PERGAMON

Automatica 36 (2000) 1741–1746

automatica

www.elsevier.com/locate/automatica

Brief Paper

Global exponential setpoint control of wheeled mobile robots: a Lyapunov approach☆

W. E. Dixon[a,]*, Z. P. Jiang[b], D. M. Dawson[a]

[a]Department of Electrical and Computer Engineering, Clemson University, Clemson, SC 29634-0915, USA
[b]Department of Electrical Engineering, Polytechnic University, Brooklyn, NY 11201, USA

Received 26 July 1999; revised 14 March 2000; received in final form 7 April 2000

Abstract

This paper presents a new differentiable, time-varying controller for the regulation problem for wheeled mobile robots. After the WMR kinematics have been transformed into an advantageous form, a dynamic oscillator, in lieu of explicit cosine or sine terms, is constructed to promulgate a global exponential regulation property for the transformed kinematic model via a Lyapunov-type argument. In order to showcase the differentiable nature of the proposed kinematic control structure, we demonstrate how the standard backstepping technique can be applied to obtain a global exponential regulator for an exact dynamic model.

Keywords: Mobile robots; Underactuated system; Nonlinear control; Backstepping

1. Introduction

Over the past twenty years wheeled mobile robots (WMRs) have become increasingly important in settings that range from shopping centers, hospitals, warehouses, and nuclear waste facilities for applications such as security, transportation, inspection, planetary exploration, etc. This increased demand for WMRs has led to a greater research interest in the areas of electromechanical design, sensor integration techniques, path planning, and control design. As noted in Canudas de Wit and Sordalen (1992), and Fierro and Lewis (1997), control research for nonholonomic systems (i.e., the family of mechanical systems which includes WMRs as special cases) has been centered around the tracking problem (which includes the geometric "path-planning" problem as a subset) and the stabilization problem. It has also been noted that the tracking problem can be solved with standard nonlinear control techniques; however, many researchers (d'Andréa-Novel, Campion & Bastin, 1995; Canudas de Wit & Sordalen, 1992; Lamiraux & Laumond, 1998; Samson, 1990) have pointed out that the problem of stabilization about a fixed point is more challenging due to the structure of the governing differential equations (i.e., the control problem cannot be solved via a smooth, time-invariant state feedback law due to the implications of Brockett's (1983) condition. With this technical obstacle in mind, researchers have proposed controllers that utilize discontinuous control laws, piecewise continuous control laws, smooth time-varying control laws, or a hybrid form of the previous controllers to achieve setpoint regulation. For an in-depth review of the previous work, the interested reader is referred to Kolmanovsky and McClamroch (1995), M'Closkey and Murray (1997), Samson (1990), and the references therein. For brevity and for illuminating the motivation for this work, we confine our review to a much smaller subset of papers.

In Samson (1990), a smooth time-varying feedback controller that could be utilized to asymptotically stabilize a mobile robot about a point was presented. The

☆This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor M. Tomizuka under the direction of Editor M. Araki. This work is supported in part by the U.S. NSF Grants DMI-9457967, CMS-9634796, ECS-9619785, DMI-9813213, EPS-9630167, DOE Grant DE-FG07-96ER14728, ONR Grant N0 0014-99-1-0589, and a DOC Grant.

* Corresponding author. Tel.: + 1-864-656-5924.
E-mail addresses: wdixon@eng.clemson.edu (W. E. Dixon), ddawson@eng.clemson.edu (D. M. Dawson), zjiang@control.poly.edu (Z. P. Jiang).

