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Balanced containment control and cooperative timing of a multiagent system over random communication graphs

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Summary

A multiagent system is considered, which is tasked with the objective of approaching a predetermined target from a desired region to minimize detection and then simultaneously converge at the target. The considered cooperative timing problem consists of 2 stages: navigation and simultaneous arrival. During the navigation stage, the agents are driven from a distant starting point toward the target while restricting their motion within a desired area. Only a few agents (ie, leaders) are equipped with the desired bearing information to the target, whereas the remaining agents (ie, followers) may only have local feedback with neighboring agents to coordinate their headings. No range information to the target and no absolute or other relative position information among agents are available. The arrival stage begins when agents enter a neighborhood of the target (ie, range information becomes available during the arrival stage), and agents coordinate their motion to perform simultaneous arrival. The agents could experience random loss of communication with immediate neighbors, which results in a stochastic communication network. On the basis of the random communication network, balanced containment control is developed, which almost surely restricts the motion of the group within a desired region while equally spacing the agents. An almost sure consensus algorithm is designed for agents to coordinate the simultaneous arrival time by achieving a consensus on informed agents during the arrival stage. Simulation results demonstrate the performance of the developed approach.

KEYWORDS

balanced containment control, cooperative timing, random communication graphs

1 | INTRODUCTION

In multiagent systems, agents communicate their position and/or velocity information with others to coordinate their behaviors.[1-5] The availability of the position information of the agents is always assumed in navigation and coordination. Continuous information exchange among agents over a reliable communication network is also a widely adopted assumption in these applications. However, position information may not be available in all cases, eg, autonomous vehicles operating in GPS-denied environments. For some applications, because of obstacles and interference, agents can experience random loss of communication with other team members. Hence, developing a cooperative controller to perform collective tasks for a multiagent system over random communication networks can be challenging.

A multiagent cooperative timing problem is considered in this paper. Example military applications of cooperative timing include reaching the boundary of a radar detection area by unmanned aerial vehicles at the same time to complete desired military missions[6] or performing simultaneous strike by munitions with the objective of simultaneous target arrival.[7] Cooperative timing problems are also applicable to civilian applications such as refueling scenarios and hazardous material monitoring.[8] Motivated by cooperative timing applications, a multiagent system is tasked in this paper with the objective of simultaneous arrival at a predefined target while approaching the target from a desired region. The main challenges in the considered problem include intermittent communication and inconsistent information exchange among agents due to the terrain and obstacles, limited availability of team members' states within an uncertain complex environment, and the coordination of agents motion in performing simultaneous arrival.

Numerous results on rendezvous problems can be found in the literature.[9-15] Synchronized and unsynchronized strategies are developed for a group of autonomous mobile agents to ensure convergence at a common point by only using position feedback from sensing regions.[9-11] The rendezvous problem for mobile agents with nonholonomic constraints is considered by Dimarogonas and Kyriakopoulos.[12] A hybrid dynamic rendezvous protocol is designed to address finite-time rendezvous problems.[13] However, the agents in the literature[9-13] can only converge to a common setpoint determined by the initial deployment of the group rather than a predefined destination. Moreover, cooperative timing is not considered in the literature[9-13] which implies that the agents may not arrive at the common setpoint at the same time. To achieve simultaneous arrival at the desired destination, a distributed potential field-based approach is developed for a leader-follower network by requiring followers to achieve consensus with leaders, where leaders are the only agents with knowledge of the destination.[14] The coordinated arrival problem is studied, where agents are controlled to reach the targets either simultaneously or in a given time sequence.[16] Despite the consideration of simultaneous arrival, the aforementioned results are all developed on the basis of the deterministic communication networks that allow consistent information exchange between agents. Such results do not consider random failures of interagent communication when operating in an uncertain complex environment where communication may be actively inhibited.

Consensus problems over random graphs are the focus of the current research. Assuming that the communication links between any pair of agents are activated independently with a common probability, an almost sure consensus is established by Hatano and Mesbahi,[17] which is then extended from undirected random graphs to a general class of directed random graphs.[18,19] Necessary and sufficient conditions for consensus are developed by Tahbaz-Salehi and Jadbabaie[20,21] for graphs that are generated by ergodic and stationary random processes. Mean square average consensus and almost sure consensus over time-varying topologies with communication noise are investigated in the works of Tahbaz-Salehi and Jadbabaie[22] and Kar and Moura.[23] When considering Markovian switching topologies, sufficient conditions for mean square and almost sure consensus are developed for linear multiple-input–multiple-output multiagent systems.[24] However, most of the aforementioned convergence results are developed for leaderless networks, and only a few results consider constrained consensus (eg, balanced containment control as in this work).

On the basis of the preliminary efforts,[25] the cooperative timing problem in this paper consists of 2 stages: navigation and simultaneous arrival. The navigation stage aims to drive the group of agents from a distant starting point toward the target while restricting their motion within a desired area. The simultaneous arrival stage is then activated to coordinate the agents' motion to perform simultaneous arrival when the agents enter a neighborhood of the target. During the navigation stage, it is assumed that agents are only aware of the orientation to the target. No range information to the target and no absolute or other relative position information among agents are available. Only a few agents (ie, leaders) are equipped with the desired bearing information to the target, whereas the remaining agents (ie, followers) may only have local feedback with neighboring agents to coordinate their headings. It is further assumed that the agents may experience random loss of communication with immediate neighbors, which results in a stochastic communication network. When approaching the target within a desired area (eg, a circular sector), the agents are also required to be equally spaced around the sector to optimize mission performance. The arrival stage begins when agents are close enough to use short-range sensors (eg, camera or Lidar) to estimate the distance to the target (ie, range information becomes available during the arrival stage), and their speed with neighboring agents are coordinated over the random communication network to perform simultaneous arrival.

Compared with the classical containment control over deterministic communication networks[26-30] that focus on driving the follower agents to a desired region (eg, a convex hull) determined by the leaders, this work aims to achieve balanced containment control over stochastic communication networks. Rather than assuming constant interagent information exchange, the random network considered in the current work can be used to model a large class of real-world networks to reflect the unpredictable and time-varying nature of the underlying communication network in a complex environment. A decentralized control algorithm is developed to not only ensure that the agents move within the desired circular

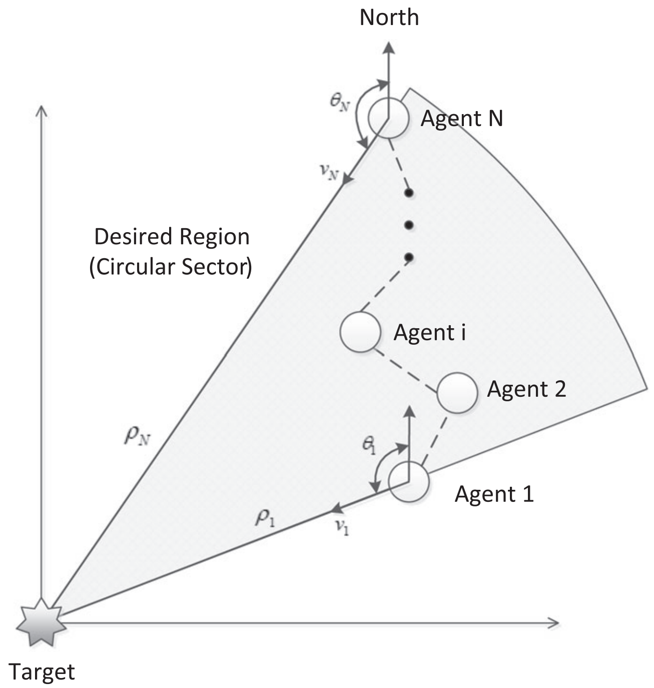

FIGURE 1 The problem scenario describing simultaneous arrival of N agents at a predefined target. The group of agents is required to be equally spaced in the sense that the relative orientations between 2 agents is equal and move within a desired circular sector toward the target. The interaction of agents is modeled as a directed graph, where the edges are indicated by dashed lines

sector but also to ensure that the agents are equally spaced within the sector. To achieve simultaneous arrival, under the constraint that only leaders are informed of the desired common arrival time, a consensus algorithm is designed for agents to coordinate their arrival time. Moreover, the developed controller allows the agents to simultaneously arrive at any desired destination versus at any arbitrary destination without cooperative timing.[9-13,31,32]

2 | PROBLEM FORMULATION

A group of N agents is tasked to simultaneously arrive at a predefined stationary destination as illustrated in Figure 1. Let $\mathcal{P}$ denote an inertial polar coordinate reference frame. Without loss of generality, the target is assumed to be at the origin of $\mathcal{P}$. Consider a time sequence $t_k = k\Delta$, $k \in \mathbb{Z}^+$, with $\Delta \in \mathbb{R}^+$ being a sampling period. The agents' states evolve according to the discrete kinematics* as

$$\begin{aligned} \rho_i(k+1) &= \rho_i(k) - v_i(k)\Delta, \\ \theta_i(k+1) &= \theta_i(k) + \omega_i(k)\Delta, \end{aligned} \quad i = 1, \cdots, N, \tag{1}$$

where $\rho_i(k) \in \mathbb{R}^+$ denotes the distance from agent i to the target, $\theta_i(k) \in [0, 2\pi)$ denotes its orientation measured with respect to the target and the common north, and $v_i(k) \in \mathbb{R}^+$ and $\omega_i(k) \in \mathbb{R}$ are the control inputs.

