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Brief paper

Adaptive boundary control of store induced oscillations in a flexible aircraft wing[☆]

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ABSTRACT

An adaptive boundary control strategy is developed for the suppression of store induced oscillations in the bending and twisting deflections of an uncertain flexible aircraft wing. A Lyapunov-based stability analysis is used to show that the total energy in the system, and hence the distributed states of the system, remains bounded and decays asymptotically to zero. Simulation results illustrate the performance of the developed controller.

1. Introduction

Store induced oscillations commonly described as Limit Cycle Oscillations (LCO) occur on current high performance fighter aircraft and are expected to remain an issue for next generation aircraft (Beran, Strganac, Kim, & Nichkawde, 2004). Store induced oscillations are characterized by antisymmetric, non-divergent periodic motion of the wings. Asymmetry in the wing oscillations cause a lateral motion in the fuselage that hinders a pilot's ability to read cockpit instruments and heads-up display which can lead to the premature termination of the mission or avoidance of a region of the flight envelope crucial to combat survivability. Furthermore, questions have been raised regarding the safe release of wing stores, the target acquisition of smart munitions, and the accuracy of unguided ordnance (Bunton & Denegri, 2000). These concerns necessitate the development of a control strategy designed to suppress store induced oscillations.

In a wide range of Mach numbers (Sheta, Harrand, Thompson, & Strganac, 2002), store induced oscillations are prominent. Store induced oscillations in the subsonic range provide additional acceleration and result in additional force on the aircraft, which affects its performance. Experimental investigations of oscillation of nonlinear aeroelastic systems in the subsonic airflow region are found in Platanitis and Strganac (2004), Sheta et al. (2002) and Strganac, Ko, Thompson, and Kurdila (2000). Experimental results in Sheta et al. (2002) indicate the importance of addressing oscillations in the subsonic airflow region.

Previously developed control strategies have focused on suppressing oscillation behavior in a two-dimensional airfoil system. Several of these control strategies require knowledge of the system dynamics, including linear–quadratic regulator (Block & Strganac, 1999; Prime, Cazzolato, Doolan, & Strganac, 2010; Zhang & Ye, 2007), feedback linearization (Ko, Strganac, & Kurdila, 1998), linear reduced order model-based control approaches (Danowsky et al., 2010; Thompson et al., 2011), a Nissim aerodynamic energy-based control approach (Cavagna, Ricci, & Scotti, 2009), and state-dependent Riccati equation and sliding mode control approaches (Elhami & Narab, 2012). Many adaptive control strategies have been developed for uncertainties in the torsional stiffness model such as adaptive feedback control for linear-in-the-parameter uncertainties (Ko, Strganac, & Kurdila, 1999; Strganac et al., 2000). Most recently, a RISE control structure was used to ensure asymptotic tracking of a two-dimensional airfoil section with modeling uncertainties in the structural and aerodynamic properties (Bialy, Pasiliao, Dinh, & Dixon, 2012), and then extended to compensate for actuator saturation (Bialy, Andrews, Curtis, & Dixon, 2013).

Previously, research on control strategies for the suppression of oscillation has been concerned with a two-dimensional airfoil section rather than a full flexible aircraft wing. This work develops an adaptive boundary controller for the suppression of store induced oscillations in a full three-dimensional flexible aircraft

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wing. The dynamics of a flexible aircraft wing can be modeled, using Hamilton's principle (Hodges & Dowell, 1974; Hodges & Ormiston, 1976; Martins, Mohamed, Tokhi, Sá da Costa, & Botto, 2003; Morita et al., 2002; Zhang, Xu, Nair, & Chellaboina, 2005; Ziabari & Ghadiri, 2010), as a set of partial differential equations (PDEs) and associated boundary conditions.

There are two boundary control methodologies that have been developed for a system described by a set of PDEs. The first method approximates the PDE system with a finite number of ordinary differential equations (ODE) using operator theoretic tools (Bucci & Lasiecka, 2010; Byrnes, Laukó, Gilliam, & Shubov, 2000; Luo, 1993; Luo & Guo, 1995) or Galerkin and Rayleigh–Ritz methods (Christofides & Daoutidis, 1997; Meirovitch, 1967; Shawky, Ordys, & Grimble, 2002). A boundary controller is then designed using the resulting reduced-order model. The primary concern with using a reduced-order model for the control design is the potential for spillover instabilities (Balas, 1978; Meirovitch & Baruh, 1983), in which the controller excites higher-order modes that were neglected in the approximation. In special cases, the placement of actuators and sensors can guarantee the neglected modes are not excited (Balas, 1982). Specifically, placing actuators at known zero locations of the higher-order modes will alleviate spillover instabilities; however, this can conflict with the desire to place actuators away from the zeros of the controlled modes.

The alternative approach is to design the controller based on the full PDE system where model reduction techniques are only required for implementation purposes. A PDE backstepping strategy, described in Krstic and Smyshlyaev (2008), constructs a state transformation using an invertible Volterra integral. The transformation maps the original system to an exponentially stable target system. Due to the invertibility of the transform, stability of the target system translates to stability of the closed-loop system consisting of the original PDE and boundary feedback control. While this method avoids spillover instabilities, it is limited to linear PDEs and nonlinear PDEs of a particular form. The boundary control strategy described in de Queiroz, Dawson, Nagarkatti, and Zhang (2000) and de Queiroz and Rahn (2002) uses Lyapunov-based design and analysis arguments to stabilize PDE systems. The essence of the analysis is the assumption that for a real physical system, if the energy of the system is bounded, then the states that compose the energy are also bounded. Based on this assumption, a Lyapunov-based stability analysis is used to show that the energy in the closed-loop system remains bounded. A PDE-based boundary control approach has been previously used to stabilize fluid flow through a channel (Vazquez & Krstic, 2007), maneuver flexible robotic arms (de Queiroz, Dawson, Agarwal, & Zhang, 1999), control the bending in an Euler beam (Fard & Sagatun, 2001; He, Ge, How, Choo, & Hong, 2011; Siranosian, Krstic, Smyshlyaev, & Bement, 2011), regulate a flexible rotor system (de Queiroz & Rahn, 2002; Nagarkatti, Dawson, de Queiroz, & Costic, 2001), and track the net aerodynamic force or moment of a flapping wing aircraft (Paranjape, Guan, Chung, & Krstic, 2013).

Many PDE-based and ODE-based control strategies have been developed to stabilize the bending of a flexible beam such as Fard and Sagatun (2001), Luo (1993), Luo and Guo (1995) and Siranosian et al. (2011); however, this collection of work is focused on structural beams and robotic arms and therefore do not encounter the closed-loop interactions between the structural dynamics and aerodynamics intrinsic to aircraft systems. Recently, the work in Paranjape et al. (2013) used the PDE-backstepping method described in Krstic and Smyshlyaev (2008) to track the net aerodynamic forces on a flapping wing UAV whose dynamics are represented by linear PDEs. The control objective in Paranjape et al. (2013) was not concerned with the performance of the distributed states, rather it focused on controlling the spatial integral of the state variables.

