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RESEARCH ARTICLE

# Controller Synthesis for Multi-Agent Systems With Intermittent Communication and Metric Temporal Logic Specifications

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**ABSTRACT** This paper investigates the controller synthesis problem for a multi-agent system (MAS) with intermittent communication. We adopt a *relay-explorer* scheme, where a mobile *relay agent* with absolute position sensors switches among a set of *explorers* with relative position sensors to provide intermittent state information. We model the MAS as a switched system where the explorers' dynamics can be either fully-actuated or under-actuated. The objective of the explorers is to reach *approximate consensus* to a predetermined goal region. To guarantee the uniform boundedness of the state estimation errors and the approximate consensus of the explorers, we derive *maximum dwell-time conditions* to constrain the length of time each explorer goes without state feedback (from the relay agent). Furthermore, the relay agent needs to satisfy practical constraints such as charging its battery and staying in specific regions of interest. Both the maximum dwell-time conditions and these practical constraints can be expressed by *metric temporal logic* (MTL) specifications. We iteratively compute the optimal control inputs for the relay agent to satisfy the MTL specifications, while guaranteeing the uniform boundedness of the state estimation errors and approximate consensus of the explorers. We implement the proposed method on a case study with the CoppeliaSim robot simulator.

**INDEX TERMS** Metric temporal logic, multi-agent systems, intermittent communication, switched systems.

## I. INTRODUCTION

Traditionally, coordination strategies for multi-agent systems (MASs) have been designed under the assumption that state feedback is continuously available and each agent can continuously communicate with its neighbors over a network. This assumption is often impractical, especially in mobile robot applications where shadowing and fading in the wireless communication can cause unreliability, and each agent has limited energy resources [1]. Due to these constraints, there is a strong interest in developing MAS coordination methods that rely on intermittent information over a communication network. In [2], [3], [4], [5], [6], and [7], the authors develop *event-triggered* and *self-triggered* controllers to only utilize sampled data from networked agents when triggered by conditions that ensure desired stability and performance properties. However, these results usually require a network, represented by a connected graph, to enable agent

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coordination. In [8], the authors develop a framework that enables a MAS to achieve position consensus at a known and common location without needing a global communication network, i.e., there is no communication graph coupling all agents. In addition, the framework does not require all agents to measure their global position or inter-agent displacements to achieve position consensus. A relay-explorer strategy is adopted, where the MAS is divided into $N$ *explorers* and a single *relay agent*. Each explorer uses dead-reckoning-based navigation and intermittent feedback to maneuver towards the desired rendezvous point, while the relay agent visits each explorer to provide said intermittent feedback. By introducing a relay agent, the explorers are able to navigate towards the desired location without performing additional maneuvers to obtain position information, such as moving towards known feedback zones. This relay-explorer strategy provides an alternative solution to the position consensus or rendezvous problem, relative to graph-based methods like [3], [4], and [5], that eliminates the need of a global communication topology and for all agents to be equipped with global position or inter-agent position sensors.

In this article, we develop a distributed and graphless control strategy that solves the position consensus problem and improves upon our previous work in [8] and [9]. The result leverages dead-reckoning-based navigation, intermittent state feedback, a Lyapunov-based switched systems analysis, and Metric Temporal Logic (MTL) to achieve *approximate consensus* relative to a known and common location. As an illustrative example, Fig. 1 shows three explorers working towards reaching the green goal region, while one relay agent provides intermittent state information to each explorer. To facilitate navigation via dead reckoning and the provision of intermittent state feedback, we develop model-based estimators that generate position estimates of the explorers. In addition, for each explorer, we derive a *maximum dwell-time condition* that constrains the length of time it can go without state feedback to ensure a desired degree of accuracy for the position estimate. Specifically, satisfaction of the maximum dwell-time condition for each explorer guarantees the uniform boundedness of its state estimation error by a user-defined bound.

The maximum dwell-time conditions can be encoded using MTL specifications, as in [10]. Such specifications have also been used in robotic applications for time-related constraints [11]. Since the relay agent is typically more energy-consuming than the explorers, due to high-quality communication hardware and superior kinematic capability, the relay agent is likely required to satisfy additional MTL specifications, such as charging its battery and staying within specific regions of the workspace. In the example shown in Fig. 1, the relay agent needs to satisfy an MTL specification "*reach the charging station* $G_1$ *or* $G_2$ *every 6 time units and always stay in the purple region* $D$". Under the proposed control design, the explorers achieve approximate consensus relative to a desired rendezvous location provided the maximum dwell-time conditions are satisfied for all time,

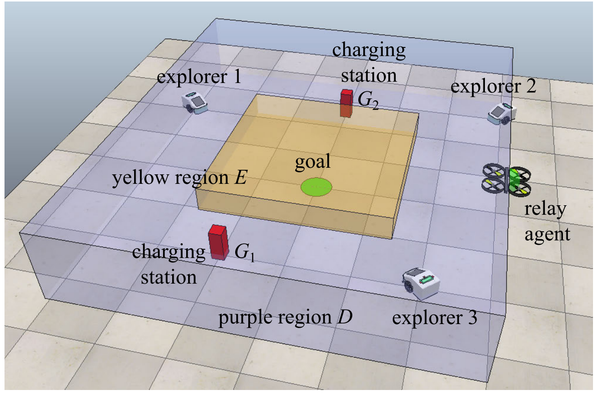


**FIGURE 1. Illustrative example of an MAS with a relay agent (quadrotor) and three explorers (mobile robots). The two red structures ($G_1$ and $G_2$) represent charging stations that the relay agent can utilize. The purple region $D$ describes a space in which the relay agent must always reside, and the yellow region $E$ depicts a restricted space from which the relay agent must intermittently exit. The green goal region defines a feedback zone, where, once within the goal region, an explorer can obtain state information about itself through, for example, wireless communication with a positioning system.**

which ensure the desired uniform boundedness of the position estimation errors needed to yield sufficiently accurate dead-reckoning-based navigation. Then, we synthesize the relay agent's controller to satisfy the MTL specifications that encode the maximum dwell-time conditions and the additional practical constraints. There is a rich literature on controller synthesis subject to temporal logic specifications [12], [13], [14], [15], [17], [18]. For linear or switched linear systems, the controller synthesis problem can be converted into a mixed-integer linear programming (MILP) problem [14], [15]. However, rather than solve a single MILP problem, we solve a sequence of MILP problems (iteratively) due to the jumps exhibited by the position estimates of the explorers, due to their resetting with the intermittent global position feedback provided by the relay agent.

This paper provides additional insights and generalizes our previous work in [19]. (a) The proposed approach in [19] only applies to fully-actuated or over-actuated dynamics for the explorers, while we extend the approach to under-actuated dynamics (e.g., unicycle dynamics) for the explorers in this paper. (b) We used both maximum and minimum dwell-time conditions to achieve stability and approximate consensus for the explorers in [19], while in this paper we only rely on the maximum dwell-time conditions, i.e., the minimum dwell-time conditions are not necessary to enable the result. (c) This paper provides additional evidence of the approach through CoppeliaSim robot simulators with multiple MTL specifications in the case studies. We implemented the developed method in a simulation case study with three mobile robots as explorers and one quadrotor as the relay agent. The results in three different scenarios show that the synthesized controller can lead to satisfaction of the MTL specifications, while ensuring the uniform boundedness of the state estimation errors and achieving the approximate consensus objective.

## II. PROBLEM FORMULATION

### A. AGENT DYNAMICS

Consider a multi-agent system (MAS) consisting of $Q \in \mathbb{Z}_{>0}$ explorers indexed by $F \triangleq \{1, 2, \ldots, Q\}$ and a relay agent indexed by 0. Let $\mathbb{T} \triangleq \mathbb{R}_{\geq 0}$ represent the continuous-time set and $\mathbb{T}_d \triangleq \{t[0], t[1], \ldots\}$ represent the discrete-time set, where $t[j] = jT_s$ is the time instant at time index $j \in \mathbb{I} = \{0, 1, \ldots\}$ and $T_s \in \mathbb{R}_{>0}$ is the sampling period. The state, control input, and position of explorer $i \in F$ are denoted by $x_i \in \mathbb{R}^m$, $u_i \in \mathbb{R}^n$, and $y_i \in \mathbb{R}^z$, respectively. The state, control input, and position of the relay agent are denoted by $x_0 \in \mathbb{R}^l$, $u_0 \in \mathbb{R}^n$, and $y_0 \in \mathbb{R}^z$, respectively. The known continuous linear time-invariant (LTI) motion model of explorer $i$ and discrete LTI motion model of the relay agent are[1]

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + d_i, \quad x_0^{j+1} = A_0 x_0^j + B_0 u_0^j, \\ y_i &= Cx_i, \quad y_0^j = C_0 x_0^j, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{m\times m}$, $B \in \mathbb{R}^{m\times n}$, $C \in \mathbb{R}^{z\times m}$, $A_0 \in \mathbb{R}^{l\times l}$, $B_0 \in \mathbb{R}^{l\times n}$, $C_0 \in \mathbb{R}^{z\times l}$, and $x_0^j \triangleq x_0(t[j])$ denotes the value of $x_0$ at time $t[j]$. Similarly, we have $u_0^j \triangleq u_0(t[j])$ and $y_0^j \triangleq y_0(t[j])$. Furthermore, for each $i \in F$, let $d_i\colon \mathbb{T} \to \mathbb{R}^m$ be a bounded and locally Lipschitz function representing an exogenous disturbance. Since $d_i$ is bounded, there exists a $\overline{d}_i \in \mathbb{R}_{>0}$ such that $\|d_i(t)\| \leq \overline{d}_i$ for all $t \in \mathbb{T}$[2]. We assume the pair $(A, B)$ is controllable.