The considered cooperative timing problem consists of 2 stages: navigation and simultaneous arrival. During navigation, the agents are assumed to move with constant linear velocities v_i. Although the orientation to the target (ie, θ_i) is available to the agents, no range information to the target (ie, ρ_i) and no absolute or other relative position information among agents are available during the navigation stage. It is further assumed that only a small set of agents (ie, leaders) are informed of the desired bearing information to the target, where the remaining agents (ie, followers) can only use local feedback with neighboring agents to coordinate their headings. In addition, agents are required to approach the target within a desired area defined by a circular sector while equally spaced in terms of their relative orientations (ie, $\theta_2 - \theta_1 = \ldots = \theta_N - \theta_{N-1}$) to optimize the mission performance. During the simultaneous arrival stage, after the agents are driven

*The primary contribution of this work is in the cooperative timing, and known methods can be used to compensate for more complex kinematics/dynamics of the agent. To specifically focus on this contribution, the methods (like other such results in literature) are developed on the basis of the single-integrator kinematics.

to the neighborhood of the target where the agents can estimate the range information to the target, cooperative timing among agents is performed by communicating the estimated arrival time and controlling the velocity of the agents.

2.1 | Directed random graph

The group of agents is assumed to communicate and exchange θ_i over a wireless network. Since the agents may experience unexpected communication failure with neighboring agents in a complex and unreliable environment, the wireless network is modeled as a time-varying graph $\mathcal{G}_\theta(t) = (\mathcal{V}, \mathcal{E}_\theta(t))$, where the node set $\mathcal{V}$ represents agents and the edge set $\mathcal{E}_\theta(t) \subseteq \mathcal{V} \times \mathcal{V}$ indicates the directed orientation exchange between agents at time t. At each time instant, suppressing the time dependence, the graph $\mathcal{G}_\theta$ is a directed random graph. The time-varying graph $\mathcal{G}_\theta(t)$ consists of a time sequence of directed random graphs in which the edge connections vary randomly with time. In particular, associated with each agent i, let there be a Bernoulli random variable δ_i such that $\delta_i = 1$ indicates that agent i is able to communicate with its neighbors and $\delta_i = 0$ indicates failed communication. The stochastic processes $\{\delta_i(t)\}$ are assumed to evolve according to a 2-state continuous-time homogeneous Markov process (see chapter 6 in the work of Grimmett and Stirzaker[33]), where the transition rates between the state $\delta_i = 0$ and $\delta_i = 1$ at agent i can be specified as $\lambda_{0,1}^{(i)} > 0$. It is further assumed that, for different agents, the $\{\delta_i(t)\}$ are statistically independent.

Assumption 1. The random processes $\{\delta_i(t)\}$ do not change infinitely fast, and the sampling time Δ is selected such that $\delta_i(t) = \delta_i(t + t_0)$ if $0 \leq t_0 < \Delta, \forall i \in \mathcal{V}$.

Note that Assumption 1 will be true in many real systems. For example, let T_0 and T_1 denote the expected dwell times in states 0 and 1 for the Markov process $\delta_i(t)$, respectively. Then, the probability of staying in the same state during an observation period can be made arbitrarily large by selecting an appropriate Δ (eg, the probability of remaining in state 0 during an interval of length Δ is $e^{-\Delta/T_0}$).

Assumption 1 indicates that the graph $\mathcal{G}_\theta(t)$ remains constant over each time interval $[t_k, t_{k+1})$. Let $\mathcal{G}_\theta(k)$ denote the random graph $\mathcal{G}_\theta(t)$ at $t = t_k$. Note that $\mathcal{G}_\theta(k)$ is drawn from a finite sample space $\bar{\mathcal{G}}_\theta \triangleq \{\mathcal{G}_\theta^1, \dots, \mathcal{G}_\theta^M\}$, $M \in \mathbb{Z}^+$, and $|\bar{\mathcal{G}}_\theta| \leq 2^{|\mathcal{V}|}$, which is determined by the power set of $\mathcal{V}$. Note that $|\mathcal{V}|$ and $|\bar{\mathcal{G}}_\theta|$ denote the cardinality of the set $\mathcal{V}$ and $\bar{\mathcal{G}}_\theta$, respectively. Note that the graphs $\mathcal{G}_\theta^i \in \bar{\mathcal{G}}_\theta$, $i = \{1, \dots, M\}$, are directed graphs, which share a common node set $\mathcal{V}$ and differ in the edge set due to the random variable δ_i. The subsequent development is based on the availability of 2 leaders.[†] Specifically, the leader set is defined as $\mathcal{V}_L = \{1, N\}$, and the follower set is defined as $\mathcal{V}_F = \{2, \dots, N-1\}$, where the leaders move with desired immutable orientations, θ_1 and θ_N, such that $\theta_1 < \theta_N$, which determines the desired region as shown in Figure 1. The followers can only communicate with neighboring agents and update their orientations over the random communication graph $\mathcal{G}_\theta(k)$. Let $\mathcal{G}_\theta^* \in \bar{\mathcal{G}}_\theta$ be the graph where all agents are able to communicate with their immediate neighboring agents (ie, $\delta_i = 1$ for $\forall i \in \mathcal{V}$). To ensure that the followers approach the target within a desired region determined by the leaders over the directed random network, the following assumptions are made.

Assumption 2. $\mathcal{G}_\theta^* \in \bar{\mathcal{G}}_\theta$ and $\Pr(\mathcal{G}_\theta(k) = \mathcal{G}_\theta^*) > 0$ for all k, where $\Pr(\diamond)$ denotes the probability that the event $\diamond$ occurs.

Assumption 3. In $\mathcal{G}_\theta^*$, the leaders have directed paths to every follower and each follower can exchange orientation information with its immediate neighbors (ie, node $i-1$ and $i+1$ for follower i).

Assumption 2 will be true in many real systems if the Markov processes $\{\delta_i(t)\}$ for the different nodes are statistically independent. Assumption 3 indicates that the underlying communication graph contains a line graph, where leaders in $\mathcal{V}_L$ act as the roots in $\mathcal{G}_\theta^*$ and their states can be delivered to all the nodes through a connected path. In our work, since one objective is to achieve balanced heading for each agent i with respect to its immediate neighbors, such a line graph assumption enables information exchange of follower i with its neighbors $i-1$ and $i+1$ and is necessary for mission completion. The line graph assumption is not a strong assumption for multiagent systems and has been widely used in the literature. For example, the line graph assumption has been applied in the works of Marshall et al[34] and Zheng et al[35] to perform cyclic pursuit or circular formation. Assumption 3 does not limit the underlying communication graph to be an exact line graph. A general communication graph could also be applied as long as the general communication graph contains a line graph as described in Assumption 3.

[†]The developed approach is not limited to the particular case that the convex hull is a line segment $[\theta_1, \theta_N]$, which only requires 2 leaders. If a multidimensional convex hull is considered,[26-30] our approach can be extended to achieve balanced deployment of followers within the convex hull formed by more than 2 leaders.