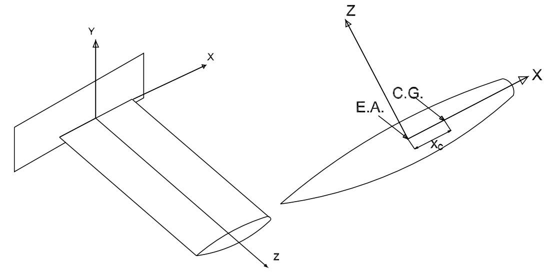

Fig. 1. Schematic of the wing section., where E.A. denotes the elastic axis and C.G. denotes the center of gravity.

The focus of the current work is the design of a controller to suppress store induced oscillations in an aircraft wing described by uncertain coupled nonlinear PDEs via regulation of the state variables. An adaptive boundary controller is designed to ensure the distributed states of the flexible wing are regulated asymptotically. The challenge in this problem is that the uncertain nonlinear PDE cannot be transformed into an exponentially stable target system using the Voltera integral strategy in Krstic and Smyshlyaev (2008). As a result, the controller is developed through a Lyapunov-based analysis. The Lyapunov analysis is facilitated by examining the energy in the aircraft wing and through the development of novel auxiliary terms introduced to yield favorable outcomes from the derivative of the wing energy. Simulation results demonstrate the open-loop oscillation and how the controller is applied to damp out the oscillation.

2. Flexible aircraft wing model

Consider a flexible wing of length $l \in \mathbb{R}$, mass per unit span of $\rho \in \mathbb{R}$, moment of inertia per unit length of $I_w \in \mathbb{R}$, and bending and torsional stiffnesses of $EI \in \mathbb{R}$ and $GJ \in \mathbb{R}$, respectively, with a store of mass $m_s \in \mathbb{R}$ and moment of inertia $J_s \in \mathbb{R}$ attached at the wing tip. The bending and twisting dynamics of the flexible wing are described by the following PDE system[1]

$$\begin{aligned}\bar{L}_w\varphi(y,t) &= \rho\omega_{tt}(y,t) - \rho x_c \sin(\varphi(y,t))\,\varphi_t^2(y,t) \\ &\quad + \rho x_c \cos(\varphi(y,t))\,\varphi_{tt}(y,t) + EI\omega_{yyyy}(y,t), \qquad (1)\end{aligned}$$

$$\begin{aligned}\bar{M}_w\varphi(y,t) &= \left(I_w + \rho x_c^2\right)\varphi_{tt}(y,t) \\ &\quad + \rho x_c \cos(\varphi(y,t))\,\omega_{tt}(y,t) - GJ\varphi_{yy}(y,t), \qquad (2)\end{aligned}$$

where $\omega : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and $\varphi : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ denote the bending and twisting displacements, respectively, $y \in [0, l]$ denotes spanwise location on the wing, $x_c \in \mathbb{R}$ represents the distance from the wing elastic axis to the wing center of gravity (as shown in Fig. 1), and $\bar{L}_w \in \mathbb{R}$ and $\bar{M}_w \in \mathbb{R}$ denote aerodynamic lift and moment coefficients, respectively.

Remark 1. *EI*, *GJ* and other wing parameters are considered to be spatially invariant, although this work may be extended to include spatially varying wing parameters by incorporating a similar approach as in Ishihara and Nguyen (2014).

In (1) and (2), the subscripts t and y denote partial derivatives with respect to time or the spanwise position along a wing. The boundary conditions for tip-based control are

$$\omega(0,t) = \omega_y(0,t) = \omega_{yy}(l,t) = \varphi(0,t) = 0, \qquad (3)$$

$$L_{tip}(t) = m_s\omega_{tt}(l,t) - m_s x_s \sin(\varphi(l,t))\,\varphi_t^2(l,t)$$

[1] Damping terms (e.g., Kelvin–Voigt damping Kangsheng & Zhuangyi, 1998) could be added to the model; however, the subsequent development illustrates how to mitigate the oscillation through the closed-loop control.

$$- EI\omega_{yyy}(l,t) + m_s x_s \cos(\varphi(l,t))\varphi_{tt}(l,t), \quad (4)$$

$$M_{tip}(t) = \left(m_s x_s^2 + J_s\right)\varphi_{tt}(l,t) + GJ\varphi_y(l,t) + m_s x_s \cos(\varphi(l,t))\omega_{tt}(l,t), \quad (5)$$

where $L_{tip} : \mathbb{R} \to \mathbb{R}$ and $M_{tip} : \mathbb{R} \to \mathbb{R}$ denote the aerodynamic lift and moment at the wing tip, and $x_s \in \mathbb{R}$ represents the distance from the wing elastic axis to the store center of gravity. To simplify the notation, dependency of φ and ω on y and t will be suppressed in the rest of the paper, i.e. $\varphi(y,t) \equiv \varphi$, $\omega(y,t) \equiv \omega$, $\varphi_t(y,t) \equiv \varphi_t$, $\omega_t(y,t) \equiv \omega_t$, and all time and spatial derivatives of ω and φ. All model parameters are assumed to be uncertain constants. Furthermore, based on Remark 5.1 in de Queiroz et al. (2000), the system is assumed to have following properties.

Assumption 1. Potential energy of the system, $E_P(t) \triangleq \frac{1}{2}\int_0^l \left(EI\omega_{yy}^2 + GJ\varphi_y^2\right) dy$ is assumed to be bounded $\forall t \in [0,\infty)$, and $\frac{\partial^n \omega}{\partial y^n}$ and $\frac{\partial^m \varphi}{\partial y^m}$ are assumed to be bounded, uniformly in y $\forall t \in [0,\infty)$ for $n = 2, 3, 4$ and $m = 1, 2$.

Assumption 2. The kinetic energy of the system,

$$E_K(t) \triangleq \frac{1}{2}\int_0^l \left(\rho\omega_t^2 + 2\rho x_c \cos(\varphi)\varphi_t\omega_t\right) dy + \frac{1}{2}m_s\omega_t^2(l,t) + \frac{1}{2}J_s\varphi_t^2(l,t) + \frac{1}{2}\int_0^l \left(\left(I_w + \rho x_c^2\right)\varphi_t^2\right) dy,$$

is assumed to be bounded $\forall t \in [0,\infty)$, and $\frac{\partial^q \omega}{\partial t^q}$ and $\frac{\partial^q \varphi}{\partial t^q}$ are assumed to be bounded, uniformly in t $\forall y \in [0,l]$ for $q = 1, 2, 3$.