### B. NAVIGATION AND COMMUNICATION

Each explorer is equipped with a relative position sensor and hardware to enable communication with the relay agent and a home base within the goal region. Since the explorers lack absolute position sensors, they are unable to locate themselves within the global coordinate system. Nevertheless, the explorers can use their relative position sensors to self-locate relative to their initially known locations, which are expressed in the global coordinate frame. However, relative position sensors, like encoders and inertial measurement units (IMUs), can produce unreliable position information since wheels of mobile robots may slip and IMUs may generate noisy data. Hence, the $d_i$ term in (1) models the inaccurate position measurements from the relative position sensor of explorer $i$ as well as any external influences from the environment. Navigating with relative position sensors results in dead-reckoning, which becomes increasingly inaccurate with time if not corrected. On the other hand, the relay agent is equipped with an absolute position sensor and hardware to enable communication with each explorer. Note that, for each $i \in F$, communication is distance limited where explorer $i$ and the relay agent can communicate provided their Euclidean distance is below a fixed communication radius. Unlike a relative position sensor, an absolute position sensor allows the agent to locate itself within the global coordinate frame.

Let $x_g \in \mathbb{R}^m$ be a fixed user-defined state. A goal region (e.g., see Fig. 1) centered at the position $Cx_g \in \mathbb{R}^z$ with radius $R_f \in \mathbb{R}_{>0}$ defines a feedback zone. If $\|y_i - Cx_g\| \leq R_f$, then explorer $i$ can obtain state information about itself, i.e., $x_i$, via communication with a home base. Let $R \in \mathbb{R}_{>0}$ denote the communication radius of the relay agent and each explorer. Within this work, the relay agent has full knowledge of its own state $x_0$ for all time and the initial state $x_i(0)$ for each explorer $i \in F$. The relay agent provides state information to explorer $i$ (i.e., *services* explorer $i$) if and only if $\|y_i - y_0\| \leq R$ and the communication channel of explorer $i$ is on. We define the *communication switching signal* $\zeta_i$ for explorer $i$ as $\zeta_i = 1$ if the communication channel is *on* for explorer $i$, and $\zeta_i = 0$ if the communication channel is *off* for explorer $i$. In general, the relay agent is unable to continuously service all explorers simultaneously as would be the case when explorers are dispersed over an expanse. In addition, we do not model the communication topology with a graph since we analyze each subsystem individually rather than analyze the ensemble dynamics. Communication between explorers does not occur, and communication between the relay agent and each explorer takes place at isolated times. Hence, let $\{t_s^i\}_{s=0}^{\infty} \subset \mathbb{T}$ be an increasing sequence of servicing times for explorer $i$, where $t_s^i$ denotes the $s^{\text{th}}$ servicing instant for explorer $i$.

### C. APPROXIMATE CONSENSUS

Given a goal region centered at $Cx_g$ with radius $R_f$, one objective is to design distributed controllers for all explorers that achieve approximate consensus within the goal region. This objective is decomposed into two tasks, where it is the task of the explorers to dead-reckon towards $Cx_g$, and it is the task of the relay agent to intermittently service each explorer. Since a single relay agent must intermittently service $Q$ explorers, the MAS can be modeled as a switched system, where the relay agent has $2^Q$ modes of operation, i.e., the relay agent can service no, a single, multiple[3], or all explorers at an instance depending on the configuration of the explorers. Let $\sigma : \mathbb{T} \to 2^F$ be a piecewise constant switching signal that determines the mode of operation of the relay agent. The switching signal $\sigma$ also determines the servicing times for all explorers, i.e., $\{t_s^i\}_{s=0}^{\infty}$. To quantify the consensus objective, let the tracking error of explorer $i$ be

$$e_i \triangleq x_g - x_i \in \mathbb{R}^m. \tag{2}$$

To facilitate the analysis, let the state estimation error of explorer $i$ be

$$e_{1,i} \triangleq \hat{x}_i - x_i \in \mathbb{R}^m, \tag{3}$$

[1] We consider discrete-time system dynamics for the relay agent to avoid discretization error induced by conversion from the continuous-time domain to the discrete-time domain when synthesizing control inputs for the relay agent through optimization with MTL specifications (see Section IV). We note that using discrete-time system dynamics for the relay agent and continuous-time system dynamics for the explorer agents results in a hybrid multi-agent system [20], [21].

[2] $\|\cdot\|$ denotes the 2-norm.

[3] Whenever there is more than one explorer within the communication region of the relay agent, those explorers are serviced by the relay agent.

where $\hat{x}_i \in \mathbb{R}^m$ denotes the state estimate of explorer $i$. For each $i \in F$, the state estimate of explorer $i$ is synchronized between explorer $i$ and the relay agent, which is generated by an estimator presented in the following section. Let the estimated tracking error of explorer $i$ be

$$e_{2,i} \triangleq x_g - \hat{x}_i \in \mathbb{R}^m. \tag{4}$$

Using (3) and (4), (2) can be alternatively expressed as

$$e_i = e_{2,i} + e_{1,i}. \tag{5}$$

Given the tracking error in (2), approximate consensus is achieved within the goal region whenever $\|e_i\| \leq R_f / S_{\max}(C)$ for all $i \in F$, where $S_{\max}(C) \in \mathbb{R}_{>0}$ is the maximum singular value of $C$.

### *D. STATE ESTIMATOR AND CONTROLLER DEVELOPMENT*

The state estimate of explorer $i \in F$ is generated by the following model-based estimator

$$\dot{\hat{x}}_i = -Ae_{2,i} + Bu_i, \ t \in [t_s^i, t_{s+1}^i),$$
$$\hat{y}_i = C\hat{x}_i, \tag{6}$$

where, for each servicing instant $t_s^i$, the state estimate of explorer $i$ is reset according to $\hat{x}_i(t_s^i) = x_i(t_s^i)$. Moreover, the state estimate $\hat{x}_i$ is initialized as $\hat{x}_i(0) = x_i(0)$ for all $i \in F$. The controller of explorer $i$ is

$$u_i \triangleq B^{\mathrm{T}} P e_{2,i}, \tag{7}$$

where $P \in \mathbb{R}^{m \times m}$ is the positive definite solution to the algebraic Riccati equation (ARE)

$$A^{\mathrm{T}}P + PA - 2PBB^{\mathrm{T}}P + kI_m = 0_{m\times m}. \tag{8}$$

In (8), $k > 0$ is a user-defined parameter, $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix, and $0_{m\times m} \in \mathbb{R}^{m\times m}$ is the zero matrix. Since the pair $(A, B)$ is controllable by assumption, there exists a unique positive definite solution $P$ to the ARE in (8). Substituting (4) and (7) into (6) yields the closed-loop estimator

$$\dot{\hat{x}}_i = -(A - BB^{\mathrm{T}}P)(x_g - \hat{x}_i), \ t \in [t_s^i, t_{s+1}^i). \tag{9}$$

The estimator in (9) enables the computation of $\hat{x}_i$ over each period of no communication, i.e., $[t_s^i, t_{s+1}^i)$, where $\hat{x}_i$ is employed by explorer $i$ and the relay agent in two distinct ways. Explorer $i$ utilizes $\hat{x}_i$ to compute its control input with (4) and (7), while the relay agent uses $\hat{x}_i$ to locate explorer $i$ with (9). In practice, explorer $i$ and the relay agent will have their own copy of the estimator in (9), where synchronization is achieved by numerically integrating (9) while using the same initial condition. Therefore, both agents can determine $\hat{x}_i$ without the need for continuous communication.

We now derive the relevant closed-loop systems. Substituting (1) and (6) into the time derivative of (3) yields

$$\dot{e}_{1,i} = Ae_{1,i} - Ax_g - d_i, \ t \in [t_s^i, t_{s+1}^i), \tag{10}$$

where, for each servicing instant $t_s^i$, $e_{1,i}(t_s^i) = 0_m$ and $0_m \in \mathbb{R}^m$ is the zero vector. Substituting (6) and (7) into the time derivative of (4) yields

$$\dot{e}_{2,i} = (A - BB^{\mathrm{T}}P)e_{2,i}, \ t \in [t_s^i, t_{s+1}^i), \tag{11}$$

where, for each servicing instant $t_s^i$, $e_{2,i}(t_s^i) = x_g - x_i(t_s^i)$. Substituting (1), (5), and (7) into the time derivative of (2) yields

$$\dot{e}_i = (A - BB^{\mathrm{T}}P)e_i + BB^{\mathrm{T}}Pe_{1,i} - Ax_g - d_i. \tag{12}$$

Let $\xi_i \triangleq [e_i^{\mathrm{T}}, e_{1,i}^{\mathrm{T}}, \nu]^{\mathrm{T}} \in \mathbb{R}^{2m+1}$ be an auxiliary state for explorer $i$, where $\nu \in \mathbb{T}$ is a timer variable that evolves according to $\dot{\nu} = 1$ with $\nu(0) = 0$. Using $\dot{\nu} = 1$, (10), and (12), the closed-loop dynamics of $\xi_i$ during flows, i.e., over each $[t_s^i, t_{s+1}^i)$, is $\dot{\xi}_i = h_i(\xi_i)$, where

$$h_i(\xi_i) \triangleq \begin{bmatrix} (A - BB^{\mathrm{T}}P)e_i + BB^{\mathrm{T}}Pe_{1,i} - Ax_g - d_i(\nu) \\ Ae_{1,i} - Ax_g - d_i(\nu) \\ 1 \end{bmatrix}. \tag{13}$$

Whenever $t = t_s^i$, $s \in \mathbb{Z}_{\geq 0}$, the auxiliary state $\xi_i$ jumps such that $\xi_i^+ = [e_i^{\mathrm{T}}, 0_m^{\mathrm{T}}, \nu]^{\mathrm{T}}$, where $\xi_i^+$ denotes the value of $\xi_i$ after a jump. Hence, for each jump time $t_s^i$, the value of $e_i$ after the jump is set equal to the value of $e_i$ before the jump, the value of $e_{1,i}$ after the jump is set equal to $0_m$, and the value of $\nu$ after the jump is set equal to the value of $\nu$ before the jump.

