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Asymptotic Tracking for Uncertain Dynamic Systems Via a Multilayer Neural Network Feedforward and RISE Feedback Control Structure

Parag M. Patre, William MacKunis, Kent Kaiser, and Warren E. Dixon

***Abstract*—The use of a neural network (NN) as a feedforward control element to compensate for nonlinear system uncertainties has been investigated for over a decade. Typical NN-based controllers yield uniformly ultimately bounded (UUB) stability results due to residual functional reconstruction inaccuracies and an inability to compensate for some system disturbances. Several researchers have proposed discontinuous feedback controllers (e.g., variable structure or sliding mode controllers) to reject the residual errors and yield asymptotic results. The research in this paper describes how a recently developed continuous robust integral of the sign of the error (RISE) feedback term can be incorporated with a NN-based feedforward term to achieve semi-global asymptotic tracking. To achieve this result, the typical stability analysis for the RISE method is modified to enable the incorporation of the NN-based feedforward terms, and a projection algorithm is developed to guarantee bounded NN weight estimates.**

***Index Terms*—Adaptive control, asymptotic stability, Lyapunov methods, neural network, nonlinear systems, RISE feedback, robust control.**

I. Introduction

Control researchers have extensively investigated the use of neural networks (NNs) as a feedforward control element over the last fifteen years. The focus on NN-based control methods is spawned from the ramifications of the fact that NNs are universal approximators [1]. That is, NNs can be used as a black-box estimator for a general class of systems. Examples include: nonlinear systems with parametric uncertainty that do not satisfy the linear-in-the-parameters assumption required in most adaptive control methods; systems with deadzones or discontinuities; and systems with backlash. Typically, NN-based controllers yield global uniformly ultimately bounded (UUB) stability results (e.g., see [2]–[4] for examples and reviews of literature) due to residual functional reconstruction inaccuracies and an inability to compensate for some system disturbances. Motivated by the desire to eliminate the residual steady-state errors, several researchers have obtained asymptotic tracking results by combining the NN feedforward element with discontinuous feedback methods such as variable structure controllers (VSC) (e.g., [5] and [6]) or sliding mode (SM) controllers (e.g., [6] and [7]). A clever VSC-like controller was also proposed in [8], where the controller is not initially discontinuous, but exponentially becomes discontinuous as an exogenous control element exponentially vanishes. Well known limitations of VSC and SM controllers include a requirement for infinite control bandwidth and chattering. Unfortunately, ad hoc fixes for these effects result in a loss of asymptotic stability (i.e.,

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The authors are with the Department of Mechanical and Aerospace Engineering, University of Florida, MAE-A, 32608 Gainesville, FL 32611-6250 USA (e-mail: parag.patre@gmail.com; mackunis@gmail.com; michael.kaiser@eglin.af.mil; wdixon@ufl.edu).

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UUB typically results). Motivated by issues associated with discontinuous controllers and the typical UUB stability result, an innovative continuous NN-based controller was recently developed in [9] to achieve partial asymptotic stability for a particular class of systems. The contribution in this paper is motivated by the question: *Can a NN feedforward controller be modified by a continuous feedback element to achieve an asymptotic tracking result for a general class of systems?* Despite the pervasive development of NN controllers in literature and the widespread use of NNs in industrial applications, the answer to this fundamental question has remained an open problem.

To provide an answer to the fundamental motivating question, the result in this paper focuses on augmenting a multilayer NN-based feedforward method with a recently developed [10] high gain control strategy coined the robust integral of the sign of the error (RISE) in [11], [12]. The RISE control structure is advantageous because it is a differentiable control method that can compensate for additive system disturbances and parametric uncertainties under the assumption that the disturbances are C^2 with bounded time derivatives. Due to the advantages of the RISE control structure, a flurry of results have recently been developed (e.g., [13]–[17]).

A RISE feedback controller can be directly applied to yield asymptotic stability for the class of systems described in this paper. However, the RISE method is a high-gain feedback tool, and hence, clear motivation exists (as with any other feedback controller) to combine a feedforward control element with the feedback controller for potential gains such as improved transient and steady-state performance, and reduced control effort. That is, it is well accepted that a feedforward component can be used to cancel out some dynamic effects without relying on high-gain feedback. Given this motivation, some results have already been developed that combine the RISE feedback element with feedforward terms. In [18], a remark is provided regarding the use of a constant best-guess feedforward component in conjunction with the RISE method to yield a UUB result. In [11] and [12], the RISE feedback controller was combined with a standard gradient feedforward term for systems that satisfy the linear-in-the-parameters assumption. The experimental results in [11] illustrate significant improvement in the root-mean-squared tracking error with reduced root-mean-squared control effort. However, for systems that do not satisfy the linear-in-the-parameters assumption, motivation exists to combine the RISE controller with a new feedforward method such as the NN.

