' communication delays; customization of κ_1 and κ_2 for each neighbor based on the delay estimates; distributed communication-based algorithms which allow each agent to predict the leader trajectory, similar to that in [37], thereby reducing the impact of the propagation of communication delays through agents' dynamics; and extension of the developed approach to the framework of containment control.

Appendix

For notational brevity, the networked systems' dynamics are grouped into block matrices and composite vectors as

$$M \triangleq \operatorname{diag}(M_1, \ldots, M_{\mathcal{F}}) \in \mathbb{R}^{\mathcal{F}m \times \mathcal{F}m}$$
$$C \triangleq \operatorname{diag}(C_1, \ldots, C_{\mathcal{F}}) \in \mathbb{R}^{\mathcal{F}m \times \mathcal{F}m}$$
$$F \triangleq \left[F_1^T, \ldots, F_{\mathcal{F}}^T\right]^T \in \mathbb{R}^{\mathcal{F}m}$$

$$G \triangleq \left[G_1^T, \ldots, G_{\mathcal{F}}^T\right]^T \in \mathbb{R}^{\mathcal{F}m}$$
$$U \triangleq \left[u_1^T, \ldots, u_{\mathcal{F}}^T\right]^T \in \mathbb{R}^{\mathcal{F}m}$$
$$d \triangleq \left[d_1^T, \ldots, d_{\mathcal{F}}^T\right]^T \in \mathbb{R}^{\mathcal{F}m}$$
$$Q_{\mathcal{F}} \triangleq \left[q_1^T, \ldots, q_{\mathcal{F}}^T\right]^T \in \mathbb{R}^{\mathcal{F}m}$$

such that

$$U + d(t) = M(Q_{\mathcal{F}})\ddot{Q}_{\mathcal{F}} + C\big(Q_{\mathcal{F}}, \dot{Q}_{\mathcal{F}}\big)\dot{Q}_{\mathcal{F}} + F\big(\dot{Q}_{\mathcal{F}}\big) + G(Q_{\mathcal{F}}). \quad (6)$$

Additionally, nonimplemented error signals $E \triangleq Q_L - Q_{\mathcal{F}} \in \mathbb{R}^{\mathcal{F}m}$ and $R \triangleq \dot{E} + \lambda E \in \mathbb{R}^{\mathcal{F}m}$ are introduced to develop a network-wide closed-loop error system, where E is used to denote the network-wide synchronization errors, R is used to help create a first-order differential equation representation of the second-order closed-loop error dynamics, and $Q_L \triangleq \mathbf{1}_{\mathcal{F}} \otimes q_L$. Clearly, if $\|E\| \to 0$, then the control objective is achieved.

To facilitate the description of the normalized neighbor feedback, let the matrix $\mathscr{A} \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$ be defined as $\mathscr{A} \triangleq [\mathfrak{a}_{ij}]$, where $\mathfrak{a}_{ij} \triangleq (a_{ij}/|\mathcal{N}_{\mathcal{F}i}|)$. Additionally, let the matrix $\mathscr{D} \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$ be defined as $\mathscr{D} \triangleq \operatorname{diag}\{\mathfrak{d}_1, \ldots, \mathfrak{d}_{\mathcal{F}}\}$, where $\mathfrak{d}_i \triangleq \sum_{j\in\mathcal{N}_{\mathcal{F}i}} \mathfrak{a}_{ij}$. For convenience in describing the effects of the heterogeneous communication delays individually in the closed-loop system, let the constant matrix $\mathcal{A}_{ij} \in \mathbb{R}^{\mathcal{F}m\times\mathcal{F}m}$ be defined as $\mathcal{A}_{ij} \triangleq (\mathscr{A} \circ \mathbf{1}_{ij}) \otimes I_m$, where $\circ$ denotes the Hadamard product and $\mathbf{1}_{ij} \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$ denotes an indicator matrix, which has all zero entries except for the ith row and jth column, which has a value of 1. Similarly, let the constant matrix $\mathcal{D}_{ij} \in \mathbb{R}^{\mathcal{F}m\times\mathcal{F}m}$ be defined as $\mathcal{D}_{ij} \triangleq \mathfrak{a}_{ij}\mathbf{1}_{ii} \otimes I_m$. Note that $\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}} \mathcal{A}_{ij} = \mathscr{A} \otimes I_m$ and $\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}} \mathcal{D}_{ij} = \mathscr{D} \otimes I_m$. Also, let $\mathcal{B}_i \in \mathbb{R}^{\mathcal{F}\times\mathcal{F}}$ be defined as $\mathcal{B}_i \triangleq B \circ \mathbf{1}_{ii}$; note that $\sum_{(L,i)\in\mathcal{E}_L} \mathcal{B}_i = B$. Finally, let the vectors $Q_{\tau_{ji}}, Q_{\hat{\tau}_{ji}}, Q_{L\tau_{Li}} : \mathbb{R} \to \mathbb{R}^{\mathcal{F}m}$ be defined as $Q_{\tau_{ji}}(t) \triangleq Q_{\mathcal{F}}(t - \tau_{ji}(t))$, $Q_{\hat{\tau}_{ji}}(t) \triangleq Q_{\mathcal{F}}(t - \hat{\tau}_{ji}(t))$, $Q_{L\tau_{Li}}(t) \triangleq Q_L(t - \tau_{Li}(t))$.