0005-1098/00/$ - see front matter
PII: S0005-1098(00)00099-6

work in Samson (1990) led to the development of smooth time-varying controllers for a more general class of system (see e.g., Coron & Pomet, 1992; Pomet, 1992; Teel, Murray & Walsh, 1995). In Bloch, Reyhanoglu and McClamroch (1992), a piecewise analytic control structure for regulating nonholonomic systems to a setpoint was developed. In Canudas de Wit and Sordalen (1992), a piecewise smooth controller was constructed to exponentially stabilize a WMR to a setpoint; however, due to the control structure, the orientation of the WMR is not arbitrary. More recently, in Samson (1997), globally asymptotically stabilizing feedback controllers for a general class of nonholonomic systems in chained form were developed and a detailed discussion on the convergence issue was provided. In M'Closkey and Murray (1997), a set of sufficient conditions for generating Lipschitz continuous ρ-exponential stabilizers from smooth asymptotic stabilizers for a general class of driftless systems was developed. In Godhavn and Egeland (1997), a local, continuous feedback control law with time-periodic terms that ρ-exponentially stabilized nonholonomic systems in the power form was constructed. Motivated by the desire to remove the exact model knowledge dependence of the aforementioned controllers, Dong and Huo (1997) proposed an adaptive control solution for chained nonholonomic systems with unknown constant inertia effects (also see Jiang & Pomet, 1996). In Jiang and Nijmeijer (1997), a reference robot tracking controller was proposed; however, the tracking control problem does not reduce to the regulation problem. Recently, Escobar, Ortega and Reyhanoglu (1998) illustrated how the field-oriented controller which has been derived for induction motors can be redesigned to exponentially stabilize a nonholonomic double integrator control problem (e.g., Heisenberg flywheel). Unfortunately, the controller presented in Escobar et al. (1998) exhibited singularities in either the double integrator state or the output variable.

In this paper, we illustrate how the previously designed controller for the induction motor control problem given in Dawson, Hu and Vedagharba (1995) can be reconfigured to globally exponentially regulate a WMR to any constant setpoint. In contrast with Escobar et al. (1998), the proposed controller does not exhibit any singularities. In contrast, with much of the previous work on nonholonomic systems, the proposed controller does not utilize explicit sinusoidal terms in the feedback controller; rather, a dynamic oscillator with a tunable frequency of oscillation is constructed. Roughly speaking, the frequency of oscillation is used as auxiliary control input to cancel odious terms during the Lyapunov analysis. While the control synthesis and the error system development are slightly more involved than some of the previously designed controllers for the WMR problem, the stability analysis is straightforward, which involves simple Lyapunov arguments and yields a global exponential result for the transformed kinematic model. Since the proposed kinematic controller is differentiable, the standard backstepping technique can be directly applied to incorporate the dynamic model into the overall control design. It is worth noting that the exponential kinematic controllers given in Astolfi (1996), Bloch et al. (1992), and Escobar et al. (1998) are not differentiable and, therefore, it is unclear how they can also be extended to incorporate the dynamic model via the standard backstepping procedure. The paper is organized as follows. In Section 2, we transform the kinematic model of the WMR into a form which resembles the induction motor model. In Section 3, we present the differentiable, time-varying control law, the corresponding closed-loop error system, and the stability analysis for the kinematic model developed in Section 2. Concluding remarks are presented in Section 4.

2. Kinematic problem formulation

The kinematic equations of motion of the center of mass (COM) of a WMR under the nonholonomic constraint of *pure rolling* and *nonslipping* can be written as follows:

$$\dot{q} = S(q)v, \tag{1}$$

where $\dot{q}(t) \in \Re^3$, defined as

$$\dot{q}(t) = [\dot{x}_c \quad \dot{y}_c \quad \dot{\theta}]^T, \tag{2}$$

represents the time derivative of $q(t) \in \Re^3$, $v(t) \in \Re^2$ is a vector of linear and angular velocities of the WMR denoted by $v_l(t) \in \Re^1$ and $\dot{\theta}(t) \in \Re^1$, respectively, as follows:

$$v = [v_1 \quad v_2]^T = [v_l \quad \dot{\theta}]^T, \tag{3}$$

$\dot{x}_c(t)$, $\dot{y}_c(t) \in \Re^1$ represent the time derivative of the Cartesian position of the COM denoted by $x_c(t)$, $y_c(t) \in \Re^1$, and the transformation matrix $S(q) \in \Re^{3\times 2}$ is defined as

