Event-Triggered Control of Multiagent Systems for Fixed and Time-Varying Network Topologies

Teng-Hu Cheng, Zhen Kan, Justin R. Klotz, John M. Shea, and Warren E. Dixon

***Abstract*—A decentralized controller that uses event-triggered communication scheduling is developed for the leader-follower consensus problem under fixed and switching communication topologies. To eliminate continuous interagent communication, state estimates of neighboring agents are designed for control feedback and are updated via communication to reset growing estimate errors. The communication times are based on an event-triggered approach and are adapted based on the trade-off between the control system performance and the desire to minimize the amount of communication. An important aspect of the developed event trigger strategy is that communication is not required to determine when a state update is needed. Since the control strategy produces switched dynamics, analysis is provided to show that Zeno behavior is avoided by developing a positive constant lower bound on the minimum inter-event interval. A Lyapunov-based convergence analysis is also provided to indicate bounded convergence of the developed control methodology.**

***Index Terms*—Event-triggered control, limited communication, multi-agent systems, networked control, systems switching topologies.**

I. Introduction

Networks of autonomous agents are often tasked with collaborative objectives. To achieve such collaborative goals, each agent typically requires knowledge of the dynamics of neighboring agents, where the states of network neighbors are assumed to be continuously available (cf., [1]–[4]). Continuous communication of state information facilitates the ability to configure the network topology; however, such feedback has an associated cost (e.g., bandwidth consumption, transmit power, large signature that can be detected/monitored).

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Intermittent feedback strategies have been investigated to relax the requirements of continuous state feedback from the network. For instance, event-triggered feedback strategies are developed in [5]–[17], in which control inputs are updated via communication from neighbors only when a triggering condition is met. Typically, the triggering condition is met when the ratio of the norm of some error exceeds a predefined threshold. A dynamic event-trigger mechanism was recently developed in [18] to enhance system stability by introducing an internal dynamic variable; however, verifying the triggering condition requires continuous communication. But the approach is not designed for network systems, and verifying the triggering condition also requires continuous communication. Variable communication delay was considered in [19] using an event-based control strategy for networked embedded systems, but whether and how this can be extended to multiagent network systems is unclear.

Some earlier results that apply event-triggered strategies to multiagent systems include [7] and [8]. However, the potential bandwidth minimizing advantages are compromised because verifying the event triggering condition requires constant communication. These results were later extended to directed and undirected graphs in [9] and [10], but in these works the triggering condition requires a priori knowledge of the Fiedler value and the final consensus value. These requirements were relaxed in [11] and [12] by designing a new trigger function using the sum of relative states from neighbors. In [13], a time-based triggering function (i.e., a time-dependent threshold) is introduced, and a similar time-varying threshold is applied in [14] for a directed time-varying communication topology. Advanced event-triggered approaches for fixed network topologies were designed in [20], [21], and [21] to eliminate the need for continuous communication for verifying triggering events, and the state disagreement was proven to be bounded with guaranteed interevent times. The consensus problem with communication delay was also considered, and solutions were developed in [22]. However, the strategies in [7]–[14], [20]–[22] solve the leaderless average consensus problem. The more challenging leader-follower consensus control problem is investigated in [23], but the leader state is assumed to be stationary, which limits applicability; additionally, constant neighbor communication is used to detect the trigger condition, which mitigates the benefits of the event-triggered control strategy. An event-triggered control method was developed in [24] for a network of nonlinear dynamical systems, where the considered network is switching and the edge weights are time-varying. However, computing the event at which the controller is updated requires continuous communication, which may limit its applicability in networks with intermittent communication.

In practice, unpredictable physical constraints (e.g., random sensor/device failure, obstacles/interference in a complex environment) can cause intermittent communication[16], [25]–[27]. When interagent communication is absent, the unavailability of the recent states from neighboring agents can impose significant control design challenges for network systems with collaborative objectives, since most control strategies assume constant information exchange between agents. Additionally, since intermittent communication can result in switching network topologies, designing trigger functions with respect to switching network topologies can be challenging. Motivated to design a uniform trigger condition for agents under arbitrarily switching topologies, a uniform control matrix is developed in [28] by jointly using Riccati inequalities associated with dynamics of the follower agents and network topologies, which facilitates the design of a uniform trigger condition for the agents and ensures a uniform convergence rate of the tracking error under arbitrarily switching graphs. Riccati inequalities were also used to facilitate the convergence analysis in [29], although not for developing an event-triggered controller to reduce interagent communication. An event-triggered control approach considering limited communication for leader–follower consensus with fixed and switching topologies was developed in [30]; however, the trigger function requires neighboring information via continuous communication.

The methods developed in this paper eliminate the need for continuous communication, and thereby reduce the required communication bandwidth by using an open-loop estimate of the neighbor's state. An advantage of the open-loop estimate is that no interagent communication is required between triggering events. The challenge is that open-loop state estimation can be unstable, resulting in a failure to achieve the cooperative network objective. To overcome such challenges, an event triggering strategy that provides sufficient intermittent feedback is developed to ensure leader–follower consensus. Specifically, the estimates of the neighbors' states are used as a substitute for the neighbor's true states, resulting in less interagent communication. We analyze the interplay between the control gains and the dynamics of the estimate error to obtain a trigger function that determines the next required estimate update without using inter-agent communication. A Lyapunov-based analysis is provided that ensures bounded leader–follower consensus.

