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Predictor-based control for an uncertain Euler–Lagrange system with input delay☆

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ABSTRACT

Controlling a nonlinear system with actuator delay is a challenging problem because of the need to develop some form of prediction of the nonlinear dynamics. Developing a predictor-based controller for an uncertain system is especially challenging. In this paper, tracking controllers are developed for an Euler–Lagrange system with time-delayed actuation, parametric uncertainty, and additive bounded disturbances. The developed controllers represent the first input delayed controllers developed for uncertain nonlinear systems that use a predictor to compensate for the delay. The results are obtained through the development of a novel predictor-like method to address the time delay in the control input. Lyapunov–Krasovskii functionals are used within a Lyapunov-based stability analysis to prove semi-globally uniformly ultimately bounded tracking. Experimental results illustrate the performance and robustness of the developed control methods.

1. Introduction

Time delay in the control input (also known as dead time, or input delay) is a pervasive problem in various control applications. Chemical and combustion processes, telerobotic systems, vehicle platoons, communication networks, and biological systems (Bequette, 1991; Evesque, Annaswamy, Niculescu, & Dowling, 2003; Huang & Lewis, 2003; Riener & Fuhr, 1998; Yanakiev & Kanellakopoulos, 2001) often encounter delays in the control input. Such delays are often attributed to transport lags, communication delays, task prioritization or slow biological response, and can lead to poor performance and potential instability.

Motivated by performance and stability problems, various methods have been developed for linear systems with input delays (cf. Artstein, 1982; Bresch-Pietri & Krstic, 2009; Fiagbedzi & Pearson, 1986; Gu, Kharitonov, & Chen, 2003; Jankovic, 2008; Krstic, 2008; Krstic & Bresch-Pietri, 2009; Krstic & Smyshlyaev, 2008; Kwon & Pearson, 1980; Manitius & Olbrot, 1979; Mondié & Michiels, 2003; Richard, 2003; Roh & Oh, 1999, and the references therein). As discussed in Gu et al. (2003) and Richard (2003), an outcome of these results is the development and use of prediction techniques such as Artstein model reduction (Artstein, 1982), finite spectrum assignment (Manitius & Olbrot, 1979), and continuous pole placement (Michiels, Engelborghs, Vansevenant, & Roose, 2002). The concept of predictive control originated from classic Smith predictor methods (Smith, 1959). The Smith predictor requires a plant model for output prediction and has been widely studied and modified for control purposes (cf. Chien, Peng, & Liu, 2002; Garcia & Albertos, 2008; Majhi & Atherton, 1999, 2000; Matausek & Micic, 1996; Nortcliffe & Love, 2004; Roca et al., 2009; Zhang & Sun, 1996, and references therein). However, the Smith predictor may not yield desirable closed-loop performance in the presence of model mismatch and can only be applied for stable plants (Gu et al., 2003; Huang & Lewis, 2003). Contrary to the Smith predictor, finite spectrum assignment or Artstein model reduction techniques and their extensions (cf. Artstein, 1982; Fiagbedzi & Pearson, 1986; Jankovic, 2008; Kwon & Pearson, 1980; Manitius & Olbrot, 1979; Mondié & Michiels, 2003; Roh & Oh, 1999; Wang, 2009, 2008; Wang, Hu, & Zhang, 2008; Xiang, Cao, Wang, & Lee, 2008, and references therein) can be applied to unstable or multivariable linear plants. These predictor-based methods utilize finite integrals over past control values to transform the delayed system to a delay free system. Discrete predictor-based techniques have also been developed for linear systems with time varying input delay in Lozano, Castillo, Garcia, and Dzul (2004), where small bounded uncertainties in the system parameters, delay, and sampling instants are considered.

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Another approach to develop predictive controllers is based on the fact that input delay systems can be represented by hyperbolic partial differential equations (cf. Gu et al., 2003; Richard, 2003, and references therein). This fact is exploited in Bresch-Pietri and Krstic (2009), Krstic (2008), Krstic and Bresch-Pietri (2009), Krstic and Smyshlyaev (2008) and Krstic (2010) to design controllers for actuator delayed linear systems. These novel methods model the time delayed system as an ordinary differential equation (ODE)–partial differential equation (PDE) cascade where the non-delayed input acts at the PDE boundary. The controller is then designed by employing a backstepping type approach for PDE control (Krstic & Smyshlyaev, 2008).

Predictor techniques have also been extended to adaptive control of unknown linear plants in Bresch-Pietri and Krstic (2009) Evesque et al. (2003) and Niculescu and Annaswamy (2003). In Evesque et al. (2003) and Niculescu and Annaswamy (2003), a modified Smith predictor type structure is used to achieve a semi-global result. In Bresch-Pietri and Krstic (2009) (and the companion paper Krstic & Bresch-Pietri, 2009), a global adaptive controller is developed that compensates for uncertain plant parameters and a possibly large unknown delay. Adaptive Posicast Controllers have also been developed for uncertain linear systems with delays and for automotive applications in Yildiz, Annaswamy, Kolmanovsky, and Yanakiev (2010), Yildiz, Annaswamy, Yanakiev, and Kolmanovsky (2010) and Yildiz, Annaswamy, Yanakiev, and Kolmanovsky (2011).

In comparison to input delayed linear systems, fewer results are available for nonlinear systems. Approaches for input delayed nonlinear systems such as Kravaris and Wright (1989) and Henson and Seborg (1994) utilize a Smith predictor-based globally linearizing control method and require a known nonlinear plant model for time delay compensation. In Huang and Lewis (2003), a specific technique is developed for a telerobotic system with constant input and feedback delays where a Smith predictor for a locally linearized subsystem is used in combination with a neural network controller for a remotely located uncertain nonlinear plant to drive the position coordinates of the slave robot to a delayed trajectory. A similar type of control objective is defined in Chopra, Spong, and Lozano (2008), where the aim is to drive the position of the slave robot to the delayed position of the master robot and the position of the master robot to the delayed position of the slave robot (i.e., a different control objective and problem formulation than considered in the current result). Also, the control design and stability analysis assume the system dynamics to be linearly parameterizable. In Mazenc and Bliman (2006), an approach is provided to construct Lyapunov–Krasovskii (LK) functionals for the input delayed nonlinear system in feedback form. In Francisco, Mazenc, and Mondié (2007), bounded state feedback and output-based controllers are developed to stabilize the origin of the dynamic system describing a PVTOL aircraft with delay in the input. The developed control laws are extensions of approach developed for the feedforward system with delays in the input (Mazenc et al., 2003b,a). Although the work in Francisco et al. (2007) and Mazenc et al. (2003b,a) provide fundamental contributions to the input delay problem in feedforward systems, its applicability to general uncertain mechanical systems modeled by Euler–Lagrange dynamics is not clear. The dynamic model considered in Francisco et al. (2007) is a simplified model of equations where dependency on parameters (e.g., mass of the system, lengths, etc.) does not exist. Development was provided in Mazenc and Bowong (2003) to design a tracking controller for a cart-pendulum system (a typical example of Euler–Lagrange system). After some transformations, the Euler–Lagrange dynamics of the cart-pendulum system are converted into a feedforward system. These transformations require exact model knowledge, thus the technique is not applicable when the system parameters are unknown or dynamics are uncertain, which implies that methods developed for feedforward systems with delay may not be applicable to uncertain Euler–Lagrange dynamics. The control method in Jankovic (2006) utilizes a composite Lyapunov function containing an integral cross term and an LK functional for stabilizing nonlinear cascade systems, where delay can enter the system through the input or the states. The robustness of input to state stabilizability is proven in Teel (1998) for nonlinear finite-dimensional control systems in presence of small input delays by utilizing a Razumikhin-type theorem. In Krstic (2008) and Krstic (2010), the backstepping approach that utilizes the ODE–PDE cascade transformation for input delayed systems is extended to nonlinear control systems with an actuator delay of unrestricted length. In Ailon, Segev, and Arogeti (2004), Ailon (2004) and Ailon and Gil (2000), delay dependent sufficient conditions are established to prove local exponential or asymptotic stability of the zero/stationary solution of uncertain nonlinear systems (linearized about a setpoint). Specifically, a velocity-free controller is developed for attitude regulation of spacecraft in the presence of constant feedback delay in Ailon et al. (2004), while in Ailon (2004) and Ailon and Gil (2000), output-based controllers are developed for uncertain flexible-joint robots with multiple input delays and rigid robots with time delay, respectively. Unlike the current paper that develops a predictor-based method to compensate for the input delay in uncertain nonlinear systems (without linearization), the results in Ailon et al. (2004), Ailon (2004) and Ailon and Gil (2000) utilize controllers developed for delay-free systems and prove robustness to the delay provided certain delay dependent conditions hold true. No prior results address stabilizing an uncertain/disturbed nonlinear system with input delays (for the typical tracking problem without linearization).