$$S(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}. \tag{4}$$

In order to express the WMR model in a form that is more amenable to the subsequent control design and stability analysis, we define the following new variables, denoted by $z(t) = [z_1(t) \;\; z_2(t)]^T \in \Re^2$ and $w(t) \in \Re^1$, which are related to the Cartesian position/orientation of the COM via the following transformation:

$$[z(t) \quad w(t)]^T = T[\tilde{x}(t) \quad \tilde{y}(t) \quad \tilde{\theta}(t)]^T, \tag{5}$$

where the transformation matrix $T \in \Re^{3 \times 3}$ is defined as

$$T = \begin{bmatrix} 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \\ -\tilde{\theta}\cos\theta + 2\sin\theta & -\tilde{\theta}\sin\theta - 2\cos\theta & 0 \end{bmatrix}. \quad (6)$$

$\tilde{x}(t)$, $\tilde{y}(t)$, $\tilde{\theta}(t) \in \Re^1$ are defined as the difference between the actual Cartesian position/orientation of the COM and the desired constant position/orientation setpoints, denoted by x_{cd}, y_{cd}, $\theta_d \in \Re^1$, as follows

$$\tilde{x} = x_c - x_{cd}, \qquad \tilde{y} = y_c - y_{cd}, \qquad \tilde{\theta} = \theta - \theta_d. \quad (7)$$

After taking the time derivative of (5), and using (1)–(7), we have the following transformed kinematic equations:

$$\begin{bmatrix} \dot{w} \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 u_1 - z_1 u_2 \\ u_1 \\ u_2 \end{bmatrix}, \quad (8)$$

where the new variable, denoted by $u(t) = [u_1(t), u_2(t)]^T \in \Re^2$, is defined as follows:

$$u_1 = v_2, \qquad u_2 = v_1 - v_2(\tilde{x}\sin\theta - \tilde{y}\cos\theta). \quad (9)$$

Finally, we rewrite the expression given in (8) in the following compact form:

$$\dot{w} = u^T J^T z, \qquad \dot{z} = u, \quad (10)$$

where $J \in \Re^{2 \times 2}$ is a constant, skew-symmetric matrix defined as follows:

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \quad (11)$$

3. Kinematic control development

Our control objective is to design an exponentially regulating controller for the WMR kinematic model given by (10). To this end, we define an auxiliary error signal $\tilde{z}(t) \in \Re^2$ as the difference between the subsequently designed auxiliary signal $z_d(t) \in \Re^2$ and the transformed variable $z(t)$ defined in (5) as follows:

$$\tilde{z} = z_d - z. \quad (12)$$

3.1. Control formulation

Based on the kinematic equations given in (10) and the subsequent stability analysis, we design $u(t)$ given in (10) as follows:

$$u = u_a - kz, \quad (13)$$

where $k > 0$ is a design parameter and the control term $u_a(t) \in \Re^2$ is defined as

$$u_a = \left(\frac{kw}{\delta_d^2}\right) J z_d + \Omega_1 z_d. \quad (14)$$

The oscillator-like signal $z_d(t)$ in (12) and (14) is generated by the following initial-value differential equation:

$$\dot{z}_d = \frac{\dot{\delta}_d}{\delta_d} z_d + \left(\frac{kw}{\delta_d^2} + w\Omega_1\right) J z_d, \quad (15)$$

$$z_d^T(0) z_d(0) = \delta_d^2(0)$$

and the auxiliary terms $\Omega_1(t)$, $\delta_d(t) \in \Re^1$ in (14) and (15) are defined as

$$\Omega_1 = k + \frac{\dot{\delta}_d}{\delta_d} + \frac{kw^2}{\delta_d^2}, \quad (16)$$

$$\delta_d = \alpha_0 \exp(-\alpha_1 t) \quad (17)$$

where $\alpha_0, \alpha_1 > 0$ are design parameters.