To blend the NN and RISE methods, several technical challenges must be addressed. One (lesser) challenge is that the NN must be constructed in terms of the desired trajectory instead of the actual trajectory (i.e., a DCAL-based NN structure [8]) to remove the dependence on acceleration. The development of a DCAL-based NN structure is challenging for a multilayer NN because the adaptation law for the weights is required to be state-dependent. Straightforward application of the RISE method would yield an acceleration dependent adaptation law. One method to resolve this issue is to use a "dirty derivative" (as in the UUB result in [19]; see also [17]). In lieu of a dirty derivative, the result in this paper uses a Lyapunov-based stability analysis approach for the design of an adaptation law that is only velocity dependent. In comparison with the efforts in [11], [12], a more significant challenge arises from the fact that since a multilayer NN includes the first layer weight estimate inside of a nonlinear activation function, the previous methods (e.g., [11] and [12]) cannot be applied. That is, because of the unique manner in which the NN weight estimates appear, the stability analysis and sufficient conditions developed in previous works are violated. Previous RISE methods have a restriction (encapsulated by a sufficient gain condition) that terms in the stability analysis that are upper bounded by a constant must also have time derivatives that are upper bounded by a constant (these terms are usually denoted by $N_d(t)$ in RISE control literature; see [18]). The norm of the NN weight estimates can be bounded by a constant (due to a projection algorithm) but the time derivative is state-dependent (i.e., the norm of $N_d(t)$ can be bounded by a constant but the norm of $\dot{N}_d(t)$ is state dependent). To address this issue, modified RISE stability analysis techniques are developed that result in modified (but not more restrictive) sufficient gain conditions. By addressing this issue through stability analysis methods, the standard NN weight adaptation law does not need to be modified. Through unique modifications to the stability analysis that enable the RISE feedback controller to be combined with the NN feedforward term, the result in this paper provides an affirmative answer for the first time to the aforementioned motivating question.

Since the NN and the RISE control structures are model independent (black box) methods, the resulting controller is a universal reusable controller [8] for continuous systems. Because of the manner in which the RISE technique is blended with the NN-based feedforward method, the structure of the NN is not altered from textbook examples [4] and can be considered a somewhat modular element in the control structure. Hence, the NN weights and thresholds are automatically adjusted on-line, with no off-line learning phase required. Compared to standard adaptive controllers, the current asymptotic result does not require linearity in the parameters or the development and evaluation of a regression matrix.

For systems with linear-in-the-parameters uncertainty, an adaptive feedforward controller has the desirable characteristics that the controller is continuous, can be proven to yield global asymptotic tracking, and includes the specific dynamics of the system in the feedforward path. Continuous feedback NN controllers don't include the specific dynamics in a regression matrix and have a degraded steady-state stability result (i.e., UUB tracking); however, they can be applied when the uncertainty in the system is unmodeled, cannot be linearly parameterized, or the development and implementation of a regression matrix is impractical. Sliding mode feedback NN controllers have the advantage that they can achieve global asymptotic tracking at the expense of implementing a discontinuous feedback controller (i.e., infinite bandwidth, exciting structural modes, etc.). In comparison to these controllers, the development in this paper has the advantage of asymptotic tracking with a continuous feedback controller for a general class of uncertainty; however, these advantages are at the expense of semi-global tracking instead of the typical global tracking results.

II. Dynamic Model and Properties

The class of nonlinear dynamic systems considered in this paper is assumed to be modeled by the following Euler-Lagrange formulation that describes the behavior of a large class of engineering systems (e.g., robot manipulators, satellites, vehicular systems):

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d(t) = \tau(t). \qquad (1)$$

In (1), $M(q) \in \mathbb{R}^{n\times n}$ denotes the inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes the gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ denotes friction, $\tau_d(t) \in \mathbb{R}^n$ denotes a general nonlinear disturbance (e.g., unmodeled effects), $\tau(t) \in \mathbb{R}^n$ represents the torque input control vector, and $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration vectors, respectively. The subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable and that $M(q)$, $V_m(q,\dot{q})$, $G(q)$, $F(\dot{q})$ and $\tau_d(t)$ are unknown. Moreover, the following properties and assumptions will be exploited in the subsequent development.