By taking the time-derivative of R, premultiplying by the block inertia matrix M, using the Fundamental Theorem of Calculus, using the fact that $(\dot{Q}_L + \lambda Q_L) \in \operatorname{Null}(\mathcal{L}_{\mathcal{F}} \otimes I_m)$ due to the structure of the Laplacian matrix, and adding and subtracting terms, the closed-loop error system is represented using (4) and (6) as

$$\begin{aligned} M\dot{R} &= C\dot{Q}_{\mathcal{F}} + F + G - d + M\ddot{Q}_L + \lambda M\dot{E} - kL_B R \\ &\quad + k\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\left[\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)\int_{t-\tau_{ji}}^{t}\dot{R}(\sigma)d\sigma - \kappa_2\mathcal{D}_{ij}\int_{t-\hat{\tau}_{ji}}^{t-\tau_{ji}}\dot{R}(\sigma)d\sigma\right] \\ &\quad - k\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)\int_{t-\tau_{ji}}^{t}\left(\ddot{Q}_L(\sigma) + \lambda\dot{Q}_L(\sigma)\right)d\sigma \\ &\quad + k\kappa_2\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\mathcal{D}_{ij}\int_{t-\hat{\tau}_{ji}}^{t-\tau_{ji}}\left(\ddot{Q}_L(\sigma) + \lambda\dot{Q}_L(\sigma)\right)d\sigma \\ &\quad - k\kappa_2\sum_{(L,i)\in\mathcal{E}_L}(\mathcal{B}_i\otimes I_m)\int_{t-\hat{\tau}_{Li}}^{t}\dot{R}(\sigma)d\sigma \\ &\quad - k\kappa_1\sum_{(L,i)\in\mathcal{E}_L}(\mathcal{B}_i\otimes I_m)\int_{t-\tau_{Li}}^{t}\left(\ddot{Q}_L(\sigma) + \lambda\dot{Q}_L(\sigma)\right)d\sigma \\ &\quad + k\kappa_2\sum_{(L,i)\in\mathcal{E}_L}(\mathcal{B}_i\otimes I_m)\int_{t-\hat{\tau}_{Li}}^{t-\tau_{Li}}\left(\ddot{Q}_L(\sigma) + \lambda\dot{Q}_L(\sigma)\right)d\sigma \end{aligned} \quad (7)$$

where $\mathcal{L}_{ij} \triangleq \mathcal{D}_{ij} - \mathcal{A}_{ij} \in \mathbb{R}^{\mathcal{F}m\times\mathcal{F}m}$, and $L_B \triangleq (B + \mathscr{D} - \mathscr{A}) \otimes I_m \in \mathbb{R}^{\mathcal{F}m\times\mathcal{F}m}$, which is symmetric and positive definite by Assumption 1 and [55].