This paper (and its preliminary version Sharma, Bhasin, Wang, & Dixon, 2010) focuses on the development of tracking controllers for an uncertain nonlinear Euler–Lagrange system with input delay. The input time delay is assumed to be a known constant and can be arbitrarily large. The dynamics are assumed to contain parametric uncertainty and additive bounded disturbances. A modified proportional integral derivative (PID) controller is first developed based on the desire to include integral feedback. To develop the controller the inertia matrix is assumed to be known. For the purpose of eliminating the assumption of a known inertia matrix, a second controller is developed. The second control structure is based on the same design approach (i.e., develop a predictor to transform the dynamics to a new system that does not have an input delay); however, the resulting modified PD controller has a completely different structure (i.e., the modified PID does not simplify to the modified PD controller if the integral is removed). For both controllers, the key contributions are the design of a delay compensating auxiliary signal to obtain a time delay free open-loop error system, and the construction of LK functionals to cancel the time delayed terms. The auxiliary signal leads to the development of a predictor-based controller that contains a finite integral of past control values. This delayed state to delay free transformation is analogous to the Artstein model reduction approach, where a similar predictor-based control is obtained. LK functionals containing finite integrals of control input values are used in a Lyapunov-based analysis that proves that the tracking errors are semi-globally uniformly ultimately bounded. Experimental results are obtained for a two-link direct drive robot. The results illustrate the robustness and added value of the developed predictor-based controllers.

2. Dynamic model and properties

Consider the following input delayed Euler–Lagrange dynamics

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + d(t) = u(t-\tau). \tag{1}$$

In (1), $M(q) \in \mathbb{R}^{n\times n}$ denotes a generalized inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes a generalized centripetal–Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes a generalized gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ denotes generalized friction, $d(t) \in \mathbb{R}^n$ denotes an exogenous disturbance (e.g., unmodeled effects), $u(t-\tau) \in \mathbb{R}^n$ represents the generalized delayed input control vector, where $\tau \in \mathbb{R}$ is a constant time delay, and $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ denote the generalized states. The subsequent development is based on the assumptions that $q(t)$ and $\dot{q}(t)$ are measurable, $V_m(q,\dot{q}), G(q), F(\dot{q}), d(t)$ are unknown, the time delay constant $\tau \in \mathbb{R}$ is known,[2] and the control input vector $u(t)$ and its past values (i.e., $u(t-\theta)\forall\theta \in [0\,\tau]$) are measurable. For the controller developed in Section 3.2, $M(q)$ is assumed to be known to illustrate the development of a PID-like controller. In Section 3.3, this assumption is removed and a PD-like controller is developed. Throughout the paper, a time dependent delayed function is denoted as $x(t-\tau)$ (or as x_τ) and a time dependent function (without time delay) is denoted as $x(t)$ (or as x). The following assumptions are used in the subsequent development.

Assumption 1. The inertia matrix $M(q)$ is symmetric, positive definite, and satisfies the following inequality $\forall\xi(t) \in \mathbb{R}^n$:

$$m_1\|\xi\|^2 \le \xi^T M\xi \le m_2\|\xi\|^2, \tag{2}$$

where $m_1, m_2 \in \mathbb{R}^+$ are known constants and $\|\cdot\|$ denotes the standard Euclidean norm.

Assumption 2. The desired trajectory $q_d(t)$ is designed such that $q_d(t), q_d^{(i)}(t) \in \mathcal{L}_\infty$, where $q_d^{(i)}(t)$ denotes the ith time derivative for $i = 1, 2, 3$.

Assumption 3. If $q(t), \dot{q}(t) \in \mathcal{L}_\infty$, then $M(q), V_m(q,\dot{q}), G(q)$, and $F(\dot{q})$ are bounded. Moreover, if $q(t), \dot{q}(t), \ddot{q}(t) \in \mathcal{L}_\infty$, then the first time derivatives of $M(q), V_m(q,\dot{q}), G(q), F(\dot{q})$ exist and are bounded. The infinity norm of $M(q)$ and its inverse can be upper bounded as

$$\|M(q)\|_\infty \le \zeta_1 \qquad \|M^{-1}(q)\|_\infty \le \zeta_2, \tag{3}$$

where $\zeta_1, \zeta_2 \in \mathbb{R}^+$ are known constants.

Assumption 4. The nonlinear disturbance term and its first time derivative are bounded, i.e., $d(t), \dot{d}(t) \in \mathcal{L}_\infty$.

3. Control development

3.1. Objective

The objective is to develop a controller that will enable the input delayed system in (1) to track a desired trajectory, denoted by $q_d(t) \in \mathbb{R}^n$. To quantify the objective, a position tracking error, denoted by $e_1(t) \in \mathbb{R}^n$, is defined as

$$e_1 = q_d(t) - q(t). \tag{4}$$

3.2. Control development given a known inertia matrix

To facilitate the subsequent analysis, a filtered tracking error, denoted by $e_{2_a}(t) \in \mathbb{R}^n$, is defined as

$$e_{2_a} = \dot{e}_1 + \alpha_1 e_1, \tag{5}$$

where $\alpha_1 \in \mathbb{R}^+$ denotes a constant.

[2] Experimental results (where the time delay is artificially injected in a desired manner) illustrate the performance of the developed controllers when the time delay has as much as 100% error between the assumed and the actual delay.

One approach to develop a delay compensating control law for the input delayed system in (1), is to reduce the system to an input delay free system. As an example (Artstein, 1982), consider a linear system with input delay $\dot{x}(t) = Ax(t) + Bu(t-\tau)$. A simple transformation $z(t) = x(t) + \int_{t-\tau}^{t} e^{A(t-\theta-\tau)}Bu(\theta)$ converts the linear system to $\dot{z}(t) = Az(t) + e^{-A\tau}Bu(t)$, which is a delay free system. A state feedback law of the form $Kz(t)$ for the transformed system will generate a distributed delay control term for the original system. This control approach is often described as an Artstein model reduction predictor-like controller. Inspired from this approach, an auxiliary signal denoted by $r(t) \in \mathbb{R}^n$, is also defined as

$$r = \dot{e}_{2_a} + \alpha_2 e_{2_a} + M^{-1}(q(t))(u(t-\tau) - u(t)), \tag{6}$$

where $\alpha_2 \in \mathbb{R}^+$. The auxiliary signal $r(t)$ is only introduced to facilitate the subsequent analysis, and is not used in the control design since the expression in (6) depends on the unmeasurable generalized state $\ddot{q}(t)$.