Property 1: The inertia matrix $M(q)$ is symmetric, positive definite, and satisfies the following inequality $\forall\, y(t) \in \mathbb{R}^n$:

$$m_1 \|y\|^2 \leq y^T M(q) y \leq \bar{m}(q)\|y\|^2 \qquad (2)$$

where $m_1 \in \mathbb{R}$ is a known positive constant, $\bar{m}(q) \in \mathbb{R}$ is a known positive function, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: If $q(t), \dot{q}(t) \in \mathcal{L}_\infty$, then $V_m(q,\dot{q})$, $F(\dot{q})$ and $G(q)$ are bounded. Moreover, if $q(t), \dot{q}(t) \in \mathcal{L}_\infty$, then the first and second partial derivatives of the elements of $M(q), V_m(q,\dot{q}), G(q)$ with respect to $q(t)$ exist and are bounded, and the first and second partial derivatives of the elements of $V_m(q,\dot{q})$, $F(\dot{q})$ with respect to $\dot{q}(t)$ exist and are bounded.

Property 3: The nonlinear disturbance term and its first two time derivatives are bounded, i.e., $\tau_d(t), \dot{\tau}_d(t), \ddot{\tau}_d(t) \in \mathcal{L}_\infty$.

III. Control Objective

The control objective is to ensure that the system tracks a desired time-varying trajectory, denoted by $q_d(t) \in \mathbb{R}^n$, despite uncertainties in the dynamic model. To quantify this objective, a position tracking error, denoted by $e_1(t) \in \mathbb{R}^n$, is defined as

$$e_1 \triangleq q_d - q. \tag{3}$$

The subsequent development is based on the assumption that the desired trajectory designed such that $q_d^{(i)}(t) \in \mathbb{R}^n \ (i = 0, 1, \ldots, 4)$ exist and are bounded. To facilitate the subsequent analysis, filtered tracking errors, denoted by $e_2(t), r(t) \in \mathbb{R}^n$, are also defined as

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1 \tag{4}$$

$$r \triangleq \dot{e}_2 + \alpha_2 e_2 \tag{5}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}$ denote positive constants. The filtered tracking error $r(t)$ is not measurable since the expression in (5) depends on $\ddot{q}(t)$.

IV. Feedforward NN Estimation

NN-based estimation methods are well suited for control systems where the dynamic model contains unstructured nonlinear disturbances as in (1). The main feature that empowers NN-based controllers is the universal approximation property. Let $\mathbb{S}$ be a compact simply connected set of $\mathbb{R}^{N_1+1}$. With map $f : \mathbb{S} \to \mathbb{R}^n$, define $\mathbb{C}^n(\mathbb{S})$ as the space where f is continuous. There exist weights and thresholds such that some function $f(x) \in \mathbb{C}^n(\mathbb{S})$ can be represented by a three-layer NN as [3], [4]

$$f(x) = W^T \sigma\left(V^T x\right) + \varepsilon(x) \tag{6}$$

for some given input $x(t) \in \mathbb{R}^{N_1+1}$. In (6), $V \in \mathbb{R}^{(N_1+1)\times N_2}$ and $W \in \mathbb{R}^{(N_2+1)\times n}$ are bounded constant ideal weight matrices for the first-to-second and second-to-third layers respectively, where N_1 is the number of neurons in the input layer, N_2 is the number of neurons in the hidden layer, and n is the number of neurons in the third layer. The activation function[1] in (6) is denoted by $\sigma(\cdot) \in \mathbb{R}^{N_2+1}$, and $\varepsilon(x) \in \mathbb{R}^n$ is the functional reconstruction error. Note that, augmenting the input vector $x(t)$ and activation function $\sigma(\cdot)$ by "1" allows us to have thresholds as the first columns of the weight matrices [3], [4]. Thus, any tuning of W and V then includes tuning of thresholds as well.

Remark 1: If $\varepsilon = 0$, then $f(x)$ is in the functional range of the NN. In general for any positive constant real number $\varepsilon_N > 0$, $f(x)$ is within ε_N of the NN range if there exist finite hidden neurons N_2, and constant weights so that for all inputs in the compact set, the approximation holds with $\|\varepsilon\| < \varepsilon_N$. For various activation functions, results such as the Stone-Weierstrass theorem indicate that any sufficiently smooth function can be approximated by a suitable large network. Therefore, the fact that the approximation error ε is bounded follows from the *Universal Approximation Property* of the NNs [1].