*Remark 1: The control strategy allows the explorers to dead-reckon to a common goal location while only intermittently communicating with a relay agent to obtain state feedback. In the future, one could leverage inter-explorer communication and distributed state estimation to accommodate more general communication topologies and develop more accurate state estimates for each explorer.*

### *E. METRIC TEMPORAL LOGIC (MTL)*

To achieve the uniform boundedness of the state estimation errors and approximate consensus of the explorers while satisfying the practical constraints of the relay agent, the requirements of the MAS can be specified in MTL specifications (see details in Section IV). In this subsection, we briefly review MTL interpreted over discrete-time trajectories [22]. The domain of the position $y$ of a certain agent is denoted by $\mathcal{Y} \subset \mathbb{R}^z$. The Boolean domain is $\mathbb{B} = \{\text{True}, \text{False}\}$, and the time index set is $\mathbb{I} = \{0, 1, \dots\}$. With slight abuse of notation, we use $y$ to denote the discrete-time trajectory as a function from $\mathbb{I}$ to $\mathcal{Y}$. A set $AP$ is a set of atomic propositions, each of which maps $\mathcal{Y}$ to $\mathbb{B}$. The syntax of MTL is defined recursively as

$$\phi := \top \mid \pi \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \phi_1 \mathcal{U}_{\mathcal{I}} \phi_2$$

where $\top$ stands for the Boolean constant True, $\pi \in AP$ is an atomic proposition, $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction) are standard Boolean connectives, $\mathcal{U}$ is a temporal operator representing "until" and $\mathcal{I}$ is a time interval of the form $\mathcal{I} = [j_1, j_2]$ ($j_1 \leq j_2, j_1, j_2 \in \mathbb{I}$). We can also derive two useful temporal operators from "until" ($\mathcal{U}$), which are "eventually" $\Diamond_{\mathcal{I}}\phi \triangleq \top\mathcal{U}_{\mathcal{I}}\phi$ and "always" $\Box_{\mathcal{I}}\phi \triangleq \neg\Diamond_{\mathcal{I}}\neg\phi$. We define the set of states that satisfy the atomic proposition $\pi$ as $\mathcal{O}(\pi) \subset \mathcal{Y}$. The subscript $\mathcal{I}$ in the temporal operators $\Diamond_{\mathcal{I}}$, $\Box_{\mathcal{I}}$, and $\mathcal{U}_{\mathcal{I}}$ refers to the *bounded* temporal operators. When the subscript $\mathcal{I}$ is not used in the temporal operators, this implies the use of unbounded temporal operators, where the time interval $\mathcal{I} = [0, \infty)$ by default.

Next, we introduce the Boolean semantics of MTL for trajectories of finite length in the strong and the weak view, which are modified from the literature of temporal logic model checking and monitoring [23], [24], [25]. We use $t[j]$ to denote the time instant at time index $j \in \mathbb{I}$ and $y^j \triangleq y(t[j])$ to denote the value of $y$ at time $t[j]$. In the following, $(y^{0:H}, j) \models_S \phi$ (resp. $(y^{0:H}, j) \models_W \phi$) means the trajectory $y^{0:H} \triangleq y^0 \dots y^H$ ($H \in \mathbb{Z}_{\geq 0}$) strongly (resp. weakly) satisfies $\phi$ at time index $j$, $(y^{0:H}, j) \not\models_S \phi$ (resp. $(y^{0:H}, j) \not\models_W \phi$) means $y^{0:H}$ fails to strongly (resp. weakly) satisfy $\phi$ at time index $j$.

*Definition 1: The Boolean semantics of MTL for trajectories of finite length in the strong view is defined recursively as follows [16]:*

$$
\begin{aligned}
&(y^{0:H}, j) \models_S \pi \text{ iff } j \leq H \text{ and } y^j \in \mathcal{O}(\pi),\\
&(y^{0:H}, j) \models_S \neg\phi \text{ iff } (y^{0:H}, j) \not\models_W \phi,\\
&(y^{0:H}, j) \models_S \phi_1 \wedge \phi_2 \text{ iff } (y^{0:H}, j) \models_S \phi_1\\
&\quad\text{and } (y^{0:H}, j) \models_S \phi_2,\\
&(y^{0:H}, j) \models_S \phi_1 \vee \phi_2 \text{ iff } (y^{0:H}, j) \models_S \phi_1\\
&\quad\text{or } (y^{0:H}, j) \models_S \phi_2,\\
&(y^{0:H}, j) \models_S \phi_1\mathcal{U}_{\mathcal{I}}\phi_2 \text{ iff } \exists j' \in j+\mathcal{I}, \text{s.t.} (y^{0:H}, j') \models_S \phi_2,\\
&(y^{0:H}, j'') \models_S \phi_1, \forall j'' \in [j, j'].
\end{aligned}
$$

*Definition 2: The Boolean semantics of MTL for trajectories of finite length in the weak view is defined recursively as follows [16]:*

$$
\begin{aligned}
&(y^{0:H}, j) \models_W \pi \text{ iff either of the following holds}:\\
&\quad 1)\, j \leq H \text{ and } y^j \in \mathcal{O}(\pi);\ 2)\, j > H,\\
&(y^{0:H}, j) \models_W \neg\phi \text{ iff } (y^{0:H}, j) \not\models_S \phi,\\
&(y^{0:H}, j) \models_W \phi_1 \wedge \phi_2 \text{ iff } (y^{0:H}, j) \models_W \phi_1\\
&\quad\text{and } (y^{0:H}, j) \models_W \phi_2,\\
&(y^{0:H}, j) \models_W \phi_1 \vee \phi_2 \text{ iff } (y^{0:H}, j) \models_W \phi_1\\
&\quad\text{or } (y^{0:H}, j) \models_W \phi_2,\\
&(y^{0:H}, j) \models_W \phi_1\mathcal{U}_{\mathcal{I}}\phi_2 \text{ iff } \exists j' \in j+\mathcal{I}, \text{s.t.} (y^{0:H}, j') \models_W \phi_2,\\
&(y^{0:H}, j'') \models_W \phi_1, \forall j'' \in [j, j'].
\end{aligned}
$$

Intuitively, if a trajectory of finite length can be extended to infinite length, then the strong view indicates that the truth value of the formula on the infinite-length trajectory is already "determined" on the trajectory of finite length, while the weak view indicates that it may not be "determined" yet [25]. As an example, a trajectory $y^{0:3} = y^0y^1y^2y^3$ is not possible to strongly satisfy $\phi = \Box_{[0,5]}\pi$ at time 0, but $y^{0:3}$ is possible to strongly violate $\phi$ at time 0, i.e., $(y^{1:3}, 0) \models_S \neg\phi$ is possible.

For an MTL formula $\phi$, the necessary length $L(\phi)$ is defined recursively as follows [26]:

$$
\begin{aligned}
L(\pi) &:= 0,\ L(\neg\phi) := L(\phi),\\
L(\phi_1 \wedge \phi_2) &:= \max(L(\phi_1), L(\phi_2)),\\
L(\phi_1\mathcal{U}_{[j_1,j_2]}\phi_2) &:= \max(L(\phi_1), L(\phi_2)) + j_2.
\end{aligned}
$$

### F. PROBLEM STATEMENT

We now present the problem formulation for the control of the MAS with intermittent communication and MTL specifications.

*Problem 1: Design the control inputs for the relay agent* $\mathbf{u}_0 = [u_0^0, u_0^1, \cdots]$ *($u_0^j$ denotes the control input at time index $j$) such that the following characteristics are satisfied while minimizing the control effort* $\sum_{j=0}^{\infty}\|u_0^j\|^2$*:*
*Correctness: A given MTL specification $\phi$ is weakly satisfied by the trajectory of the relay agent.*
*Uniform Boundedness: For all $i \in F$ and $t \in \mathbb{T}$, the error $e_{1,i}(t)$ is bounded, i.e., $\|e_{1,i}(t)\| \leq V_T$, where $V_T > 0$ is a user-defined constant.*
*Approximate Consensus: The states of the explorers in $\{x_i\}_{i\in F}$ reach approximate consensus within the goal region centered at $Cx_g$ with radius $R_f$.*

## III. STABILITY ANALYSIS

In this section, we provide conditions that generate a stable switched system and enable approximate consensus for the explorers. Let $V_T \in \mathbb{R}_{>0}$ be a user-defined parameter that quantifies the maximum tolerable state estimation error, i.e., it is desirable to ensure $\|e_{1,i}(t)\| \leq V_T$ for all $t \in \mathbb{T}$ and each $i \in F$. We now derive a maximum dwell-time condition that ensures $\|e_{1,i}(t)\| \leq V_T$ for all $t \in [t_s^i, t_{s+1}^i]$ and each $s \in \mathbb{Z}_{\geq 0}$.