Remark 1. Based on the definition of $\delta_d(t)$ in (17), there appear to be potential singularities in the auxiliary terms given by (14), (15), and (16). That is, since $\delta_d(t)$ goes to zero exponentially fast, the terms contained in (14)–(16),

$$\frac{kw}{\delta_d^2} J z_d, \qquad \frac{kw^2}{\delta_d^2} z_d, \qquad \frac{kw^3}{\delta_d^2} J z_d, \quad (18)$$

appear to be unbounded as $t \to \infty$. However, in the subsequent stability analysis we demonstrate that the potential singularities are always avoided provided certain gain conditions are met.

Remark 2. Motivation for the structure of (15) is obtained by taking the time derivative of $z_d^T(t) z_d(t)$ as follows:

$$\frac{d}{dt}(z_d^T z_d) = 2 z_d^T \left(\frac{\dot{\delta}_d}{\delta_d} z_d + \left(\frac{kw}{\delta_d^2} + w\Omega_1\right) J z_d\right), \quad (19)$$

where (15) has been utilized. After noting that the matrix J of (11) is skew symmetric, we can rewrite (19) as follows

$$\frac{d}{dt}(z_d^T z_d) = 2 \frac{\dot{\delta}_d}{\delta_d} z_d^T z_d. \quad (20)$$

As a result of the selection of the initial conditions given in (15), the unique solution to (20) must be $\delta_d^2(t)$, i.e.,

$$z_d^T(t) z_d(t) = \|z_d(t)\|^2 = \delta_d^2(t) \quad \forall\, t \geq 0, \quad (21)$$

where $\|\cdot\|$ stands for the standard Euclidean norm.

3.2. Error system development

We begin the closed-loop error system formulation by substituting (13) for $u(t)$ in the open-loop expression for $w(t)$ given in (10) and then adding/subtracting $u_a^{\mathrm{T}} J z_{\mathrm{d}}$ to the right-hand side of the resulting expression to obtain

$$\dot{w} = -u_a^{\mathrm{T}} J z_{\mathrm{d}} + u_a^{\mathrm{T}} J \tilde{z}, \tag{22}$$

where (12) and the skew symmetry of J defined in (11) have been used. After substituting (14) for only the first occurrence of $u_a(t)$ in (22), we obtain the final closed-loop expression for $w(t)$ as follows:

$$\dot{w} = -kw + u_a^{\mathrm{T}} J \tilde{z}, \tag{23}$$

where (21), the skew symmetry of J defined in (11), and the fact that $J^{\mathrm{T}} J = I_2$ have been used. (Note that I_2 denotes the two by two identity matrix.) To determine the closed-loop error system for $\tilde{z}(t)$, we take the time derivative of (12) and then substitute (10) and (15) to obtain

$$\dot{\tilde{z}} = \frac{\dot{\delta}_{\mathrm{d}}}{\delta_{\mathrm{d}}} z_{\mathrm{d}} + \left(\frac{kw}{\delta_{\mathrm{d}}^2} + w\Omega_1\right) J z_{\mathrm{d}} - u. \tag{24}$$

After substituting (13) for $u(t)$ and then substituting (14) into the resulting expression, we have

$$\dot{\tilde{z}} = \frac{\dot{\delta}_{\mathrm{d}}}{\delta_{\mathrm{d}}} z_{\mathrm{d}} + \left(\frac{kw}{\delta_{\mathrm{d}}^2} + w\Omega_1\right) J z_{\mathrm{d}} - \left(\frac{kw}{\delta_{\mathrm{d}}^2}\right) J z_{\mathrm{d}} - \Omega_1 z_{\mathrm{d}} + kz. \tag{25}$$