To avoid notational confusion, let $\theta(k) \triangleq [\theta_1(k), \dots, \theta_N(k)]^T$ and $\Theta(k) \triangleq [\Theta_1(k), \dots, \Theta_N(k)]^T$ denote the stacked deterministic states $\theta_i(k)$ and the stacked corresponding random variables $\Theta_i(k)$, respectively, for all $i \in \mathcal{V}$ at time k. Because of the random variable $\delta_i(k)$ associated with each agent, the second equation in (1) is rewritten for each follower as

$$\Theta_i(k+1) = \Theta_i(k) + \Delta\delta_i(k)\omega_i(k), \quad \forall i \in \mathcal{V}_F. \tag{2}$$

To coordinate the arrival time, similar to $\mathcal{G}_\theta(t)$, interagent communication is modeled as a directed random graph $\mathcal{G}_\rho(t) = (\mathcal{V}, \mathcal{E}_\rho(t))$, where the directed edge $(j, i) \in \mathcal{E}_\rho$ indicates that node i is able to access the states of node j. After completing the orientation control objective, velocity control is activated to perform a simultaneous arrival where short-range sensors (eg, camera or Lidar) can be used to estimate the range to the target. Because of the random communication network, kinematics in (1) can be simplified and written in a stochastic manner as

$$P_i(k+1) = P_i(k) - v_i(k)\Delta \tag{3}$$

for $k \geq T_a$, where T_a denotes the time instant that velocity control is activated and $v_i(k)$ is the input. Instead of using the x and y position of the agents, only range information ρ_i to the target is required in (3). In (3), $P_i(k)$ and $P_i(k+1)$ are corresponding random variables of $\rho_i(k)$ and $\rho_i(k+1)$, respectively. Similar to $\bar{\mathcal{G}}_\theta$, let $\bar{\mathcal{G}}_\rho \triangleq \{\mathcal{G}_\rho^1, \dots, \mathcal{G}_\rho^S\}$ denote the sample space of $\mathcal{G}_\rho$ and $\mathcal{G}_\rho^* \in \bar{\mathcal{G}}_\rho$ denote a particular element graph in which every agent is able to communicate with its immediate neighbors. Different from the navigation stage where 2 leaders (ie, $\mathcal{V}_L = \{1, N\}$) are required to determine the desired circular area when approaching the target, only one informed agent (ie, leader) with the desired arrival time is required to perform simultaneous target arrival. The following assumption is made to facilitate the subsequent development.

Assumption 4. The directed graph $\mathcal{G}_\rho^* \in \bar{\mathcal{G}}_\rho$ has a directed spanning tree with leader Node 1 as the root so that Node 1 always has directed influence to every other follower agent, and $\Pr(\mathcal{G}_\rho(k) = \mathcal{G}_\rho^*) > 0$, for all k.

3 | BALANCED CONTAINMENT CONTROL

3.1 | Control design

To equally space the agents in the angular sector (θ_1, θ_N), the orientation controller $\omega(k) \triangleq [\omega_1(k), \dots, \omega_N(k)]^T \in \mathbb{R}^N$ is designed as

$$\omega(k) = -K_g Q D \theta(k), \tag{4}$$

where $K_g \in \mathbb{R}^+$ is a control gain and $Q \in \mathbb{R}^{N\times(N-1)}$ and $D \in \mathbb{R}^{(N-1)\times N}$ are defined as

$$Q \triangleq \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ 1 & -1 & 0 & \cdots & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & -1 \\ 0 & \cdots & \cdots & 0 & 0 \end{bmatrix} \tag{5}$$

and

$$D \triangleq \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}, \tag{6}$$

respectively. Let the orientation difference between 2 adjacent agents be defined as

$$e_i(k) \triangleq \theta_{i+1}(k) - \theta_i(k) \tag{7}$$

for $i = 1, \dots, N-1$. The term $D\theta(k)$ in (4) is the group orientation difference

$$e(k) = D\theta(k), \tag{8}$$

where $e(k) \triangleq [e_1(k), \dots, e_{N-1}(k)] \in \mathbb{R}^{N-1}$. The matrix Q in (4) is designed to achieve consensus by allowing each follower to update its own orientation by using the orientation differences from the adjacent nodes (ie, e_{i-1} and e_i for node i). Since the leaders are informed of the desired orientation to the target and have directed paths to the followers, the leaders can influence the followers through local communication but not vice versa. Hence, matrix Q is designed with $Q_{1i} = 0$ and $Q_{Ni} = 0$, for all $i = 1, \dots, N-1$, which indicates that θ_1 and θ_N are immutable from (4).

The balanced deployment of the agents' orientations is achieved if each e_i achieves consensus, ie, $e_1 = \cdots = e_{N-1}$. Let $E_i(k) \in \mathbb{R}$ be the corresponding random variable of $e_i(k)$ in (7) and $E_k \triangleq [E_1(k), \dots, E_{N-1}(k)] \in \mathbb{R}^{N-1}$ be the stacked vector. Using (2) and (4), the deterministic system in (8) is written in a stochastic form as

$$E_{k+1} = D\Theta_{k+1} = \left(I_{N-1} - K_g \Delta D \Xi Q\right) E_k, \tag{9}$$

where $\Xi(k) \in \mathbb{R}^{N\times N}$ is a diagonal random matrix with $\Xi_{ii}(k) = \delta_i(k)$ and $\Xi_{ij} = 0$ for $\forall i \neq j$. Let $\Phi \triangleq D\Xi Q \in \mathbb{R}^{(N-1)\times(N-1)}$, which is computed from (5) and (6) as

$$\Phi = \begin{bmatrix} \delta_2 & -\delta_2 & 0 & \cdots & 0 \\ -\delta_2 & \delta_2+\delta_3 & -\delta_3 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -\delta_{N-2} & \delta_{N-2}+\delta_{N-1} & -\delta_{N-1} \\ 0 & \cdots & 0 & -\delta_{N-1} & \delta_{N-1} \end{bmatrix}. \tag{10}$$

From (10), it is clear that the random matrix Φ is a symmetric matrix with zero row sums, ie, $\Phi \mathbf{1}_{N-1} = 0$, where $\mathbf{1}_{N-1} = [1, \dots, 1]^T \in \mathbb{R}^{N-1}$. Moreover, since the diagonal entries are nonnegative, matrix Φ is a positive semidefinite matrix from the Gersgorin circle theorem.[36] Let $\Psi \triangleq I_{N-1} - K_g \Delta \Phi$, then (9) can be represented in a compact form as

$$E_{k+1} = \Psi E_k, \tag{11}$$

where Ψ is a symmetric random matrix.

Definition 1. The agreement space $\mathcal{A} \in \mathbb{R}^{N-1}$ is defined as the subspace spanned by $\{\mathbf{1}_{N-1}\}$, ie, $\mathcal{A} = \{c\mathbf{1}_{N-1} | c \in \mathbb{R}\}$.

Lemma 1. *The agreement space $\mathcal{A}$ is the equilibrium set of (11). Moreover, $\sum_{i=1}^{N-1} E_i(k)$ for $\forall k$ is invariant.*

Proof. Consider a random sequence $\{E_k\}$ evolving according to (11). If $E_{k_0} = c\mathbf{1}_{N-1}$ for $\forall k_0 \in \mathbb{Z}^+$, it must have $E_{k_0+1} = c\mathbf{1}_{N-1}$ since

$$\begin{aligned} E_{k_0+1} &= \Psi E_{k_0} \\ &= \left(I_{N-1} - K_g \Delta \Phi\right) c\mathbf{1}_{N-1} \\ &= c\mathbf{1}_{N-1}, \end{aligned} \tag{12}$$

where $\Phi\mathbf{1} = 0$ is used. Similar to (12), it can be shown that $E_{k+1}^T \mathbf{1}_{N-1} = E_k^T \Psi \mathbf{1}_{N-1} = E_k^T \mathbf{1}_{N-1}$, which indicates that $\sum_{i=1}^{N-1} E_i(k)$ is invariant (ie, an equilibrium set). □

3.2 | Convergence analysis

Since the consensus of all e_i (ie, $e_1 = \cdots = e_{N-1}$) indicates a balanced deployment of the agents, an almost sure convergence of the orientation difference e_i's ($i = 1, \dots, N-1$) to a common value in the agreements space $\mathcal{A}$ is established in this section for the agreement protocol in (4) over the directed random graph $\mathcal{G}_\theta$. To facilitate the subsequent convergence analysis, the definitions of almost sure convergence[17] and supermartingale[33] are introduced.

Definition 2. The random sequence $\{Z(k)\}$ in $\mathbb{R}^{N-1}$ almost surely converges to an agreement $Z^* \in \mathcal{A}$ if

$$\lim_{k_0\to\infty} \Pr \left\{ \sup_{k \geq k_0} \inf_{Z^* \in \mathcal{A}} \|Z(k) - Z^*\| > \epsilon \right\} = 0, \tag{13}$$

for every $\epsilon > 0$. An almost sure convergence is also called convergence with probability one.