*Theorem 1: Suppose $\{t_s^i\}_{s=0}^{\infty} \subset \mathbb{T}$ is an increasing sequence of servicing times, such that $t_{s+1}^i - t_s^i > 0$ for all $s \in \mathbb{Z}_{\geq 0}$. If $\|e_{1,i}(t_s^i)\| = 0$ and the relay agent services explorer $i$ at time $t_{s+1}^i$ such that the maximum dwell-time condition*

$$t_{s+1}^i - t_s^i \leq \frac{1}{S_{\max}(A)} \ln\left(\frac{V_T S_{\max}(A)}{\kappa_i} + 1\right) \triangleq \Delta t_i \tag{14}$$

*is satisfied, then $\|e_{1,i}(t)\| \leq V_T$ for all $t \in [t_s^i, t_{s+1}^i]$ and each $s \in \mathbb{Z}_{\geq 0}$, where $\kappa_i \triangleq S_{\max}(A)\|x_g\| + \overline{d}_i \in \mathbb{R}_{>0}$.*

*Proof:* Consider the interval $[t_s^i, t_{s+1}^i)$ and the common Lyapunov-like function candidate $V_1 : \mathbb{R}^m \to \mathbb{R}_{\geq 0}$,

$$V_1(e_{1,i}) \triangleq \frac{1}{2}e_{1,i}^{\mathrm{T}}e_{1,i}. \tag{15}$$

Substituting (10) into the time derivative of (15) yields $\dot{V}_1(e_{1,i}) = e_{1,i}^{\mathrm{T}}(Ae_{1,i} - Ax_g - d_i)$, which implies

$$\dot{V}_1(e_{1,i}) \leq S_{\max}(A)\|e_{1,i}\|^2 + \kappa_i\|e_{1,i}\|. \tag{16}$$

Using (15), (16) can be expressed as

$$\dot{V}_1(e_{1,i}) \leq 2S_{\max}(A)V_1(e_{1,i}) + \kappa_i\sqrt{2V_1(e_{1,i})}. \tag{17}$$

Over the flow interval $[t_s^i, t_{s+1}^i)$, both $V_1$ and $\dot{V}_1$ are continuous, and, therefore, integrable[4]. Consequently, integrating (17) over $[t_s^i, t_{s+1}^i]$ yields

$$V_1(e_{1,i}(t)) \leq \left( \frac{\sqrt{2}\kappa_i}{2S_{\max}(A)} \left( \exp(S_{\max}(A)(t - t_s^i)) - 1 \right) \right)^2, \tag{18}$$

where integration of (17) over $[t_s^i, t_{s+1}^i)$ and $[t_s^i, t_{s+1}^i]$ produces the same result. Substituting (15) into (18) yields $\|e_{1,i}(t)\| \leq \Phi_i(t)$ over $[t_s^i, t_{s+1}^i]$, where

$$\Phi_i(t) \triangleq \frac{\kappa_i}{S_{\max}(A)} \left( \exp(S_{\max}(A)(t - t_s^i)) - 1 \right). \tag{19}$$

Moreover, $\Phi_i(t_{s+1}^i) \leq V_T$ by (14) and (19). Since $\Phi_i(t)$ is an increasing function, $\Phi_i(t) \leq \Phi_i(t_{s+1}^i)$ over $[t_s^i, t_{s+1}^i]$. Hence, $\|e_{1,i}(t)\| \leq \Phi_i(t) \leq \Phi_i(t_{s+1}^i) \leq V_T$ over $[t_s^i, t_{s+1}^i]$. $\square$

*Remark 2: For each explorer $i \in F$, if the relay agent services explorer $i$ at time $t_s^i$ while satisfying the maximum dwell-time condition in (14) for all $s \in \mathbb{Z}_{\geq 0}$, then $\|e_{1,i}(t)\| \leq V_T$ for all $t \in \mathbb{T}$, which follows by mathematical induction.*

We now show the tracking error in (2) is globally uniformly ultimately bounded (GUUB). Given a symmetric and real-valued matrix $P$, the maximum and minimum eigenvalues of $P$ are denoted by $\lambda_{\max}(P) \in \mathbb{R}$ and $\lambda_{\min}(P) \in \mathbb{R}$, respectively.

*Theorem 2: Suppose $\{t_s^i\}_{s=0}^{\infty} \subset \mathbb{T}$ is an increasing sequence of servicing times, such that $t_{s+1}^i - t_s^i > 0$ for all $s \in \mathbb{Z}_{\geq 0}$. If the relay agent services explorer $i$ while satisfying the maximum dwell-time condition in (14) for each $s \in \mathbb{Z}_{\geq 0}$ and $e_{1,i}(t_0^i) = 0_m$, then the estimator in (6) and controller in (7) ensure the tracking error in (2) is GUUB for all $t \in \mathbb{T}$ in the sense that*

$$\|e_i(t)\| \leq \frac{\rho_i \mathrm{c}_1}{k} + \sqrt{\mathrm{c}_1}\|e_i(0)\| \exp(-\mathrm{c}_2 t) \triangleq \Psi_i(t), \tag{20}$$

*where $\mathrm{c}_1 \triangleq \lambda_{\max}(P)/\lambda_{\min}(P)$, $\mathrm{c}_2 \triangleq k/(2\lambda_{\max}(P))$, and $\rho_i \triangleq 2\overline{d}_i\lambda_{\max}(P) + 2V_T\lambda_{\max}(PBB^T P) + 2S_{\max}(PA)\|x_g\|$. Moreover, $e_{2,i}(t)$ and $u_i(t)$ are bounded for all $t \in \mathbb{T}$.*

*Proof:* Consider the common Lyapunov-like function $V_2 : \mathbb{R}^{2m+1} \to \mathbb{R}_{\geq 0}$, such that

$$V_2(\xi_i) \triangleq e_i^{\mathrm{T}} P e_i. \tag{21}$$

Recall that $P$ is the symmetric and positive definite solution to the ARE in (8), which exists since the pair $(A, B)$ is controllable. Using the Rayleigh quotient, (21) can be bounded as

$$\lambda_{\min}(P)\|e_i\|^2 \leq V_2(\xi_i) \leq \lambda_{\max}(P)\|e_i\|^2. \tag{22}$$

The change in $V_2(\xi_i)$ is given by $\dot{V}_2(\xi_i) = \langle \nabla V_2(\xi_i), h_i(\xi_i)\rangle$ during flows, where $h_i(\xi_i)$ is the vector field provided in (13). Therefore,

$$\dot{V}_2(\xi_i) = e_i^{\mathrm{T}}(A^{\mathrm{T}}P + PA - 2PBB^{\mathrm{T}}P)e_i + 2e_i^{\mathrm{T}}P(BB^{\mathrm{T}}Pe_{1,i} - Ax_g - d_i). \tag{23}$$

Using the ARE in (8), (23) can be bounded as

$$\dot{V}_2(\xi_i) \leq -k\|e_i\|^2 + 2\lambda_{\max}(PBB^{\mathrm{T}}P)\|e_i\|\|e_{1,i}\| + 2S_{\max}(PA)\|e_i\|\|x_g\| + 2\lambda_{\max}(P)\|e_i\|\|d_i\|. \tag{24}$$

Since the relay agent satisfies the maximum dwell-time condition in (14) for each $s \in \mathbb{Z}_{\geq 0}$, $\|e_{1,i}(t)\| \leq V_T$ for all $t \in \mathbb{T}$ using Theorem 1. Recall that $\|d_i\| \leq \overline{d}_i$. Hence, (24) can be bounded as

$$\dot{V}_2(\xi_i) \leq -k\|e_i\|^2 + \rho_i\|e_i\|, \tag{25}$$

where the auxiliary constant $\rho_i$ is defined in Theorem 2. Using (22), (25) implies

$$\dot{V}_2(\xi_i) \leq -\frac{k}{\lambda_{\max}(P)}V_2(\xi_i) + \rho_i\sqrt{\frac{V_2(\xi_i)}{\lambda_{\min}(P)}}. \tag{26}$$

During jumps, i.e., when $t = t_s^i$ for $s \in \mathbb{Z}_{\geq 0}$, the change in $V_2(\xi_i)$ is computed using $V_2(\xi_i^+) - V_2(\xi_i)$. Because $e_i$ evolves continuously under $h_i(\xi_i)$ in (13), $e_i^+ = e_i$, and $V_2(\xi_i) = e_i^{\mathrm{T}}Pe_i$, it follows that

$$V_2(\xi_i^+) - V_2(\xi_i) = 0. \tag{27}$$

Let $\phi_i$ be a maximal solution to $\dot{\xi}_i = h_i(\xi_i)$ that satisfies the jump condition. Since the vector field $h_i(\xi_i)$ is locally Lipschitz and $\xi_i^+ = [e_i^{\mathrm{T}}, 0_m^{\mathrm{T}}, \nu]^{\mathrm{T}}$ at each jump, the maximal solution exists, is unique, and is discontinuous only at the points $\{t_s^i\}_{s=0}^{\infty}$. In fact, the maximal solution $\phi_i$ is only discontinuous along the $e_{1,i}$–coordinate at the points $\{t_s^i\}_{s=0}^{\infty}$. Writing $\xi_i(t) = \phi_i(t)$, integrating (26) over flow intervals, stitching the solutions of (26) for adjacent flow intervals using (27), and employing (22) yields (20). By (20), $e_i$ is bounded. Since $e_i$ is bounded and $e_{1,i}$ is bounded by inductive use of Theorem 1, (5) implies $e_{2,i}$ is bounded. Hence, $u_i$ is bounded given (7). $\square$

*Remark 3: From (20), we see that*

$$\limsup_{t\to\infty} \|e_i(t)\| \leq \frac{\Lambda_{\max}(P)}{\Lambda_{\min}(P)}\frac{\rho_i}{k} \triangleq \Lambda(\rho_i),$$