Consider the candidate Lyapunov function $V_L : \mathbb{R}^{2\mathcal{F}m+6} \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ defined as

$$V_L \triangleq \frac{c}{2}E^T E + \frac{1}{2}R^T M R + \Psi_{1a} + \Psi_{1b} + \Psi_{2a} + \Psi_{2b} + \Psi_{3a} + \Psi_{3b} \quad (8)$$

which satisfies the inequalities

$$\min\left\{\frac{c}{2}, \frac{\underline{m}}{2}, 1\right\}\|y\|^2 \leq V_L(y, t) \leq \max\left\{\frac{c}{2}, \frac{\bar{m}}{2}, 1\right\}\|y\|^2$$

for all $y \in \mathbb{R}^{2\mathcal{F}m+6}$ and $t \in \mathbb{R}$, where $c \in \mathbb{R}_{>0}$ denotes a tunable constant parameter, the block inertia matrix M is interpreted as a function of time, the state $y \in \mathbb{R}^{2\mathcal{F}m+6}$ is defined as the composite vector[2] $y \triangleq [Z^T, \Psi_{1a}^{(1/2)}, \Psi_{1b}^{(1/2)}, \Psi_{2a}^{(1/2)}, \Psi_{2b}^{(1/2)}, \Psi_{3a}^{(1/2)}, \Psi_{3b}^{(1/2)}]^T$, where $Z \in \mathbb{R}^{2\mathcal{F}m}$ is the composite error vector $Z \triangleq [\,E^T\ R^T\,]^T$, and $\Psi_{1a}, \Psi_{1b}, \Psi_{2a}, \Psi_{2b}, \Psi_{3a}, \Psi_{3b}$ denote LK functionals defined as

$$\Psi_{1a} \triangleq \frac{\phi\iota_2|\mathcal{E}_{\mathcal{F}}|}{k^2}\int_{t-\bar{\tau}}^{t}\int_{s}^{t}\left\|\dot{R}(\sigma)\right\|^2 d\sigma ds$$

$$\Psi_{1b} \triangleq \frac{\phi(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)}{k^2}\int_{t-\bar{\tau}-\bar{\tilde{\tau}}}^{t}\int_{s}^{t}\left\|\dot{R}(\sigma)\right\|^2 d\sigma ds$$

$$\Psi_{2a} \triangleq \frac{2\phi|\mathcal{E}_{\mathcal{F}}|\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau} + \bar{\tilde{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)}{\omega\underline{m}^2} \times \sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\int_{t-\tau_{ji}}^{t}\left(\left\|\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)R(\sigma)\right\|^2 + \kappa_2^2\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2\right)d\sigma$$

$$\Psi_{2b} \triangleq \frac{2\phi\kappa_2^2|\mathcal{E}_{\mathcal{F}}|\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau} + \bar{\tilde{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)}{\hat{\omega}\underline{m}^2} \times \int_{t-\hat{\tau}_{ji}}^{t}\left(|\mathcal{E}_{\mathcal{F}}|\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2 + |\mathcal{E}_L|\sum_{(L,i)\in\mathcal{E}_L}\|(\mathcal{B}_i\otimes I_m)R(\sigma)\|^2\right)d\sigma$$

$$\Psi_{3a} \triangleq \frac{2\phi|\mathcal{E}_{\mathcal{F}}|\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau} + \bar{\tilde{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)}{\omega\underline{m}^2} \times \sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\int_{t-\tau_{ji}}^{t}\int_{s}^{t}\left(\left\|\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)R(\sigma)\right\|^2 + \kappa_2^2\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2\right)d\sigma ds$$

[2]The LK functionals are interpreted as time-varying signals and are incorporated into the overall system state to facilitate the convergence analysis.