After multiplying (6) by $M(q)$ and utilizing the expressions in (1), (4) and (5), the transformed open-loop tracking error system can be expressed in an input delay free form as

$$\begin{aligned} M(q)r &= M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + d \\ &\quad + \alpha_1 M(q)\dot{e}_1 + \alpha_2 M(q)e_{2_a} - u(t). \end{aligned} \tag{7}$$

Based on (7) and the subsequent stability analysis, the control input $u(t) \in \mathbb{R}^n$ is designed as

$$\begin{aligned} u = k_a\bigg(e_{2_a} + \int_0^t \alpha_2 e_{2_a}(\theta) + M^{-1}(\theta)(u(\theta-\tau) \\ - u(\theta))\mathrm{d}\theta\bigg) - k_a e_{2_a}(0), \end{aligned} \tag{8}$$

where $k_a \in \mathbb{R}^+$ is a known constant that can be expanded as

$$k_a = k_{a_1} + k_{a_2} + 1, \tag{9}$$

to facilitate the subsequent stability analysis, where $k_{a_1}, k_{a_2} \in \mathbb{R}^+$ are known constants. The controller $u(t)$ in (8) is a proportional integral derivative (PID) controller modified by a predictor like feedback term for time delay compensation.

Although the control input $u(t)$ is present in the open-loop error system in (7), an additional derivative is taken to facilitate the subsequent stability analysis. The time derivative of (7) can be expressed as

$$M(q)\dot{r} = -\frac{1}{2}\dot{M}(q)r + N + \dot{d} - k_a r, \tag{10}$$

where $N(e_1, e_{2_a}, r, t) \in \mathbb{R}^n$ is an auxiliary term defined as

$$\begin{aligned} N &= -\frac{1}{2}\dot{M}(q)r + M(q)\dddot{q}_d + \dot{M}(q)\ddot{q}_d + \dot{V}_m(q,\dot{q})\dot{q} \\ &\quad + V_m(q,\dot{q})\ddot{q} + \dot{G}(q) + \dot{F}(\dot{q}) + (\alpha_1+\alpha_2)\,(M(q)r \\ &\quad - (u_\tau - u)) - \alpha_1\alpha_2 M(q)e_{2_a} - \alpha_1^2 M(q)(e_1 + \dot{e}_1) \\ &\quad - \alpha_2^2 M(q)e_{2_a} + \alpha_1\dot{M}(q)\dot{e}_1 + \alpha_2\dot{M}(q)e_{2_a}, \end{aligned} \tag{11}$$

and (6) is used to write the time derivative of (8) as

$$\dot{u} = k_a r.$$

After adding and subtracting the auxiliary function $N_d(q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d, t) \in \mathbb{R}^n$ defined as

$$\begin{aligned} N_d &= M(q_d)\dddot{q}_d + \dot{M}(q_d)\ddot{q}_d + \dot{V}_m(q_d,\dot{q}_d)\dot{q}_d + V_m(q_d,\dot{q}_d)\ddot{q}_d \\ &\quad + \dot{G}(q_d) + \dot{F}(\dot{q}_d), \end{aligned}$$

to (10), the following expression is obtained:

$$M(q)\dot{r} = -\frac{1}{2}\dot{M}(q)r + \tilde{N} + S - e_{2_a} - k_a r, \tag{12}$$

where the auxiliary functions $\tilde{N}(e_1, e_{2_a}, r, t) \in \mathbb{R}^n$ and $S(q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d, t) \in \mathbb{R}^n$ are defined as

$$\tilde{N} = N - N_d + e_{2_a}, \qquad S = N_d + \dot{d}. \tag{13}$$

Some terms in the closed-loop dynamics in (12) are segregated into auxiliary terms in (13) because of differences in how the terms can be upper bounded. For example, Assumptions 2–4, can be used to upper bound $S(q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d, t)$ as

$$\|S\| \leq \varepsilon_1, \tag{14}$$

where $\varepsilon_1 \in \mathbb{R}^+$ is a known constant, and the Mean Value Theorem can be used to upper bound $\tilde{N}(e_1, e_{2_a}, r, t)$ as

$$\tilde{N} \leq \rho_1(\|z_a\|)\|z_a\|, \tag{15}$$

where $z_a \in \mathbb{R}^{4n}$ is defined as

$$z_a = \begin{bmatrix} e_1^T & e_{2_a}^T & r^T & e_a^T \end{bmatrix}^T, \tag{16}$$

and the bounding function $\rho_1(\|z_a\|) \in \mathbb{R}$ is a known positive globally invertible nondecreasing function. In (16), $e_a \in \mathbb{R}^n$ is defined as

$$e_a \triangleq u - u_\tau = \int_{t-\tau}^{t} \dot{u}(\theta)\mathrm{d}\theta,$$

based on the Leibniz–Newton formula.

Theorem 1. *The controller given in (8) ensures semi-globally uniformly ultimately bounded (SUUB) tracking in the sense that*

$$\|e_1(t)\| \leq \epsilon_0 \exp(-\epsilon_1 t) + \epsilon_2, \tag{17}$$

where $\epsilon_0, \epsilon_1, \epsilon_2 \in \mathbb{R}^+$ denote constants, provided the control gains α_1, α_2, and k_a introduced in (5), (6) and (8), respectively are selected according to the following sufficient conditions:

$$\alpha_1 > \frac{1}{2}, \qquad \alpha_2 > 1 + \frac{\zeta_2^2\gamma^2}{4}, \qquad k_a^2 < \frac{1}{\omega\tau}, \qquad \omega\gamma^2 > 2\tau, \tag{18}$$

where $\omega, \gamma \in \mathbb{R}^+$ are subsequently defined control gains.

Proof. Let $y_a(t) \in \mathcal{D} \subset \mathbb{R}^{3n+1}$ be defined as

$$y_a(t) \triangleq \begin{bmatrix} e_1^T & e_{2_a}^T & r^T & \sqrt{Q} \end{bmatrix}^T, \tag{19}$$

where $Q(t) \in \mathbb{R}$ is defined as Mazenc and Bliman (2006) and Richard (2003)

$$Q = \omega \int_{t-\tau}^{t} \left(\int_{s}^{t} \|\dot{u}(\theta)\|^2 \, \mathrm{d}\theta \right) \mathrm{d}s, \tag{20}$$

where $\omega \in \mathbb{R}^+$ is a known constant. A positive definite Lyapunov functional candidate $V(y_a, t) : \mathcal{D} \times [0 \quad \infty) \to \mathbb{R}$ is defined as

$$V(y_a, t) \triangleq e_1^T e_1 + \frac{1}{2} e_{2_a}^T e_{2_a} + \frac{1}{2} r^T M(q) r + Q, \tag{21}$$

and satisfies the following inequalities

$$\lambda_1 \|y_a\|^2 \leq V \leq \lambda_2 \|y_a\|^2, \tag{22}$$

where $\lambda_1, \lambda_2 \in \mathbb{R}^+$ are known constants defined as

$$\lambda_1 = \frac{1}{2}\min[m_1, 1], \qquad \lambda_2 = \max\left[\frac{1}{2}m_2, 1\right], \tag{23}$$

where m_1 and m_2 are defined in (2).