II. Preliminaries and Problem Statement

A. Algebraic Graph Theory Preliminaries

A directed graph $\bar{\mathcal{G}}$ consists of a finite node set $\mathcal{V}$ and an edge set $\mathcal{E}$, where $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$. An edge, denoted as (j, i), implies that node i can obtain information from node j, but not vice versa. On the contrary, the graph $\mathcal{G}$ is undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$, and vice versa. The neighbor set of agent i is defined as $\mathcal{N}_i \triangleq \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}, j \neq i\}$.

A directed path is a sequence of edges that connect consecutive vertices in a directed graph. An undirected path of the undirected graph is defined analogously. An undirected graph is connected if there exists an undirected path between any two distinct nodes in the graph. An adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ of the directed graph is given by $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. For the undirected graph, $a_{ij} = a_{ji}$. For both the directed and undirected graph, $a_{ii} = 0$ and $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$. The Laplacian matrix of the graph $\mathcal{G}$ is defined as $L = [l_{ij}] \in \mathbb{R}^{N \times N}$, where $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

B. Agent Dynamics

Consider N follower agents, defined as $\mathcal{V} \triangleq \{1, 2, \ldots, N\}$, with a network topology modeled by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Let $\bar{\mathcal{G}}$ denote a directed graph with the node set $\mathcal{V} \cup \{0\}$ and the edge set that contains all edges in $\mathcal{E}$ and the edges connecting leader agent 0 and follower agent $j \in \mathcal{V}$. The dynamics of the followers and the leader are described by

$$\dot{x}_i = Ax_i + Bu_i \quad (1)$$

$$\dot{x}_0 = Ax_0 \quad (2)$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the state and control input of follower agent $i \in \mathcal{V}$, respectively, $x_0 \in \mathbb{R}^n$ denotes the leader's state, $A \in \mathbb{R}^{n \times n}$ is a state matrix, and $B \in \mathbb{R}^{n \times m}$ is a full column rank matrix.

Remark 1: A disturbance $\xi \in \mathbb{R}^n$ can also be added to the leader's dynamics as $\dot{x}_0 = Ax_0 + \xi$, where $\|\xi\| \leq \bar{\xi}$ and $\bar{\xi} \in \mathbb{R}^+$ is a positive constant. Furthermore, given the same trigger condition, introducing the noise does not cause system instability even though the followers keep using the same controller.

Assumption 1: The dynamics of the follower agents are controllable, i.e., the pair (A, B) is stabilizable.

Assumption 2: The leader trajectory, x_0, is bounded (i.e., $\exists M < \infty \ni \|x_0(t)\| < M, \forall t$).

Definition 1: A directed graph is connected if each follower has a directed path from the leader.

C. Control Objective

The objective is to achieve bounded leader–follower consensus under the constraint that feedback from neighboring follower agents is only intermittently available. To quantify the control objective, let the leader–follower tracking error for agent i be defined as $\varepsilon_i \triangleq x_i - x_0 \in \mathbb{R}^n$ and ε denotes the stacked form of ε_i.

III. Leader–Follower Consensus Under Fixed Topologies

Consider N follower agents with a fixed network topology that satisfies the following two assumptions.

Assumption 3: $\mathcal{G}$ is connected and at least one follower is connected to the leader.

Assumption 4: The followers that are connected to the leader can continuously receive information from the leader.

Note that the communication between followers is undirected, and the communication from the leader to the subset of the followers is directed.

A. Intermittent Feedback Approach

Based on the subsequent convergence analysis, a decentralized event-triggered controller for agent $i \in \mathcal{V}$ is designed to reduce follower-to-follower communication as

$$u_i = K\hat{z}_i + K(\hat{x}_i - x_i) \quad (3)$$

$$\hat{z}_i = \sum_{j \in \mathcal{N}_i} (\hat{x}_j - \hat{x}_i) + d_i(x_0 - \hat{x}_i) \quad (4)$$

where $K \in \mathbb{R}^{m \times n}$ is a control gain, $\hat{z}_i \in \mathbb{R}^n$ is the relative state estimate, $\hat{x}_i, \hat{x}_j \in \mathbb{R}^n$ are state estimates defined in the subsequent analysis, and $d_i = 1$ if agent i is connected to the leader, $d_i = 0$ otherwise. In (3), the control gain K can be designed as [28]

$$K = B^T P \quad (5)$$

where $P \in \mathbb{R}^{n \times n}$ satisfies the following Riccati inequality:

$$\mathrm{PA} + A^T P - 2\delta_{\min} \mathrm{PBB}^T P + \delta_{\min} I_n < 0. \quad (6)$$

In (6), $\delta_{\min} \in \mathbb{R}_{>0}$ denotes the minimum eigenvalue of $H \triangleq L + D \in \mathbb{R}^{N \times N}$, where L is a previously defined Laplacian matrix, and $D \in \mathbb{R}^{N \times N}$ is defined as $D \triangleq \operatorname{diag}(d_1, d_2, \ldots, d_N)$. The minimum eigenvalue $\delta_{\min}$ is positive because H is a positive definite matrix based on Assumption 3 and [31, Lemma 1]. In (4), the computation of $\hat{z}_i$ only requires $\hat{x}_i$ (i.e., the estimate of agent i's state) and $\hat{x}_j$, $j \in \mathcal{N}_i$, (i.e., the estimates of the neighboring followers' states), instead of using their true states x_j, $j \in \mathcal{N}_i$, via continuous communication. When the leader is a neighbor, the true state x_0 is used since the leader state is available according to Assumption 4. The estimate $\hat{x}_j$ in (4) evolves according to the dynamics

$$\dot{\hat{x}}_j(t) = A\hat{x}_j(t),\ j \in \mathcal{N}_i,\ t \in \left[t_k^j, t_{k+1}^j\right) \tag{7}$$