$$\Psi_{3b} \triangleq \frac{2\phi\kappa_2^2|\mathcal{E}_\mathcal{F}|\left(\bar{\tau}\iota_2|\mathcal{E}_\mathcal{F}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)\right)}{\hat{\omega}\underline{m}^2} \times \int_{t-\hat{\tau}_{ji}}^{t}\int_{s}^{t}\left(|\mathcal{E}_\mathcal{F}|\sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2 + |\mathcal{E}_L|\sum_{(L,i)\in\mathcal{E}_L}\|(\mathcal{B}_i\otimes I_m)R(\sigma)\|^2\right)d\sigma\, ds$$

where the constants $\iota_2, \iota_3, \iota_4 \in \mathbb{R}_{>0}$ are positive constants that depend on the network topology and $\phi \in \mathbb{R}_{>0}$ denotes a tunable constant parameter (see [50, Ch. 4]). The LK functionals are instrumental in obtaining an upper bound on the derivative of the Lyapunov function that is independent of expressions evaluated at a delay time (e.g., $t - \tau_{ji}$), such as in (7). LK functionals Ψ_{1a}, Ψ_{1b}, Ψ_{2a}, and Ψ_{2b} provide compensation of expressions in the closed-loop error system which are a function of a delay time. However, because the LK functionals Ψ_{1a}, Ψ_{1b}, Ψ_{2a}, and Ψ_{2b} are included in the state of the Lyapunov function V_L, the LK functionals Ψ_{3a}, Ψ_{3b} are used to provide negative feedback of LK functionals in the upper bound of the derivative of the Lyapunov function, at the cost of also injecting additional disturbance terms. Negative definite feedback of each LK functional helps provide full-state negative definite feedback in the upper bound of the derivative of the Lyapunov function, which facilitates the convergence analysis. Specifically, the derivatives of the LK functionals can be computed or upper-bounded using the Leibniz rule as

$$\dot{\Psi}_{1a} = \frac{\phi\iota_2|\mathcal{E}_\mathcal{F}|}{\underline{k}^2}\left(\bar{\tau}\left\|\dot{R}\right\|^2 - \int_{t-\bar{\tau}}^{t}\left\|\dot{R}(\sigma)\right\|^2 d\sigma\right) \tag{9}$$

$$\dot{\Psi}_{1b} = \frac{\phi(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)}{\underline{k}^2}\left(\left(\bar{\tau} + \bar{\bar{\tau}}\right)\left\|\dot{R}\right\|^2 - \int_{t-\bar{\tau}-\bar{\bar{\tau}}}^{t}\left\|\dot{R}(\sigma)\right\|^2 d\sigma\right) \tag{10}$$

$$\dot{\Psi}_{2a} \le \frac{2\phi|\mathcal{E}_\mathcal{F}|\left(\bar{\tau}\iota_2|\mathcal{E}_\mathcal{F}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)\right)}{\underline{m}^2} \times \sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left(\frac{1}{\omega}\left(\left\|\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)R\right\|^2 + \kappa_2^2\left\|\mathcal{D}_{ij}R\right\|^2\right) - \left(\left\|\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)R\left(t-\tau_{ji}\right)\right\|^2 + \kappa_2^2\left\|\mathcal{D}_{ij}R\left(t-\tau_{ji}\right)\right\|^2\right)\right) \tag{11}$$

$$\dot{\Psi}_{2b} \le \frac{2\phi\kappa_2^2|\mathcal{E}_\mathcal{F}|\left(\bar{\tau}\iota_2|\mathcal{E}_\mathcal{F}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)\right)}{\underline{m}^2} \times \left(\frac{|\mathcal{E}_\mathcal{F}|}{\hat{\omega}}\sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left\|\mathcal{D}_{ij}R\right\|^2 + \frac{|\mathcal{E}_L|}{\hat{\omega}}\sum_{(L,i)\in\mathcal{E}_L}\|(\mathcal{B}_i\otimes I_m)R\|^2 - |\mathcal{E}_\mathcal{F}|\sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left\|\mathcal{D}_{ij}R\left(t-\hat{\tau}_{ji}\right)\right\|^2 - |\mathcal{E}_L|\sum_{(L,i)\in\mathcal{E}_L}\left\|(\mathcal{B}_i\otimes I_m)R\left(t-\hat{\tau}_{ji}\right)\right\|^2\right) \tag{12}$$