After utilizing (5), (6) and (12) and canceling common terms, the time derivative of (21) is

$$\begin{aligned} \dot{V} &= 2e_1^T e_{2_a} - 2\alpha_1 e_1^T e_1 - \alpha_2 e_{2_a}^T e_{2_a} - k_a r^T r + r^T S \\ &\quad + e_{2_a}^T M^{-1}(q) e_a + r^T \tilde{N} + \omega\tau \|\dot{u}\|^2 \\ &\quad - \omega \int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta, \end{aligned} \tag{24}$$

where the Leibniz integral rule was applied to determine the time derivative of $Q(t)$ in (20). The expression in (24) can be upper bounded by using (3), (14) and (15) as

$$\begin{aligned} \dot{V} &\leq -(2\alpha_1 - 1)\|e_1\|^2 - (\alpha_2 - 1)\|e_{2_a}\|^2 - k_a\|r\|^2 \\ &\quad + \zeta_2 \|e_{2_a}\|\|e_a\| + \omega\tau\|\dot{u}\|^2 + \varepsilon_1 \|r\| \\ &\quad + \rho_1(\|z_a\|)\|z_a\|\|r\| - \omega \int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta. \end{aligned} \tag{25}$$

The following term in (25) can be upper bounded by using Young's inequality:

$$\zeta_2 \|e_{2_a}\|\|e_a\| \leq \frac{\zeta_2^2\gamma^2}{4}\|e_{2_a}\|^2 + \frac{1}{\gamma^2}\|e_a\|^2, \tag{26}$$

where $\gamma \in \mathbb{R}^+$ is a known constant. Further, by using the Cauchy–Schwarz inequality, the following term in (26) can be upper bounded as

$$\|e_a\|^2 \leq \tau \int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta. \tag{27}$$

Adding and subtracting $\frac{\tau}{\gamma^2}\int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta$ in (25) yields

$$\begin{aligned} \dot{V} &\leq -(2\alpha_1 - 1)\|e_1\|^2 - (\alpha_2 - 1)\|e_{2_a}\|^2 - k_a\|r\|^2 \\ &\quad + \zeta_2 \|e_{2_a}\|\|e_a\| + \omega\tau\|\dot{u}\|^2 + \varepsilon_1 \|r\| \\ &\quad + \rho_1(\|z_a\|)\|z_a\|\|r\| - \left(\omega - \frac{\tau}{\gamma^2}\right) \int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta \\ &\quad - \frac{\tau}{\gamma^2} \int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta. \end{aligned} \tag{28}$$

Utilizing (9) and the bounds given in (26) and (27), the inequality in (28) can be upper bounded as

$$\begin{aligned} \dot{V} &\leq -(2\alpha_1 - 1)\|e_1\|^2 - \left(\alpha_2 - 1 - \frac{\zeta_2^2\gamma^2}{4}\right)\|e_{2_a}\|^2 \\ &\quad - (1 - \omega k_a^2 \tau)\|r\|^2 - \frac{1}{\tau}\left(\omega - \frac{2\tau}{\gamma^2}\right)\|e_a\|^2 + \varepsilon_1\|r\| \\ &\quad + \rho_1(\|z_a\|)\|z_a\|\|r\| - k_{a_2}\|r\|^2 - k_{a_1}\|r\|^2 \\ &\quad - \frac{\tau}{\gamma^2} \int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta. \end{aligned} \tag{29}$$

After completing the squares, the inequality in (29) can be upper bounded as

$$\begin{aligned} \dot{V} &\leq -\beta_1\|z_a\|^2 - \frac{\tau}{\gamma^2}\int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta \\ &\quad + \frac{\rho_1^2(\|z_a\|)}{4k_{a_1}}\|z_a\|^2 + \frac{\varepsilon_1^2}{4k_{a_2}}, \end{aligned} \tag{30}$$

where $\beta_1 \in \mathbb{R}^+$ is defined as

$$\beta_1 = \min\left[\left(\alpha_2 - 1 - \frac{\zeta_2^2\gamma^2}{4}\right), (2\alpha_1 - 1), (1 - \omega k_a^2\tau), \frac{1}{\tau}\left(\omega - \frac{2\tau}{\gamma^2}\right)\right].$$

Since

$$\begin{aligned} \int_{t-\tau}^{t} \left(\int_{s}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta \right) \mathrm{d}s &\leq \tau \sup_{s\in[t,t-\tau]} \left[\int_{s}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta \right] \\ &= \tau \int_{t-\tau}^{t} \|\dot{u}(\theta)\|^2 \mathrm{d}\theta, \end{aligned}$$

the expression in (30) can be rewritten as

$$\dot{V} \le -\left(\beta_1 - \frac{\rho_1^2(\|z_a\|)}{4k_{a_1}}\right)\|z_a\|^2 - \frac{1}{\gamma^2}\int_{t-\tau}^{t}\left(\int_s^t \|\dot{u}(\theta)\|^2 d\theta\right) + \frac{\varepsilon_1^2}{4k_{a_2}}. \tag{31}$$

Using the definition of $z_a(t)$ in (16) and $y_a(t)$ in (19), the expression in (31) can be expressed as

$$\dot{V} \le -\bar{\beta}_1\|y_a\|^2 - \left(\beta_1 - \frac{\rho_1^2(\|z_a\|)}{4k_{a_1}}\right)\|e_a\|^2 + \frac{\varepsilon_1^2}{4k_{a_2}}, \tag{32}$$

where $\bar{\beta}_1(\|z_a\|) \in \mathbb{R}^+$ is defined as

$$\bar{\beta}_1 = \min\left[\left(\beta_1 - \frac{\rho_1^2(\|z_a\|)}{4k_{a_1}}\right), \frac{1}{\omega\gamma^2}\right].$$

By further utilizing (22), the inequality in (32) can be upper bounded as

$$\dot{V} \le -\frac{\bar{\beta}_1}{\lambda_2}V + \frac{\varepsilon_1^2}{4k_{a_2}}. \tag{33}$$

Consider a set $\mathcal{S}$ defined as

$$\mathcal{S} \triangleq \{z_a(t) \in \mathbb{R}^{4n} \mid \|z_a\| < \rho_1^{-1}(2\sqrt{\beta_1 k_{a_1}})\}. \tag{34}$$

In $\mathcal{S}$, $\bar{\beta}_1(\|z_a\|)$ can be lower bounded by a constant $\delta_1 \in \mathbb{R}^+$ as

$$\delta_1 \le \bar{\beta}_1(\|z_a\|). \tag{35}$$

Based on (35), the linear differential equation in (33) can be solved as

$$V(y_a, t) \le V(0)e^{-\frac{\delta_1}{\lambda_2}t} + \frac{\varepsilon_1^2\lambda_2}{4k_{a_2}\delta_1}[1 - e^{-\frac{\delta_1}{\lambda_2}t}], \tag{36}$$

provided $\|z_a\| \le \rho_1^{-1}(2\sqrt{\beta_1 k_{a_1}})$. From (36), if $z_a(0) \in \mathcal{S}$ then k_a can be chosen according to the sufficient conditions in (18) (i.e. a semi-global result) to yield the result in (17). Based on definition of $y_a(t)$, it can be concluded that $e_1(t), e_{2_a}(t), r(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$. Given that $e_1(t), e_{2_a}(t), q_d(t), \dot{q}_d(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$, (4) and (5) indicate that $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$. Since $r(t), e_{2_a}(t), q(t), \dot{q}(t), \dot{q}_d(t), \ddot{q}_d(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$, and $u(t) - u(t-\tau) = \int_{t-\tau}^{t}\dot{u}(\theta)d\theta = k_a\int_{t-\tau}^{t} r(\theta)d\theta \in \mathcal{L}_\infty$ in $\mathcal{S}$ (where the Leibniz–Newton formula was used), then (6) and Assumption 3 indicate that $\ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$. Given that $r(t), e_{2_a}(t)$, $q(t), \dot{q}(t), \dot{q}_d(t)\ddot{q}_d(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$, (7) and Assumptions 3 and 4 indicate that $u(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$. □

3.3. *Control development with an unknown inertia matrix*

The modified PID controller in the previous section included integral feedback provided the inertia matrix is known. The development in this section is motivated by the desire to eliminate knowledge of the inertia matrix. The design structures are similar in the sense that both are based on the strategy of creating a predictor based on transforming the input delayed system into an input delay free system. However, as a result of changes in the stability analysis to eliminate knowledge of the inertia matrix, the resulting controller in this section has a unique structure. Specifically, the controller in this section is a modified PD controller, where the modified PD controller is not a subset of the modified PID controller in the previous section with the integral removed.