Definition 3. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, where Ω denotes the sample space, $\mathcal{F}$ denotes the set of events, and $\mathbb{P}$ denotes the probabilities associated with events. A filtration $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \mathcal{F}_2 \cdots \subseteq \mathcal{F}_n$ is an increasing subsequence of sub-σ-algebras of $\mathcal{F}$. A sequence of random variables $Z(k)$ is adapted to a filtration $\mathcal{F}_k$ if $Z(k)$ is $\mathcal{F}_k$-measurable for all k. The pair $(Z, \mathcal{F})$ is called supermartingale if, for all $k \geq 0$,

$$\mathbb{E}[Z(k)] < \infty \text{ and } \mathbb{E}\left[Z(k+1)|\mathcal{F}_k\right] \leq Z(k), \tag{14}$$

where $\mathbb{E}[Z(k)]$ denotes the expectation of the random variable $Z(k)$ and $\mathbb{E}[Z(k+1)|\mathcal{F}_k]$ denotes the conditional expectation of $Z(k+1)$ under the condition that the event $\mathcal{F}_k$ occurs.

The supermartingale sequence $\{Z(k)\}$ in (14) indicates that $\lim_{k\to\infty} Z(k)$ exists and is finite with probability 1. In addition, if the sequence $\{Z(k)\}$ is a nonnegative supermartingale, $Z(k)$ converges with probability 1 to a limit.[37]

Let $B \triangleq \begin{bmatrix} b_1 & b_2 & \cdots & b_{n-2} \end{bmatrix} \in \mathbb{R}^{(N-1)\times(N-2)}$ denote a matrix, where $b_i \in \mathbb{R}^{N-1}(i = 1, \ldots, N-2)$ and $\frac{1}{\sqrt{N-1}}\mathbf{1}_{N-1}$ form an orthonormal basis of $\mathbb{R}^{N-1}$, with properties $B^T\mathbf{1}_{N-1} = 0$, $B^TB = I_{N-2}$ and

$$BB^T = I_{N-1} - \frac{1}{N-1}\left(\mathbf{1}_{N-1}\mathbf{1}_{N-1}^T\right). \tag{15}$$

The error vector $\xi_k \in \mathbb{R}^{N-2}$ at time t_k is defined as

$$\xi_k \triangleq B^T E_k. \tag{16}$$

For the case where ξ_k and E_k are deterministic states in (16), $\xi_k = 0$ indicates that the consensus is achieved for all entries in E_k. Since ξ_k and E_k are random variables in this work, the following Lemma 2 and Theorem 1 indicate that $\xi_k \to 0$ almost surely (ie, with probability one), which indicates that all $E_i(k)$ achieve consensus almost surely.

Lemma 2. *An almost sure convergence of the random sequence $\{E_k\}$ to a value in the agreement space $\mathcal{A}$ in (13) is equivalent to*

$$\lim_{k_0\to\infty} \Pr\left\{\sup_{k\geq k_0} \|\xi_k\| > \epsilon\right\} = 0, \tag{17}$$

for every $\epsilon > 0$.

Proof. Given a consensus value $c\mathbf{1}_{N-1} \in \mathcal{A}$ for $\{E_k\}$, where $c \in \mathbb{R}$ is a constant, (17) is equivalent to (13) if

$$\|\xi_k\| = \inf_{c\mathbf{1}_{N-1}\in\mathcal{A}} \|E_k - c\mathbf{1}_{N-1}\| \quad .$$

Note that

$$\begin{aligned}\|E_k - c\mathbf{1}_{N-1}\|^2 &= E_k^TE_k - 2c\mathbf{1}_{N-1}^TE_k + c^2(N-1)\\ &= (N-1)\left(c - \frac{\mathbf{1}_{N-1}^TE_k}{N-1}\right)^2 + E_k^TE_k - \frac{\left(\mathbf{1}_{N-1}^TE_k\right)^2}{N-1},\end{aligned}$$

which is minimized when $c = \frac{\mathbf{1}_{N-1}^TE_k}{N-1}$. That is,

$$\inf_{c\mathbf{1}_{N-1}\in\mathcal{A}} \|E_k - c\mathbf{1}_{N-1}\| = \sqrt{E_k^TE_k - \frac{1}{N-1}\left(\mathbf{1}_{N-1}^TE_k\right)^2}. \tag{18}$$

From (15) and (16)

$$\begin{aligned}\|\xi_k\|^2 &= E_k^T\left(I_{N-1} - \frac{1}{N-1}\left(\mathbf{1}_{N-1}\mathbf{1}_{N-1}^T\right)\right)E_k\\ &= E_k^TE_k - \frac{1}{N-1}\left(\mathbf{1}_{N-1}^TE_k\right)^2,\end{aligned}$$

which is equivalent to (18). □

Theorem 1. *Given a directed random graph $\mathcal{G}_\theta$ composed of N agents and provided that Assumption 2 holds and K_g in (4) is selected sufficiently small such as $K_g\Delta < \frac{1}{2}$, the controller designed in (4) yields an almost sure orientation convergence to a balanced distribution in the desired angular sector (θ_1, θ_N), in the sense that $e_1 = \cdots = e_{N-1} = \frac{\theta_N-\theta_1}{N-1}$ almost surely as $k \to \infty$.*

Proof. Consider the function $V_k : \mathbb{R}^{N-2} \times \mathbb{R}^{N-2} \to \mathbb{R}$ defined as

$$V_k \triangleq \xi_k^T\xi_k, \tag{19}$$

where the random variable ξ_k is defined in (16). Let

$$g(k) \triangleq \mathbb{E}\left[V_{k+1} - V_k \,|\, \xi_k = B^T e(k)\right],$$

where $\mathbb{E}\left[V_{k+1} - V_k | \xi_k = B^T e(k)\right]$ represents the conditional expectation of $V_{k+1} - V_k$ given that the random variable $\xi_k = B^T e(k)$, where $e(k)$ is the deterministic state at time k. Using (11) and (16),

$$\begin{aligned} g(k) &= \mathbb{E}\left[\xi_{k+1}^T \xi_{k+1} - \xi_k^T \xi_k \middle| \xi_k = B^T e(k)\right] \\ &= e(k)^T \mathbb{E}\left[\Psi^T B B^T \Psi - B B^T\right] e(k). \end{aligned} \tag{20}$$

The term $\Psi^T B B^T \Psi - B B^T$ in (20) can be simplified by using the definition of Ψ and the properties of B in (15) as

$$\Psi^T B B^T \Psi - B B^T = \Upsilon \Phi, \tag{21}$$

where $\Upsilon \triangleq (-2I_{N-1} + K_g \Delta \Phi^T) K_g \Delta \in \mathbb{R}^{(N-1)\times(N-1)}$.

From the definition of Φ in (10), the ith row of Υ is specified by $\Upsilon_{i,i} = (-2 + K_g\Delta(\delta_i + \delta_{i+1}))K_g\Delta$, $\Upsilon_{i,(i-1)} = -K_g^2\Delta^2\delta_i$, $\Upsilon_{i,(i+1)} = -K_g^2\Delta^2\delta_{i+1}$, and $\Upsilon_{i,j} = 0$ for $j \notin \{i-1, i, i+1\}$. By the Gersgorin circle theorem,[36] the eigenvalues $\{\lambda_i\}$ of Υ satisfy

$$\left|\lambda_i - \left[-2K_g\Delta + K_g^2\Delta^2\left(\delta_i + \delta_{i+1}\right)\right]\right| \le K_g^2\Delta^2(\delta_i + \delta_{i+1}).$$

Thus,

$$\begin{aligned} \lambda_i &\le \left(-2 + 2K_g\Delta\left(\delta_i + \delta_{i+1}\right)\right) K_g\Delta \\ &\le \left(-2 + 4K_g\Delta\right) K_g\Delta. \end{aligned} \tag{22}$$

From (22), a sufficient condition for Υ to be negative definite is $K_g\Delta < 0.5$. Since Φ is positive semidefinite and $\Upsilon\Phi$ is a symmetric matrix,[38] then $\Upsilon\Phi$ is a negative semidefinite matrix provided that $K_g\Delta < 0.5$.