Assumption 3. The subsequent control development is based on the assumption that $\omega_t(l,\cdot)$, $\omega_{yyy}(l,\cdot)$, $\varphi_t(l,\cdot)$, $\varphi_y(l,\cdot)$, $\varphi_{ty}(l,\cdot)$, $\omega_{tyyy}(l,\cdot)$ and $\varphi(l,\cdot)$ are measurable.

Remark 2. In practice, time variation of both wing tip bending and twisting deflection can be measured by transducers. Spatial variation of bending deflection can be measured by strain gauges (as mentioned in Fard and Sagatun (2001)) or shear sensors (as discussed in de Queiroz et al. (1999)), based on the order of differentiation. Time variations of these sensor measurements can be obtained through numerical methods. Such measurements and numerical methods can introduce noise. While motivation exists for additional research to eliminate these higher-order measurements, the subsequent simulation section includes measurement noise that provides insight of the controller's robustness. Advances in fiber optic sensing (both Long Period Fiber Gratings and Fiber Bragg Grating) can also be used to measure the deformation of the wing. For example, fiber optic strain data from a ground load test of a full-scale aircraft wing can be used to measure the deflection of the wing and corrugated long-period fiber grating can be used to measure strain, bending and torsion of the wing as in Durana et al. (2009).

3. Adaptive boundary control development

The control objective is to ensure the wing bending and twisting deformations are regulated in the sense that $\omega \to 0$ and $\varphi \to 0 \forall y \in [0,l]$ as $t \to \infty$ via boundary control at the wing tip. To facilitate the subsequent stability analysis, let the auxiliary signal $e : [0,\infty) \to \mathbb{R}^2$ and $\bar{M}(t) : [0,\infty) \to \mathbb{R}^{2\times 2}$ be defined as

$$e(t) \triangleq \begin{bmatrix} \omega_t(l,t) - \omega_{yyy}(l,t) \\ \varphi_t(l,t) + \varphi_y(l,t) \end{bmatrix}, \quad (6)$$

$$\bar{M}(t) \triangleq \begin{bmatrix} m_s & m_s x_s \cos(\varphi(l,t)) \\ m_s x_s \cos(\varphi(l,t)) & m_s x_s^2 + J_s \end{bmatrix}.$$

The open-loop dynamics of the auxiliary signal are obtained by multiplying the time derivative of e by $\bar{M}$ to yield

$$\bar{M}(t)\dot{e}(t) = \bar{M}(t)\begin{bmatrix} \omega_{tt}(l,t) \\ \varphi_{tt}(l,t) \end{bmatrix} + \bar{M}(t)\begin{bmatrix} -\omega_{tyyy}(l,t) \\ \varphi_{ty}(l,t) \end{bmatrix}. \quad (7)$$

Substituting the boundary conditions in (4) and (5) into (7) and after some algebraic manipulation, (7) can be expressed as

$$\bar{M}(t)\dot{e}(t) = U(t) - \frac{1}{2}\dot{\bar{M}}(t)e(t) + Y(t)\theta, \quad (8)$$

where $U(t) \triangleq \left[L(t) \quad M(t)\right]^T : [0,\infty) \to \mathbb{R}^2$, $\theta \in \mathbb{R}^5$ is a vector of unknown parameters, and $Y : [0,\infty) \to \mathbb{R}^{2\times 5}$ is a regression matrix of known quantities. Specifically, Y and θ are defined as

$$Y(t) \triangleq \begin{bmatrix} Y_{11} & \omega_{yyy}(l,t) & -\omega_{tyyy}(l,t) & 0 & 0 \\ Y_{21} & 0 & 0 & -\varphi_y(l,t) & \varphi_{ty}(l,t) \end{bmatrix},$$

$$\theta \triangleq \left[m_s x_s \quad EI \quad m_s \quad GJ \quad \left(m_s x_s^2 + J_s\right)\right]^T,$$

[2]where $Y_{11} = \frac{1}{2}\sin(\varphi(l,t))\left(\varphi_t^2(l,t) - \varphi_t(l,t)\varphi_y(l,t)\right) + \cos\left(\varphi(l,t)\right)\varphi_{ty}(l,t)$ and $Y_{21} = \frac{1}{2}\sin(\varphi(l,t))\varphi_t(l,t)\left(\omega_{yyy}(l,t) - \omega_t(l,t)\right) - \cos(\varphi(l,t))\omega_{tyyy}(l,t)$.

Based on the open-loop dynamics in (8), the boundary control is designed as

$$U(t) = -Ke(t) - Y(t)\hat{\theta}, \quad (9)$$

where $K \in \mathbb{R}$ is a positive constant control gain, and $\hat{\theta} : [0,\infty) \to \mathbb{R}^5$ is a vector of estimates of the uncertain parameters in θ. The vector of parameter estimates $\hat{\theta}(t)$ is updated according to the gradient update law defined as

$$\dot{\hat{\theta}}(t) = \Gamma Y(t)^T e(t), \quad (10)$$

where $\Gamma \in \mathbb{R}^{5\times 5}$ is a positive definite control gain.[3] Substituting (9) into (8) yields

$$\bar{M}(t)\dot{e}(t) = -\frac{1}{2}\dot{\bar{M}}(t)e(t) - Ke(t) + Y\tilde{\theta}(t), \quad (11)$$

where $\tilde{\theta}(t) \triangleq \theta - \hat{\theta}(t)$.

4. Lyapunov-based stability analysis

To facilitate the subsequent stability analysis, let the auxiliary terms $E_T : [0,\infty) \to \mathbb{R}$ and $E_c : [0,\infty) \to \mathbb{R}$ be defined as

$$E_T(t) \triangleq \frac{1}{2}\int_0^l \left(\rho\omega_t^2 + 2\rho x_c \cos(\varphi)\varphi_t\omega_t + EI\omega_{yy}^2\right) dy + \frac{1}{2}\int_0^l \left(\left(I_w + \rho x_c^2\right)\varphi_t^2 + GJ\varphi_y^2\right) dy, \quad (12)$$

$$E_c(t) \triangleq \beta_1 \int_0^l \rho\omega_y y\left(\omega_t + x_c\cos(\varphi)\varphi_t\right) dy + \beta_1\int_0^l \varphi_y y\left(I_w + \rho x_c^2\right)\varphi_t dy + \beta_1\int_0^l \varphi_y y\rho x_c \cos(\varphi)\omega_t dy, \quad (13)$$