$$\hat{x}_j\left(t_k^j\right) = x_j\left(t_k^j\right) \tag{8}$$

for $k = 0, 1, 2, \ldots$, where $\hat{x}_j$ is an open-loop estimate that flows along the leader drift dynamics during $t \in [t_k^j, t_{k+1}^j)$, where feedback is used to reset the estimate at the discrete times t_k^j, for $j \in \mathcal{N}_i$, where t_k^j is the event-triggered time. Although agent i does not communicate the estimate $\hat{x}_i$, agent i maintains $\hat{x}_i$ for implementation in (4). The estimate $\hat{x}_i$ is updated continuously with the dynamics in (7) and discretely at time instances described in (8). Therefore, u_i is a piecewise continuous signal, where communication is required when state information is transmitted to, or received from, neighboring agents for estimate updates; otherwise, u_i flows continuously during the interevent intervals.

B. Dynamics of Estimate Error

The controller in (3) and (4) uses feedback of agent i's true state and the estimated state. This strategy facilitates the development of a triggering condition that does not rely on communication with neighboring agents. Specifically, the mismatch between the true state and the open-loop estimate state defined as the estimate error $e_i \in \mathbb{R}^n$ as

$$e_i \triangleq \hat{x}_i - x_i,\ i \in \mathcal{V},\ t \in \left[t_k^i, t_{k+1}^i\right) \tag{9}$$

is used in the subsequent event-triggered design. Using (1), (3), and (7), the stack form of the time-derivative of (9) can be expressed as

$$\dot{e} = \Big(I_N \otimes (A - BK)\Big)e + (H \otimes BK)\,\varepsilon + (H \otimes BK)\,e, \tag{10}$$

where $e \in \mathbb{R}^{nN}$ is defined as $e \triangleq [e_1^T, e_2^T, \ldots, e_N^T]^T$, $\otimes$ denotes the Kronecker product, and $\varepsilon \in \mathbb{R}^{nN}$.

C. Closed-Loop Error System

Using (9), a nonimplementable form[1] (to facilitate the subsequent analysis) of (3) can be expressed as

$$u_i = K\sum_{j\in\mathcal{N}_i}\left[(x_j - x_i) + (e_j - e_i)\right] + Kd_i\,(x_0 - x_i) - Kd_ie_i + Ke_i. \tag{11}$$

Substituting (11) into the open-loop dynamics in (1) and using the definition of ε_i yields a stack form of the closed-loop consensus error system:

$$\dot{\varepsilon} = (I_N \otimes A)\,\varepsilon - (H \otimes BK)\,\varepsilon - \Big((H - I_N) \otimes BK\Big)e. \tag{12}$$

To facilitate the subsequent analysis, a relationship between ε and $\hat{z}$ is developed, where $\hat{z} \triangleq [\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_N]^T \in \mathbb{R}^{nN}$ is defined as

$$\hat{z} \triangleq (H \otimes I_n)\,[1_N \otimes x_0 - \hat{x}] \tag{13}$$

where $\hat{x} \triangleq [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N]^T \in \mathbb{R}^{nN}$, and 1_N is the ones vector with denoted dimension. Using the relationship

$$\varepsilon_i = x_i - x_0 = (\hat{x}_i - e_i) - x_0$$

the useful expression

$$\hat{x} - (1_N \otimes x_0) = \varepsilon + e \tag{14}$$

can be obtained. Combining (13) and (14) yields

$$\varepsilon = -\left(H^{-1} \otimes I_n\right)\hat{z} - e \tag{15}$$

where $\hat{z}$ is governed by the dynamics

$$\dot{\hat{z}} = (I_N \otimes A)\,\hat{z}, \tag{16}$$

where (2) and (7) were used.

D. Convergence Analysis

In this section, leader–follower consensus with the event-triggered controller designed in (3) is examined using a Lyapunov-based analysis. To facilitate the subsequent convergence analysis, an event time t_k^i is explicitly defined as follows.

Definition 2: An event time t_k^i for the follower agent $i \in \mathcal{V}$ is defined as

$$t_k^i \triangleq \min\left\{t > t_{k-1}^i \,\middle|\, \|e_i(t)\|^2 \geq c_i^2\,\|\hat{z}_i(t)\|^2 + \epsilon^2\right\} \tag{17}$$

for $k = 0, 1, 2\ldots$, where $c_i \in \mathbb{R}_{>0}$ is a positive constant defined as

$$c_i = \sqrt{\frac{(\gamma_1 - \frac{\gamma_3}{\beta})}{\gamma_2 + \gamma_3\beta}}, \epsilon \in \mathbb{R}_{>0}$$

is a positive constant, and $\beta \in \mathbb{R}_{>0}$ satisfies

$$\beta > \frac{\gamma_3}{\gamma_1} \tag{18}$$

where γ_i, $i = 1, 2, 3$, are positive constants defined as

$$1\gamma_1 \triangleq \delta_1\lambda_{\min}\left(H^{-2}\right)$$

$$\gamma_2 \triangleq S_{\max}\Big((H - I_N) \otimes \left(2\mathrm{PBB}^TP\right)\Big) - \delta_1$$

$$\gamma_3 \triangleq \frac{1}{2}S_{\max}\Big((H - I_N)\,H^{-} \otimes \left(2\mathrm{PBB}^TP\right) - H^{-1} \otimes 2\delta_1 I_n\Big)$$

where, based on the structure of γ_2 and γ_3, $\gamma_2 \neq 0$ and $\gamma_3 \neq 0$, $\delta_1 \in \mathbb{R}_{>0}$ satisfies $0 < \delta_1 < \delta_{\min}$, and $\lambda_{\min}(\cdot)$ and $S_{\max}(\cdot)$ denote the minimum eigenvalue and the maximum singular value of a matrix argument, respectively.