Using (21), the term $g(k)$ in (20) can be rewritten as

$$g(k) = e(k)^T \mathbb{E}\left[\Upsilon\Phi\right] e(k). \tag{23}$$

Since $\Upsilon\Phi$ in (21) is a negative semidefinite matrix, the random sequence $\{V_k\}$ is a supermartingale from Definition 3. By invoking Theorem 1 of chapter 8 in the work of Kushner,[37] $e(k)^T\mathbb{E}\left[\Upsilon\Phi\right] e(k) \to 0$ as $k \to \infty$ with probability 1. Note that

$$\mathbb{E}\left[\Upsilon\Phi\right] = \sum_{i=1}^{M} \left(\Upsilon\Phi_i\right) q_i,$$

where M is the cardinality of $\bar{\mathcal{G}}_\theta$, q_i denotes the probability that the graph $\mathcal{G}_\theta^i$ exists in $\bar{\mathcal{G}}_\theta$, and Φ_i denotes the state matrix associated with $\mathcal{G}_\theta^i$ satisfying $\Phi_i \mathbf{1}_{N-1} = 0$ from (10). Let q^* and Φ^* denote the probability and the state matrix associated with the graph $\mathcal{G}_\theta^*$, respectively. The definition of $\mathcal{G}_\theta^*$ ensures that the null space of Φ^* is the agreement space $\mathcal{A}$ only. Since $\mathcal{G}_\theta^* \in \bar{\mathcal{G}}_\theta$ exists with $q^* > 0$ from Assumption 2, $e(k)^T\mathbb{E}\left[\Upsilon\Phi\right] e(k) \to 0$ indicates that $\{E_k\} \to \mathcal{A}$ almost surely, which indicates that a consensus is achieved in the sense that $E_1(k) = \cdots = E_{N-1}(k)$ almost surely as $k \to \infty$. Given that $\sum E_i$ is invariant from Lemma 1 and $e_i = \theta_{i+1} - \theta_i$ in (7), each agent will almost surely converge to the equilibrium point,

$$\theta_2 - \theta_1 = \cdots = \theta_N - \theta_{N-1} = \frac{\theta_N - \theta_1}{N-1},$$

provided that θ_1 and θ_N are prespecified and immutable. □

Remark 1. The controller in (4) is not limited to a fixed number of agents. Because of the decentralized nature of (4), where each agent updates its orientation by communicating with the adjacent nodes, if any agents are added or removed, the remaining agents will correspondingly alter their orientations to achieve balanced spacing.

Remark 2. The convergence rate and convergence time of consensus are investigated in the works of Olshevsky and Tsitsiklis.[39,40] Given a network of n nodes, let $s(t) \triangleq [s_1^T(t), \dots, s_n^T(t)]^T$ represent the stacked node states, where s_i

represents the state of node i, $s(0)$ represents the initial states, and $s^* = \lim_{t\to\infty} s(t)$ denotes the final consensus. The convergence time $\tau(n,\epsilon)$ of consensus is defined on the basis of the system initial error $e_\tau(0)$ in the result[39] as

$$\tau(n,\epsilon) = \min\left\{ t : \frac{\|e_\tau(t)\|_2}{\|e_\tau(0)\|_2} \le \epsilon,\ s(0) \ne s^* \right\}, \tag{24}$$

which indicates the first time t when each node is within an ϵ-neighborhood of the final consensus. In (24), $e_\tau(t) \triangleq s(t) - s^*$, $e_\tau(0) \triangleq s(0) - s^*$, and $\epsilon \in \mathbb{R}^+$ denotes the desired error bound. The worst-case consensus convergence time is developed for the particular case of fixed line graphs, where the nodes are connected sequentially like a line.[40] In this paper, the closed-loop system in (11) is indeed a consensus algorithm over a line graph since the matrix Ψ in (11) indicates that each node updates its state by using its own states e_i and its immediate neighbors' states (ie, e_{i-1} and e_{i+1}). If there exists an interval $I_G \in \mathbb{Z}^+$ such that, for all k, the union of the graphs $\mathcal{G}_u = \left(\mathcal{V}, \bigcup_{m=0}^{I_G-1} \mathcal{E}(k+m)\right)$ is a connected line graph, the worst-case convergence time of consensus is developed as[40] $\tau(n,\epsilon) \ge \frac{n^2 I_G}{30} \log\frac{1}{\epsilon}$. Because of the considered Markov process, there does not exist a fixed I_G in this paper such that $\mathcal{G}_u$ is guaranteed to be a connected line graph. However, on the basis of Assumption 2, there will always be an interval I_G such that $\mathcal{G}_u$ is connected with arbitrarily high probability q_I because increasing I_G increases the probability that at least one interval contains the connected graph $\mathcal{G}^*_\theta$. For instance, in the case that the channels are independent across time intervals, a sufficient I_G can be selected to satisfy $q_I \ge 1 - [1 - \Pr(\mathcal{G}_\theta(k) = \mathcal{G}^*_\theta)]^{I_G}$. If a consensus is required to be achieved with a desired probability q_d, the worst-case convergence time $\tau(n,\epsilon)$ indicates that convergence will happen with probability q_d if there exist more than $\frac{n^2}{30}\log\frac{1}{\epsilon}$ connected intervals of length I_G, where I_G can be selected sufficiently large on the basis of the transition probabilities for the agent dynamics. Again, if the agents states are independent across time, a sufficient value of I_G can be selected to satisfy

$$\left[1 - \left[1 - \Pr\left(\mathcal{G}_\theta(k) = \mathcal{G}^*_\theta\right)\right]^{I_G}\right]^{\frac{n^2}{30}\log\frac{1}{\epsilon}} > q_d. \tag{25}$$

Since the group size n, the tolerant error ϵ, the desired consensus probability q_d, and $\Pr(\mathcal{G}_\theta(k) = \mathcal{G}^*_\theta)$ are known initially and the interval I_G can be selected from (25), limited global information is required to obtain the worst-case convergence time.

Since an only almost sure asymptotic convergence is established in Theorem 1, orientation control cannot be completed in finite time. Practically, the worst-case convergence time $\tau(n,\epsilon)$ discussed in Remark 2 provides an estimated time frame for orientation control within a predefined error bound ϵ. Such a time frame $\tau(n,\epsilon)$ indicates that, given an operating time no shorter than $\tau(n,\epsilon)$, reaching the ϵ-neighborhood of equal relative orientation between agents is guaranteed.

4 | COOPERATIVE TIMING

4.1 | Control design

In this section, a consensus-based coordination algorithm is developed for the multiagent system to simultaneously arrive at the prespecified target. The available information exchange between agents on $\mathcal{G}_\rho(t) = (\mathcal{V}, \mathcal{E}_\rho(t))$ is captured by the adjacency matrix $A = [a_{ij}]_{N\times N}$, where $a_{ij} = 1$ if there exists a directed edge from node j to i and $a_{ij} = 0$ if otherwise. Note that v_1 acts as the root node in $\mathcal{G}_\rho$ with immutable states (ie, the desired arrival time) from Assumption 4, which indicates that $a_{1j} = 0$ for all $j = 1, \dots, N$. Similar to $\mathcal{G}_\theta$, the graph $\mathcal{G}_\rho(k)$ evolves according to the 2-state Markov process, and $\mathcal{G}_\rho(k)$ remains constant over the time interval $[t_k, t_{k+1})$. Because of the random variable δ_i associated with agent i, the existing edge between agent i and its neighbors j can be either connected or disconnected at t_k, and the time-varying neighbor set of agent i is denoted by $\mathcal{N}_i(k) = \{v_j | (v_i, v_j) \in \mathcal{E}_\rho(k)\}$.

As described in Section 3.1, the orientation controller in (4) is applied to equally space the agents in the desired sector. When each agent has converged to the desired orientation (ie, $t \ge \tau(n,\epsilon)$ as discussed in Remark 2), which indicates that the agents are in a balanced deployment, the agents are driven along a line towards the target. Once the agents are close enough to estimate the distance to the target (ie, range information becomes available), their speed with neighboring agents are coordinated over the random communication network $\mathcal{G}_\rho(t)$ to perform simultaneous arrival.