$$\dot{\Psi}_{3a} \le \frac{2\phi|\mathcal{E}_\mathcal{F}|\left(\bar{\tau}\iota_2|\mathcal{E}_\mathcal{F}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)\right)}{\underline{m}^2} \times \sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left(-\int_{t-\tau_{ji}}^{t}\left(\left\|\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)R(\sigma)\right\|^2 + \kappa_2^2\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2\right)d\sigma + \frac{\bar{\tau}}{\omega}\left(\left\|\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)R\right\|^2 + \kappa_2^2\left\|\mathcal{D}_{ij}R\right\|^2\right)\right) \tag{13}$$

$$\dot{\Psi}_{3b} \le \frac{2\phi\kappa_2^2|\mathcal{E}_\mathcal{F}|\left(\bar{\tau}\iota_2|\mathcal{E}_\mathcal{F}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)\right)}{\underline{m}^2} \times \left(-\int_{t-\hat{\tau}_{ji}}^{t}\left(|\mathcal{E}_\mathcal{F}|\sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2 + |\mathcal{E}_L|\sum_{(L,i)\in\mathcal{E}_L}\|(\mathcal{B}_i\otimes I_m)R(\sigma)\|^2\right)d\sigma + \frac{\bar{\bar{\tau}}|\mathcal{E}_\mathcal{F}|}{\hat{\omega}}\sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left\|\mathcal{D}_{ij}R\right\|^2 + \frac{\bar{\bar{\tau}}|\mathcal{E}_L|}{\hat{\omega}}\sum_{(L,i)\in\mathcal{E}_L}\|(\mathcal{B}_i\otimes I_m)R\|^2\right) \tag{14}$$

where the uncertain constants $\omega, \hat{\omega} \in \mathbb{R}$ are defined as $\omega \triangleq 1 - \sup_{t\in\mathbb{R},(j,i)\in\mathcal{E}_\mathcal{F}\cup\mathcal{E}_L}\dot{\tau}_{ji}$ and $\hat{\omega} \triangleq 1 - \sup_{t\in\mathbb{R},(j,i)\in\mathcal{E}_\mathcal{F}\cup\mathcal{E}_L}\dot{\hat{\tau}}_{ji}$, and are positive by Assumptions 6 and 7. Feedback of the LK functionals from their derivatives is made possible by the inequalities

$$\frac{\Psi_{1a}}{\bar{\tau}} \le \frac{\phi\iota_2|\mathcal{E}_\mathcal{F}|}{\underline{k}^2}\int_{t-\bar{\tau}}^{t}\left\|\dot{R}(\sigma)\right\|^2 d\sigma \tag{15}$$

$$\frac{\Psi_{1b}}{\left(\bar{\tau} + \bar{\bar{\tau}}\right)} \le \frac{\phi(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)}{\underline{k}^2}\int_{t-\bar{\tau}-\bar{\bar{\tau}}}^{t}\left\|\dot{R}(\sigma)\right\|^2 d\sigma \tag{16}$$

$$\frac{\omega}{2}\Psi_{2a} + \frac{\omega}{2\bar{\tau}}\Psi_{3a} \le \frac{2\phi|\mathcal{E}_\mathcal{F}|\left(\bar{\tau}\iota_2|\mathcal{E}_\mathcal{F}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)\right)}{\underline{m}^2} \times \sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\int_{t-\tau_{ji}}^{t}\left(\left\|\left(\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right)R(\sigma)\right\|^2 + \kappa_2^2\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2\right)d\sigma \tag{17}$$

$$\frac{\hat{\omega}}{2}\Psi_{2b} + \frac{\hat{\omega}}{2\bar{\bar{\tau}}}\Psi_{3b} \le \frac{2\phi\kappa_2^2|\mathcal{E}_\mathcal{F}|\left(\bar{\tau}\iota_2|\mathcal{E}_\mathcal{F}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_\mathcal{F}| + \iota_4|\mathcal{E}_L|)\right)}{\underline{m}^2} \times \int_{t-\hat{\tau}_{ji}}^{t}\left(|\mathcal{E}_\mathcal{F}|\sum_{(j,i)\in\mathcal{E}_\mathcal{F}}\left\|\mathcal{D}_{ij}R(\sigma)\right\|^2 + |\mathcal{E}_L|\sum_{(L,i)\in\mathcal{E}_L}\|(\mathcal{B}_i\otimes I_m)R(\sigma)\|^2\right)d\sigma \tag{18}$$

which can be demonstrated using the methods developed in [50, Ch. 4].