Remark 2: A cost for the event-triggered method, compared to distributed networks that assume continuous communication, is that some knowledge of the graph Laplacian is required. Specifically, the triggering condition in (17) only requires spectral knowledge (i.e., the minimum eigenvalue $\lambda_{\min}(\cdot)$ and the maximum singular value $S_{\max}(\cdot)$) of the Laplacian matrix. While the exact Laplacian may not always be available, various results (e.g., [32]–[34]) can be used to derive the bound of such spectral knowledge based on limited knowledge of the Laplacian. A priori information about the network (i.e., the set of possible switching topologies) can also be utilized to facilitate the development of the spectral knowledge bound. This idea is analogous to assuming knowledge of a disturbance bound in classic control problems, where the exact disturbance is unknown.

Theorem 1: Provided agents receive feedback at the event times t_k^i defined in (17), then the controller designed in (3) and (4) ensures that

[1]The expression in (11) is nonimplementable because e_j is unavailable without continuous interagent communication.

the network system achieves bounded leader–follower consensus as

$$\|\varepsilon(t)\| \leq \sqrt{\frac{N(\gamma_2+\gamma_3\beta)\epsilon^2\lambda_{\max}(P)}{\delta_2\lambda_{\min}(P)}} \quad as \quad t\to\infty. \tag{19}$$

Proof: Consider a Lyapunov function candidate $V:\mathbb{R}^{nN}\to\mathbb{R}$ as

$$V \triangleq \varepsilon^T(I_N\otimes P)\varepsilon \tag{20}$$

where P is a symmetric positive-definite matrix satisfying (6). Using (5) and (12), the time derivative of (20) can be expressed as

$$\dot{V} = \varepsilon^T\left[I_N\otimes\left(PA+A^TP\right)-H\otimes\left(2\text{PBB}^TP\right)\right]\varepsilon - e^T\left[(H-I_N)\otimes\left(2\text{PBB}^TP\right)\right]\varepsilon. \tag{21}$$

Since H is symmetric and positive definite, (6) can be used to upper bound (21) as

$$\dot{V} \leq -\delta_{\min}\varepsilon^T\varepsilon - e^T\left[(H-I_N)\otimes\left(2\text{PBB}^TP\right)\right]\varepsilon. \tag{22}$$

Using (15), (22) can be upper bounded by

$$\begin{aligned}\dot{V} \leq & -\delta_1\hat{z}^T\left(H^{-2}\otimes I_n\right)\hat{z}-\delta_1e^Te-2\delta_1e^T\left(H^{-1}\otimes I_n\right)\hat{z}\\ &+e^T\left[(H-I_N)H^{-1}\otimes\left(2\text{PBB}^TP\right)\right]\hat{z}\\ &+e^T\left[(H-I_N)\otimes\left(2\text{PBB}^TP\right)\right]e\\ &-\delta_2\varepsilon^T\varepsilon\end{aligned} \tag{23}$$

where $\delta_{\min}=\delta_1+\delta_2$ for $\delta_1,\ \delta_2\in\mathbb{R}_{>0}$. By using the inequality $x^Ty\leq\frac{\beta}{2}\|x\|^2+\frac{1}{2\beta}\|y\|^2$, (23) can be upper bounded by

$$\begin{aligned}\dot{V} \leq & -\gamma_1\|\hat{z}\|^2+2\gamma_3\left(\frac{\beta}{2}\|e\|^2+\frac{1}{2\beta}\|\hat{z}\|^2\right)+\gamma_2\|e\|^2\\ &-\delta_2\varepsilon^T\varepsilon\\ \leq & -\sum_{i\in\mathcal{V}}\left[\left(\gamma_1-\frac{\gamma_3}{\beta}\right)\|\hat{z}_i\|^2-(\gamma_2+\gamma_3\beta)\|e_i\|^2\right.\\ &\left.+(\gamma_2+\gamma_3\beta)\epsilon^2\right]-\delta_2\varepsilon^T\varepsilon+N(\gamma_2+\gamma_3\beta)\epsilon^2\end{aligned} \tag{24}$$

where $\gamma_i,\ \forall i=1,2,3$ are positive constants defined in Definition 2. Provided feedback is available at each event time t_k^i as defined in (17) then the inequality in (24) can be upper bounded as

$$\dot{V} \leq -\delta_2\varepsilon^T\varepsilon+N(\gamma_2+\gamma_3\beta)\epsilon^2. \tag{25}$$

Given (20) and (25), the inequality in (19) can be obtained.

Based on (20) and (25), $\varepsilon\in\mathcal{L}_\infty$. Based on the definition of ε and Assumption 2, it is straightforward to show that $x_i\in\mathcal{L}_\infty$. Since $\hat{x}_i$ follows the same dynamics as the leader according to (7), $x_0\in\mathcal{L}_\infty$ implies $\hat{x}_i\in\mathcal{L}_\infty$. Therefore, the fact that $\hat{z}_i\in\mathcal{L}_\infty$ from (4) can be used to show $u_i\in\mathcal{L}_\infty$. ■

Remark 3: A cost function, $J\in\mathbb{R}$, consisting of the tracking error and number of communication events can be defined as

$$J=\frac{1}{|\mathcal{V}|}\sum_{i\in\mathcal{V}}\left(N_{c_i}+\int_0^\infty e_i^TQe_i\right) \tag{26}$$

where $N_{c_i}\in\mathbb{N}$ represents the number of communication events for agent i over the time window of $T\in\mathbb{R}_{>0}$, and $Q\in\mathbb{R}^{n\times n}$ denotes the positive definite weighting matrix of the tracking error. This cost function can be used for arbitrating between convergence rate and number of communication events through simulation trials based on the user's preference.