After substituting (16) for only the second occurrence of $\Omega_1(t)$, cancelling common terms, and then rearranging the resulting expression, we have

$$\dot{\tilde{z}} = -k\tilde{z} + wJ\left[\left(\frac{kw}{\delta_{\mathrm{d}}^2}\right) J z_{\mathrm{d}} + \Omega_1 z_{\mathrm{d}}\right], \tag{26}$$

where (12) and the fact that $JJ = -I_2$ have been used. Since the bracketed term in (26) is equal to $u_a(t)$ defined in (14), we can now obtain the closed-loop error system for $\tilde{z}(t)$ as follows:

$$\dot{\tilde{z}} = -k\tilde{z} + wJu_a. \tag{27}$$

3.3. Stability analysis

Theorem 1. *Given the closed-loop system of* (23) *and* (27), *the position/orientation setpoint error defined in* (7) *is globally exponentially regulated in the sense that*

$$|\tilde{x}(t)|, |\tilde{y}(t)|, |\tilde{\theta}(t)| \leq \beta_0 \exp(-\beta_1 t) \tag{28}$$

provided the control parameters α_1 and k are selected as follows:

$$k > \alpha_1, \tag{29}$$

where $\beta_0 \in \Re^1$ is a positive constant that depends on the initial conditions of the system, and $\beta_1 \in \Re^1$ is a positive constant that is independent of the initial conditions of the system.

Proof. To prove Theorem 1, we define the following nonnegative and radially unbounded function:

$$V(w(t), \tilde{z}(t)) = \tfrac{1}{2} w^2 + \tfrac{1}{2} \tilde{z}^{\mathrm{T}} \tilde{z}. \tag{30}$$

After taking the time derivative of (30), making substitutions for (23) and (27) and then cancelling common terms, we obtain

$$\dot{V}(w(t), \tilde{z}(t)) = -kw^2 - k\tilde{z}^{\mathrm{T}}\tilde{z} + u_a^{\mathrm{T}} J \tilde{z} w + \tilde{z}^{\mathrm{T}} J u_a w. \tag{31}$$

After noting that $J^{\mathrm{T}} = -J$ (see (11)), we can rewrite $\dot{V}(w(t), \tilde{z}(t))$ of (31) as follows:

$$\dot{V}(w(t), \tilde{z}(t)) = -2kV(w(t), \tilde{z}(t)). \tag{32}$$

Based on (30) and (32), it is evident that standard Lyapunov arguments (Slotine & Li, 1991) can be utilized to conclude that

$$V(w(t), \tilde{z}(t)) = V(w(0), \tilde{z}(0)) \exp(-2kt) \tag{33}$$

and thus,

$$\|\Psi(w(t), \tilde{z}(t))\| = \|\Psi(w(0), \tilde{z}(0))\| \exp(-kt), \tag{34}$$

where $\Psi(w(t), \tilde{z}(t)) \in \Re^3$ is defined as

$$\Psi = [w(t), \quad \tilde{z}(t)]^{\mathrm{T}}. \tag{35}$$

In view of (12), (21), (34), and (35), we can conclude that $w(t)$, $\tilde{z}(t)$, $z(t)$, $z_{\mathrm{d}}(t) \in \mathscr{L}_\infty$. Since $w(t)$ is driven to zero within the exponential envelope given in (34), it can be clearly seen that if the sufficient condition given in (29) holds then the potential singularities discussed in Remark 1 are always avoided. Specifically, if the condition given in (29) is satisfied then the terms given in (18) can be upper bounded by the following exponentially decaying bounds:

$$\begin{aligned} &k\zeta_0 \exp(-(k-\alpha_1)t), \\ &k\zeta_0^2 \exp(-(2k-\alpha_1)t), \\ &k\zeta_0^2 \exp(-(3k-\alpha_1)t), \end{aligned} \tag{36}$$

respectively. Based on this fact, we can now use standard signal chasing arguments to show that $u(t)$, $u_a(t)$, $\dot{z}_d(t)$, $\Omega_1(t) \in \mathscr{L}_\infty$.