By using the closed-loop error system in (7), the time derivative of (8) can be expressed as

$$\dot{V}_L = cE^T(R-\lambda E) + \frac{1}{2}R^T\dot{M}R + R^T M\dot{R} + \dot{\Psi}_{1a} + \dot{\Psi}_{1b} + \dot{\Psi}_{2a} + \dot{\Psi}_{2b} + \dot{\Psi}_{3a} + \dot{\Psi}_{3b} \tag{19}$$

where the expression for the closed-loop error system $M\dot{R}$ is given in (7). After using the derivatives and given inequalities associated with the LK functionals in (9)–(18), Young's inequality, the Cauchy-Schwarz inequality, the inequality $\|\sum_{i=1}^{n}\xi\|^2 \le n\sum_{i=1}^{n}\|\xi\|^2$ for $\xi \in \mathbb{R}^{\mathcal{F}m}$, expanding the resulting expressions that contain $\|\dot{R}\|^2$, and canceling terms, (19) can be upper-bounded as (see [50, Ch. 4])

$$\begin{aligned}\dot{V}_L \le & -k\left(\lambda_{\min}(L_B) - \frac{\left(\bar{\tau}+\bar{\bar{\tau}}\right)k^3}{\phi} - \frac{c}{2k}\right)\|R\|^2 \\ & - c\left(\lambda - \frac{1}{2}\right)\|E\|^2 - \frac{\Psi_{1a}}{2\bar{\tau}} - \frac{\Psi_{1b}}{2\left(\bar{\tau}+\bar{\bar{\tau}}\right)} - \frac{\omega}{2}\Psi_{2a} \\ & - \frac{\hat{\omega}}{2}\Psi_{2b} - \frac{\omega}{2\bar{\tau}}\Psi_{3a} - \frac{\hat{\omega}}{2\bar{\bar{\tau}}}\Psi_{3b} + \left(N_{d0}+\tilde{N}_0\right)\|R\| \\ & + \phi\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right) \\ & \times \left(\left(N_{d1}+\tilde{N}_1\right)\|R\| + N_{d2} + \tilde{N}_2\right) + k\iota_1\|R\| \end{aligned} \tag{20}$$

where the functions $\tilde{N}_0, \tilde{N}_1, \tilde{N}_2 : \Pi_{p=1}^{6}\mathbb{R}^{\mathcal{F}m} \to \mathbb{R}$ contain contributions from state-dependent terms in the agents' dynamics, have the arguments $Q_{\mathcal{F}}$, $\dot{Q}_{\mathcal{F}}$, Q_L, $\dot{Q}_L$, E, R, and can be, respectively, upper-bounded with the strictly increasing, radially unbounded functions $\rho_0, \rho_1, \rho_2 : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ such that $|\tilde{N}_k| \le \rho_k(\|Z\|)\|Z\|$ for $k \in \{1,2,3\}$ (see [56, Lemma 5]), and the constants N_{d0}, N_{d1}, $N_{d2} \in \mathbb{R}_{\ge 0}$ contain contributions from bounded terms in the agents' dynamics and are bounded above, and the positive constant $\iota_1 \in \mathbb{R}_{\ge 0}$ is defined as

$$\begin{aligned}\iota_1 \triangleq \left(\bar{\ddot{Q}}_L + \lambda\bar{\dot{Q}}_L\right)\Bigg(&\bar{\tau}\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\left\|\kappa_1\mathcal{A}_{ij} - \kappa_2\mathcal{L}_{ij}\right\| \\ & + \kappa_2\bar{\bar{\tau}}\sum_{(j,i)\in\mathcal{E}_{\mathcal{F}}}\left\|\mathcal{D}_{ij}\right\| + \left(\kappa_1\bar{\tau} + \kappa_2\bar{\bar{\tau}}\right)\sum_{(L,i)\in\mathcal{E}_L}\|\mathcal{B}_i\|\Bigg)\end{aligned}$$