Based on (6), the typical three-layer NN approximation for $f(x)$ is given as [3], [4]

$$\hat{f}(x) \triangleq \hat{W}^T \sigma(\hat{V}^T x) \tag{7}$$

where $\hat{V}(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ and $\hat{W}(t) \in \mathbb{R}^{(N_2+1)\times n}$ are subsequently designed estimates of the ideal weight matrices. The estimate mismatch for the ideal weight matrices, denoted by $\tilde{V}(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ and $\tilde{W}(t) \in \mathbb{R}^{(N_2+1)\times n}$, are defined as

$$\tilde{V} \triangleq V - \hat{V}, \quad \tilde{W} \triangleq W - \hat{W}$$

and the mismatch for the hidden-layer output error for a given $x(t)$, denoted by $\tilde{\sigma}(x) \in \mathbb{R}^{N_2+1}$, is defined as

$$\tilde{\sigma} \triangleq \sigma - \hat{\sigma} = \sigma(V^T x) - \sigma(\hat{V}^T x). \tag{8}$$

Property 4: (*Boundedness of the Ideal Weights*) The ideal weights are assumed to exist and be bounded by known positive values so that

$$\|V\|_F^2 = tr(V^T V) \le \bar{V}_B \tag{9}$$

$$\|W\|_F^2 = tr(W^T W) \le \bar{W}_B \tag{10}$$

where $\|\cdot\|_F$ is the Frobenius norm of a matrix, and $tr(\cdot)$ is the trace of a matrix.

V. RISE Feedback Control Development

The contribution of this paper is the control development and stability analysis that illustrates how the aforementioned textbook (e.g., [4]) NN feedforward estimation strategy can be fused with a RISE feedback control method as a means to achieve an asymptotic stability result for general Euler-Lagrange systems described by (1). In this section, the open-loop and closed-loop tracking error is developed for the combined control system.

A. Open-Loop Error System

The open-loop tracking error system can be developed by premultiplying (5) by $M(q)$ and utilizing the expressions in (1), (3), and (4) to obtain the following expression:

$$M(q)r = f_d + S + \tau_d - \tau \tag{11}$$

where the auxiliary function $f_d(q_d, \dot{q}_d, \ddot{q}_d) \in \mathbb{R}^n$ is defined as

$$f_d \triangleq M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F(\dot{q}_d) \tag{12}$$

and the auxiliary function $S(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d) \in \mathbb{R}^n$ is defined as

$$\begin{aligned} S \triangleq\ & M(q)(\alpha_1 \dot{e}_1 + \alpha_2 e_2) + M(q)\ddot{q}_d - M(q_d)\ddot{q}_d \\ & + V_m(q,\dot{q})\dot{q} - V_m(q_d,\dot{q}_d)\dot{q}_d \\ & + G(q) - G(q_d) + F(\dot{q}) - F(\dot{q}_d). \end{aligned} \tag{13}$$

The expression in (12) can be represented by a three-layer NN as

$$f_d = W^T \sigma(V^T x_d) + \varepsilon(x_d). \tag{14}$$

In (14), the input $x_d(t) \in \mathbb{R}^{3n+1}$ is defined as $x_d(t) \triangleq [1 \quad q_d^T(t) \quad \dot{q}_d^T(t) \quad \ddot{q}_d^T(t)]^T$ so that $N_1 = 3n$ where N_1 was

[1] A variety of activation functions (e.g., sigmoid, hyperbolic tangent or radial basis) could be used for the control development in this paper.

introduced in (6). Based on the assumption that the desired trajectory is bounded, the following inequalities hold:

$$\|\varepsilon(x_d)\| \leq \varepsilon_{b_1}, \quad \|\dot{\varepsilon}(x_d, \dot{x}_d)\| \leq \varepsilon_{b_2}$$
$$\|\ddot{\varepsilon}(x_d, \dot{x}_d, \ddot{x}_d)\| \leq \varepsilon_{b_3} \tag{15}$$

where $\varepsilon_{b_1}, \varepsilon_{b_2}, \varepsilon_{b_3} \in \mathbb{R}$ are known positive constants.