E. Minimal Interevent Interval

The event times defined in (17) indicate the maximum time between events allowed to ensure bounded convergence given the open-loop estimate in (3) and (4). The development in this section determines the positive lower-bound on the minimum interevent interval. If the event time intervals were zero, communication would need to occur infinitely fast (i.e., Zeno behavior). To prove Zeno behavior is avoided, the subsequent development establishes a positive lower bound on $t_k^i-t_{k+1}^i$, thereby establishing the worse-case maximum communication rate that needs to be available.

Theorem 2: The event time defined in (17) ensures that the minimum interevent interval $\tau\triangleq t_{k+1}^i-t_k^i\in\mathbb{R}_{>0}$ is lower bounded by

$$\tau\geq\frac{\ln\left(\frac{\epsilon}{c}+1\right)}{\|A-BK\|} \tag{27}$$

where $c\in\mathbb{R}_{>0}$ is defined as $c\triangleq\frac{\|BK\|}{\|A-BK\|}\bar{\hat{z}}_i$, $\bar{\hat{z}}_i$ is the upper bound of $\|\hat{z}_i\|$, which is proven to be bounded in Theorem 1.

Proof: Taking time derivative on both sides of (9) and using (7) and (3), the dynamics of $\dot{e}_i$ can be obtained as

$$\dot{e}_i=(A-BK)e_i-BK\hat{z}_i$$

which can be upper bounded as

$$\dot{y}\leq\|A-BK\|y+\|BK\|\bar{\hat{z}}_i \tag{28}$$

where $y:[0,\infty)\to\mathbb{R}_{\geq 0}$ is a non-negative, piecewise continuous function, which is differentiable in the interevent interval and is defined as

$$y\left(t-t_k^i\right)\triangleq\|e_i(t)\|,\ \text{for}\ t\in\left[t_k^i,\ t_{k+1}^i\right) \tag{29}$$

for $k=0,1,2\ldots$. Based on (28), a non-negative function $\phi:[0,\infty)\to\mathbb{R}_{\geq 0}$, satisfying

$$\dot{\phi}=\|A-BK\|\phi+\|BK\|\bar{\hat{z}}_i,\quad \phi(0)=y_0 \tag{30}$$

can be lower bounded by y as

$$y\leq\phi,\ \text{for}\ t\in[0,\tau) \tag{31}$$

where $y_0\in\mathbb{R}_{\geq 0}$ is the initial condition of y, which is 0 since $e\left(t_k^i\right)=0$ for $k=0,1,2\ldots$. An analytical solution to (30) with initial condition $\phi(0)=0$ can be solved as

$$\phi(t)=c\left(e^{\|A-BK\|t}-1\right) \tag{32}$$

where c is defined in Theorem 2. Using the trigger condition defined in (17) along with (31) and (32) with $t\to\tau$ yields

$$c^2\left(e^{\|A-BK\|\tau}-1\right)^2\geq c_i^2\|\hat{z}_i\|^2+\epsilon^2. \tag{33}$$

Since the first term of RHS in (33) is positive, (33) can be further bounded by

$$c^2\left(e^{\|A-BK\|\tau}-1\right)^2\geq\epsilon^2. \tag{34}$$

The lower bound of τ defined in (27) can be obtained by solving (34).

Remark 4: This lower bound implies that Zeno behavior is excluded. However, there is a tradeoff between the minimum interevent interval and the error convergence rate. A higher value of the control gain K can yield a smaller value of τ, which increases the convergence rate, and vice versa.

IV. Leader–Follower Consensus Under Switching Topologies

In this section, an event-triggered-based decentralized control approach is developed to extend the results developed in Section III to achieve leader–follower consensus under switching network topologies. To address the switching topologies, the following definition and assumptions are made.

A. Definitions and Assumptions

The time-varying interaction topology of the N followers described in (1) can be modeled by a switched undirected graph $\mathcal{G}_\sigma$, where the piecewise constant switching signal $\sigma : [0, \infty) \to \mathcal{P}$ indicates an underlying graph from a finite set $\mathcal{P} \triangleq \{1, 2, \ldots, M\}$ at time t, such that $\{\mathcal{G}_p : p \in \mathcal{P}\}$ includes all graphs in $\{\cup_{t\geq 0}\mathcal{G}\}$.

Similarly, the time-varying interaction topology of the leader–follower system described in (1)–(2) is modeled by a directed switching graph denoted as $\bar{\mathcal{G}}_\sigma$, which consists of the node set $\mathcal{V} \cup \{0\}$ and the edge set that contains all edges in $\mathcal{G}_\sigma$ and the edges connecting node 0 and the followers that have a directed edge from the leader.

Assumption 5: $\bar{\mathcal{G}}_p$ is connected for each $p \in \mathcal{P}$.

Assumption 6: The switching signal σ has a finite number of switches in a finite-time interval. Specifically, σ switches at t_q and is invariant during a nonvanishing interval $[t_q, t_{q+1})$, $q = 0, 1, \ldots$, with $t_0 = 0$, $0 < \Omega < t_{q+1} - t_q < T$, where $\Omega, T \in \mathbb{R}$ are positive constants, and Ω is a nonvanishing dwelltime. Additionally, the switching sequence of σ is arbitrary.