2 Unknown parameters EI and GJ can be upper bounded by known constants $\bar{EI}$ and $\bar{GJ}$ respectively.

3 The subsequent stability analysis ensures the parameter estimate will remain bounded. In practice, a projection algorithm (cf. Dixon, Behal, Dawson, & Nagarkatti, 2003; Krstic, Kanellakopoulos, & Kokotovic, 1995) can be used to keep the parameter estimates within a specified bound.

where $\beta_1 \in \mathbb{R}$ is a positive weighting constant. Note that E_T is analogous to the energy in the wing and E_c contains cross terms used to facilitate the stability analysis. Using Young's inequality, an upper bound on E_T can be expressed as

$$E_T(t) \le \frac{1}{2}\max\Big\{(\rho+\rho\,|x_c|)\,,\,\left(I_w+\rho x_c^2+\rho\,|x_c|\right),EI,GJ\Big\}E_b(t),$$

where $E_b : [0, \infty) \to \mathbb{R}$ is defined as

$$E_b(t) \triangleq \int_0^l \left(\omega_t^2+\omega_{yy}^2+\varphi_t^2+\varphi_y^2\right)dy. \tag{14}$$

In a similar manner, E_T can be lower bounded as

$$E_T(t) \ge \frac{1}{2}\min\Big\{(\rho-\rho\,|x_c|)\,,\,\left(I_w+\rho x_c^2-\rho\,|x_c|\right),EI,GJ\Big\}E_b(t). \tag{15}$$

Remark 3. Provided that $|x_c| < 1$ and $I_w > \rho x_c^2 - \rho\,|x_c|$, E_T will be non-negative. The conditions $|x_c| < 1$ and $I_w > \rho x_c^2 - \rho|x_c|$ are engineering design considerations that ensure the store is mounted sufficiently close to the wing center of mass Thompson and Strganac (2005).

After using Young's inequality, the cross term E_c can be upper bounded as

$$\begin{aligned}|E_c(t)| \le{}& \beta_1\rho l\,(1+|x_c|)\int_0^l \omega_t^2 dy\\ &+\beta_1 l\left(I_w+\rho x_c^2+\rho\,|x_c|\right)\int_0^l\left(\varphi_t^2+\varphi_y^2\right)dy\\ &+\beta_1\rho l\,(1+|x_c|)\int_0^l \omega_y^2 dy.\end{aligned} \tag{16}$$

Lemma A.12 in de Queiroz et al. (2000) can be applied to the third integral in (16) to yield

$$\begin{aligned}|E_c(t)| \le{}& \beta_1\rho l\,(1+|x_c|)\int_0^l \omega_t^2 dy\\ &+\beta_1\rho l^3\,(1+|x_c|)\int_0^l \omega_{yy}^2 dy\\ &+\beta_1 l\left(I_w+\rho x_c^2+\rho\,|x_c|\right)\int_0^l\left(\varphi_t^2+\varphi_y^2\right)dy\\ \le{}& \beta_1 l\max\Big\{(\rho+\rho\,|x_c|)\,,\,l^2\,(\rho+\rho\,|x_c|)\,,\\ &\left(I_w+\rho x_c^2+\rho\,|x_c|\right)\Big\}E_b(t).\end{aligned} \tag{17}$$

From (17), E_c can be lower bounded as

$$\begin{aligned}E_c(t) \ge{}& -\beta_1 l\max\Big\{(\rho+\rho\,|x_c|)\,,\,l^2\,(\rho+\rho\,|x_c|)\,,\\ &\left(I_w+\rho x_c^2+\rho\,|x_c|\right)\Big\}E_b(t).\end{aligned} \tag{18}$$

From (15) and (18), if β_1 is selected as $\beta_1 < \frac{\delta_1}{2l\delta_2}$, where

$$\delta_1 \triangleq \min\left\{(\rho-\rho\,|x_c|)\,,\,\left(I_w+\rho x_c^2-\rho\,|x_c|\right),EI,GJ\right\},$$

$$\delta_2 \triangleq \max\Big\{(\rho+\rho\,|x_c|)\,,\,l^2(\rho+\rho|x_c|)\,,\left(I_w+\rho x_c^2+\rho|x_c|\right)\Big\}$$

then

$$\zeta_1 E_b(t) \le E_T(t)+E_c(t) \le \zeta_2 E_b(t) \tag{19}$$

where the positive constants ζ_1 and ζ_2 are defined as

$$\zeta_1 \triangleq \frac{1}{2}\delta_1-\beta_1 l\delta_2, \qquad \zeta_2 \triangleq \frac{1}{2}\delta_2+\beta_1 l\delta_2.$$

Theorem 1. *The boundary control law in (9) along with the adaptive update law in (10) ensure the system states $\omega \to 0$ and $\varphi \to 0$ $\forall y \in [0, l]$ as $t \to \infty$ provided the following sufficient gain conditions are satisfied:*

$$K > \frac{1}{2}\max\left\{\bar{EI}+\beta_1\bar{EI}l,\,K_\beta\bar{GJ}\right\}, \tag{20}$$

$$\beta_1 l < K_\beta, \tag{21}$$

$$\beta_1\rho-\beta_1\rho x_c-\bar{L}_w > 0, \tag{22}$$

$$\frac{3\bar{EI}}{2}-\frac{\bar{L}_w l^3}{2} > 0, \tag{23}$$

$$\beta_1\left(I_w+\rho x_c^2\right)-\beta_1\rho x_c-\bar{M}_w > 0, \tag{24}$$

$$\beta_1\bar{GJ}-\beta_1\bar{M}_w l^3-\beta_1\bar{M}_w l-\beta_1\bar{L}_w l^3-\left(\bar{M}_w+\bar{L}_w\right)l^2 > 0, \tag{25}$$

$$\beta_1\bar{EI}l+\bar{EI}-\beta_1\rho-\beta_1\rho x_c l > 0, \tag{26}$$

$$\bar{GJ}-\beta_1 l\left(I_w+\rho x_c^2\right)-\beta_1\rho x_c l > 0. \tag{27}$$

Remark 4. The sufficient gain conditions in (20)–(27) can be satisfied by a combination of gain selection and engineering design consideration. Selection of the wing aerodynamic properties can be done to satisfy aircraft performance criteria (e.g., minimum takeoff distance, maximum range, etc.). The structural properties of the wing can then be selected to satisfy the sufficient conditions. Increasing the stiffness and mass of the wing or mounting the store closer to the wing center of mass will satisfy the sufficient conditions. A set of wing and store parameters satisfying these conditions are listed in Thompson and Strganac (2005).

Remark 5. The gain conditions in (20)–(27) may be further simplified and potentially less conservative by using the Wirtinger bound and the inequalities in Mitrinovic, Pecaric, and Fink (1992). Furthermore, an extension of this work can be done by incorporating spatial variation of wing parameters, and the use of the Wirtinger bound for such an extension may provide more flexibility in choosing control gains.