B. Closed-Loop Error System

Based on the open-loop error system in (11), the control torque input is composed of a three-layer NN feedforward term plus the RISE feedback terms as

$$\tau \triangleq \hat{f}_d + \mu. \tag{16}$$

Specifically, the RISE feedback control term $\mu(t) \in \mathbb{R}^n$ is defined as [13]

$$\mu(t) \triangleq (k_s+1)e_2(t) - (k_s+1)e_2(0) + \int_0^t [(k_s+1)\alpha_2 e_2(\sigma) + \beta_1 \mathrm{sgn}(e_2(\sigma))]d\sigma \tag{17}$$

where $k_s, \beta_1 \in \mathbb{R}$ are positive constant control gains. The feedforward NN component in (16), denoted by $\hat{f}_d(t) \in \mathbb{R}^n$, is generated as

$$\hat{f}_d \triangleq \hat{W}^T \sigma(\hat{V}^T x_d). \tag{18}$$

The estimates for the NN weights in (18) are generated on-line (there is no off-line learning phase) as

$$\dot{\hat{W}} \triangleq \mathrm{proj}\left(\Gamma_1 \hat{\sigma}' \hat{V}^T \dot{x}_d e_2^T\right) \tag{19}$$
$$\dot{\hat{V}} \triangleq \mathrm{proj}\left(\Gamma_2 \dot{x}_d \left(\hat{\sigma}'^T \hat{W} e_2\right)^T\right) \tag{20}$$

where $\Gamma_1 \in \mathbb{R}^{(N_2+1)\times(N_2+1)}$, $\Gamma_2 \in \mathbb{R}^{(N_1+1)\times(N_1+1)}$ are constant, positive definite, symmetric control gain matrices.[2]

The closed-loop tracking error system can be developed by substituting (16) into (11) as

$$M(q)r = f_d - \hat{f}_d + S + \tau_d - \mu. \tag{21}$$

To facilitate the subsequent stability analysis, the time derivative of (21) is determined as

$$M(q)\dot{r} = -\dot{M}(q)r + \dot{f}_d - \dot{\hat{f}}_d + \dot{S} + \dot{\tau}_d - \dot{\mu}. \tag{22}$$

Remark 2: Taking the time derivative of the closed-loop error system is typical of the RISE stability analysis. In our case, the time differentiation also facilitates the design of NN weight adaptation laws instead of using the typical (as in [3] and [4]) Taylor series approximation method to obtain a linear form for the estimation error $\tilde{V}$.

Using (14) and (18), the closed-loop error system in (22) can be expressed as

$$\begin{aligned} M(q)\dot{r} = &-\dot{M}(q)r + W^T\sigma'\left(V^T x_d\right)V^T\dot{x}_d \\ &- \dot{\hat{W}}^T\sigma(\hat{V}^T x_d) - \hat{W}^T\sigma'(\hat{V}^T x_d)\dot{\hat{V}}^T x_d \\ &- \hat{W}^T\sigma'(\hat{V}^T x_d)\hat{V}^T\dot{x}_d + \dot{\varepsilon} + \dot{S} + \dot{\tau}_d - \dot{\mu} \end{aligned} \tag{23}$$

[2]The use of the smooth projection algorithm in (19) and (20) is to ensure that $\hat{W}(t)$ and $\hat{V}(t)$ remain bounded. This fact will be exploited in the subsequent stability analysis.

where $\sigma'(\hat{V}^T x) \equiv d\sigma\left(V^T x\right)/d\left(V^T x\right)|_{V^T x = \hat{V}^T x}$. After adding and subtracting the terms $W^T\hat{\sigma}'\hat{V}^T\dot{x}_d + \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d$ to (23), the following expression can be obtained:

$$\begin{aligned} M(q)\dot{r} = &-\dot{M}(q)r + \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d + \tilde{W}^T\hat{\sigma}'\hat{V}^T\dot{x}_d \\ &+ W^T\sigma'V^T\dot{x}_d - W^T\hat{\sigma}'\hat{V}^T\dot{x}_d - \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d \\ &+ \dot{S} - \dot{\hat{W}}^T\hat{\sigma} - \hat{W}^T\hat{\sigma}'\dot{\hat{V}}^T x_d + \dot{\tau}_d + \dot{\varepsilon} - \dot{\mu} \end{aligned} \tag{24}$$

where the notation $\hat{\sigma}$ is introduced in (8). Using the NN weight tuning laws in (19) and (20); the expression in (24) can be rewritten as

$$M(q)\dot{r} = -\frac{1}{2}\dot{M}(q)r + \tilde{N} + N - e_2 - (k_s+1)r - \beta_1\mathrm{sgn}(e_2) \tag{25}$$