B. Controller Design

In the previous section, the decentralized controller received intermittent feedback at discrete points determined from an error-based triggering condition. In this section, events that trigger communication also include switches in the topology.

Motivated by the development in the previous section and the subsequent convergence analysis, a decentralized event-triggered controller for agent $i \in \mathcal{V}$ is designed with the same structure as in (3) and (4), where $\mathcal{N}_i$ is a time-varying neighbor set, and the estimate $\hat{x}_j$ evolves according to the dynamics

$$\hat{x}_j\left(t_E^j\right) = x_j\left(t_E^j\right) \tag{35}$$

$$\dot{\hat{x}}_j = A\hat{x}_j,\ t \in \left[t_E^j, t_{E+1}^j\right),\ j \in \{i\} \cup \mathcal{N}_i \tag{36}$$

$$t_E^j = \begin{cases} t_q, & \text{if } j \text{ is a new neighbor} \\ t_k^j, & \text{otherwise} \end{cases} \tag{37}$$

for $E, k = 0, 1, 2 \ldots$, where $\hat{x}_j$ is an open-loop estimate that flows along the leader dynamics during $t \in [t_E^j, t_{E+1}^j)$, where feedback is used to reset the estimate at the discrete time t_E^i, for $j \in \mathcal{N}_i$. In (37), t_q is the time when $\bar{\mathcal{G}}_\sigma$ switches. Although agent i does not communicate the estimate $\hat{x}_i$, agent i maintains $\hat{x}_i$ for implementation. The estimate $\hat{x}_i$ is updated continuously with the dynamics in (36) and discretely at time instances described in (35). Therefore, u_i is a piecewise continuous signal, where communication is required when any new one-hop neighbor is connected or when state information is transmitted to, or received from, neighboring agents for estimate updates; otherwise, u_i flows during the interevent intervals.

Remark 5: In (37), since the link between two follower neighbors is undirected, $j \in \mathcal{N}_i$ implies $i \in \mathcal{N}_j$. That is, mutual communication is conducted at t_q if $j \in \mathcal{N}_i$ is a new neighbor.

C. Estimate Error Dynamics

Following the same procedure developed in (9), the dynamics of the estimate error of the switched system can be expressed as

$$\dot{e} = \Big(I_N \otimes (A - BK)\Big)e + (H_\sigma \otimes BK)\,\varepsilon + (H_\sigma \otimes BK)\,e \tag{38}$$

where σ is the switching signal defined in Section IV-A. To facilitate the subsequent stability analysis, a symmetric positive definite matrix P is solved from the following Riccati inequalities:

$$PA + A^T P - 2\delta_p PBB^T P + \delta_p I_n < 0 \quad \forall p \in \mathcal{P} \tag{39}$$

where K is the control gain defined in (5) with the P matrix solved from the Riccati inequalities in (39). In (39), $\delta_p \in \mathbb{R}_{>0}$ is the minimum eigenvalue of H_p and is a positive constant based on Assumption 5 and [31]. To facilitate proving the consensus convergence of the switched system with an arbitrary switching sequence, a minimum value of a finite set composed of δ_p, denoted by $\delta_{\min}$, is defined as

$$\delta_{\min} \triangleq \min\{\delta_p \mid p \in \mathcal{P}\}. \tag{40}$$

D. Closed-Loop Error System

Following the development in (12), the stack form of the closed-loop consensus error system for the switched system can be expressed as

$$\dot{\varepsilon} = (I_N \otimes A)\,\varepsilon - (H_\sigma \otimes BK)\,\varepsilon - \Big((H_\sigma - I_N) \otimes BK\Big)e \tag{41}$$

where ε is defined as

$$\varepsilon = -\left(H_\sigma^{-1} \otimes I_n\right)\hat{z} - e \tag{42}$$

where $\hat{z}$ is governed by the dynamics defined in (16).

E. Convergence Analysis

Since communication can fail in an unpredictable manner, a convergence analysis for the switched system with an arbitrary switching sequence is required. To this end, the objective in this section is to prove the existence of a common Lyapunov function which considers an arbitrary switching signal σ.

Definition 3: An event-triggered time t_k^i for the switched system is defined as in (17), where the positive constants k_i, $i = 1, 2, 3$ are associated with the switched system parameters and are redefined as

$$1\gamma_1 \triangleq \min_{p\in\mathcal{P}}\left\{\delta_{m1}\lambda_{\min}\left(H_p^{-2}\right)\right\} \tag{43}$$

$$\gamma_2 \triangleq \max_{p\in\mathcal{P}}\left\{S_{\max}\Big((H_p - I_N) \otimes \left(2\text{PBB}^T P\right)\Big) - \delta_{m1}\right\} \tag{44}$$

$$\gamma_3 \triangleq \frac{1}{2}\max_{p\in\mathcal{P}}\Big\{S_{\max}\Big((H_p - I_N)\,H_p^{-1} \otimes \left(2\text{PBB}^T P\right) - H_p^{-1} \otimes 2\delta_{m1} I_n\Big)\Big\} \tag{45}$$

where $\delta_{m1} \in \mathbb{R}_{>0}$ satisfies $0 < \delta_{m1} < \delta_{\min}$ such that $\gamma_2 > 0$ and $\gamma_3 > 0$.