Proof. Let $V_L : [0, \infty) \to \mathbb{R}^+$, continuously differentiable function defined as

$$V_L(t) \triangleq E_T(t)+E_c(t)+\frac{e(t)^T\bar{M}(t)e(t)}{2}+\frac{\tilde{\theta}(t)^T\Gamma^{-1}\tilde{\theta}(t)}{2}. \tag{28}$$

Note that the nontrivial cases for which V_L is zero (e.g., $\tilde{\theta} = 0$, and when φ and ω are constants) will not occur because of the essential boundary conditions in (3). Based on the structure of V_L in (28) and the inequalities in (19), V_L can be bounded as

$$V_L(t) \ge \zeta_1 E_b(t)+\frac{\lambda_{min}}{2}\left(\bar{M}(t)\,\|e(t)\|^2+\Gamma^{-1}\right)\left\|\tilde{\theta}(t)\right\|^2, \tag{29}$$

$$V_L(t) \le \zeta_2 E_b(t)+\frac{\lambda_{max}}{2}\left(\bar{M}(t)\,\|e(t)\|^2+\Gamma^{-1}\right)\left\|\tilde{\theta}(t)\right\|^2, \tag{30}$$

where $\lambda_{min}(\xi)$ and $\lambda_{max}(\xi)$ denote the minimum and maximum eigenvalue of ξ, respectively. Differentiating (28) and substituting (10) and (11) into the resulting expression yields

$$\dot{V}_L(t) = \dot{E}_T(t)+\dot{E}_c(t)-e(t)^T Ke(t). \tag{31}$$

In (31), $\dot{E}_T$ is determined by differentiating (12) with respect to time to obtain

$$\begin{aligned}\dot{E}_T(t) ={}& \int_0^l \omega_t\left(\rho\omega_{tt}+\rho x_c\cos(\varphi)\,\varphi_{tt}-\rho x_c\sin(\varphi)\,\varphi_t^2\right)dy\\ &+\int_0^l\left(EI\omega_{yy}\omega_{tyy}+GJ\varphi_y\varphi_{ty}\right)dy\\ &+\int_0^l\left(\left(I_w+\rho x_c^2\right)\varphi_{tt}+\rho x_c\cos(\varphi)\,\omega_{tt}\right)\varphi_t dy.\end{aligned} \tag{32}$$

Substituting (1) and (2) into the first two integrals of (32) yields

$$\dot{E}_T(t) = \int_0^l \left(\bar{L}_w\varphi\omega_t + \bar{M}_w\varphi\varphi_t\right) dy - \int_0^l EI\omega_t\omega_{yyyy}dy + \int_0^l EI\omega_{yy}\omega_{tyy}dy + \int_0^l GJ\varphi_t\varphi_{yy}dy + \int_0^l GJ\varphi_y\varphi_{ty}dy. \quad (33)$$

Integrating the third and fifth integrals in (33) by parts and applying the boundary conditions of the PDE system results in

$$\int_0^l EI\omega_{yy}\omega_{tyy}dy = -EI\omega_{yyy}(l,t)\omega_t(l,t) + \int_0^l EI\omega_t\omega_{yyyy}dy, \quad (34)$$

$$\int_0^l GJ\varphi_y\varphi_{ty}dy = GJ\varphi_y(l,t)\varphi_t(l,t) - \int_0^l GJ\varphi_t\varphi_{yy}dy. \quad (35)$$

Using the expressions in (34) and (35) and using the auxiliary signal definition in (6), (33) can be rewritten as

$$\dot{E}_T(t) = \int_0^l (\bar{L}_w\varphi\omega_t + \bar{M}_w\varphi\varphi_t)dy + e(t)^T \begin{bmatrix} \frac{EI}{2} & 0 \\ 0 & \frac{k_\beta GJ}{2} \end{bmatrix} e(t) - \frac{EI}{2}\left(\omega_t^2(l,t) + \omega_{yyy}^2(l,t)\right) - \frac{K_\beta GJ}{2}\left(\varphi_y^2(l,t) + \varphi_t^2(l,t)\right). \quad (36)$$

After integrating and using Young's inequality and Lemma A.12 from de Queiroz et al. (2000), $\dot{E}_c$ can be upper bounded as

$$\begin{aligned}\dot{E}_c(t) \le & -(1-x_c)\frac{\beta_1\rho}{2}\int_0^l \omega_t^2 dy + \frac{\beta_1 EIl}{2}e_1^2 \\ & - \left(\frac{3EI}{2} - \frac{\bar{L}_w l^3}{2}\right)\beta_1\int_0^l \omega_{yy}^2 dy \\ & - \left(I_w + \rho x_c^2 - \rho x_c\right)\frac{\beta_1}{2}\int_0^l \varphi_t^2 dy \\ & + \frac{1}{2}\beta_1 l\left(I_w + \rho x_c^2\right)\varphi_t^2(l,t) - \frac{\beta_1 EIl}{2}\omega_y^2(l,t) \\ & - \left(GJ - \bar{M}_w l^3 - \bar{M}_w l - \bar{L}_w l^3\right)\frac{\beta_1}{2}\int_0^l \varphi_y^2 dy \\ & - \frac{\beta_1 EIl}{2}\omega_{yyy}^2(l,t) + \frac{1}{2}\beta_1 GJl\varphi_y^2(l,t) \\ & + \frac{1}{2}\beta_1\rho l\omega_t^2(l,t) + \beta_1\rho x_c l\varphi_t(l,t)\,\omega_t(l,t), \end{aligned} \quad (37)$$

where e_1 denotes the first element of the vector e, (i.e., $e_1(t) \triangleq \omega_t(l,t) - \omega_{yyy}(l,t)$). Inserting (36) and (37) into (31) and using Young's inequality yields

$$\begin{aligned}\dot{V}_L(t) \le & -\frac{1}{2}\left(\beta_1\rho - \beta_1\rho x_c - \bar{L}_w\right)\int_0^l \omega_t^2 dy - \frac{EI}{2}\beta_1 l\omega_{yyy}^2(l,t) \\ & - \frac{1}{2}\left(-\beta_1\bar{L}_w l^3 - \left(\bar{M}_w + \bar{L}_w\right)l^2\right)\int_0^l \varphi_y^2 dy \\ & - \frac{1}{2}\left(\beta_1\left(I_w + \rho x_c^2\right) - \beta_1\rho x_c - \bar{M}_w\right)\int_0^l \varphi_t^2 dy \\ & - \frac{1}{2}\left(\beta_1 GJ - \beta_1\bar{M}_w l^3 - \beta_1\bar{M}_w l\right)\int_0^l \varphi_y^2 dy \\ & - \frac{GJ}{2}\left(K_\beta - \beta_1 l\right)\varphi_y^2(l,t) - \frac{EI}{2}\omega_{yyy}^2(l,t) \\ & - \left(\frac{3EI}{2} - \frac{\bar{L}_w l^3}{2}\right)\beta_1\int_0^l \omega_{yy}^2 dy \\ & - \frac{1}{2}\left(\beta_1 EIl + EI - \beta_1\rho l - \beta_1\rho x_c l\right)\omega_t^2(l,t) \\ & - \frac{1}{2}\left(K_\beta GJ - \beta_1 l\left(I_w + \rho x_c^2\right) - \beta_1\rho x_c l\right)\varphi_t^2(l,t) \\ & - \left(K - \frac{1}{2}\max\left\{EI + \beta_1 EIl, K_\beta GJ\right\}\right)\|e(t)\|^2. \end{aligned} \quad (38)$$