where the fact that the time derivative of (17) is given as

$$\dot{\mu}(t) = (k_s+1)r + \beta_1\mathrm{sgn}(e_2) \tag{26}$$

was utilized, and where the unmeasurable auxiliary terms $\tilde{N}(e_1, e_2, r, t), N(\hat{W}, \hat{V}, x_d, \dot{x}_d, t) \in \mathbb{R}^n$ are defined as

$$\begin{aligned} \tilde{N}(t) \triangleq &-\frac{1}{2}\dot{M}(q)r - \mathrm{proj}(\Gamma_1\hat{\sigma}'\hat{V}^T\dot{x}_d e_2^T)^T\hat{\sigma} \\ &- \hat{W}^T\hat{\sigma}'\mathrm{proj}(\Gamma_2\dot{x}_d(\hat{\sigma}'^T\hat{W}e_2)^T)^T x_d + \dot{S} + e_2 \end{aligned} \tag{27}$$

and

$$N \triangleq N_d + N_B. \tag{28}$$

In (28), $N_d(x_d, \dot{x}_d, t) \in \mathbb{R}^n$ is defined as

$$N_d \triangleq W^T\sigma'V^T\dot{x}_d + \dot{\varepsilon} + \dot{\tau}_d \tag{29}$$

while $N_B(\hat{W}, \hat{V}, x_d, \dot{x}_d, t) \in \mathbb{R}^n$ is further segregated as

$$N_B \triangleq N_{B_1} + N_{B_2} \tag{30}$$

where $N_{B_1}(\hat{W}, \hat{V}, x_d, \dot{x}_d, t) \in \mathbb{R}^n$ is defined as

$$N_{B_1} \triangleq -W^T\hat{\sigma}'\hat{V}^T\dot{x}_d - \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d \tag{31}$$

and the term $N_{B_2}(\hat{W}, \hat{V}, x_d, \dot{x}_d, t) \in \mathbb{R}^n$ is defined as

$$N_{B_2} \triangleq \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d + \tilde{W}^T\hat{\sigma}'\hat{V}^T\dot{x}_d. \tag{32}$$

Motivation for segregating the terms in (28) is derived from the fact that the different components in (28) have different bounds. Segregating the terms as in (28)–(32) facilitates the development of the NN weight update laws and the subsequent stability analysis. For example, the terms in (29) are grouped together because the terms and their time derivatives can be upper bounded by a constant and rejected by the RISE feedback, whereas the terms grouped in (30) can be upper bounded by a constant but their derivatives are state dependent. The state dependency of the time derivatives of the terms in (30) violates the assumptions given in previous RISE-based controllers (e.g., [11]–[17]), and requires additional consideration in the adaptation law design and stability analysis. The terms in (30) are further segregated because $N_{B_1}(\hat{W}, \hat{V}, x_d)$ will be rejected by the RISE feedback, whereas $N_{B_2}(\hat{W}, \hat{V}, x_d)$ will be partially rejected by the RISE feedback and partially canceled by the adaptive update law for the NN weight estimates.

In a similar manner as in [13], the Mean Value Theorem can be used to develop the following upper bound:

$$\left\|\tilde{N}(t)\right\| \leq \rho(\|z\|)\|z\| \tag{33}$$

where $z(t) \in \mathbb{R}^{3n}$ is defined as

$$z(t) \triangleq [\, e_1^T \quad e_2^T \quad r^T \,]^T \tag{34}$$

and the bounding function $\rho(\|z\|) \in \mathbb{R}$ is a positive globally invertible nondecreasing function. The following inequalities can be developed based on Property 3, (9), (10), (15), (30)–(32):

$$\|N_d\| \leq \zeta_1, \quad \|N_B\| \leq \zeta_2, \quad \left\|\dot{N}_d\right\| \leq \zeta_3. \tag{35}$$

By using (19) and (20), the time derivative of $N_B(\hat{W}, \hat{V}, x_d)$ can be bounded as

$$\left\|\dot{N}_B\right\| \leq \zeta_4 + \zeta_5 \|e_2\|. \tag{36}$$

In (35) and (36), $\zeta_i \in \mathbb{R}$, $(i = 1, 2, \ldots, 5)$ are known positive constants.