Theorem 3: Provided agents receive feedback at the event times t_E^i defined in (37), then the controller using the same structure as (3) and (4) ensures that the network system in (1) and (2) modeled by the switching graph $\bar{\mathcal{G}}_\sigma$ achieves bounded leader–follower consensus defined

$$\|\varepsilon(t)\| \leq \sqrt{\frac{N(\gamma_2 + \gamma_3\beta)\,\epsilon^2\lambda_{\max}(P)}{\delta_{m2}\lambda_{\min}(P)}} \quad as \quad t \to \infty. \tag{46}$$

Proof: Consider a common Lyapunov function candidate V defined as in (20) with the P matrix defined in (39). Following the proof of

Theorem 1, the following inequality can be developed

$$\dot{V} \leq -\sum_{i\in\mathcal{V}}\Big[\left(\gamma_1 - \frac{\gamma_3}{\beta}\right)\|\hat{z}_i\|^2 - (\gamma_2 + \gamma_3\beta)\|e_i\|^2 + (\gamma_2 + \gamma_3\beta)\,\epsilon^2\Big] - \delta_{m2}\varepsilon^T\varepsilon + N(\gamma_2 + \gamma_3\beta)\,\epsilon^2 \quad (47)$$

where $\delta_{\min} = \delta_{m1} + \delta_{m2}$ for $\delta_{m1}, \delta_{m2} \in \mathbb{R}_{>0}$, and δ_{m1} is included in γ_1, γ_2, and γ_3 defined in Definition 3. If the trigger condition in (17) is satisfied and β is selected according to (18), then (47) and (20) can be used to show (46). ■

F. Minimal Interevent Interval

Based on Assumption 6, graph switches never cause Zeno behavior (i.e., $\Omega < t_{q+1} - t_q$). Therefore, only the following three intervals smaller than μ are analyzed.

Case 1: Consider any interevent interval $[t_k^j, t_{k+1}^j)$, where $0 < t_{k+1}^j - t_k^j < \Omega$. This interval is proven to be lower bounded by Theorem 4.

Case 2: Consider any interevent interval $[t_q, t_{k+1}^j)$, for $0 < t_{k+1}^j - t_q < \Omega$. By (37), a new neighbor agent $j \in \mathcal{N}_i$ has a mutual communication at t_q, at which time e_j is reset to zero. Therefore, t_q can be considered as the instant t_k^j, which implies *Case 1*and *Case 2* are equivalent.

Case 3: Consider any interevent interval $[t_k^j, t_q)$, for $0 < t_q - t_k^j < \Omega$. Then, the next cycle must be $[t_q, t_{k+1}^j)$ since t_{k+1}^j comes earlier than t_{q+1}. Therefore, proving a positive lower bound of the interval $[t_q, t_{k+1}^j)$ implies no Zeno execution since infinite switches cannot happen in the finite interval. Moreover, finding the lower bound of $[t_q, t_{k+1}^j)$ is equivalent to proving *Case 2*.

Theorem 4: The event-triggered time defined in (17) that the minimum interevent interval $\tau \triangleq t_{k+1}^i - t_k^i \in \mathbb{R}_{>0}$ is lower bounded by

$$\tau \geq \frac{\ln\left(\frac{\epsilon}{c} + 1\right)}{\|A - BK\|}.$$

Proof: The proof is similar to the proof of Theorem 2 and is thus omitted.

V. Discussion

A decentralized event-triggered control scheme for leader–follower network consensus under fixed and time-varying network topologies is developed to reduce communication with neighboring agents while ensuring the stability of the system. The estimate-based decentralized controller along with the decentralized trigger function reduces the number of interagent communications and prevents potential communication channel overload. A Lyapunov-based stability analysis indicates that the network system achieves bounded leader–follower consensus under this event-triggered control scheme in the presence of noise. Moreover, the trigger function is proven to never exhibit Zeno behavior. Based on our previous result in [35], additional research will focus on event-triggered leader–follower consensus for a network of heterogeneous agents.

References

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[2] L. Navaravong, Z. Kan, J. M. Shea, and W. E. Dixon, “Formation reconfiguration for mobile robots with network connectivity constraints,” *IEEE Netw.*, vol. 26, no. 4, pp. 18–24, Jul./Aug. 2012.

[3] Z. Li, W. Ren, X. Liu, and M. Fu, “Distributed containment control of multi-agent systems with general linear dynamics in the presence of multiple leaders,” *Int. J. Robust Nonlinear Control*, vol. 23, pp. 534–547, Mar. 2011.

[4] Z. Kan, A. Dani, J. M. Shea, and W. E. Dixon, “Network connectivity preserving formation stabilization and obstacle avoidance via a decentralized controller,” *IEEE Trans. Autom. Control*, vol. 57, no. 7, pp. 1827–1832, Jul. 2012.

[5] P. Tabuada, “Event-triggered real-time scheduling of stabilizing control tasks,” *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, Sep. 2007.

[6] X. Wang and M. Lemmon, “Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability,” *IEEE Trans. Autom. Control*, vol. 54, no. 3, pp. 452–467, Mar. 2009.

[7] D. V. Dimarogonas and K. H. Johansson, “Event-triggered control for multi-agent systems,” in *Proc. IEEE Conf. Decision Control*, Dec. 2009, pp. 7131–7136.

[8] D. V. Dimarogonas and K. H. Johansson, “Event-triggered cooperative control,” in *Proc. Eur. Control Conf.*, Budapest, Hungary, Aug. 2009, pp. 3015–3020.

[9] L. Zhongxin and C. Zengqiang, “Event-triggered average-consensus for multi-agent systems,” in *Proc. Chin. Control Conf.*, Beijing, China, Jul. 2010, pp. 4506–4511.

[10] Z. Liu, Z. Chen, and Z. Yuan, “Event-triggered average-consensus of multi-agent systems with weighted and direct topology,” *J. Syst. Sci. Complex.*, vol. 25, pp. 845–855, Oct. 2012.