where the constant upper bounds $\bar{\dot{Q}}_L, \bar{\ddot{Q}}_L \in \mathbb{R}_{\ge 0}$ are defined such that $\sup_{t\in\mathbb{R}}\|\dot{Q}_L(t)\| \le \bar{\dot{Q}}_L$ and $\sup_{t\in\mathbb{R}}\|\ddot{Q}_L(t)\| \le \bar{\ddot{Q}}_L$. After using the inequality $k\iota_1\|R\| \le k^2\iota_1\|R\|^2 + (\iota_1/4)$, the feedback gain of the signal $\|R\|^2$ can be expressed as $\underline{k} \triangleq k(\lambda_{\min}(L_B) - [((\bar{\tau}+\bar{\bar{\tau}})k^3)/2\phi] - \iota_1 k) - (c/2)$. Provided that the bounds $\bar{\tau}$ and $\bar{\bar{\tau}}$ are sufficiently small, the gain k can be selected such that $\underline{k} > 0$, where the tuning parameter c is selected such that $0 < c < 2k(\lambda_{\min}(L_B) - [(\bar{\tau}+\bar{\bar{\tau}})/2\phi]k^3 - \iota_1 k)$. By implementing the bounding functions ρ_0, ρ_1, ρ_2, using a fraction of the feedback $-\underline{k}\|R\|^2$ to perform nonlinear damping on the other terms multiplied by $\|R\|$, and using the inequality $\rho_2(\|Z\|)\|Z\| \le \rho_2^2(\|Z\|)\|Z\|^2 + (1/4)$, (20) can be upper-bounded as

$$\begin{aligned}\dot{V}_L \le & -\eta\|Z\|^2 - \frac{\Psi_{1a}}{2\bar{\tau}} - \frac{\Psi_{1b}}{2\left(\bar{\tau}+\bar{\bar{\tau}}\right)} \\ & - \frac{\omega}{2}\Psi_{2a} - \frac{\hat{\omega}}{2}\Psi_{2b} - \frac{\omega}{2\bar{\tau}}\Psi_{3a} - \frac{\hat{\omega}}{2\bar{\bar{\tau}}}\Psi_{3b} \\ & - \|Z\|^2\Bigg(\eta - \frac{3\rho_0^2(\|Z\|)}{2\underline{k}} \\ & - \frac{3\phi^2\rho_1^2(\|Z\|)}{2\underline{k}}\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)^2 \\ & - \phi\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)\rho_2^2(\|Z\|)\Bigg) \\ & + \frac{3N_{d0}^2}{2\underline{k}} + \frac{3\phi^2N_{d1}^2}{2\underline{k}}\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)^2 \\ & + \phi\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)\left(\frac{1}{4} + N_{d2}\right) + \frac{\iota_1}{4}\end{aligned} \tag{21}$$

where $\eta \triangleq \min\{[(c(\lambda - (1/2)))/2], (\underline{k}/6)\}$ is the feedback gain of the signal $\|Z\|^2$. The expression in (21) can be upper-bounded as

$$\begin{aligned}\dot{V}_L \le & -\theta\|y\|^2 + \frac{\iota_1}{4} + \frac{3N_{d0}^2}{2\underline{k}} + \frac{3\phi^2N_{d1}^2}{2\underline{k}} \\ & \times \left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)^2 \\ & + \phi\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)\left(\frac{1}{4} + N_{d2}\right)\end{aligned} \tag{22}$$

where $\theta \in \mathbb{R}_{>0}$ is the feedback gain of the signal $\|y\|^2$ defined as $\theta \triangleq (1/2)\min\{2\eta, [1/(\bar{\tau}+\bar{\bar{\tau}})], \omega, (\omega/\bar{\tau}), \hat{\omega}, (\hat{\omega}/\bar{\bar{\tau}})\}$, provided that the gains η and $\underline{k}$ are sufficiently large; i.e., (22) is valid for all $y \in \mathfrak{D}$, where the set $\mathfrak{D} \subset \mathbb{R}^{2\mathcal{F}m+6}$ is defined as

$$\mathfrak{D} \triangleq \left\{\xi \in \mathbb{R}^{2\mathcal{F}m+6} \mid \|\xi\| < \inf\left\{\rho^{-1}\left([\sqrt{\eta}, \infty)\right)\right\}\right\}$$