*where $\Lambda(\rho_i)$ can be made small by making $\rho_i$ small, i.e., selecting a small $V_T \in \mathbb{R}_{>0}$ and setting the desired state as the origin. A change of coordinate transformation can be used to make the desired state the origin.*

*Remark 4: Observe that $\|Cx_g - y_i\| \leq S_{\max}(C)\|e_i\|$ by (1) and (2). Suppose the radius of the goal region is selected such that $S_{\max}(C)\Lambda(\rho_i) < R_f$ holds. Then, (20) implies that there exists a time $T_i > 0$ where $\Psi_i(T_i) \leq R_f/S_{\max}(C)$ and $\|e_i(t)\| \leq R_f/S_{\max}(C)$ for all $t \geq T_i$. Consequently, $\|Cx_g - y_i(t)\| \leq R_f$ for all $t \geq T_i$, which allows explorer $i$ to receive continuous position feedback from the goal region. Hence, $\hat{x}_i(t) = x_i(t)$ for all $t \geq T_i$, and (3) implies $\|e_{1,i}(t)\| = 0$ for all $t \geq T_i$. Furthermore, (24) implies $\Lambda(\rho_i)$ can be reduced to $\Lambda(\rho_i^*)$ once $t \geq T_i$, where $\rho_i^* \triangleq 2\overline{d}_i\lambda_{\max}(P) + 2S_{\max}(PA)\|x_g\|$.*

[4]Recall that $\hat{x}_i$ is reset at each service time $t_s^i$, where the reset occurs in zero continuous-time. Hence, $e_{1,i}$ evolves continuously over $[t_{s-1}^i, t_s^i)$, resets to $0_m$ at time $t_s^i$, evolves continuously over $[t_s^i, t_{s+1}^i)$, etc.

## IV. CONTROLLER SYNTHESIS WITH INTERMITTENT COMMUNICATION AND MTL SPECIFICATIONS

In this section, we provide the framework and algorithms for controller synthesis of the relay agent to satisfy the maximum dwell-time conditions and the practical constraints. The controller synthesis for the relay agent is conducted iteratively as the state estimates for the explorers are reset to the true state values whenever they are serviced by the relay agent, and thus the control inputs need to be recomputed with the reset values.

The maximum dwell-time outlined by the right-hand side of (14) for explorer $i$ ($i \in F$) is in the interval $[n_iT_s, (n_i+1)T_s)$ for some non-negative integer $n_i$. We use the following MTL specifications for encoding the maximum dwell-time condition. In the following MTL specification, $\eta$ is a user-defined parameter that determines an upper bound on the distance between the estimated position of explorer agent $i$ ($\hat{y}_i$) and the position of the relay agent ($y_0$). We also require that $n_iT_s \leq \Delta t_i$ so that the maximum-dwell time condition as stated in (14) is satisfied.

$$\phi_{\mathrm{m}} = \bigwedge_{1\leq i\leq Q} \big(\Box\Diamond_{[0,n_i]}\|y_0 - \hat{y}_i\| \leq \eta\big),$$

where $\phi_{\mathrm{m}}$ means "for any explorer $i$, the relay agent needs to be within $\eta$ distance from the estimated position of explorer $i$ at least once in any $n_iT_s$ time periods".

The relay agent also needs to satisfy an MTL specification $\phi_{\mathrm{p}}$ for the practical constraints. One example of $\phi_{\mathrm{p}}$ is as follows.

$$\phi_{\mathrm{p}} = \Box\Diamond_{[0,c]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge \Box(y_0 \in D),$$

which means "the relay agent needs to reach the charging station $G_1$ or $G_2$ at least once in any $cT_s$ time periods, and it should always remain in the region $D$" ($c$ is a positive integer).

Combining $\phi_{\mathrm{m}}$ and $\phi_{\mathrm{p}}$, the MTL specification for the relay agent is $\phi = \phi_{\mathrm{m}} \wedge \phi_{\mathrm{p}}$. We use $[\phi]_j^\ell$ to denote the formula modified from the MTL formula $\phi$ when $\phi$ is evaluated at time index $j$ and the current time index is $\ell$. $[\phi]_j^\ell$ can be calculated recursively as follows (we use $\pi_j$ to denote the atomic proposition $\pi$ evaluated at time index $j$):

$$\begin{aligned}
[\pi]_j^\ell &:= \begin{cases} \pi_j, & \text{if } j > \ell \\ \top, & \text{if } j \leq \ell \text{ and } y^j \in \mathcal{O}(\pi) \\ \bot, & \text{if } j \leq \ell \text{ and } y^j \notin \mathcal{O}(\pi) \end{cases} \\
[\neg\phi]_j^\ell &:= \neg[\phi]_j^\ell \\
[\phi_1 \wedge \phi_2]_j^\ell &:= [\phi_1]_j^\ell \wedge [\phi_2]_j^\ell \\
[\phi_1 \mathcal{U}_{\mathcal{I}} \phi_2]_j^\ell &:= \bigvee_{j'\in(j+\mathcal{I})} \Big([\phi_2]_{j'}^\ell \wedge \bigwedge_{j\leq j''<j'} [\phi_1]_{j''}^\ell\Big), \qquad (28)
\end{aligned}$$

where $\bot$ stands for the Boolean constant False. If the MTL formula $\phi$ is evaluated at the initial time index (which is the usual case when the task starts at the initial time), then the modified formula is $[\phi]_0^\ell$.

**Algorithm 1** Controller Synthesis of MASs With Intermittent Communication and MTL Specifications

1: **Inputs:** $x_0^0, x_i^0, \phi, x_g, R_f, V_T, \eta$
2: $\ell \leftarrow 0, \ell^* \leftarrow 0$
3: **Solve** MILP-sol to obtain optimal inputs $u_0^{*q}$ $(q = 0, 1, \ldots, N-1)$
4: **while** $\|Cx_g - y_i(t[\ell])\| > R_f$ for some $i \in F$ **do**
5: $\quad\mathcal{W} = \{i \mid \|y_0 - \hat{y}_i(t[\ell])\| \leq \eta\}$
6: $\quad$**if** $\mathcal{W} \neq \emptyset$ or $\ell \geq \ell^* + N$ **then**
7: $\quad\quad\forall i \in \mathcal{W}$, update $\hat{x}_i^\ell$ in constraint (29) and change constraint (29) as follows:
$\quad\quad\quad\hat{x}_i^{j+1} \leftarrow \bar{Z}(x_g - \hat{x}_i^j),\ \forall i \in F,$
$\quad\quad\quad\forall j = \ell, \ell+1, \ldots, \ell+N-1,$
$\quad\quad\quad\hat{x}_i^\ell \leftarrow x_i^\ell,\ \forall i \in \mathcal{W}$
8: $\quad\quad$**Re-solve** MILP-sol to obtain optimal inputs $u^{*\ell+q}$ $(q = 0, 1, \ldots, N-1)$
9: $\quad\quad$**if** MILP-sol is infeasible **then**
$\quad\quad\quad$**Return** *Infeasible*
10: $\quad\quad$**end if**
11: $\quad\quad u_0^{*\ell+q} \leftarrow u^{*\ell+q}$ $(q = 0, 1, \ldots, N-1)$, $\ell^* \leftarrow \ell$
12: $\quad$**end if**
13: $\quad\ell \leftarrow \ell + 1$
14: **end while**
15: **Return** $\mathbf{u}_0^* = [u_0^{*0}, u_0^{*1}, \ldots]$

Algorithm 1 shows the controller synthesis approach with intermittent communication and MTL specifications. The controller synthesis problem can be formulated as a sequence of mixed integer linear programming (MILP) problems, denoted as MILP-sol in Line 3, and expressed as follows:

$$\begin{aligned}
&\underset{\mathbf{u}_0^{\ell:\ell+N-1}}{\arg\min} \sum_{j=\ell}^{\ell+N-1} \|u_0^j\|^2 \\
&\text{subject to: } x_0^{j+1} = A_0x_0^j + B_0u_0^j,\ y_0^j = C_0x_0^j, \\
&\qquad \forall j = \ell, \ldots, \ell+N-1; \\
&\qquad \hat{x}_i^{j+1} = \bar{Z}(x_g - \hat{x}_i^j),\ \hat{y}_i^j = \bar{C}\hat{x}_i^j, \\
&\qquad \forall i \in F, \forall j = \ell, \ldots, \ell+N-1; \qquad (29) \\
&\qquad u_{0,\min} \leq u_0^j \leq u_{0,\max}, \\
&\qquad \forall j = \ell, \ldots, \ell+N; \\
&\qquad (\tilde{y}^{0:\ell+N-1}, 0) \models_{\mathrm{w}} [\phi]_0^\ell, \qquad (30)
\end{aligned}$$