[11] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, “Distributed event-triggered control for multi-agent systems,” *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1291–1297, May 2012.

[12] X. Meng and T. Chen, “Event based agreement protocols for multi-agent networks,” *Automatica*, vol. 49, pp. 2125–2132, Jul. 2013.

[13] G. S. Seybotha, D. V. Dimarogonas, and K. H. Johansson, “Event-based broadcasting for multi-agent average consensus,” *Automatica*, vol. 49, pp. 245–252, Jan. 2013.

[14] G. Shi and K. H. Johansson, “Multi-agent robust consensus–Part II: Application to distributed event-triggered coordination,” in *Proc. IEEE Conf. Decision Control*, Orlando, FL, USA, Dec. 2011, pp. 5738–5743.

[15] H. Li, X. Liao, G. Chen, D. Hill, Z. Dong, and T. Huang, “Event-triggered asynchronous intermittent communication strategy for synchronization in complex networks,” *Neural Netw.*, vol. 66, pp. 1–10, 2015.

[16] T.-H. Cheng, Z. Kan, J. Klotz, J. Shea, and W. E. Dixon, “Decentralized event-triggered control of networked systems-Part 1: Leader-follower consensus under switching,” in *Proc. Am. Control Conf.*, 2015, pp. 5438–5443.

[17] T.-H. Cheng, Z. Kan, J. Klotz, J. Shea, and W. E. Dixon, “Decentralized event-triggered control of networked systems-Part 2: Containment control,” in *Proc. Am. Control Conf.*, 2015, pp. 5444–5448.

[18] A. Girard, “Dynamic triggering mechanisms for event-triggered control,” *IEEE Trans Autom. Contol*, vol. 60, no. 7, pp. 1992–1997, Jul. 2015.

[19] S. Al-Areqi, D. Gorges, and S. Liu, “Event-based control and scheduling codesign: Stochatic and robust approaches,” *IEEE Trans. Autom. Control*, vol. 60, no. 5, pp. 1291–1303, May 2015.

[20] E. Garcia, Y. Cao, H. Yu, P. J. Antsaklis, and D. Casbeer, “Decentralized event-triggered cooperative control with limited communication,” *Int. J. Control*, vol. 86, no. 9, pp. 1479–1488, 2013.

[21] E. Garcia, Y. Cao, and D. W. Casbeer, “Decentralized event-triggered consensus with general linear dynamics,” *Automatica*, vol. 50, no. 10, pp. 2633–2640, 2014.

[22] E. Garcia, Y. Cao, and D. W. Casbeer, “Event-triggered cooperative control with general linear dynamics and communication delays,” in *Proc. IEEE Conf. Decision Control*, Dec. 2014, pp. 2914–2919.

[23] D. Xie, D. Yuan, J. Lu, and Y. Zhang, “Consensus control of second-order leader-follower multi-agent systems with event-triggered strategy,” *Trans. Inst. Meas. Control*, vol. 35, no. 4, pp. 426–436, Jun. 2013.

[24] A. Adaldo et al., “Event-triggered pinning control of switching networks,” *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 2, pp. 204–213, Jun. 2015.

[25] J. Qin and C. Yu, “Exponential consensus of general linear multi-agent systems under directed dynamic topology,” *Automatica*, vol. 50, no. 9, pp. 2327–2333, 2014.

[26] T.-H. Cheng, Z. Kan, J. A. Rosenfeld, and W. E. Dixon, “Decentralized formation control with connectivity maintenance and collision avoidance under limited and intermittent sensing,” in *Proc. Am. Control Conf.*, 2014, pp. 3201–3206.

[27] H. Li, X. Liao, T. Huang, and W. Zhu, “Second-order global consensus in multiagent networks with random directional link failure,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 3, pp. 565–575, Feb. 2015.

[28] W. Ni and D. Cheng, "Leader-following consensus of multi-agent systems under fixed and switching topologies," *Syst. Control Lett.*, vol. 59, pp. 209–217, 2010.
[29] J. Qin, C. Yu, and H. Gao, "Collective behavior for group of generic linear agents interacting under arbitrary network topology," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 3, pp. 288–297, Sep. 2015.
[30] H. Li, X. Liao, T. Huang, and W. Zhu, "Event-triggering sampling based leader-following consensus in second-order multi-agent systems," *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1998–2003, Jul. 2015.
[31] P. Barooah and J. Hespanha, "Graph effective resistance and distributed control: Spectral properties and applications," in *Proc. IEEE Conf. Decision Control*, 2006, pp. 3479–3485.
[32] M. Franceschelli, A. Gasparri, A. Giua, and C. Seatzu, "Decentralized laplacian eigenvalues estimation for networked multi-agent systems," in *Proc. IEEE Conf. Decision Control Chin. Control Conf.*, Shanghai, P. R. China, Dec. 2009, pp. 2717–2722.
[33] R. Araguesa, G. Shib, D. V. Dimarogonasc, C. Sagüésd, K. H. Johanssonc, and Y. Mezouara, "Distributed algebraic connectivity estimation for undirected graphs with upper and lower bounds," *Automatica*, vol. 50, no. 12, pp. 3253–3259, Dec. 2014.
[34] P. D. Lorenzo and S. Barbarossa, "Distributed estimation and control of algebraic connectivity over random graphs," *IEEE Trans. Signal Process.*, vol. 62, no. 21, pp. 5612–5628, Nov. 2014.
[35] J. R. Klotz, "Decentralized control of networks of uncertain dynamical systems," Ph.D. dissertation, Dept. Mech. Aerosp. Eng., Univ. Florida, Gainesville, FL, USA, 2015.