Provided the sufficient conditions in (20)–(23) are satisfied, (38) can be expressed as

$$\dot{V}_L(t) \le -\lambda_1 E_b(t) - \lambda_2 e^2(t) \triangleq -g(t), \quad (39)$$

where $\lambda_1 \in \mathbb{R}$ and $\lambda_2 \in \mathbb{R}$ are positive constants defined as

$$\begin{aligned}\lambda_1 \triangleq \frac{1}{2}\min\Big\{ & \beta_1\rho - \beta_1\rho x_c - \bar{L}_w,\ 3EI - \bar{L}_w l^3, \\ & \beta_1\left(I_w + \rho x_c^2\right) - \beta_1\rho x_c - \bar{M}_w, \\ & \beta_1\left(GJ - \bar{M}_w l^3 - \bar{M}_w l - \bar{L}_w l^3\right) - \left(\bar{M}_w + \bar{L}_w\right)l^2\Big\},\end{aligned}$$

$$\lambda_2 \triangleq K - \frac{1}{2}\max\left\{EI + \beta_1 EIl, GJ\right\}.$$

From (28) and (39), $V_L \in \mathcal{L}_\infty$; hence, $E_b \in \mathcal{L}_\infty$, $e \in \mathcal{L}_\infty$, and $\tilde{\theta} \in \mathcal{L}_\infty$. Since $E_b \in \mathcal{L}_\infty$, it can be concluded that $\int_0^l \omega_{yy}^2 dy \in \mathcal{L}_\infty$ and $\int_0^l \varphi_y^2 dy \in \mathcal{L}_\infty$; hence, the elastic potential energy in the wing $E_P \in \mathcal{L}_\infty$ and by Assumption 1, $\omega_{yyy}(l,\cdot) \in \mathcal{L}_\infty$ and $\varphi_y(l,\cdot) \in \mathcal{L}_\infty$. Since $e \in \mathcal{L}_\infty$, $\omega_{yyy}(l,\cdot) \in \mathcal{L}_\infty$, and $\varphi_y(l,\cdot) \in \mathcal{L}_\infty$, (6) can be used to show $\omega_t(l,\cdot) \in \mathcal{L}_\infty$ and $\varphi_t(l,\cdot) \in \mathcal{L}_\infty$. Since $\omega_t(l,\cdot) \in \mathcal{L}_\infty$, $\varphi_t(l,\cdot) \in \mathcal{L}_\infty$, and $E_b \in \mathcal{L}_\infty$, the kinetic energy of the system $E_K \in \mathcal{L}_\infty$ and by Assumption 2, $\frac{\partial^q\omega}{\partial t^q}$ and $\frac{\partial^q\phi}{\partial t^q}$ are bounded, uniformly in $t \forall y \in [0,l]$ for $q = 1, 2, 3$. Eqs. (4) and (5) can be used to show that the boundary control input, $U \in \mathcal{L}_\infty$. Differentiating g from (39) with respect to time yields

$$\dot{g}(t) = \lambda_1\dot{E}_b(t) + 2\lambda_2 e(t)\dot{e}(t), \quad (40)$$

where

$$\dot{E}_b(t) = 2\int_0^l \left(\omega_t\omega_{tt} + \omega_{yy}\omega_{tyy} + \varphi_t\varphi_{tt} + \varphi_{ty}\varphi_y\right)dy. \quad (41)$$

After integrating by parts the second and fourth terms in (41), $\dot{E}_b$ can be expressed as

$$\dot{E}_b(t) = 2\int_0^l \left(\omega_t\left(\omega_{tt} + \omega_{yyyy}\right) + \varphi_t\left(\varphi_{tt} - \varphi_{yy}\right)\right)dy - 2\omega_t(l,t)\,\omega_{yyy}(l,t) + 2\varphi_t(l,t)\,\varphi_y(l,t). \quad (42)$$

Since all system signals are bounded, (42) can be used to conclude that $\dot{E}_b \in \mathcal{L}_\infty$. Eqs. (11) and (40) can be used to show that $\dot{g} \in \mathcal{L}_\infty$. Given that $V(t)$ is a non-negative function in time and $\dot{V}(t) \le -g(t)$, where $g(t)$ is a non-negative function and $\dot{g}(t) \in L_\infty$, Lemma A.6 in de Queiroz et al. (2000) and Lemma 4.3 in Slotine and Li (1991) can be used to show that $\lim_{t\to\infty} E_b(t), e(t) \to 0$. Using (14) and Lemma A.12 in de Queiroz et al. (2000) the following inequalities can be developed

$$E_b(t) \ge \int_0^l \omega_{yy}^2 dy \ge \frac{1}{l^3}\omega^2 \ge 0, \quad (43)$$

$$E_b(t) \ge \int_0^l \varphi_y^2 dy \ge \frac{1}{l}\varphi^2 \ge 0. \quad (44)$$

Since $E_b \to 0$ as $t \to \infty$, it can be concluded from (43) and (44) that $\omega \to 0$ and $\varphi \to 0$ as $t \to \infty \forall y \in [0,l]$.