VI. Stability Analysis

Theorem: The composite NN and RISE controller given in (16)–(20) ensures that all system signals are bounded under closed-loop operation and that the position tracking error is regulated in the sense that

$$\|e_1(t)\| \to 0 \text{ as } t \to \infty$$

provided the control gain k_s introduced in (17) is selected sufficiently large (see the subsequent proof), and β_1 and β_2 are selected according to the following sufficient conditions:

$$\beta_1 > \zeta_1 + \zeta_2 + \frac{1}{\alpha_2}\zeta_3 + \frac{1}{\alpha_2}\zeta_4, \quad \beta_2 > \zeta_5 \tag{37}$$

where $\zeta_i \in \mathbb{R}$, $i = 1, 2, \ldots, 5$ are introduced in (35)–(36) and β_2 is introduced in (40).

Proof: Let $\mathcal{D} \subset \mathbb{R}^{3n+2}$ be a domain containing $y(t) = 0$, where $y(t) \in \mathbb{R}^{3n+2}$ is defined as

$$y(t) \triangleq \left[z^T(t) \quad \sqrt{P(t)}\sqrt{Q(t)} \right]^T. \tag{38}$$

In (38), the auxiliary function $P(t) \in \mathbb{R}$ is defined as

$$P(t) \triangleq \beta_1 \sum_{i=1}^{n} |e_{2i}(0)| - e_2(0)^T N(0) - \int_0^t L(\tau) d\tau \tag{39}$$

where the subscript $i = 1, 2, \ldots, n$ denotes the ith element of the vector, and the auxiliary function $L(t) \in \mathbb{R}$ is defined as

$$L(t) \triangleq r^T (N_{B_1}(t) + N_d(t) - \beta_1 \operatorname{sgn}(e_2)) + \dot{e}_2(t)^T N_{B_2}(t) - \beta_2 \|e_2(t)\|^2 \tag{40}$$

where $\beta_2 \in \mathbb{R}$ is a positive constant chosen according to the second sufficient condition in (37). The derivative $\dot{P}(t) \in \mathbb{R}$ can be expressed as

$$\begin{aligned}\dot{P}(t) = -L(t) = &-r^T (N_{B_1}(t) + N_d(t) \\ &- \beta_1 \operatorname{sgn}(e_2)) - \dot{e}_2(t)^T N_{B_2}(t) + \beta_2 \|e_2(t)\|^2.\end{aligned} \tag{41}$$

Provided the sufficient conditions introduced in (37) are satisfied, the following inequality can be obtained

$$\int_0^t L(\tau) d\tau \leq \beta_1 |e_{2i}(0)| - e_2(0)^T N(0). \tag{42}$$

Hence, (42) can be used to conclude that $P(t) \geq 0$. The auxiliary function $Q(t) \in \mathbb{R}$ in (38) is defined as

$$Q(t) \triangleq \frac{\alpha_2}{2} tr\left(\tilde{W}^T \Gamma_1^{-1} \tilde{W}\right) + \frac{\alpha_2}{2} tr\left(\tilde{V}^T \Gamma_2^{-1} \tilde{V}\right). \tag{43}$$

Since Γ_1 and Γ_2 are constant, symmetric, and positive definite matrices and $\alpha_2 > 0$, it is straightforward that $Q(t) \geq 0$.

Let $V_L(y, t) : \mathcal{D} \times [0, \infty) \to \mathbb{R}$ be a continuously differentiable positive definite function defined as

$$V_L(y, t) \triangleq e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} r^T M(q) r + P + Q \tag{44}$$

which satisfies the following inequalities:

$$U_1(y) \leq V_L(y, t) \leq U_2(y) \tag{45}$$

provided the sufficient conditions introduced in (37) are satisfied. In (45), the continuous positive definite functions $U_1(y), U_2(y) \in \mathbb{R}$ are defined as

$$U_1(y) \triangleq \lambda_1 \|y\|^2, \quad U_2(y) \triangleq \lambda_2(q) \|y\|^2 \tag{46}$$

where $\lambda_1, \lambda_2(q) \in \mathbb{R}$ are defined as

$$\lambda_1 \triangleq \frac{1}{2} \min\{1, m_1\}, \quad \lambda_2(q) \triangleq \max\left\{\frac{1}{2}\bar{m}(q), 1\right\}$$

where $m_1, \bar{m}(q)$ are introduced in (2). After utilizing (4), (5), (25), and (26), the time derivative of (44) can be expressed as