To show that the Cartesian position/orientation of the COM defined in (1) are bounded, we note that it is easy to show that the inverse transformation for (5) is given by

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}(\tilde{\theta}\sin\theta + 2\cos\theta) & \frac{1}{2}\sin\theta \\ 0 & -\frac{1}{2}(\tilde{\theta}\cos\theta - 2\sin\theta) & -\frac{1}{2}\cos\theta \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ w \end{bmatrix}. \tag{37}$$

Since $z(t)\in\mathscr{L}_\infty$, we can see from (37) that $\tilde{\theta}(t)\in\mathscr{L}_\infty$ (and hence $\theta(t)\in\mathscr{L}_\infty$). Since $w(t)$, $z(t)$, $\tilde{\theta}(t)\in\mathscr{L}_\infty$, we can see from (37) that $\tilde{x}(t)$, $\tilde{y}(t)\in\mathscr{L}_\infty$ (and hence $x_c(t)$, $y_c(t)\in\mathscr{L}_\infty$). To show that the linear and angular velocities defined in (3) are bounded, it is easy to show that the inverse transformation for (9) is given by

$$v_2 = u_1,$$
$$v_1 = u_2 + u_1(\tilde{x}\sin\theta - \tilde{y}\cos\theta). \tag{38}$$

Since $u(t)$, $\tilde{x}(t)$, $\tilde{y}(t)\in\mathscr{L}_\infty$, we can see from (38) that $v(t)\in\mathscr{L}_\infty$; therefore, it follows from (1)–(4) that $\dot{\theta}(t)$, $\dot{x}_c(t)$, $\dot{y}_c(t)\in\mathscr{L}_\infty$. Standard signal chasing arguments can now be used to show that all of the remaining signals in the control and the system are bounded during closed-loop operation.

In order to prove (28), we apply the triangle inequality to (12) to obtain the following exponential bound for $z(t)$:

$$\|z(t)\| \le \|\tilde{z}(t)\| + \|z_d(t)\|$$
$$\le \|\Psi(w(0),\tilde{z}(0))\|\exp(-kt) + \alpha_0\exp(-\alpha_1 t), \tag{39}$$

where (17), (21), and (34) have been utilized. The main result given by (28) now directly follows from (34), (37), and (39). □

Remark 3. Given (13)–(18), (21), (36) and (39), it is a trivial matter to explicitly upper bound the control input $u(t)$ by the following inequality:

$$\|u(t)\| \le \alpha_0(k+\alpha_1)\exp(-\alpha_1 t) + \frac{k\|\Psi(w(0),\tilde{z}(0))\|}{\alpha_0}\exp(-(k-\alpha_1)t) + \frac{k\|\Psi(w(0),\tilde{z}(0))\|}{\alpha_0}\exp(-(2k-\alpha_1)t) + k\|\Psi(w(0),\tilde{z}(0))\|\exp(-kt) + k\alpha_0\exp(-\alpha_1 t). \tag{40}$$

Remark 4. Note that the stability result given in (28) depends on the states of the system approaching zero exponentially fast. However, in practice the states will not equal zero due to measurement errors, noise, etc. In order to provide for robustness, the proposed controller is modified in Dixon, Dawson, Zergeroglu and Zhang, (2000): however, the modified controller forces the regulation errors to an arbitrarily small neighborhood about the origin (i.e., globally uniformly ultimately bounded) rather than exponentially forcing the regulation errors to zero as illustrated in (28).

Remark 5. Note that based on the differentiable nature of the proposed kinematic controller, it is straightforward to utilize standard backstepping techniques to incorporate the effects of the dynamic model (see Dixon, Dawson, Zhang & Zergeroglu, 1999 for explicit details). In addition, we note that to employ the standard backstepping technique the kinematic control input $u(t)$ defined in (9) is required to be differentiable, and hence, it is not clear how the non-differentiable controllers presented in Astolfi 1996), Bloch and Drakunov (1996) and Escobar et al. (1998) can be extended via standard backstepping techniques to incorporate the effects of the dynamic model in the control design.

Remark 6. A discussion and simulation results regarding a comparative analysis between the proposed kinematic controller and the kinematic controllers given in M'Closkey and Murray (1997), Samson (1997), and Teel et al. (1995) are given in Dixon et al. (1999).