5. Numerical simulation

A numerical simulation is presented to illustrate the performance of the developed controller. To approximate the simultaneous nonlinear system of PDEs that describe the bending and twisting of aircraft wing with a finite number of ODEs, a Galerkin-based method is used. The twisting and bending deflections of the wing are represented as a weighted sum of basis functions as given by

$$\varphi(y,t) = a_0(t)h_0(y) + \sum_{i=1}^{n} a_i(t)h_i(y),$$
$$\omega(y,t) = b_0(t)g_0(y) + \sum_{i=1}^{p} b_i(t)g_i(y), \tag{45}$$

where $n = 5$, $p = 4$, denote the number of basis functions used in the approximations of the wing twisting and bending deflection, respectively. Eq. (45) is a standard trail solution for Galerkin's weighted residual method. Selecting the trial solution in this way ensures that the solution satisfies the PDEs, by using principle of orthogonality between the basis functions and any arbitrary function. A set of linearly independent functions $\{h_i(y)\}_{i=0}^{n}$ and $\{g_i(y)\}_{i=0}^{p}$ is used satisfying the following boundary conditions.

$$h_0(0) = h_i(0) = 0, \quad h_{y_0}(l) = 1, \quad h_{y_i}(l) = 0,$$
$$g_0(0) = g_i(0) = 0, \quad g_{y_0}(0) = g_{y_i}(0) = 0,$$
$$g_{yy_0}(l) = g_{yy_i}(l) = 0, \quad g_{yyy_0}(l) = 1, \quad g_{yyy_i}(l) = 0.$$

First the approximation of the twisting and bending deflection given in (45) is substituted in the system of PDEs in (1) and (2), and then Taylor's approximation up to two terms is used to approximate sine and cosine terms, and the resulting equations can be written as a set of coupled nonlinear ODEs

$$G_1\ddot{b} + \dot{a}^2\left(G_{21}a + G_{22}a^3\right) + \ddot{a}\left(G_{31} + G_{32}a^2\right)$$
$$+ G_4 b + G_5 a = 0, \tag{46}$$
$$H_1\ddot{a} + H_{21}\ddot{b} + H_{22}\ddot{b}a^2 + H_3 a + H_4 a = 0. \tag{47}$$

In (46) and (47) $b(t) \triangleq \begin{bmatrix} b_0(t) & b_1(t) & \dots & b_p(t)\end{bmatrix}^T$, $a(t) \triangleq \begin{bmatrix} a_0(t) & a_1(t) & \dots & a_n(t)\end{bmatrix}^T$, $G_1 \triangleq \rho\int_0^l g(y)g^T(y)dy$, $G_{21} \triangleq -\rho x_c \int_0^l g(y)\left(h(y)h(y)^2\right)^T dy$, $G_{22} \triangleq \frac{\rho x_c}{3!}\int_0^l g(y)\left(h(y)^2h(y)^3\right)^T dy$, $G_{31} \triangleq \rho x_c\int_0^l g(y)h^T(y)dy$, $G_{32} \triangleq -\frac{\rho x_c}{2!}\int_0^l g(y)\left(h(y)h(y)^2\right)^T dy$, $G_4 \triangleq EI\int_0^l g(y)g_{yyyy}^T(y)dy$, $G_5 \triangleq -\bar{L}_w\int_0^l g(y)h^T(y)dy$, $H_1 \triangleq \left(I_w + \rho x_c^2\right)\int_0^l h(y)h^T(y)dy$, $H_{21} \triangleq \rho x_c\int_0^l h(y)g^T(y)dy$, $H_{22} \triangleq -\frac{\rho x_c}{2!}\int_0^l h(y)\left(g(y)h(y)^2\right)^T dy$, $H_3 \triangleq -GJ\int h(y)h_{yy}^T(y)dy$, $H_4 \triangleq -\bar{M}_w\int_0^l h(y)h^T(y)dy$.

The coupled nonlinear ODEs are simulated with the following initial conditions: $\omega(y,0) = 0$ m and $\varphi(y,0) = \frac{y^2}{2l^2}$ rad. Figs. 2 and 3 indicate that these initial conditions yield store induced oscillations. Figs. 4 and 5 show the twisting and bending deflection at the wing tip and wing center, respectively. Unlike twisting deflection, bending deflection is higher at the wing center compared to the wing tip, which is intuitive since there is added mass at the wing tip due to the store. Therefore, for the closed-loop response, twisting and bending deflection for both the wing tip and center are plotted in subsequent figures.

The control objective is to regulate the twisting and bending deflection of the flexible wing. For the adaptive controller designed in (9), the only user defined input parameter is the positive constant control gain K. The control gain is selected as $K = 100$ to satisfy the sufficient gain conditions. Figs. 6 and 7 indicate that the controller is capable of suppressing the store induced oscillations with measurement noise in the system. To illustrate robustness to measurement noise, standard white Gaussian noise is added to the simulation with a signal to noise ratio of 28. Figs. 8 and 9 indicate the mitigation of both twisting and bending deflection at wing tip and wing center, respectively, which justifies the previous claim

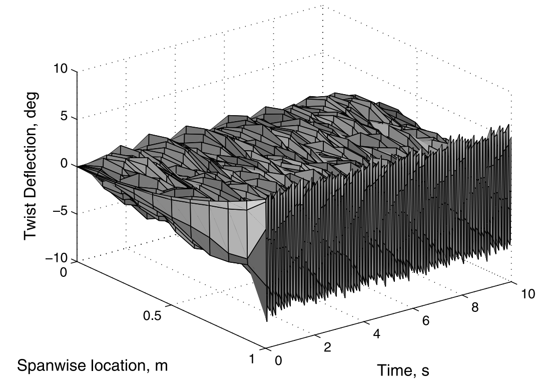

Fig. 2. Open-loop twisting deflection.

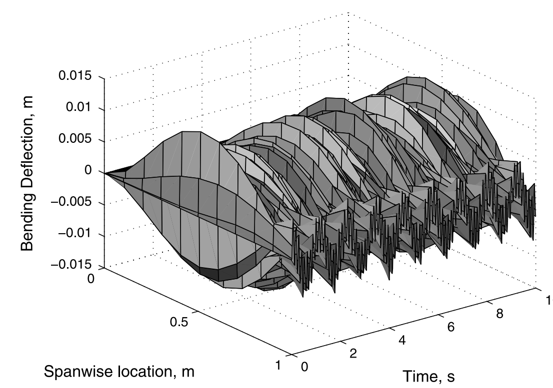

Fig. 3. Open-loop bending deflection.

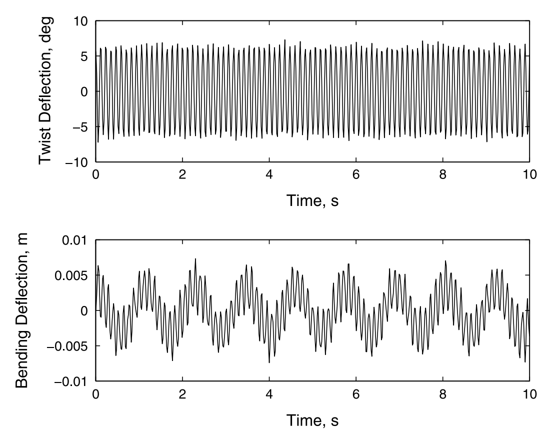

Fig. 4. Open-loop response at the wing tip.

of controlling the state variables instead of the integral of state variables over spatial interval (Paranjape et al., 2013).