where the time index $\ell$ is initially set as 0, $N \in \mathbb{Z}_{>0}$ is the number of time instants in the control horizon, $\tilde{y}^{0:\ell+N-1} = [y_0^{0:\ell+N-1}, \hat{y}_1^{0:\ell+N-1}, \ldots, \hat{y}_Q^{0:\ell+N-1}]$, $\mathbf{u}_0^{\ell:\ell+N-1} = [u_0^\ell, u_0^{\ell+1}, \ldots, u_0^{\ell+N-1}]$ are the control inputs of the relay agent, and the input values are constrained to $[u_{0,\min}, u_{0,\max}]$. Moreover, $\bar{Z}$ is converted from $Z = -(BB^TP - A)$ in $\dot{\hat{x}}_i = Z(x_g - \hat{x}_i)$ (stated in (9)) and $\bar{C}$ is converted from $C$ in (6) using a Zero-Order Hold method with a sampling period of $T_s$ which causes zero local discretization error at the sampling points for homogeneous linear system dynamics [27], [28] for a the discrete-time state-space representation. Note that we only require the trajectory $y_0^{0:\ell+N-1}$ to weakly satisfy $\phi$ as $\ell+N-1$ may be less than the necessary length $L(\phi)$. At each time index $\ell$, we check if there exists any explorer that is being serviced (Line 5). If there are such explorers, we update the state estimates of those explorers with their true state values (Line 7). Then, we modify the MTL formula as in (28). The MILP is solved for time index $\ell$ with the updated state values and the modified MTL formula $[\phi]_0^\ell$ (Line 8). The previously computed relay agent control inputs are replaced by the newly computed control inputs from time index $\ell$ to $\ell+N-1$ (Line 11). In Line 6, $\ell^*$ refers to the last time index at which the controller input $u_0$ is calculated. Initially, $\ell^*$ is set to 0 (Line 2). $\ell^*$ is updated whenever the MILP-sol needs to be resolved (i.e., either of the two conditions is met). Specifically, both $\ell$ and $\ell^*$ are set to 0 in the beginning, and the MILP-sol is solved at $\ell = 0$ (Lines 2 and 3). As an illustrative example, if at $\ell = 1, 2, 3$, neither of the conditions at Line 6 is met, then $\ell^*$ remains to be 0. If at $\ell = 4$ at least one of the conditions is met, then we re-solve the MILP-sol (Line 8) and set $\ell^* = 4$ (Line 11). In order to guarantee the weak satisfaction of the MTL formula $\phi$ (Constraint (30)), we use the big-$M$ formulation, where we employ a sufficiently large positive number $M$ and binary variables [29] (as there may be disjunction and eventually operators in the MTL formula) to enforce the weak satisfaction of a MTL formula $\phi$.

*Example 1: As an illustrative example, we want to guarantee the weak satisfaction of the MTL formula $\phi := \Box\Diamond_{\mathcal{I}}((y_0 \in G_1) \vee (y_0 \in G_2))$ evaluated at time index 0 (we use $[\phi]_0^\ell$ to denote $\phi$ evaluated at time index 0 and the current time index is $\ell$) using the trajectory $y_0^{0:\ell+N-1}$. We denote the length of the time interval $\mathcal{I} = [0, c]$ with $|\mathcal{I}|$, where c is a positive integer. We also denote the length of the time horizon $H = [0 : \ell + N - 1]$ with $|H|$. The atomic proposition $y_0 \in G_1$ represents the interior area of the region $G_1$. The interior area of region $G_1$ can be indicated using six inequalities representing the boundaries of the three-dimensional region $G_1$. The matrix representation of these inequalities is denoted by $A_1 y_0 \leq b_1$, where $A_1 \in \mathbb{R}^{6\times 3}$, $y_0 \in \mathbb{R}^{3\times 1}$, and $b_1 \in \mathbb{R}^{6\times 1}$. Similarly, we represent the proposition $y_0 \in G_2$ using $A_2 y_0 \leq b_2$, where $A_2 \in \mathbb{R}^{6\times 3}$, $y_0 \in \mathbb{R}^{3\times 1}$, and $b_2 \in \mathbb{R}^{6\times 1}$. In addition, we use $\mathbf{1}$ to denote a vector of appropriate dimension with all the entries to be one. We enforce the weak satisfaction of the MTL formula $[\phi]_\ell^0$ by defining the constraints in (31), which ensures that the relay agent reaches at least one charging station at least once in any $cT_s$ sampling period. In (31), superscript j refers to the time index at which we enforce the weak satisfaction of the MTL formula $[\phi]_\ell^0$ in the control horizon N using the trajectory $y_0^{0:\ell+N-1}$. In (31), $p_1^j$ and $p_2^j$ are binary variables associated with $G_1$ and $G_2$ at time index j, respectively.*

$$A_1 y_0^j \leq b_1 + M(1-p_1^j)\mathbf{1},\ \forall j = \ell, \ldots, \ell+N-1$$
$$A_2 y_0^j \leq b_2 + M(1-p_2^j)\mathbf{1},\ \forall j = \ell, \ldots, \ell+N-1$$
$$\sum_{k'=j}^{j+|\mathcal{I}|-1} p_1^{k'} + \sum_{k'=j}^{j+|\mathcal{I}|-1} p_2^{k'} \geq 1,$$
$$\forall j = \max\{1, \ell - |\mathcal{I}| + 1\}, \ldots, \ell + |H| - |\mathcal{I}|. \quad (31)$$

**Complexity:** The computational complexity of the MILPs can be characterized using the number of variables and the constraints involved in the optimization problem. The main measure of the problem size is the number of binary variables and continuous variables introduced in the problem. We denote the set of atomic propositions used in the MTL formula $\phi$ by $P$ and the size of the set $P$ by $|P|$. Also, we denote the number of the operators (both logical and temporal) used in the MTL $\phi$ by $|\phi|$. For $F$ explorer agents, the control horizon $N$, a trajectory $y$ of length $|y|$, and the $\mathcal{L}$ required iterations for reaching the approximate consensus (by solving MILP-sol at each iteration), the computation complexity of Algorithm 1 is $\mathcal{O}(|y| \cdot |P| \cdot |\phi| \cdot F \cdot N \cdot \mathcal{L})$.

Let $\{\hat{t}_s^i\}_{s=0}^\infty \subset \mathbb{T}_d$ be an increasing sequence for explorer $i$, iteratively generated by Algorithm 1, where $\hat{t}_s^i$ is the $s^{\text{th}}$ time instant that $\|\hat{y}_i - y_0\| \leq \eta$ holds. Moreover, let the communication switching signal $\zeta_i$ be

$$\zeta_i(t) \triangleq \begin{cases} 1, & \text{if } t = \hat{t}_s^i \text{ for some } s \in \mathbb{I}, \\ 0, & \text{otherwise.} \end{cases} \quad (32)$$

Finally, we present Theorem 3, which provides theoretical guarantees for achieving correctness, uniform boundedness, and approximate consensus (in Problem 1).

*Theorem 3: Given the estimator in (6), controller in (7), and communication switching signal in (32) for each explorer $i \in F$, if each optimization in Algorithm 1 is feasible, $\eta \in [0, R)$, $V_T \in (0, (R-\eta)/S_{\max}(C)]$, and $S_{\max}(C)\Lambda(\rho_i) < R_f$, then Algorithm 1 terminates in finite time, the MTL specification $\phi$ is weakly satisfied, and the explorers achieve approximate consensus within the goal region, i.e., $\limsup_{t\to\infty}\|e_i(t)\| \leq \Lambda(\rho_i^*)$, where $\rho_i^* = 2\overline{d}_i\lambda_{\max}(P) + 2S_{\max}(PA)\|x_g\|$. In addition, for each explorer $i \in F$ and servicing instance $s \in \mathbb{Z}_{\geq 0}$, $t_{s+1}^i - t_s^i \geq T_s$, where $T_s > 0$ is the sampling period in Algorithm 1.*

*Proof:* We first utilize mathematical induction to prove $\hat{t}_s^i = t_s^i$ for each explorer $i \in F$ and each servicing instance $s \in \mathbb{Z}_{\geq 0}$. First, set $\hat{t}_0^i = t_0^i = 0$ for each $i \in F$. Next, fix $i \in F$, and suppose $\hat{t}_s^i = t_s^i$ for some $s \in \mathbb{Z}_{>0}$. We now show $\hat{t}_{s+1}^i = t_{s+1}^i$. Recall that $t_{s+1}^i$ is the next time instant explorer $i$ is serviced by the relay agent, which occurs if and only if $\|y_i - y_0\| \leq R$ and the communication switching signal $\zeta_i$ is on, i.e., $\zeta_i = 1$. Note that $R$ is the communication radius of the agents. On the other hand, $\hat{t}_{s+1}^i \in \mathbb{T}_d = \{t[0], t[1], \ldots\}$ represents the next discrete time instant that $\|\hat{y}_i - y_0\| \leq \eta$ holds, where $\eta \in [0, R)$. Since each optimization in Algorithm 1 is feasible, it follows that the MTL specification $\phi_m$ is weakly satisfied, which results in $\hat{t}_{s+1}^i - \hat{t}_s^i = \hat{t}_{s+1}^i - t_s^i \leq n_i T_s$. Recall that $n_i \in \mathbb{Z}_{>0}$ and $T_s > 0$ is the sampling period. Therefore, as $n_i T_s \leq \Delta t_i$ ($\Delta t_i$ is defined in (14)), we can derive that $\hat{t}_{s+1}^i - \hat{t}_s^i = \hat{t}_{s+1}^i - t_s^i \leq n_i T_s \leq \Delta t_i$. Since $\hat{t}_{s+1}^i - t_s^i \leq \Delta t_i$, Theorem 1 implies that $\|e_{1,i}(\hat{t}_{s+1}^i)\| \leq V_T$. Using (1), (3), (6), and the triangle inequality, it follows that

$$\begin{aligned}\|y_i(\hat{t}^i_{s+1}) - y_0(\hat{t}^i_{s+1})\| &\leq \|Cx_i(\hat{t}^i_{s+1}) - C\hat{x}_i(\hat{t}^i_{s+1})\| \\ &\quad + \|C\hat{x}_i(\hat{t}^i_{s+1}) - Cx_0(\hat{t}^i_{s+1})\| \\ &\leq S_{\max}(C)V_T + \eta,\end{aligned}$$

where $V_T \leq (R-\eta)/S_{\max}(C)$ implies that $\|y_i(\hat{t}^i_{s+1}) - y_0(\hat{t}^i_{s+1})\| \leq R$. According to (32), $\zeta_i(\hat{t}^i_{s+1}) = 1$. Thus, the relay agent services explorer $i$ at discrete time $\hat{t}^i_{s+1}$, and $\hat{t}^i_{s+1} = t^i_{s+1}$ holds. Therefore, $\hat{t}^i_s = t^i_s$ for each $i \in F$ and $s \in \mathbb{Z}_{\geq 0}$ by mathematical induction. Moreover, since Algorithm 1 generates the sequence of discrete servicing times $\{\hat{t}^i_s\}^{\infty}_{s=0}$ and employs the sampling period $T_s > 0$, then $t^i_{s+1} - t^i_s = \hat{t}^i_{s+1} - \hat{t}^i_s \geq T_s$ for each $(i,s) \in F \times \mathbb{Z}_{\geq 0}$.