4. Conclusion

In this paper, we utilized our previous experience in the field of induction motor control to design a new differentiable time-varying controller for the stabilization problem for the wheeled mobile robot. Through the use of a dynamic oscillator and a Lyapunov stability analysis, we prove that the controller exponentially regulates the WMR to any fixed setpoint. It should be noted that in addition to the WMR problem, the kinematic portion of the proposed controller can be applied to other nonholonomic systems (for several other examples of kinematic systems the proposed controller can exponentially stabilize about a point see, Bloch et al. (1992)). It should also be pointed out that the exponential result presented in this paper allows one to utilize existing Lyapunov control design tools to provide enhancements to the proposed control structure. Specifically, in Dixon et al. (2000), we illustrate how the controller can be redesigned to provide for robustness against uncertainty in the dynamic model.

References

d'Andréa-Novel, B., Campion, G., & Bastin, G. (1995). Control of nonholonomic wheeled mobile robots by state feedback linearization. *International Journal of Robotics Research*, *14*(6), 543–559.

Astolfi, A. (1996). Discontinuous control of nonholonomic systems. *Systems & Control Letters*, *27*, 37–45.

Bloch, A. M., & Drakunov, S. (1996). Stabilization and tracking in the nonholonomic integrator via sliding modes. *Systems & Control Letters*, *29*, 91–99.

Bloch, A. M., Reyhanoglu, M., & McClamroch, N. (1992). Control and stabilization of nonholonomic dynamic systems. *IEEE Transactions on Automatic Control*, *37*(11), 1746–1757.

Brockett, R. (1983). Asymptotic stability and feedback stabilization. In: R. Brockett, R. Millman, & H. Sussmann, *Differential geometric control theory* (pp. 181–191).

Canudas de Wit, C., & Sordalen, O. J. (1992). Exponential stabilization of mobile robots with nonholonomic constraints. *IEEE Transactions on Automatic Control*, *37*(11), 1791–1797.

Coron, J. M., & Pomet, J. B. (1992). A remark on the design of time-varying stabilizing feedback laws for controllable systems without drift, *Proceedings of the IFAC symposium on nonlinear control systems design (NOLCOS)* (pp. 413–417).

Dawson, D., Hu, J., & Vedagharba, P. (1995). An adaptive control for a class of induction motor systems, *Proceedings of the IEEE conference on decision and control* (pp. 1567–1572).

Dixon, W. E., Dawson, D. M., Zergeroglu, E., & Zhang, F. (2000). Robust tracking and regulation control for a mobile robot. *International Journal of Robust and Nonlinear Control: Special Issue on Control of Underactuated Nonlinear Systems* (pp. 199–216).

Dixon, W. E., Dawson, D. M., Zhang, F., & Zergeroglu, E. (1999). Global exponential setpoint control of mobile robots. *Proceedings of the 1999 IEEE/ASME international conference on advanced intelligent mechatronics* (pp. 683–688).

Dong, W., & Huo, W. (1997). Adaptive stabilization of dynamic nonholonomic chained systems with uncertainty, *Proceedings of the 36th IEEE conference on decision and control* (pp. 2362–2367).

Escobar, G., Ortega, R., & Reyhanoglu, M. (1998). Regulation and tracking of the nonholonomic double integrator: A field-oriented control approach. *Automatica*, *34*(1), 125–131.

Fierro, R., & Lewis, F. L. (1997). Control of a nonholonomic mobile robot: Backstepping kinematics into dynamics. *Journal of Robotic Systems*, *14*(3), 149–163.

Godhavn, J. M., & Egeland, O. (1997). A Lyapunov approach to exponential stabilization of nonholonomic systems in power form. *IEEE Transactions on Automatic Control*, *42*(7), 1028–1032.

Jiang, Z. P., & Nijmeijer, H. (1997). Tracking control of mobile robots: A case study in backstepping. *Automatica*, *33*(7), 1393–1399.

Jiang, Z. P., & Pomet, J. -B. (1996). Global stabilization of parametric chained-form systems by time-varying dynamic feedback. *International Journal of Adaptive Control Signal Processing*, *10*, 47–59.