Control actuation is applied at the store attached at the wing tip. Figs. 10 and 11 illustrate the time variation of the applied control force and moment.

6. Conclusion

This paper presents the development of a boundary control strategy for suppressing store induced oscillations in an uncertain flexible aircraft wing. The boundary control strategy retains the full PDE system, thereby avoiding potential spillover instabilities, and ensures asymptotic regulation of the distributed states in the presence of parametric uncertainties. Numerical simulations illustrate the performance of the developed adaptive controller. Although a finite difference method is used in the simulation

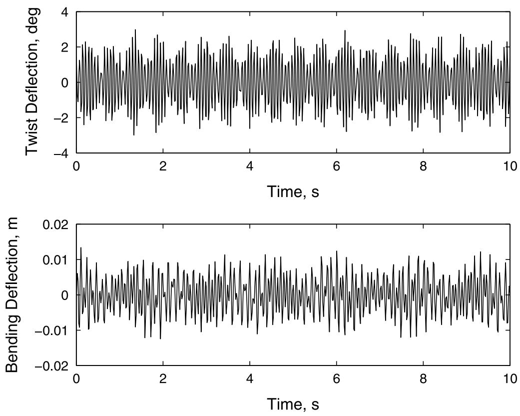

Fig. 5. Open-loop response at the wing center.

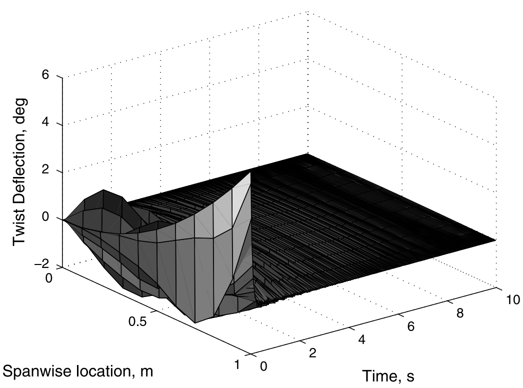

Fig. 6. Closed-loop twisting deflection (with added measurement noise).

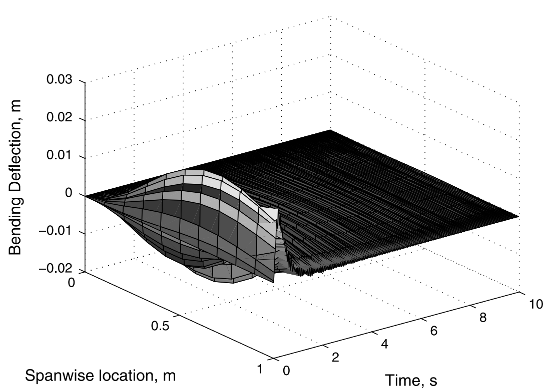

Fig. 7. Closed-loop bending deflection (with added measurement noise).

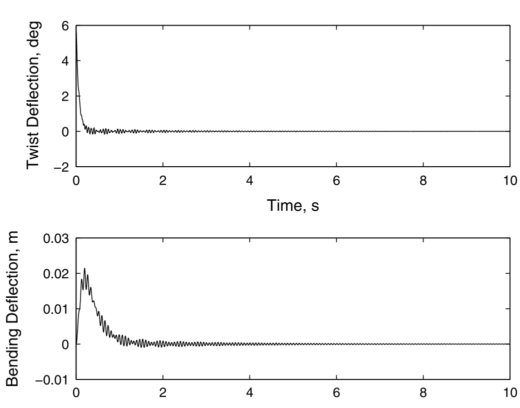

Fig. 8. Closed-loop response at the wing tip (with added measurement noise).

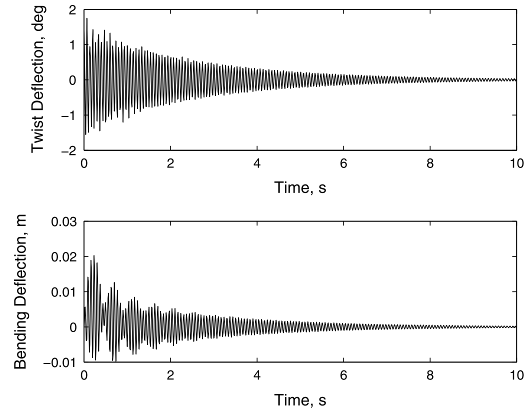

Fig. 9. Closed-loop response at the wing center (with added measurement noise).

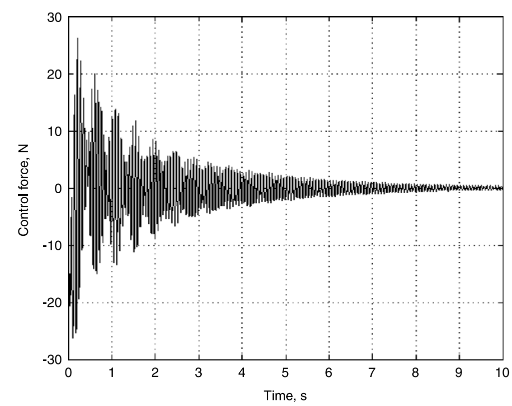

Fig. 10. Applied control force.

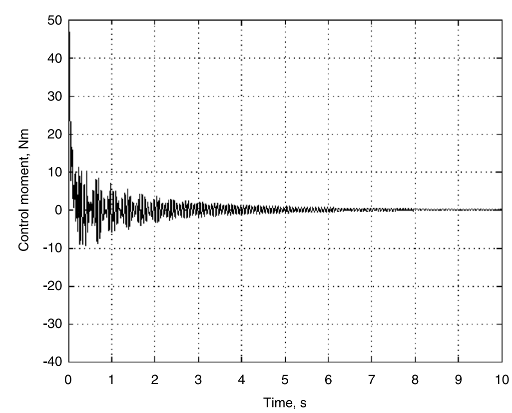

Fig. 11. Applied control moment.

section to approximate derivative terms, methods such as a Kalman-filter or extension of this type of observer can be used to reduce the noise, caused by numerical methods. A potential drawback to the developed method is the need for measurements of high-order spatial derivatives of the distributed states (e.g., $\omega_{yyy}(l,t)$). Future efforts are focused on developing PDE-based output feedback boundary control strategies that would eliminate the need for high-order spatial derivative measurements. The sufficient gain conditions require the control gains to be selected large enough based on the aerodynamics lift and moment, which varies with airspeed. The sufficient conditions also require the structural stiffness to be sufficiently larger than the lift and moment. These conditions ultimately limit the airspeed and motivate further development for higher airspeed operating conditions (e.g., transonic speed). For flight readiness implementation of the developed control method, additional robustness guarantees in terms of stability margins would need to be examined further, along with experimental validation. These tasks are the focus of future efforts.

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