If each optimization is feasible in Algorithm 1, then $[\phi]^{\ell}_0$ is weakly satisfied for each $\ell$ as constraint (30) is satisfied; hence, the MTL specification $\phi$ is weakly satisfied. With $\hat{t}^i_s = t^i_s$, the maximum dwell-time condition in (14) is satisfied for all $t^i_s$ and $i \in F$. From Theorem 2 and Remarks 3 and 4, if $S_{\max}(C)\Lambda(\rho_i) < R_f$ holds, then, for each $i \in F$, there exists a time $T_i > 0$ such that explorer $i$ will be inside the goal region for $t \geq T_i$. Thus, at time $\tilde{t} = \max_{i\in F}\{T_i\}$, $\|Cx_g - y_i(\tilde{t})\| < R_f$ holds for all $i \in F$, i.e., Algorithm 1 is guaranteed to terminate within finite time. Finally, if $V_T \in (0, (R-\eta)/S_{\max}(C)]$, then according to Theorem 2 and Remark 4, we have $\limsup_{t\to\infty} \|e_i(t)\| \leq \Lambda(\rho^*_i)$. □

## V. IMPLEMENTATION

We now demonstrate the controller synthesis approach in the example in Fig. 1 (in Section I). The relay agent is a quadrotor modeled as a six degrees of freedom (6-DOF) rigid body [16]. We denote the system state as $x^0_q = [p_q, \dot{p}_q, \theta_q, \Omega_q]^{\mathrm{T}} \in \mathbb{R}^{12}$, where $p_q = [x_{q,1}, x_{q,2}, x_{q,3}]^{\mathrm{T}}$ and $\dot{p}_q = [\dot{x}_{q,1}, \dot{x}_{q,2}, \dot{x}_{q,3}]^{\mathrm{T}}$ are the position and velocity vectors of the quadrotor. The vector $\theta_q = [\alpha_q, \beta_q, \gamma_q]^{\mathrm{T}} \in \mathbb{R}^3$ includes the roll, pitch and yaw Euler angles of the quadrotor. The vector $\Omega_q \in \mathbb{R}^3$ includes the angular velocities rotating around its body frame axes. The nonlinear dynamic model of such a quadrotor is given by $m_q\ddot{p}_q = r(\theta_q)T_q\mathbf{e}_3 - mg\mathbf{e}_3$, $\dot{\theta}_q = H(\theta_q)\Omega_q$, and $I\dot{\Omega}_q = -\Omega_q \times I\Omega_q + \tau_q$, where $m_q$ is the mass, $g$ is the gravitational acceleration, $I$ is the inertia matrix, $r(\theta_q)$ is the rotation matrix representing the body frame with respect to the inertial frame (which is a function of the Euler angles), $H(\theta_q)$ is a nonlinear mapping that projects the angular velocity $\Omega_q$ to the Euler angle rate $\dot{\theta}_q$, $\mathbf{e}_3 = [0,0,1]^{\mathrm{T}}$, $T_q$ is the thrust of the quadrotor, and $\tau_q \in \mathbb{R}^3$ is the torque on the three axes. The control input is $u_0 = [u_{0,1}, u_{0,2}, u_{0,3}, u_{0,4}]^{\mathrm{T}}$, where $u_{0,1}$ is the vertical velocity command, $u_{0,2}$, $u_{0,3}$ and $u_{0,4}$ are the angular velocity commands around its three body axes. We consider the state of the kinematic model of the quadrotor (relay agent) as $x_0 = [x_{q,1}, x_{q,2}, x_{q,3}, \dot{x}_{q,1}, \dot{x}_{q,2}, \alpha_q, \beta_q, \gamma_q]^{\mathrm{T}}$ and the 3-D position representation of the relay agent as $y_0 = [x_{q,1}, x_{q,2}, x_{q,3}]^{\mathrm{T}}$. By adopting the small-angle assumption and then linearizing the dynamic model around the hover state and discretizing using a Zero-Order Hold method with the sampling period $T_s$, we obtain $x^{j+1}_0 = A_0x^j_0 + B_0u^j_0$ and $y^j_0 = C_0x^j_0$, where $A_0 \in \mathbb{R}^{8\times 8}$ and $B_0 \in \mathbb{R}^{8\times 4}$.

For each $i \in F$, explorer $i$ is modeled as a planar unicycle with dynamics $\dot{x}_{i,1} = v_i\cos(\theta_i)$, $\dot{x}_{i,2} = v_i\sin(\theta_i)$, and $\dot{\theta}_i = \omega_i$. Note that $(x_{i,1}, x_{i,2})$ denotes the planar position of explorer $i$, $\theta_i$ denotes the heading, and $(v_i, \omega_i)$ denotes the control input. When $(v_i, \theta_i) \in \mathbb{R}\setminus\{0\}\times\mathbb{R}$, the unicycle model can be feedback linearized as $(\ddot{x}_{i,1}, \ddot{x}_{i,2}) = (u_{i,1}, u_{i,2})$ (see Section V of [16]), where $u_{i,1}$ and $u_{i,2}$ are the control inputs of explorer $i$ and

$$\begin{bmatrix}\ddot{x}_{i,1}\\ \ddot{x}_{i,2}\end{bmatrix} = \underbrace{\begin{bmatrix}\cos(\theta_i) & -v_i\sin(\theta_i)\\ \sin(\theta_i) & v_i\cos(\theta_i)\end{bmatrix}}_{\triangleq R(v_i,\theta_i)}\begin{bmatrix}\dot{v}_i\\ \omega_i\end{bmatrix}.$$

When $(v_i, \theta_i) \in \mathbb{R}\setminus\{0\}\times\mathbb{R}$, the control input for the double integrator model can be mapped to a control input for the unicycle using $[\dot{v}_i, \omega_i]^{\mathrm{T}} = R(v_i,\theta_i)^{-1}[u_{i,1}, u_{i,2}]^{\mathrm{T}}$.

For the state space representation of explorer $i$, $x_i \triangleq [x_{i,1}, x_{i,2}]^{\mathrm{T}}$, $u_i \triangleq [u_{i,1}, u_{i,2}]^{\mathrm{T}}$, $d_i \triangleq [d_{i,1}, d_{i,2}]^{\mathrm{T}}$ and $y_i \triangleq [x_{i,1}, x_{i,2}, 0]^{\mathrm{T}}$. Given Section II-C, $u_i$ is given by (7), which employs the estimation error in (4) and the estimator in (9). Observe that the estimator dynamics are discretized using a Zero-Order Hold method, as in Algorithm 1. The initial 3-D positions of the three explorers are $[-100,-100,0]^{\mathrm{T}}$, $[100,150,0]^{\mathrm{T}}$ and $[150,-150,0]^{\mathrm{T}}$, respectively. The initial 3-D position of the relay agent is $[-25,-150,5]^{\mathrm{T}}$. The consensus state $x_g$ is set to $[0,0,0]^{\mathrm{T}}$. For each explorer $i \in F$, the random disturbance $d_i(t)$ is a vector whose elements are drawn at each time step $t$ from a uniform distribution centered about the origin spanning $[-0.5\bar{d}_i, 0.5\bar{d}_i]$, where $\bar{d}_1 = 0.01$, $\bar{d}_2 = 0.04$, and $\bar{d}_3 = 0.01$. The randomly generated disturbance $d_i(t)$ is used when calculating $\dot{x}_i$, for each $i \in F$. Although the theoretical development applies only to deterministic MASs with continuous and bounded disturbances, random disturbances are simulated to highlight the performance of the proposed control strategy.

For approximate consensus, we consider the controller of the explorers in (7), where $P$ is as follows (computed from (8) with $k = 0.1$).

$$P = \begin{bmatrix}0.23 & 0.22\\ 0.22 & 0.52\end{bmatrix} \otimes I_2,$$

where $\otimes$ denotes the Kronecker product. We consider three different scenarios as follows.