To facilitate the subsequent control design and stability analysis for the uncertain inertia problem, the auxiliary signal, $e_{2_b}(t) \in \mathbb{R}^n$ is redefined as

$$e_{2_b}(t) = \dot{e}_1 + \alpha e_1 - B\int_{t-\tau}^{t} u(\theta)d\theta, \tag{37}$$

where $\alpha \in \mathbb{R}^+$ is a known constant, and $B \in \mathbb{R}^{n\times n}$ is a known symmetric, positive definite constant gain matrix that satisfies the following inequality

$$\|B\|_\infty \le b \tag{38}$$

where $b \in \mathbb{R}^+$ is a known constant.To facilitate the subsequent stability analysis, the error between B and $M^{-1}(q)$ is defined by

$$\eta(q) = B - M^{-1}(q), \tag{39}$$

where $\eta(q) \in \mathbb{R}^{n\times n}$ satisfies the following inequality

$$\|\eta(q)\|_\infty \le \bar{\eta}, \tag{40}$$

where $\bar{\eta} \in \mathbb{R}^+$ denotes a known constant. The open-loop tracking error system can be developed by multiplying the time derivative of (37) by $M(q)$ and utilizing the expressions in (1), (4) and (39) to obtain

$$M(q)\dot{e}_2 = M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + d + \alpha M(q)\dot{e}_1 - u(t) - M(q)\eta[u - u_\tau]. \tag{41}$$

Based on (41) and the subsequent stability analysis, the control input $u(t) \in \mathbb{R}^n$ is designed as

$$u = k_b e_{2_b}, \tag{42}$$

where $k_b \in \mathbb{R}^+$ is a known control gain that can be expanded as

$$k_b = k_{b_1} + k_{b_2} + k_{b_3}, \tag{43}$$

to facilitate the subsequent analysis, where k_{b_1}, k_{b_2}, and $k_{b_3} \in \mathbb{R}^+$ are known constants. After adding and subtracting the auxiliary term $N_d(q_d, \dot{q}_d, \ddot{q}_d, t) \in \mathbb{R}^n$ defined as

$$N_d = M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F(\dot{q}_d),$$

and using (37) and (42), the expression in (41) can be rewritten as

$$M(q)\dot{e}_{2_b} = -\frac{1}{2}\dot{M}(q)e_{2_b} + \tilde{N} + S - e_1 - k_b e_{2_b} - k_b M(q)\eta\left[e_{2_b} - e_{2\tau}\right], \tag{44}$$

where the auxiliary terms $\tilde{N}(e_1, e_{2_b}, t)$, $N(e_1, e_{2_b}, t)$, $S(q_d, \dot{q}_d, \ddot{q}_d, t) \in \mathbb{R}^n$ are defined as

$$\tilde{N} = N - N_d, \qquad S = N_d + d, \tag{45}$$

$$N = \frac{1}{2}\dot{M}(q)e_{2_b} + M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + F(q) + \alpha M(q)e_{2_b} - \alpha^2 M(q)e_1 + e_1 + \alpha M(q)B\int_{t-\tau}^{t} u(\theta)d\theta,$$

where $\tilde{N}(e_1, e_{2_b}, t)$ and $S(q_d, \dot{q}_d, \ddot{q}_d, t)$ can be upper bounded as

$$\tilde{N} \le \rho_2(\|z_b\|)\|z_b\|, \qquad \|S\| \le \varepsilon_2. \tag{46}$$

In (46), $\varepsilon_2 \in \mathbb{R}^+$ is a known constant, the bounding function $\rho_2(\|z_b\|) \in \mathbb{R}$ is a positive globally invertible nondecreasing function, and $z_b \in \mathbb{R}^{3n}$ is defined as

$$z_b = \begin{bmatrix} e_1^T & e_{2_b}^T & e_b^T \end{bmatrix}^T, \tag{47}$$

where $e_b \in \mathbb{R}^n$ is defined as

$$e_b = \int_{t-\tau}^{t} u(\theta)d\theta.$$

Theorem 2. *The controller given in (42) ensures SUUB tracking in the sense that*

$$\|e_1(t)\| \le \epsilon_0 \exp(-\epsilon_1 t) + \epsilon_2, \tag{48}$$

where $\epsilon_0, \epsilon_1, \epsilon_2 \in \mathbb{R}^+$ denote constants, provided the control gains α and k_b introduced in (37) and (42), respectively are selected according *to the sufficient conditions:*

$$\alpha > \frac{b^2\gamma^2}{4}, \qquad k_{b_3} > \frac{2\bar{\eta}m_2(k_{b_1}+k_{b_2})+\omega k_b^2\tau}{1-2\bar{\eta}m_2},$$
$$\omega\gamma^2 > 2\tau, \tag{49}$$

where $m_2, b, \in \mathbb{R}^+, \bar{\eta} \in \mathbb{R}^+$ are defined in (2), (38) and (40), respectively, and $\gamma, \omega \in \mathbb{R}^+$ are subsequently defined constants.

Remark 3. The second sufficient gain condition indicates that ω can be selected sufficiently small and k_{b_3} can be selected sufficiently large provided $1-2\bar{\eta}m_2 > 0$. The condition that $1-2\bar{\eta}m_2 > 0$ indicates that the constant approximation matrix B must be chosen sufficiently close to $M^{-1}(q)$ so that $\left\|B - M^{-1}(q)\right\|_\infty < \frac{1}{2m_2}$. Experimental results illustrate the performance/robustness of the developed controller with respect to the mismatch between B and $M^{-1}(q)$. Specifically, results indicate an insignificant amount of variation in the performance even when each element of $M^{-1}(q)$ is overestimated by as much as 100%. Different results may be obtained for different systems, but these results indicate that the gain condition is reasonable.

Proof. Let $y_b(t) \in \mathcal{D} \subset \mathbb{R}^{2n+2}$ be defined as

$$y_b(t) \triangleq \begin{bmatrix} e_1^T & e_{2_b}^T & \sqrt{P} & \sqrt{R} \end{bmatrix}^T, \tag{50}$$

where $P(t), R(t) \in \mathbb{R}$ denote LK functionals defined as Richard (2003)

$$P = \omega \int_{t-\tau}^{t}\left(\int_{s}^{t} \|u(\theta)\|^2 \, d\theta\right) ds,$$
$$R = \frac{\bar{\eta}m_2k_b}{2}\int_{t-\tau}^{t} \|e_{2_b}(\theta)\|^2 d\theta,$$

where $\omega \in \mathbb{R}^+$ is a known constant. A positive definite Lyapunov functional candidate $V(y_b, t) : \mathcal{D} \times [0 \quad \infty) \to \mathbb{R}$ is defined as

$$V(y_b,t) \triangleq \frac{1}{2}e_1^Te_1 + \frac{1}{2}e_{2_b}^T M(q) e_{2_b} + P + R, \tag{51}$$

and satisfies the following inequalities

$$\lambda_1\|y_b\|^2 \le V \le \lambda_2\|y_b\|^2, \tag{52}$$

where $\lambda_1, \lambda_2 \in \mathbb{R}^+$ are defined in (23).