$$\begin{aligned}\dot{V}_L(y, t) = &-2\alpha_1 \|e_1\|^2 + 2e_2^T e_1 + r^T \tilde{N}(t) \\ &- (k_s + 1)\|r\|^2 - \alpha_2 \|e_2\|^2 + \beta_2 \|e_2\|^2 \\ &+ \alpha_2 e_2^T \left[\hat{W}^T \hat{\sigma}' \tilde{V}^T \dot{x}_d + \tilde{W}^T \hat{\sigma}' \hat{V}^T \dot{x}_d\right] \\ &+ tr\left(\alpha_2 \tilde{W}^T \Gamma_1^{-1} \dot{\tilde{W}}\right) + tr\left(\alpha_2 \tilde{V}^T \Gamma_2^{-1} \dot{\tilde{V}}\right).\end{aligned} \tag{47}$$

Based on the fact that

$$e_2^T e_1 \leq \frac{1}{2}\|e_1\|^2 + \frac{1}{2}\|e_2\|^2$$

and using (19) and (20), the expression in (47) can be simplified as

$$\begin{aligned}\dot{V}_L(y, t) \leq &\; r^T \tilde{N}(t) - (k_s + 1)\|r\|^2 \\ &- (2\alpha_1 - 1)\|e_1\|^2 - (\alpha_2 - \beta_2 - 1)\|e_2\|^2.\end{aligned} \tag{48}$$

By using (33), the expression in (48) can be further bounded as

$$\dot{V}_L(y, t) \leq -\lambda_3 \|z\|^2 - \left(k_s \|r\|^2 - \rho(\|z\|)\|r\|\|z\|\right) \tag{49}$$

where $\lambda_3 \triangleq \min\{2\alpha_1 - 1, \alpha_2 - \beta_2 - 1, 1\}$; hence, λ_3 is positive if α_1, α_2 are chosen according to the following sufficient conditions:

$$\alpha_1 > \frac{1}{2}, \quad \alpha_2 > \beta_2 + 1.$$

After completing the squares for the second and third term in (49), the following expression can be obtained:

$$\dot{V}_L(y, t) \leq -\lambda_3 \|z\|^2 + \frac{\rho^2(z)\|z\|^2}{4k_s} \leq -U(y) \tag{50}$$

where $U(y) = c\|z\|^2$, for some positive constant $c \in \mathbb{R}$, is a continuous positive semi-definite function that is defined on the following domain:

$$D \triangleq \left\{ y \in \mathbb{R}^{3n+2} \mid \|y\| \le \rho^{-1}\left(2\sqrt{\lambda_3 k_s}\right) \right\}.$$

The inequalities in (45) and (50) can be used to show that $V_L(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, $e_1(t)$, $e_2(t)$, $r(t)$, $P(t)$, and $Q(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, standard linear analysis methods can be used to prove that $\dot{e}_1(t), \dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (4) and (5). Since $e_1(t), e_2(t), r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the assumption that $q_d(t), \dot{q}_d(t), \ddot{q}_d(t)$ exist and are bounded can be used along with (3)–(5) to conclude that $q(t), \dot{q}(t), \ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $q(t), \dot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 2 can be used to conclude that $M(q), V_m(q,\dot{q}), G(q)$, and $F(\dot{q}) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Therefore, from (1) and Property 3, we can show that $\tau(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (26) can be used to show that $\dot{\mu}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{q}(t), \ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 2 can be used to show that $\dot{V}_m(q,\dot{q})$, $\dot{G}(q)$, $\dot{F}(q)$ and $\dot{M}(q) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, (22) can be used to show that $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{e}_1(t), \dot{e}_2(t), \dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the definitions for $U(y)$ and $z(t)$ can be used to prove that $U(y)$ is uniformly continuous in $\mathcal{D}$.

Let $\mathcal{S} \subset \mathcal{D}$ denote a set defined as follows:[3]

$$\mathcal{S} \triangleq \left\{ y(t) \subset \mathcal{D} \mid U_2(y(t)) < \lambda_1 \left(\rho^{-1}\left(2\sqrt{\lambda_3 k_s}\right)\right)^2 \right\}. \quad (51)$$

[20, Theor. 8.4] can now be invoked to state that

$$c\|z(t)\|^2 \to 0 \text{ as } t \to \infty \ \forall\, y(0) \in \mathcal{S}. \quad (52)$$

Based on the definition of $z(t)$, (52) can be used to show that

$$\|e_1(t)\| \to 0 \text{ as } t \to \infty \ \forall\, y(0) \in \mathcal{S}. \quad (53)$$

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