where $\rho : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is a strictly increasing, radially unbounded function defined as

$$\begin{aligned}\rho(\|Z\|) \triangleq \Bigg(&\frac{3\rho_0^2(\|Z\|)}{2\underline{k}} + \frac{3\phi^2\rho_1^2(\|Z\|)}{2\underline{k}} \\ & \times \left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)^2 \\ & + \phi\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau}+\bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right) \\ & \times \rho_2^2(\|Z\|)\Bigg)^{\frac{1}{2}}\end{aligned} \tag{23}$$

and the inverse image $\rho^{-1}(\Theta) \subset \mathbb{R}$ for a set $\Theta \subset \mathbb{R}$ is defined as $\rho^{-1}(\Theta) \triangleq \{\xi \in \mathbb{R} \mid \rho(\xi) \in \Theta\}$. By using [57, Th. 4.18] with (22), we have that

$$\limsup_{t\to\infty}\|q_i(t) - q_L(t)\| \le \limsup_{t\to\infty}\|y(t)\| \le \varepsilon$$

uniformly in time for all $i \in \mathcal{V}_{\mathcal{F}}$ and $y(0) \in \mathcal{S}_{\mathfrak{D}}$ provided that ε is sufficiently small such that a ball of radius ε centered at the origin fits within $\mathfrak{D}$, that is

$$\inf\left\{\rho^{-1}\left([\sqrt{\eta}, \infty)\right)\right\} > \varepsilon$$

where the constant bound $\varepsilon \in \mathbb{R}$ is defined as

$$\begin{aligned}\varepsilon \triangleq \Bigg(\frac{2\max\left\{\frac{c}{2}, \frac{\bar{m}}{2}, 1\right\}}{\theta \min\left\{\frac{c}{2}, \frac{\underline{m}}{2}, 1\right\}}\Bigg(\frac{3}{2\underline{k}}\Bigg(N_{d0}^2 \\ + \phi^2 N_{d1}^2\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right)^2\Bigg) \\ + \phi\left(\bar{\tau}\iota_2|\mathcal{E}_{\mathcal{F}}| + \left(\bar{\tau} + \bar{\bar{\tau}}\right)(\iota_3|\mathcal{E}_{\mathcal{F}}| + \iota_4|\mathcal{E}_L|)\right) \\ \times \left(\frac{1}{4} + N_{d2}\right) + \frac{\iota_1}{4}\Bigg)\Bigg)^{\frac{1}{2}}\end{aligned} \tag{24}$$

and the set of stabilizing initial conditions $\mathcal{S}_{\mathfrak{D}}$ is defined as

$$\mathcal{S}_{\mathfrak{D}} \triangleq \left\{\xi \in \mathfrak{D} \mid \|\xi\| < \sqrt{\frac{\min\left\{\frac{c}{2}, \frac{\bar{m}}{2}, 1\right\}}{\max\left\{\frac{c}{2}, \frac{\underline{m}}{2}, 1\right\}}} \inf\left\{\rho^{-1}\left([\sqrt{\eta}, \infty)\right)\right\}\right\}.$$

Hence, since $y, q_L, \dot{q}_L \in \mathcal{L}_\infty$, it is clear that $q_i, \dot{q}_i \in \mathcal{L}_\infty$ for all $i \in \mathcal{V}_{\mathcal{F}}$, and each agent's control effort is bounded during the entire state trajectory. Furthermore, as the communication delay and delay estimate tend toward zero (ignoring the singularity of $\bar{\tau} \equiv 0$, which obviates the need for the LK functional-based approach taken in this paper), the effects of the delay vanish and the convergence analysis resembles that of a high-gain robust control analysis, similar to that in [4]. However, note that in the presence of delays, it may not be possible to make the synchronization error arbitrarily small, as seen in the definition of ε in (24). Note also that in the presence of no delay, the result is still UUB. This is caused by the presence of uncertain nonlinear dynamics and exogenous disturbances; a robust control term, such as a sliding-mode or RISE (see [4]) based term, would be needed to force the synchronization errors to go to zero.

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