Let $\tau_i(k)$ and $\rho_i(k)$ be the estimated arrival time for agent i and the distance to the target at t_k, respectively. Once the agents are in a balanced deployment and in the neighborhood of the target, it is assumed that the target can be sensed by all agents (ie, $\rho_i(k)$ is available to followers),‡ cooperative timing can be achieved by communicating $\tau_i(k)$ with neighboring agents and controlling the velocities of the agents for simultaneous arrival. Let $\tau(k) \triangleq \left[\tau_1(k), \ldots, \tau_N(k)\right]^T \in \mathbb{R}^N$ be the stacked deterministic state and $\Gamma_k \triangleq \left[\Gamma_1(k), \ldots, \Gamma_N(k)\right]^T \in \mathbb{R}^N$ be the corresponding random variable. Because of the random failure of each agent in communication with its neighbors, the estimated arrival time for each agent is updated according the following stochastic system:

$$\Gamma_i(k+1) = \left(1 - K_\tau \Delta\delta_i(k) \sum_{j\in\mathcal{N}_i(k)} a_{ij}\right)\Gamma_i(k) + K_\tau \Delta\delta_i(k) \sum_{j\in\mathcal{N}_i(k)} a_{ij}\Gamma_j(k), \tag{26}$$

where K_τ is the control gain, and the velocity of agent i is adjusted by

$$v_i(k) = \frac{\rho_i(k)}{\tau_i(k)}. \tag{27}$$

To show the arrival time consensus, let the disagreement $\zeta(\tau(k)) \in \mathbb{R}$ at t_k be defined as

$$\zeta(\tau(k)) \triangleq \bar{\tau}(k) - \underline{\tau}(k), \tag{28}$$

where $\bar{\tau}(k) \triangleq \max_{i=1,\ldots N} \tau_i(k)$ and $\underline{\tau}(k) \triangleq \min_{i=1,\ldots N}\tau_i(k)$. Let $\zeta_k \in \mathbb{R}$ be the corresponding random variable of $\zeta(\tau(k))$ in (28). If the random sequence $\{\zeta_k\}$ converges to zero almost surely, it is clear from (28) that the agents will also achieve arrival time consensus almost surely.

4.2 | Convergence analysis

To facilitate the subsequent lemma, the definition of a convex hull[43] is established in Definition 4.

Definition 4. For a set of points $x \triangleq \{x_1, \ldots, x_n\}$, the convex hull Co($x$) is defined as the minimal set containing all points in x, satisfying that $\text{Co}(x) \triangleq \left\{\sum_{i=1}^n \alpha_i x_i \,\middle|\, x_i \in x, \alpha_i > 0, \sum_{i=1}^n \alpha_i = 1\right\}$.

Lemma 3. *The disagreement $\zeta(\tau(k))$ is nonincreasing (ie, $\zeta(\tau(k+1)) \leq \zeta(\tau(k))$) if the state $\tau(k)$ evolves according to (26).*

Proof. The stochastic system in (26) can be written in a compact form as

$$\Gamma_{k+1} = T\Gamma_k. \tag{29}$$

In (29), $T = [T_{ij}] \in \mathbb{R}^{N\times N}$ is the state transition matrix with diagonal entries

$$T_{ii} \triangleq 1 - K_\tau \Delta\delta_i \sum_{j\in\mathcal{N}_i} a_{ij}$$

and off-diagonal entries

$$T_{ij} \triangleq K_\tau \Delta\delta_i a_{ij}$$

if $j \in \mathcal{N}_i$ and $T_{ij} = 0$ if otherwise. Note that the off-diagonal entries T_{ij} are nonnegative, and the diagonal entries T_{ii} are positive if K_τ is selected sufficiently small such that $K_\tau \Delta\delta_i \sum_{j\in\mathcal{N}_i} a_{ij} < 1$. Moreover, each row sum in T equals to one from its definition. Hence, for each agent i, given that $\Gamma_i(k) = \tau_i(k)$, where $\tau_i(k)$ is an arbitrary deterministic state at t_k, the next time state $\tau_i(k+1)$ is a convex linear combination of its current state $\tau_i(k)$, and its neighbors' current states $\tau_j(k)$ for $j \in \mathcal{N}_i(k)$ from (29) and Definition 4. The convex linear combination indicates that $\tau_i(k+1)$ will move into the convex hull formed by $\tau_i(k)$ and $\tau_j(k)$, $j \in \mathcal{N}_i(k)$, resulting in a nonincreasing disagreement (ie, $\zeta(\tau(k+1)) \leq \zeta(\tau(k))$). □

Theorem 2. *Provided that Assumption 4 is satisfied, the stochastic system in (26) will achieve arrival time consensus almost surely.*

‡For example, the range information $\rho_i(k)$ can be estimated by using the approaches developed in the works of De Luca et al[41] and Dani et al[42] if each agent knows its velocity and is equipped with a passive range sensor such as a camera.

Proof. Given the disagreement $\zeta(\tau(k))$ in (28), consider a function $f(k) \in \mathbb{R}$ as

$$f(k) \triangleq \mathbb{E}\left[\zeta_{k+1} - \zeta_k | \Gamma_k = \tau(k)\right] = \mathbb{E}\left[\zeta_{k+1} | \Gamma_k = \tau(k)\right] - \zeta(\tau(k)). \tag{30}$$

Using (29), the conditional expectation $\mathbb{E}[\zeta_{k+1}|\Gamma_k = \tau(k)]$ in (30) can be rewritten as

$$\mathbb{E}\left[\zeta_{k+1}|\Gamma_k = \tau(k)\right] = \sum_{i=1}^{S} \zeta(\tau(k+1))q_i = \sum_{i=1}^{S} \zeta(T_i\tau(k))q_i,$$

where S is the cardinality of $\bar{\mathcal{G}}_\rho$, q_i denotes the probability of the graph $\mathcal{G}_\rho^i$ exists in $\bar{\mathcal{G}}_\rho$, and T_i denotes the state transition matrix associated with $\mathcal{G}_\rho^i$. As indicated in Lemma 3,

$$\zeta\left(T_i\tau(k)\right) \leq \zeta\left(\tau(k)\right), \tag{31}$$

for all $\mathcal{G}_\rho^i \in \bar{\mathcal{G}}_\rho$. As shown in the result,[44] consensus is achieved for the particular directed graph $\mathcal{G}_\rho^* \in \bar{\mathcal{G}}_\rho$ that has a directed spanning tree as described in Assumption 4, which indicates that the disagreement decreases strictly (ie, $\zeta(T_*\tau(k)) < \zeta(\tau(k))$) for $\zeta(k) \notin \mathcal{A}$, where T_* denotes the state transition matrix associated with $\mathcal{G}_\rho^*$. Since $\Pr(\mathcal{G}_\rho(k) = \mathcal{G}_\rho^*) > 0$ from Assumption 4, the results in (31) can be used to prove that $\zeta(T_i\tau(k)) < \zeta(\tau(k))$ in expectation; hence, the random sequence $\{\zeta_k\}$ is a supermartingale. Invoking theorem 1 of chapter 8 in the work of Kushner[37] indicates that $\zeta(\Gamma_k) \to 0$ almost surely. That is, consensus is achieved almost surely in the sense that $\tau_1 = \tau_2 = \cdots = \tau_N$. Since Node 1 acts as the root node and cannot be influenced by other agents as described in Assumption 4, the arrival time of all agents will achieve consensus to the desired arrival time of τ_1, and the velocity of each agent is adjusted according to (27) for simultaneous arrival. □

In this work, only 1 agent is informed of the desired arrival time, and the other agents are required to arrive at the target exactly at the same arrival time of the informed agent. If no agents are informed of a required arrival time, alternative approaches such as max-consensus or min-consensus in the work of Nejad et al[45] can be extended to a stochastic setting and applied for simultaneous arrival. Particularly, each agent may have an estimated arrival time. Rather than arriving at a predefined time determined by a leader node, consensus could be reached on a minimum or maximum estimated arrival time of the group.

Remark 3. The consensus convergence time over a directed spanning tree is investigated in the work of Olshevsky and Tsitsiklis[39] and the references therein. Following a similar procedure in the aforementioned work[39] and the discussion in Remark 2, despite the asymptotic convergence result proved in Theorem 2, the worst-case convergence time $\tau'(n,\epsilon)$ for a given error bound ϵ can be developed for the directed tree graph $\mathcal{G}_\rho(t)$ considered in the current work. $\tau(n,\epsilon)$ and $\tau'(n,\epsilon)$ together provide a time frame for completing the task within a desired error bound ϵ (ie, within an ϵ-neighborhood of the final consensus value). As discussed in the aforementioned work,[39] the error bound ϵ indicates a shrink factor from the initial conditions (ie, $\|e_\tau(t)\|_2 \leq \epsilon\|e_\tau(0)\|_2$ in (24), for all $t \geq \tau(n,\epsilon)$). Hence, to achieve the desired error bound ϵ, the total amount of operation time should be no shorter than $\tau'(n,\epsilon) + \tau(n,\epsilon)$. In addition, to ensure that the agents have sufficient operation time to complete the orientation control and simultaneous arrival up to a given error bound ϵ, the agents are assumed to be driven from a distant starting point from the target such that the total amount of operation time is no shorter than $\tau'(n,\epsilon) + \tau(n,\epsilon)$.