**Scenario I**: The relay agent needs to reach the charging station $G_1$ or $G_2$ at least once in any $cT_s$ time (with $c$ being some positive integer), and it should always remain in region $D$, where the two charging stations $G_1$ and $G_2$ are rectangular cuboids with length, width and height being 10, 10 and 5, centered at $[-100,50,2.5]^{\mathrm{T}}$ and $[125,0,2.5]^{\mathrm{T}}$, respectively. The region $D$ is a rectangular cuboid centered at $[0,0,7]^{\mathrm{T}}$ with length, width and height being 300, 300 and 6, respectively (see Fig. 1). Scenario I can be expressed in the form

**TABLE 1.** MTL specifications $\phi_{\mathrm{p}}$ and results in Scenario I.

| MTL specification $\phi_{\mathrm{p}}$ | cumulative control effort |
|---|---|
| $\phi^1_{\mathrm{p,I}} = \Box\Diamond_{[0,20]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge\Box(y_0 \in D)$ | 70,943.32 |
| $\phi^2_{\mathrm{p,I}} = \Box\Diamond_{[0,10]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge\Box(y_0 \in D)$ | 101,079.03 |
| $\phi^3_{\mathrm{p,I}} = \Box\Diamond_{[0,6]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge\Box(y_0 \in D)$ | 165,224.55 |

**TABLE 2.** MTL specifications $\phi_{\mathrm{p}}$ and results in Scenario II.

| MTL specification $\phi_{\mathrm{p}}$ | cumulative control effort |
|---|---|
| $\phi^1_{\mathrm{p,II}} = \Box\Diamond_{[0,6]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge\Box(y_0 \in D) \wedge \neg\Diamond\Box_{[0,1]}(y_0 \in E)$ | 644,670.20 |
| $\phi^2_{\mathrm{p,II}} = \Box\Diamond_{[0,6]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge\Box(y_0 \in D) \wedge \neg\Diamond\Box_{[0,2]}(y_0 \in E)$ | 165,984.72 |
| $\phi^3_{\mathrm{p,II}} = \Box\Diamond_{[0,6]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge\Box(y_0 \in D) \wedge \neg\Diamond\Box_{[0,3]}(y_0 \in E)$ | 165,241.50 |

of the MTL formula $\phi_{\mathrm{p,I}} = \Box\Diamond_{[0,c]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge \Box(y_0 \in D)$.

**Scenario II**: The relay agent needs to reach the charging station $G_1$ or $G_2$ at least once in any $6T_s$ time, always remain in region $D$ (same as in Scenario I with $c = 6$), and never stay in region $E$ for over $c'T_s$ time (with $c'$ being some positive integer), where the region $E$ is a rectangular cuboid centered at $[0, 0, 6]^{\mathrm{T}}$ with length, width and height being 75, 75 and 4, respectively. Scenario II can be expressed in the form of the MTL formula $\phi_{\mathrm{p,II}} = \Box\Diamond_{[0,6]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge \Box(y_0 \in D) \wedge \neg\Diamond\Box_{[0,c']}(y_0 \in E)$.

**Scenario III**: The MTL specification $\phi_{\mathrm{p,III}} = \Box\Diamond_{[0,6]}\big((y_0 \in G_1) \vee (y_0 \in G_2)\big) \wedge \Box(y_0 \in D) \wedge \Box((\neg(y_0 \in O_1)) \wedge (\neg(y_0 \in O_2)))$ depicts a situation where the relay agent must always avoid two obstacles ($O_1$ and $O_2$) for safety concerns in addition to servicing the explorer agents and carrying out the other practical tasks (i.e., Scenario III is similar to Scenario I with $c = 6$ with two added obstacles). Obstacle $O_1$ is a rectangular cuboid centered at $[0, -125, 5]$ with length, width and height being 50, 50 and 10, respectively. Obstacle $O_2$ is also a rectangular cuboid centered at $[40, 60, 5]$ with length, width and height being 20, 70 and 10, respectively.

We set $R_f = R = 5$, $V_T = 1$, $\eta = 4$, $T_s = 0.5$, and $N = 120$. We consider $c = 20, 10, 6$ for Scenario I. Fig. 3 shows the simulation results in Scenario I for the case of $c = 20$. We observe that the obtained control inputs of the relay agent gradually decrease as the explorers approach the goal region (Fig. 2a), $\|e_{1,i}(t)\|$ is uniformly bounded by $V_T = 1$ (Fig 2c), and $\|e_{2,i}(t)\|$ gradually decreases and then oscillates when the explorers approach approximate consensus to the goal region (Fig. 2d). We measure the *cumulative control effort* as $\sum_{j=0}^{\bar{N}} \|u_0^j\|^2$, where $\bar{N}$ denotes the minimal time index such that $\|Cx_g - y_i(t[\bar{N}])\| \le R_f$ for all $i \in F$. In Scenario I, the obtained cumulative control effort corresponding to $c = 20$,

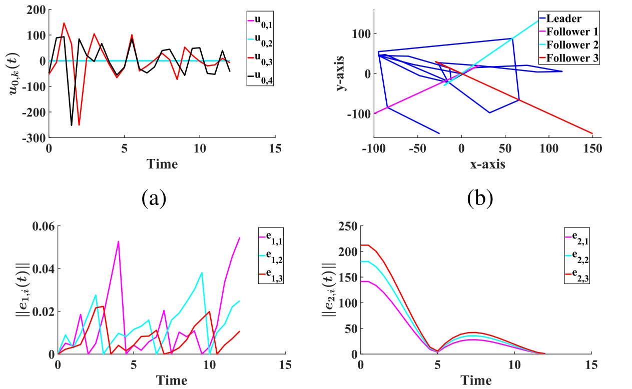


**FIGURE 2.** Results with MTL specification $\phi_{\mathrm{p,I}}$ for the practical constraints ($c = 6$): (a) the obtained optimal inputs for the relay agent; (b) 2-D planar plot of the trajectories of three explorers and a relay agent; (c) $\|e_{1,i}(t)\|$; (d) $\|e_{2,i}(t)\|$.

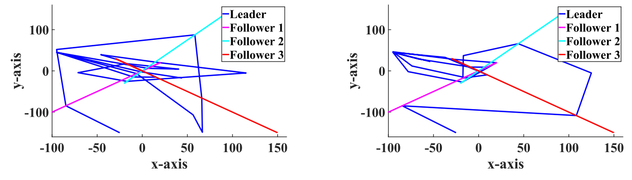


**FIGURE 3.** (a) 2-D planar plot of the trajectories of three explorers and a relay agent for Scenario II ($c = 1$); (b) 2-D planar plot of the trajectories of three explorers and a relay agent for Scenario III.

$c = 10$, and $c = 6$ is 70943.32, 101079.03, and 165224.55, respectively. We observe that the cumulative control effort for satisfying $\phi_{\mathrm{p,I}}$ with $c = 6$ is more than that for satisfying $\phi_{\mathrm{p,I}}$ with $c = 10$, which is still more than that for satisfying $\phi_{\mathrm{p,I}}$ with $c = 20$. This is consistent with the fact that $\phi_{\mathrm{p,I}}$ with $c = 10$ implies $\phi_{\mathrm{p,I}}$ with $c = 20$, and $\phi_{\mathrm{p,I}}$ with $c = 6$ implies both $\phi_{\mathrm{p,I}}$ with $c = 20$ and $c = 10$, respectively. The MTL formulas and the corresponding control effort for Scenario I are summarized in Table 1.

For Scenario II, we consider $c' = 1, 2, 3$. Similarly, for Scenario II, we obtain the same trends for $u_{0,k}(t)$, $\|e_{1,i}(t)\|$, and $\|e_{2,i}(t)\|$ as Figs. 2a, 2c, and 2d, respectively (we have not included these plots for Scenario II due to space limitations). Fig. 3a represents the 2-D planar plot of the trajectories of the relay agent and the three agents. In Scenario II, the obtained cumulative control effort corresponding to $c' = 1$, $c' = 2$, and $c' = 3$ is 644670.20, 165984.72, and 165241.50, respectively. In addition, our results show that the cumulative control effort for satisfying $\phi_{\mathrm{p,II}}$ with $c' = 1$ is more than that for satisfying $\phi_{\mathrm{p,II}}$ with $c' = 2$, which is still more than that for satisfying $\phi_{\mathrm{p,II}}$ with $c' = 3$. This is consistent with the fact that $\phi_{\mathrm{p,I}}$ with $c' = 2$ implies $\phi_{\mathrm{p,II}}$ with $c' = 3$, and $\phi_{\mathrm{p,II}}$ with $c' = 1$ implies both $\phi_{\mathrm{p,II}}$ with $c' = 2$ and $c' = 3$. We also observe that more cumulative control effort is needed in Scenario II to satisfy the MTL specifications after the explorers arrive in region $E$ as the relay agent needs to get away from $E$ after each service to the explorers. Videos of the simulations in Scenarios I and II are available in the CoppeliaSim environment[5]. The MTL formulas and the corresponding control effort for Scenario II are summarized in Table 2.

Fig. 3b represents the 2-D planar trajectories of three explorers and one relay agent in Scenario III. Similarly, for Scenario III, we obtain the same trends for $u_{0,k}(t)$, $\|e_{1,i}(t)\|$, and $\|e_{2,i}(t)\|$ as Figs. 2a, 2c, and 2d, respectively. The cumulative control effort for this scenario is 291402.4851. When comparing the cumulative control efforts associated with the MTL specification $\phi_{\mathrm{p,I}}$ with $c = 6$ and the MTL specification $\phi_{\mathrm{p,III}}$, we observe that the cumulative control effort has been increased due to the added obstacles $O_1$ and $O_2$.

## VI. CONCLUSION

We present a metric temporal logic approach for the controller synthesis of a multi-agent system with intermittent communication. We iteratively solve a sequence of mixed-integer linear programming problems for provably achieving correctness, uniform boundedness of $e_{1,i}$ with respect to time for each explorer $i \in F$, and approximate consensus in the explorers' positions. Future work may investigate scenarios where controller synthesis is conducted for the explorers with more complex specifications and motion models. In addition, it may be possible to improve the performance of the state estimators by incorporating local measurements.

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[5] CoppeliaSim videos can be found at https://tinyurl.com/y32xgmtx

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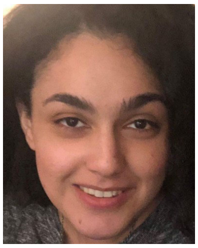

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