Taking the time derivative of (51) and using (37) and (44) yields

$$\begin{aligned}\dot{V} = &-\alpha e_1^Te_1 + e_1^T B e_b + \omega\tau\|u\|^2 + e_{2_b}^T[S + \tilde{N} \\ &- k_b e_2 - k_b M(q)\eta\left(e_{2_b} - e_{2_{b\tau}}\right)] \\ &+ \frac{\bar{\eta}m_2k_b}{2}[\|e_{2_b}\|^2 - \|e_{2_{b\tau}}\|^2] - \omega\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta,\end{aligned} \tag{53}$$

where the Leibniz integral rule was applied to determine the time derivative of $P(t)$ and $R(t)$. Using (2), (38) and (46), the terms in (53) can be upper bounded as

$$\begin{aligned}\dot{V} \le &-\alpha\|e_1\|^2 - k_b\left\|e_{2_b}\right\|^2 + \bar{\eta}m_2k_b\|e_{2_b}\|^2 + \omega\tau\|u\|^2 \\ &+ \|e_{2_b}\|\varepsilon_2 + \|e_{2_b}\|\rho_2(\|z_b\|)\|z_b\| + b\|e_1\|\|e_b\| \\ &+ \bar{\eta}m_2k_b\|e_{2_{b\tau}}\|\|e_{2_b}\| + \frac{\bar{\eta}m_2k_b}{2}[\|e_{2_b}\|^2 - \|e_{2_{b\tau}}\|^2] \\ &- \omega\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta.\end{aligned} \tag{54}$$

The following terms in (54) can be upper bounded by utilizing Young's inequality:

$$b\|e_1\|\|e_b\| \le \frac{b^2\gamma^2}{4}\|e_1\|^2 + \frac{1}{\gamma^2}\|e_b\|^2, \tag{55}$$
$$\bar{\eta}m_2k_b\|e_{2_{b\tau}}\|\|e_{2_b}\| \le \frac{\bar{\eta}m_2k_b}{2}\|e_{2_b}\|^2 + \frac{\bar{\eta}m_2k_b}{2}\|e_{2_{b\tau}}\|^2$$

where $\gamma \in \mathbb{R}^+$ is a known constant. Further, by using the Cauchy–Schwarz inequality, the following term in (55) can be upper bounded as

$$\|e_b\|^2 \le \tau\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta. \tag{56}$$

After adding and subtracting $\frac{\tau}{\gamma^2}\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta$ to (54), and utilizing (42), (43), (55) and (56), the following expression is obtained:

$$\begin{aligned}\dot{V} \le &-\left(\alpha - \frac{b^2\gamma^2}{4}\right)\|e_1\|^2 - (k_{b_3} - \omega k_b^2\tau - 2\bar{\eta}m_2k_b)\|e_{2_b}\|^2 \\ &- \frac{1}{\tau}\left(\omega - \frac{2\tau}{\gamma^2}\right)\|e_b\|^2 - k_{b_1}\|e_{2_b}\|^2 + \rho_2(\|z_b\|)\|z_b\|\|e_{2_b}\| \\ &- k_{b_2}\|e_2\|^2 + \|e_2\|\varepsilon_2 - \frac{\tau}{\gamma^2}\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta.\end{aligned} \tag{57}$$

By completing the squares, the inequality in (57) can be upper bounded as

$$\begin{aligned}\dot{V} \le &-\left\{\beta_2 - \frac{\rho_2^2(\|z_b\|)}{4k_{b_1}}\right\}\|z_b\|^2 - \frac{\tau}{\gamma^2}\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta \\ &+ \frac{\varepsilon_2^2}{4k_{b_2}},\end{aligned} \tag{58}$$

where $\beta_2 \in \mathbb{R}^+$ is denoted as

$$\beta_2 = \min\left[\left(\alpha - \frac{b^2\gamma^2}{4}\right), (k_{b_3} - 2\bar{\eta}m_2k_b - \omega k_b^2\tau), \frac{1}{\tau}\left(\omega - \frac{2\tau}{\gamma^2}\right)\right].$$

Since

$$\begin{aligned}\int_{t-\tau}^{t}\left(\int_{s}^{t}\|u(\theta)\|^2 d\theta\right) ds &\le \tau \sup_{s\in[t,t-\tau]}\left[\int_{s}^{t}\|u(\theta)\|^2 d\theta\right] \\ &= \tau\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta,\end{aligned}$$

the expression in (58) can be rewritten as

$$\begin{aligned}\dot{V} \le &-\left\{\beta_2 - \frac{\rho_2^2(\|z_b\|)}{4k_{b_1}}\right\}\|z_b\|^2 - \frac{\tau}{2\gamma^2}\int_{t-\tau}^{t}\|u(\theta)\|^2 d\theta \\ &- \frac{1}{2\gamma^2}\int_{t-\tau}^{t}\left(\int_{s}^{t}\|u(\theta)\|^2 d\theta\right) + \frac{\varepsilon_2^2}{4k_{b_2}}.\end{aligned} \tag{59}$$

Using the definitions of $z_b(t)$ in (47), $y_b(t)$ in (50), and $u(t)$ in (42), the expression in (59) can be expressed as

$$\dot{V} \le -\bar{\beta}_2\|y_b\|^2 - \left\{\beta_2 - \frac{\rho_2^2(\|z_b\|)}{4k_{b_1}}\right\}\|e_b\|^2 + \frac{\varepsilon_2^2}{4k_{b_2}}, \tag{60}$$

where $\bar{\beta}_2(\|z_b\|) \in \mathbb{R}^+$ is defined as

$$\bar{\beta}_2 = \min\left[\left(\beta_2 - \frac{\rho_2^2(\|z_b\|)}{4k_{b_1}}\right), \frac{k_b\tau}{\gamma^2\bar{\eta}m_2}, \frac{1}{2\omega\gamma^2}\right].$$

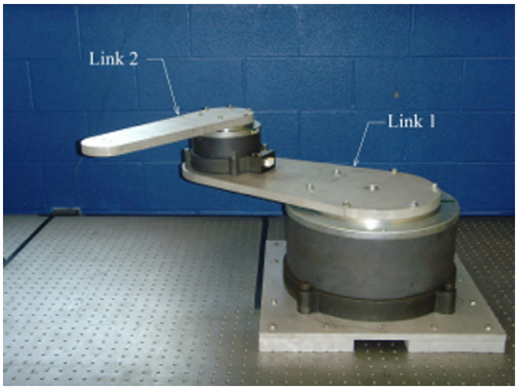

Fig. 1. Experimental testbed consisting of a two-link robot. The input delay in the system was artificially inserted in the control software.

By further utilizing (52), the inequality in (60) can be written as

$$\dot{V} \leq -\frac{\bar{\beta}_2}{\lambda_2}V + \frac{\varepsilon_2^2}{4k_{b_2}}. \tag{61}$$

Consider a set $\mathcal{S}$ defined as

$$\mathcal{S} \triangleq \left\{ z_b(t) \in \mathbb{R}^{3n} \mid \|z_b\| < \rho_2^{-1}(2\sqrt{\beta_2 k_{b_1}}) \right\}. \tag{62}$$

In $\mathcal{S}$, $\bar{\beta}_2(\|z_b\|)$ can be lower bounded by a constant $\delta_2 \in \mathbb{R}^+$ as

$$\delta_2 \leq \bar{\beta}_2(\|z_b\|). \tag{63}$$

Based on (63), the linear differential equation in (61) can be solved as

$$V \leq V(0)e^{-\frac{\delta_2}{\lambda_2}t} + \frac{\varepsilon_2^2\lambda_2}{4k_{b_2}\delta_2}[1 - e^{-\frac{\delta_2}{\lambda_2}t}], \tag{64}$$

provided $\|z_b\| < \rho_2^{-1}\left(2\sqrt{\beta_2 k_{b_1}}\right)$. From (64), if $z_b(0) \in \mathcal{S}$ then k_b can be chosen according to the sufficient conditions in (49) (i.e. a semi-global result) to yield result in (48). Based on the definition of $y_b(t)$, it can be concluded that $e_1(t), e_{2_b}(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$. Given that $e_1(t), e_{2_b}(t), q_d(t), \dot{q}_d(t) \in \mathcal{L}_\infty$ in $\mathcal{S}$, (4), (37) and (42) indicate that $q(t), \dot{q}(t), u \in \mathcal{L}_\infty$ in $\mathcal{S}$. □