Remark 4. Since range information is not exchanged between agents, collision avoidance within agents is not considered in this work. In our recent work,[46] a potential field-based approach is investigated for a multiagent system to avoid collision with other agents and/or stationary obstacles when performing collective tasks. If provided with range information to nearby agents or stationary obstacles, the current work could be extended for collision avoidance on the basis of the outcome of the aforementioned work.[46]

Remark 5. For applications that require simultaneous arrival outside of a target's sensing capabilities, our work can also be extended to perform simultaneous arrival at the target's sensing boundary. Specifically, let $r_d \in \mathbb{R}^+$ denote the radius of the target sensing neighborhood. Recall that ρ_i is the distance to the target, which is available to the agent i when the agents are close to the target. Simultaneous arrival at the boundary of the target neighborhood can be achieved by simply replacing ρ_i with $\rho_i - r_d$.

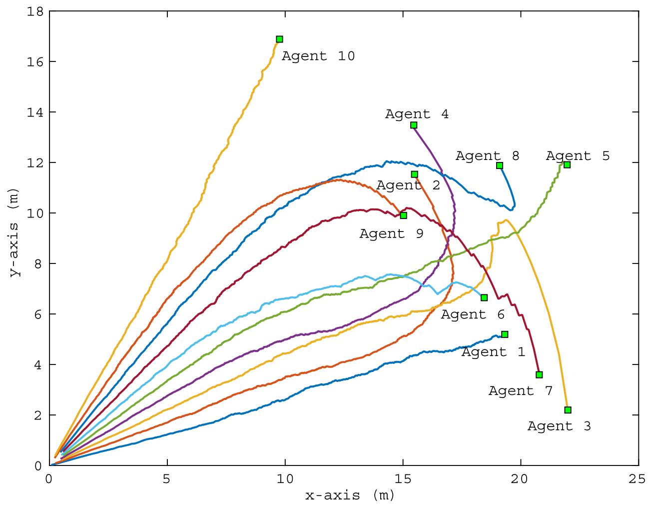

FIGURE 2 Plot of agent trajectories with squares indicating their initial positions [Colour figure can be viewed at wileyonlinelibrary.com]

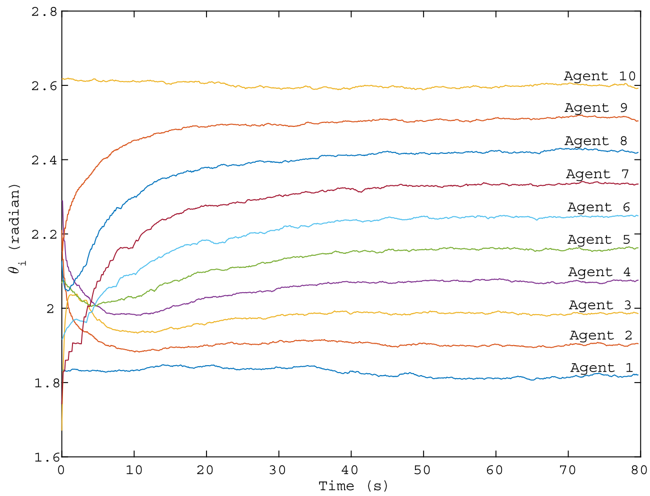

FIGURE 3 Evolution of agent orientations [Colour figure can be viewed at wileyonlinelibrary.com]

5 | SIMULATION

Numerical simulation results are provided to demonstrate the performance of the developed controller for a group of 10 agents that are tasked to be equally spaced and move within a prespecified circular sector to simultaneously arrive at the target. The target is located at the origin and the angular sector is specified by $\theta_1 = \frac{7\pi}{12}$ and $\theta_{10} = \frac{5\pi}{6}$. Zero-mean random measurement noise varying between $\left[-0.01,\ 0.01\right]$ *rad* and $\left[-0.5,\ 0.5\right]$ *m* are added to the orientation (ie, θ_i) and range (ie, ρ_i) measured by the leaders and followers, respectively, to demonstrate the robustness of the control algorithm. A 2-state Markov process is applied to model the random failure of agents communicating with neighboring agents. The agents are assumed to move with constant velocity during orientation control. Once the balanced deployment is achieved by using (4) and the target range information is available to followers, velocities are adjusted according to (27) for simultaneous arrival. On the basis of the coordinate transformation $x_i = \rho_i \cos(\theta_i - \frac{\pi}{2})$ and $y_i = \rho_i \sin(\theta_i - \frac{\pi}{2})$, where θ_i is defined in Figure 1, the agent trajectories in the Cartesian coordinate system are shown in Figure 2, where the initial positions are represented by squares and the trajectories are represented by solid lines. The evolution of orientations for each agent is plotted in Figure 3, which indicates that the orientations of the group are equally spaced in the desired sector $(\frac{7\pi}{12}, \frac{5\pi}{6})$. Figure 4 indicates that simultaneous arrival is achieved, where all agents reach a consensus to the desired arrival time of Agent 1. As shown in Figure 4, agents start to coordinate their velocities for simultaneous arrival after completion of orientation control at $t = 12.5$ seconds (ie, $\|e(t) - e^*\| < \epsilon$ for $\forall i \in \mathcal{V}$). To show the random failure of node v_i

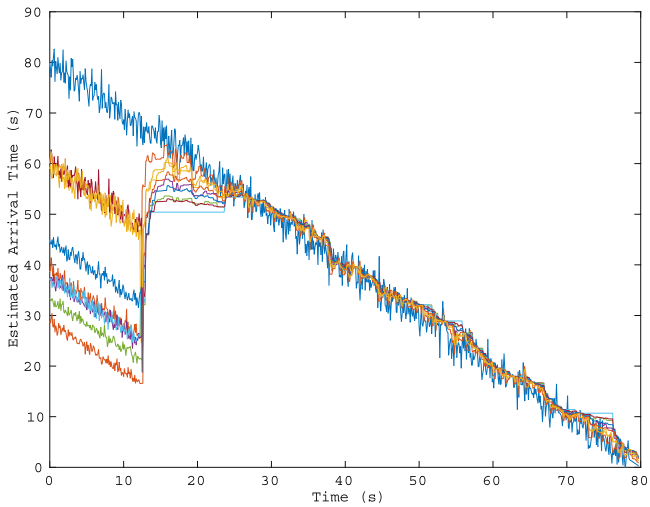

FIGURE 4 Evolution of estimated arrival time of each agent. The dashed line indicates the desired arrival time determined by Agent 1. Velocity control is activated at $t = 12.5$ after the orientation control is completed [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Percentage of time that each nodes connects with its neighboring nodes

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
46.6%	53.3%	52.8%	49.1%	50.2%	49.7%	48.1%	52.5%	50.6%	49.7%

in communication with its neighbors, the percentage that each node maintains a connection with its neighboring nodes is provided in Table 1.

6 | CONCLUSION

This paper examines the cooperative timing problem for a multiagent system over a random communication network. The underlying random network is assumed to evolve according to a 2-state Markov model. In contrast to most existing results that either require deterministic networks or assume random networks that evolve independently with their previous states, a Markov process is employed in the current work that considers the fact that the current network states can be highly dependent on its previous states. The considered Markov process–based model is true for a large class of real-world networks and practical connection models (eg, modeling the effects of channel outages caused by multipath propagation). Additional development is also required to extend the current result to consensus problems with general aperiodic sampling such as switching topologies or time-varying topologies. Compared with most results in containment control that only drive the followers' states to the desired region determined by the leaders, a balanced containment control algorithm is developed to not only drive the followers to the desired region but also equally space the agents in the desired region when driving the agents towards the target. A consensus algorithm is designed for agents to coordinate their velocities by reaching almost sure consensus on the arrival time. Following a recently developed finite-time consensus framework,[47] future work will consider extending the current consensus result to almost sure finite-time consensus to enable finite-time simultaneous target arrival. Additional development is also required to extend the current results to other tasks with multiple dynamic leaders such as flocking toward a common heading or coverage control with agents equally deployed in an area of interest, where the existing results[48] can be useful on the latter generalization to multiple dynamic leaders.