Kolmanovsky, I., & McClamroch, N. H. (1995). Developments in nonholonomic control problems. *IEEE Control Systems Magazine*, *15*, 20–36.

Lamiraux, F., & Laumond, J. P. (1998). A practical approach to feedback control for a mobile robot with trailer, *Proceedings of the international conference on robotics and automation* (pp. 3291–3296).

M'Closkey, R. T., & Murray, R. M. (1997). Exponential stabilization of driftless nonlinear control systems using homogeneous feedback. *IEEE Transactions on Automatic Control*, *42*(5), 614–628.

Pomet, J. -B. (1992). Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift. *Systems and Control Letters*, *18*(2), 147–158.

Samson, C. (1990). Velocity and torque feedback control of a nonholonomic cart. *Proceedings of the international workshop in adaptive and nonlinear control*: *Issues in robotics*.

Samson, C. (1997). Control of chained systems application to path following and time-varying point- stabilization of mobile robots. *IEEE Transactions on Automatic Control*, *40*(1), 64–77.

Slotine, J. J. E., & Li, W. (1991). *Applied nonlinear control*. Englewood Cliffs, NJ: Prentice Hall.

Teel, A. R., Murray, R. M., & Walsh, C. G. (1995). Non-holonomic control systems: From steering to stabilization with sinusoids. *International Journal Control*, *62*(4), 849–870.

Warren Dixon was born in York, Pennsylvania in 1972. He received the Bachelor of Science degree in 1994 from the Department of Electrical and Computer Engineering, Clemson University, Clemson, South Carolina. He then received the Master of Engineering degree in 1997 from the Department of Electrical and Computer Engineering from the University of South Carolina, Columbia, South Carolina. He returned to Clemson University where he is currently pursuing a Ph.D. degree. As a student in the Robotics and Mechatronics group at Clemson University, his research is focused on nonlinear based robust and adaptive control techniques with application to electromechanical systems including wheeled mobile robots and robot manipulators.

Zhong-Ping Jiang received the B.Sc. degree in mathematics from the University of Wuhan, Wuhan, China, in 1988, the M.Sc. degree in statistics from the Université de Paris-sud, Paris, France, in 1989, and Ph.D. degree in automatic control and mathematics from the École des Mines de Paris, Paris, France, in 1993. From 1993 to 1998, he held visiting researcher positions in several institutions including INRIA (Sophia-Antipolis), France, the Department of Systems Engineering in the Australian National University, Canberra and the Department of Electrical Engineering in the University of Sydney. In 1998, he also visited several U.S. universities. In January 1999, he joined the Polytechnic University at Brooklyn as an Assistant Professor of Electrical Engineering. His main research interests include stabilization, robust/adaptive nonlinear control and decentralized control with applications to nonholonomic mechanical systems. He has authored or coauthored over 80 refereed technical papers in these areas. He is a member of the IEEE and an Associate Editor for Systems & Control Letters, and the International Journal of Robust and Nonlinear Control.

Darren M. Dawson was born in 1962, in Macon, Georgia. He received an Associate Degree in Mathematics from Macon Junior College in 1982 and B.S. Degree in Electrical Engineering from the Georgia Institute of Technology in 1984. He then worked for Westinghouse as a control engineer from 1985 to 1987. In 1987, he returned to the Georgia Institute of Technology where he received Ph.D. Degree in Electrical Engineering in March 1990. During this time, he also served as a research/teaching assistant. In July 1990, he joined the Electrical and Computer Engineering Department and the Center for Advanced Manufacturing (CAM) at Clemson University where he currently holds the position of Professor. Under the CAM director's supervision, he currently leads the Robotics and Manufacturing Automation Laboratory which is jointly operated by the Electrical and Mechanical Engineering departments. His main research interests are in the fields of nonlinear based robust, adaptive, and learning control with application to electromechanical systems including robot manipulators, motor drives, magnetic bearings, flexible cables, flexible beams, and high-speed transport systems.