4. Experimental results

Experiments for the developed controllers were conducted on a two-link robot shown in Fig. 1. Each robot link is mounted on an NSK direct drive switched reluctance motor (240.0 Nm Model YS5240-GN001, and 20.0 Nm Model YS2020-GN001, respectively). The NSK motors are controlled through power electronics operating in torque control mode. Rotor positions are measured through motor resolver with a resolution of 614 400 pulses/revolution. The control algorithms were executed on a Pentium 2.8 GHz PC operating under QNX. Data acquisition and control implementation were performed at a frequency of 1.0 kHz using the ServoToGo I/O board. The input delay was artificially inserted in the system through the control software (i.e., the control commands to the motors were delayed by a value set by the user). The developed controllers were tested for various values of input delay ranging from 1 ms to 200 ms. The desired link trajectories for link 1 ($q_{d_1}(t)$) and link 2 ($q_{d_2}(t)$) were selected as (in degrees):

$$q_{d_1}(t) = q_{d_2}(t) = 20.0\sin(1.5t)(1 - \exp(-0.01t^3)).$$

The controller developed in (8) (*PID controller with delay compensation*) and the controller developed in (42) (*PD controller with delay compensation*) were compared with traditional PID and PD controllers, respectively, in the presence of input delay in the system. The input delayed two link robot dynamics are modeled as

$$\begin{aligned}\begin{bmatrix} u_{1\tau} \\ u_{2\tau} \end{bmatrix} &= \begin{bmatrix} p_1 + 2p_3\cos(q_2) & p_2 + p_3\cos(q_2) \\ p_2 + p_3\cos(q_2) & p_2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} -p_3\sin(q_2)\dot{q}_2 & -p_3\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ p_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} f_{d_1} & 0 \\ 0 & f_{d_2} \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} f_{s_1} & 0 \\ 0 & f_{s_2} \end{bmatrix}\begin{bmatrix} \tanh(\dot{q}_1) \\ \tanh(\dot{q}_2) \end{bmatrix},\end{aligned}$$

where $p_1, p_2, p_3, f_{d_1}, f_{d_2}, f_{s_1}, f_{s_2} \in \mathbb{R}^+$ are unknown constants, and $\tau \in \mathbb{R}^+$ is the user-defined time delay value. However, the following values: $p_1 = 3.473\ kg.m^2$, $p_2 = 0.196\ kg.m^2$, and $p_3 = 0.242\ kg.m^2$ were used to calculate the inverse inertia matrix for implementing the *PID controller with delay compensation* but were not used to implement the *PD controller with delay compensation.*

The control gains for the experiments were obtained by choosing gains and then adjusting based on performance (in particular, torque saturation). If the response exhibited a prolonged transient response (compared with the response obtained with other gains), the proportional gains were adjusted. If the response exhibited overshoot, derivative gains were adjusted. At a particular input delay value, the control gains were first tuned for the *PID/PD controllers with delay compensation* and then compared with traditional PID/PD controllers. Using the same control gains values as in the *PID/PD controllers with delay compensation*, the control torques for the traditional PID/PD controllers reached pre-set torque limits, leading to an incomplete experimental trial (e.g., if the control torque reaches 20 Nm, which is the set torque limit for the link-2 motor, the control software aborts the experimental trial[3]). Therefore, for each case of input delay (except at 1 ms), control gains for the traditional PID/PD controllers were retuned (i.e., lowered) to avoid torque saturation. In contrast to the above approach, the control gains could potentially have been adjusted using more methodical approaches. For example, the nonlinear system in Stefanovic, Ding, and Pavel (2007) was linearized at several operating points and a linear controller was designed for each point, and the gains were chosen by interpolating, or scheduling the linear controllers. In Fujinaka, Kishida, Yoshioka, and Omatu (2000), a neural network is used to tune the gains of a PID controller. In Nagata, Kuribayashi, Kiguchi, and Watanabe (2007) a genetic algorithm was used to fine tune the gains after initial guess were made by the controller designer. The authors in Killingsworth and Krstic (2006) provide an extensive discussion on the use of extremum seeking for tuning the gains of a PID controller. Additionally, in Kelly, Santibanez, and Loria (2005), the tuning of a PID controller for robot manipulators is discussed.

The experimental results are summarized in Table 1. The error and torque plots for the case when the input delay is 50 ms (as a representative example) are shown in Figs. 2 and 3. The *PD controller with delay compensation* was also tested to observe the sensitivity of the B gain matrix, defined in (37), where the input delay was selected as 100 ms. Each element of the B gain matrix was incremented/decremented by a certain percentage from the inverse inertia matrix (see Table 2). The purpose of this set of experiments was to show that the gain condition discussed in Remark 3 is a sufficient but not a necessary condition, and to explore the performance/robustness of the controller in (42) given inexact approximations of the inertia matrix. The controller exhibited no significant degradation, even when each element of

[3] Instead of aborting the experimental trial, the experiments could have also been performed by utilizing the saturation torque as the control torque in case the computed torque reaches or exceeds the torque limit; but for comparison purposes, the aforementioned criterion was chosen.

Table 1
Summarized experimental results of traditional PID/PD controllers and the PID/PD controllers with delay compensation. The controllers were tested for different input delay values ranging from 1 ms to 200 ms.

Controller	PID		PID + CTR[a]		PD		PD + CTR	
Time delay	Link 1 (°)	Link 2 (°)	Link 1 (°)	Link 2 (°)	Link 1 (°)	Link 2 (°)	Link 1 (°)	Link 2 (°)
RMS error								
1 ms	0.106	0.089	0.109	0.087	0.077	0.083	0.077	0.076
2 ms	0.107	0.125	0.113	0.092	0.065	0.151	0.069	0.065
5 ms	0.129	0.370	0.115	0.077	0.061	0.291	0.076	0.082
10 ms	0.089	0.285	0.131	0.091	0.057	0.505	0.089	0.088
50 ms	1.954	1.272	0.370	0.335	1.037	1.602	0.407	0.336
100 ms	3.137	6.605	1.078	0.726	3.182	5.595	1.159	0.729
200 ms	7.629	6.778	3.118	3.626	14.532	17.586	3.625	2.375
Maximum absolute peak error								
1 ms	0.164	0.173	0.169	0.178	0.124	0.158	0.127	0.150
2 ms	0.172	0.230	0.179	0.18	0.105	0.275	0.114	0.125
5 ms	0.204	0.642	0.179	0.161	0.108	0.509	0.127	0.150
10 ms	0.149	0.512	0.207	0.211	0.107	0.707	0.147	0.200
50 ms	3.430	2.068	0.671	1.196	1.776	2.998	0.774	1.193
100 ms	6.484	11.603	1.964	2.415	5.930	11.551	1.915	2.333
200 ms	14.960	12.569	6.600	10.466	24.629	32.726	5.520	6.878

[a] CTR stands for compensator.