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REFERENCES

1. Fax JA, Murray RM . Information flow and cooperative control of vehicle formations. *IEEE Trans Autom Control*. 2004;49(9):1465-1476.
2. Navaravong I, Kan Z, Shea JM, Dixon WE. Formation reconfiguration for mobile robots with network connectivity constraints. *IEEE Netw*. 2012;26(4):18-24.
3. Schwager M, Rus D, Slotine JJ. Decentralized, adaptive coverage control for networked robots. *Int J Rob Res*. 2009;28(3):357-375.
4. Cortes J, Martinez S, Karatas T, Bullo F. Coverage control for mobile sensing networks. *IEEE Trans Robot Autom*. 2004;20(2):243-255.
5. McLain TW, Beard RW. Coordination variables, coordination functions, and cooperative-timing missions. *J Guid Control Dynam*. 2005;28(1):150-161.
6. Beard RW, McLain TW, Goodrich MA, Anderson EP. Coordinated target assignment and intercept for unmanned air vehicles. *IEEE Trans Robot Autom*. 2002;18(6):911-922.
7. Beard RW, McLain TW, Nelson DB, Kingston D, Johanson D. Decentralized cooperative aerial surveillance using fixed-wing miniature UAVs. *IEEE Proc*. 2006;94(7):1306-1324.
8. Casbeer DW, Kingston DB, Beard RW, McLain TW. Cooperative forest fire surveillance using a team of small unmanned air vehicles. *Int J Syst Sci*. 2006;37(6):351-360.
9. Lin Z, Broucke M, Francis B. Local control strategies for groups of mobile autonomous agents. *IEEE Trans Autom Control*. 2004;49(4):622-629.
10. Lin J, Morse AS, Anderson BDO. The multi-agent rendezvous problem. Part 1: the synchronous case. *SIAM J Control Optim*. 2007;46(6):2096-2119.
11. Lin J, Morse AS, Anderson BDO. The multi-agent rendezvous problem. Part 2: the asynchronous case. *SIAM J Control Optim* 2007;46(6):2120-2147.
12. Dimarogonas DV, Kyriakopoulos KJ. On the rendezvous problem for multiple nonholonomic agents. *IEEE Trans Autom Control*. 2007;52(5):916-922.
13. Hui Q. Finite-time rendezvous algorithms for mobile autonomous agents. *IEEE Trans Autom Control*. 2011;56(1):207-211.
14. Gustavi T, Dimarogonas DV, Egerstedt M, Hu X. Sufficient conditions for connectivity maintenance and rendezvous in leader–follower networks. *Automatica*. 2010;46(1):133-139.
15. Kan Z, Klotz J, Cheng TH, Dixon WE. Ensuring network connectivity for nonholonomic robots during decentralized rendezvous. Paper presented at: American Control Conference; 2012; Montréal, Canada.
16. Li R, Shi YJ, Teo KL. Coordination arrival control for multi-agent systems. *Int J Robust Nonlinear Control*. 2016;26(7):1456-1474.
17. Hatano Y, Mesbahi M. Agreement over random networks. *IEEE Trans Autom Control*. 2005;50(11):1867-1872.
18. Wu CW. Synchronization and convergence of linear dynamics in random directed networks. *IEEE Trans Autom Control*. 2006;51(7):1207-1210.
19. Porfiri M, Stilwell DJ. Consensus seeking over random weighted directed graphs. *IEEE Trans Autom Control*. 2007;52(9):1767-1773.
20. Tahbaz-Salehi A, Jadbabaie A. A necessary and sufficient condition for consensus over random networks. *IEEE Trans Autom Control*. 2008;53(3):791-795.
21. Tahbaz-Salehi A, Jadbabaie A. Consensus over ergodic stationary graph processes. *IEEE Trans Autom Control*. 2010;55(1):225-230.
22. Li T, Zhang J-F. Consensus conditions of multi-agent systems with time-varying topologies and stochastic communication noises. *IEEE Trans Autom Control*. 2010;55(9):2043-2057.
23. Kar S, Moura JMF. Distributed consensus algorithms in sensor networks with imperfect communication: link failures and channel noise. *IEEE Trans Signal Process*. 2009;57(1):355-369.
24. Wang Y, Cheng L, Ren W, Hou ZG, Tan M. Seeking consensus in networks of linear agents: communication noises and Markovian switching topologies. *IEEE Trans Autom Control*. 2015;60(5):1374-1379.
25. Kan Z, Mehta S, Pasiliao E, Curtis JW, Dixon WE. Balanced containment control and cooperative timing of a multi-agent system. Paper presented at: American Control Conference; 2014; Portland, OR.
26. Notarstefano G, Egerstedt M, Haque M. Containment in leader-follower networks with switching communication topologies. *Automatica*. 2011;47(5):1035-1040.
27. Cao Y, Ren W. Containment control with multiple stationary or dynamic leaders under a directed interaction graph. Paper presented at: IEEE Conference on Decision and Control; 2009; Shanghai, China.
28. Mei J, Ren W, Ma G. Distributed containment control for Lagrangian networks with parametric uncertainties under a directed graph. *Automatica*. 2012;48(4):653-659.
29. Cao Y, Ren W, Egerstedt M. Distributed containment control with multiple stationary or dynamic leaders in fixed and switching directed networks. *Automatica*. 2012;48:1586-1597.
30. Kan Z, Klotz J, Eduardo LP, Dixon WE. Containment control for a social network with state-dependent connectivity. *Automatica*. 2015;56:86-92.
31. Cortés J, Martínez S, Bullo F. Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions. *IEEE Trans Autom Control*. 2006;51(8):1289-1298.
32. Su H, Wang X, Chen G. Rendezvous of multiple mobile agents with preserved network connectivity. *Syst Control Lett* 2010;59(5):313-322.
33. Grimmett G, Stirzaker D. *Probability and Random Processes*. New York, NY: Oxford University Press; 2001.
34. Marshall J, Broucke ME, Francis B, et al. Formations of vehicles in cyclic pursuit. *IEEE Trans Autom Control*. 2004;49(11):1963-1974.

35. Zheng R, Lin Z, Fu M, Sun D. Distributed control for uniform circumnavigation of ring-coupled unicycles. *Automatica*. 2015;53:23-29.
36. Horn PA, Johnson CR. *Matrix Analysis*. Cambridge, UK: Cambridge University Press; 1993.
37. Kushner HJ. *Introduction to Stochastic Control*. New York, NY: Holt, Reinhart and Winston; 1971.
38. Meenakshi AR, Rajian C. On a product of positive semidefinite matrices. *Lin Algebra Appl*. 1999;295(1):3-6.
39. Olshevsky A, Tsitsiklis JN. Convergence speed in distributed consensus and averaging. *SIAM J Control Optim*. 2009;48(1):33-55.
40. Olshevsky A, Tsitsiklis JN. A lower bound for distributed averaging algorithms on the line graph. *IEEE Trans Autom Control*. 2011;56(11):2694-2698.
41. De Luca A, Oriolo G, Robuffo Giordano P. Feature depth observation for image-based visual servoing: theory and experiments. *Int J Robot Res*. 2008;27(10):1093-1116.
42. Dani A, Fischer N, Kan Z, Dixon WE. Globally exponentially stable observer for vision-based range estimation. *Mechatron*. 2012;22(4):381-389.
43. Boyd S, Vandenberghe L. *Convex Optimization*. New York, NY: Cambridge University Press; 2004.
44. Ren W, Beard RW, Atkins EM. Information consensus in multivehicle cooperative control. *IEEE Control Syst Mag*. 2007;27:71-82.
45. Nejad BM, Attia SA, Raisch J. Max-consensus in a max-plus algebraic setting: The case of fixed communication topologies. Paper presented at: International Symposium on Information, Communication and Automation Technologies; 2009; Sarajevo, Bosnia and Herzegovina.
46. Kan Z, Dani A, Shea JM, Dixon WE. Network connectivity preserving formation stabilization and obstacle avoidance via a decentralized controller. *IEEE Trans Autom Control*. 2012;57(7):1827-1832.
47. Kan Z, Yucelen T, Doucette E, Pasiliao E. A finite-time consensus framework over time-varying graph topologies with temporal constraints. *J Dyn Syst Meas Contr*. 2017;139(7):1-6.
48. Cheng L, Wang Y, Ren W, Hou ZG, Tan M. Containment control of multiagent systems with dynamic leaders based on a PI^n-type approach. *IEEE Trans Cybern*. 2016;46(12):3004-3017.

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