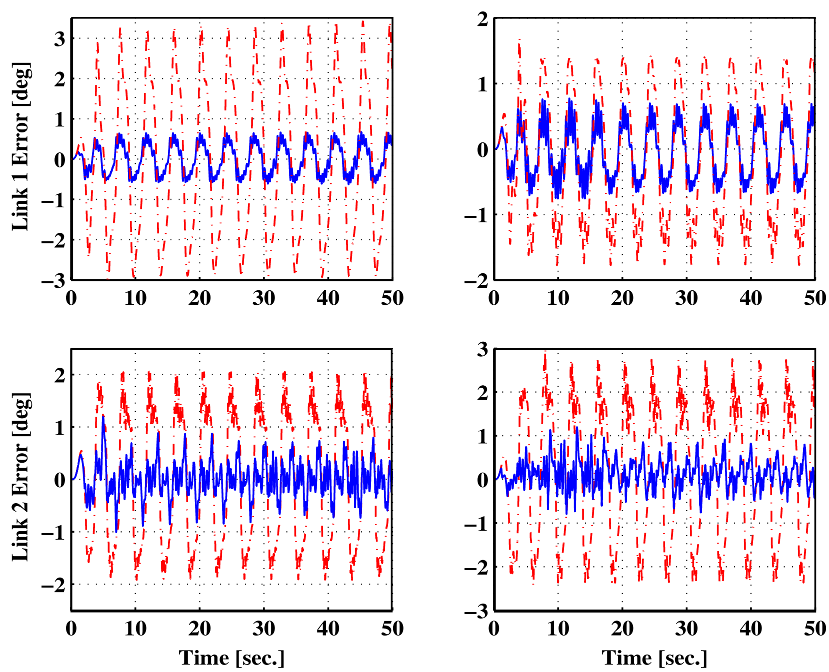

Fig. 2. The top-left and bottom-left plots show the errors of Link 1 and Link 2, respectively, obtained from the PID controller with delay compensation and a traditional PID controller. The top-right and bottom-right plots show the errors of Link 1 and Link 2, respectively, obtained from the PD controller with delay compensation and a traditional PD controller. Errors obtained from the PID/PD + delay compensator are shown as solid lines and the errors obtained from the traditional PID/PD controller are shown as dash–dot lines. The input delay was chosen to be 50 ms.

Table 2
Results compare performance of the PD controller with delay compensation, when the B gain matrix is varied from the known inverse inertia matrix. The input delay value was chosen to be 100 ms. The results indicate that large variations in the gain matrix may be possible.

Elementwise percentage change in inverse inertia matrix	RMS error	
	Link 1 (°)	Link 2 (°)
0	1.172	1.005
+10	1.246	1.168
−10	1.078	0.955
−50	1.583	1.491
+50	1.540	1.249
+100	1.191	1.086
−75	2.948	1.331

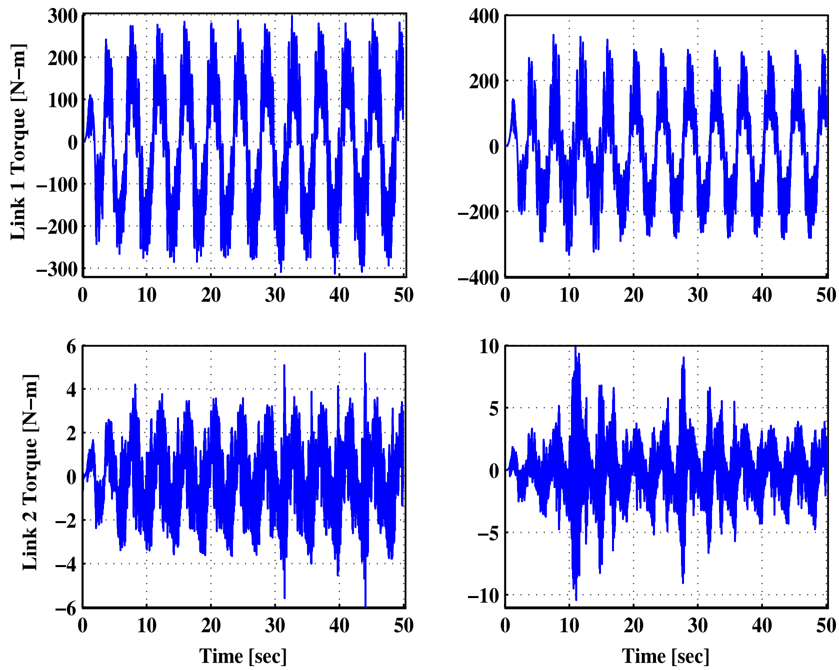

Fig. 3. The top-left and bottom-left plots show the torques of Link 1 and Link 2, respectively, obtained from the PID controller with delay compensation. The top-right and bottom-right plots show the torques of Link 1 and Link 2, respectively, obtained from the PD controller with delay compensation. The input delay was chosen to be 50 ms.

Table 3
Experimental results when the input delay has uncertainty. The input delay value was selected as 100 ms.

Uncertainty in input delay (%)	RMS error			
	PD + CTR		PID + CTR[a]	
	Link 1 (°)	Link 2 (°)	Link 1 (°)	Link 2 (°)
0	1.159	0.730	1.078	0.726
(+)10	1.234	0.966	0.937	0.910
(−)10	1.079	1.215	0.756	0.410
(+)20	1.338	1.548	1.304	1.810
(−)20	1.192	1.773	0.782	0.617
(+)30	1.451	1.761	1.498	0.659
(−)30	1.452	1.322	0.768	0.609
(+)50	1.629	2.513	2.242	1.181
(−)50	1.186	1.450	0.987	0.907
(+)80	3.528	6.819	3.092	1.510
(−)80	1.229	5.408	0.915	2.053
(+)90	4.099	12.020	3.322	1.836
(−)90	3.260	6.041	0.874	2.461
(+)100	4.331	12.445	4.219	3.101
(−)100	3.182	5.595	3.137	6.605

[a] CTR stands for compensator.

the inertia matrix is over-approximated by 100%. However, underestimating the inverse inertia matrix (particularly when deviation from the inverse inertia matrix was 75%), yielded increased tracking errors. Different results may be obtained for different systems. The third set of experiments, given in Table 3 were conducted to show that promising results can be obtained even when the input delay value is not exactly known; however, the tracking error performance degrades with increasing inaccuracy in delay value approximation (e.g., in the case of PD + compensator, the tracking error increases significantly when the delay value is overestimated by 80% or greater). For this set of experiments the input delay was chosen to be 100 ms.

A comparison of the two controllers indicates approximately equal performance. Specifically, Table 1 indicates that at smaller time delays the modified PID controller yielded slightly improved results in Link 1 RMS tracking error, whereas the modified PD controller exhibited slightly improved results for the Link 2 RMS tracking error. For delays of greater than 50 ms, the modified PID controller exhibits an overall improved trend over the modified PD controller, but not a statistically significant difference. Likewise, in terms of maximum absolute error, the modified PID controller has a consistent trend of better performance for Link 1, whereas the modified PD controller has a trend of better performance for Link 2. When comparing the two controllers with regard to robustness to uncertainty in the time delay parameter, the modified PID controller exhibits a favorable trend.

5. Conclusion

Control methods are developed for a class of unknown Euler–Lagrange systems with input delay. The designed controllers have a predictor-based structure to compensate for delays in the input. LK functionals are constructed to aid the stability analysis which yields a semi-global uniformly ultimately bounded result. The experimental results show that the developed controllers have improved performance when compared to traditional PID/PD controllers in the presence of input delay. A key contribution is the development of the first ever controllers to address delay in the input of an uncertain nonlinear system. The result has been heretofore an open challenge because of the need to develop a stabilizing predictor for the dynamic response of an uncertain nonlinear system. To develop the controllers, the time delay was required to be a known constant. While some applications have known delays (e.g., teleoperation Anderson & Spong, 1989, some network delays Liu, Mu, Rees, & Chai, 2006, time constants in biological systems Riener & Fuhr, 1998; Schauer et al., 2005), the development of more generalized results (which have been developed for some linear systems) with unknown time delays remains an open challenge. However, the experimental results illustrated some robustness with regard to the uncertainty in the time delay. The experiments also illustrated that the traditional PI and PID controllers led to control saturation, a problem that is exacerbated by the input delay resulting in a build-up of errors. The proposed controllers did not exhibit control saturation for the considered delay values; however, saturation could potentially occur for longer delays. Future studies will consider the development of predictor-based controllers for uncertain nonlinear systems under the additional constraint of actuator saturation.

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