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Robust cadence tracking for switched FES-cycling using a time-varying estimate of the electromechanical delay☆

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ABSTRACT

To combat the severity of neurological conditions (NCs), limit the complications, and reduce the cost of treatment, researchers have turned to hybrid exoskeletons such as functional electrical stimulation (FES) cycling. In this work, closed-loop FES/motor controllers are developed that compensate for time-varying, nonlinear, and uncertain dynamics, unknown disturbances, switching between actuators (e.g., between muscle groups and the motor), fatigue, and the unknown time-varying muscle delay between stimulation application and the production of muscle force, called the electromechanical delay (EMD). Control authority is maintained and efficient muscle contractions are produced through the development of FES/motor switching conditions that are both EMD and state dependent. Contributions are that the controllers implement a modular time-varying estimate of the EMD and yield exponential cadence tracking as verified by a Lyapunov-like stability analysis. An example EMD estimate is presented that varies with cycling time to account for fatigue. Furthermore, experiments were conducted to validate the developed control system, which produced an average cadence error of -0.01 ± 1.35 revolutions per minute (RPM) across five able-bodied participants and -0.05 ± 1.38 RPM across four participants with NCs.

1. Introduction

Individuals with neurological conditions (NCs) may experience paralysis, muscle weakness, partial or total loss of coordinated limb control, in addition to secondary health effects such as diabetes, obesity, muscle atrophy, cardiovascular diseases, etc. that result from a sedentary lifestyle (Benjamin et al., 2017; Rimmer & Rowland, 2008). In an effort to mitigate the severity of disability, reduce the cost of treatment of NCs, and limit the complications, researchers and clinicians have turned to technological solutions such as hybrid exoskeletons, which combine rehabilitation robots (e.g., exoskeletons, motorized stationary cycles) with functional electrical stimulation (FES) to facilitate rehabilitative therapies (Anaya, Thangavel, & Yu, 2018). However, the inherent time-varying, nonlinear, and uncertain dynamics of the cycle–rider system, the necessity to switch control between a motor and various muscle groups, unknown disturbances, and fatigue complicate the development of closed-loop FES controllers (Allen, Cousin, Rouse and Dixon, 2022; Allen, Stubbs, & Dixon, 2020a, 2020c; Allen, Stubbs and Dixon, 2022; Bellman, Downey, Parikh, & Dixon, 2017; Cousin, Rouse, Duenas, & Dixon, 2019; Downey, Merad, Gonzalez, & Dixon, 2017). Furthermore, there exists an input delay, often termed the electromechanical delay (EMD),[1] that is both unknown and time-varying, between the start/end of stimulation and the start/end of muscle force production (Allen, Stubbs, & Dixon, 2020b; Downey et al., 2017).

To prevent EMD-induced instability, closed-loop EMD-compensating FES controllers have been designed for both continuous and coordinated exercises, such as leg extensions (Karafyllis, Malisoff, de Queiroz, Krstic, & Yang, 2015; Obuz, Duenas, Downey, Klotz, & Dixon, 2020) and cycling (Allen, Cousin et al., 2022; Allen et al., 2020a, 2020c; Allen, Stubbs, & Dixon, 2020d; Allen, Stubbs et al., 2022), respectively. Coordinated exercises must consider the time latency between the start of FES and the onset of muscle contraction (i.e., the contraction delay) and the time latency between the end of FES and muscle contraction (i.e., the residual delay) (Allen, Cousin et al., 2022; Allen et al., 2020a, 2020c, 2020d; Allen, Stubbs et al., 2022). Robust FES and motor controllers were developed to achieve torque and cadence

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[1] In some literature, the EMD corresponds to the time latency between the onset of EMG activity and muscle force (Nordez et al., 2009).

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tracking in Allen et al. (2020a) and cadence tracking in Allen, Cousin et al. (2022), Allen et al. (2020c, 2020d) and Allen, Stubbs et al. (2022), where the FES controller is saturated in Allen, Stubbs et al. (2022) and Allen et al. (2020d). The EMD estimate is constant in Allen, Cousin et al. (2022), Allen et al. (2020a, 2020d) and Allen, Stubbs et al. (2022) and time-varying in Allen et al. (2020c).

Beyond FES systems, input delays have been extensively considered (cf., Bagheri, Naseradinmousavi, & Krstic, 2019, Karafyllis & Krstic, 2017; Krstic, 2009, 2010; Mazenc & Malisoff, 2020; Mazenc, Malisoff, & Ozbay, 2018; Mazenc et al., 2004; Wang, Niu, Wu, & Xie, 2018; Wang, Sun, & Mazenc, 2016 and Yang, Li, and Qiu (2019)); however, switched systems have rarely been considered (cf., Mazenc et al. (2018), Wang et al. (2018, 2016) and Yang et al. (2019)) and these studies on non-FES systems do not provide compensation for critical FES-specific factors (i.e., residual muscle torques, uncertain and state-based FES control effectiveness, etc.).

Building upon our preliminary work in Allen et al. (2020c), this work develops closed-loop FES/motor controllers that implement a modular time-varying estimate of the EMD. The EMD estimate is modular in the sense that any estimate can be used provided certain conditions (i.e., the estimate is continuous and bounded) are satisfied. However, compared to Allen et al. (2020c), this paper includes volitional effort from the participant in the dynamic model, includes comparative experiments on nine participants (including four with NCs), introduces a new switching signal, and modifies the error system, motor controller, and Lyapunov-based stability analysis to yield improved FES/motor controllers, improved gain conditions, and exponential position/cadence tracking for a delayed, switched, uncertain, and nonlinear FES-cycling system. Furthermore, FES and motor switching signals are designed to maintain control authority, to ensure efficient muscle contractions, and to mitigate contractions in antagonistic muscles.

Passive therapy (i.e., no volitional contributions) experiments were conducted on able-bodied participants to compare the developed FES/motor controllers to an alternate control method, of similar form, that was developed by assuming the EMD was negligible. Likewise, active therapy (i.e., with volitional contributions) experiments were conducted on participants with varied NCs. Experimental results show that compensating for the EMD significantly improves the cadence tracking performance, and the developed control system can safely and effectively yield cadence tracking for individuals with varied capabilities during both active and passive therapy exercises.

2. Dynamics

Throughout this paper, all switching signals are designed as piecewise right-continuous and delayed functions are denoted as $f_\tau = f(t-\tau(t))$, when $t-\tau(t) \geq t_0$, and as $f_\tau = 0$, when $t-\tau(t) < t_0$, where $\tau : \mathbb{R}_{\geq 0} \to \mathbb{S}$, $\mathbb{S} \subset \mathbb{R}_{>0}$, $t \in \mathbb{R}_{\geq 0}$, $t_0 \in \mathbb{R}_{\geq 0}$ represent the EMD, set of possible EMD values (Allen et al., 2020b; Allen, Stubbs, & Dixon, 2021), time, and initial time, respectively. The dynamics of the cycle–rider system are modeled according to Bellman et al. (2017) as[2]

$$\begin{aligned} &M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + P(q,\dot{q}) + b_c\dot{q} + d(t) \\ &\quad = \tau_{vol} + \underbrace{B_e k_e}_{B_E} u_e(t) + \underbrace{\sum_{m\in\mathcal{M}} B_m(q,\dot{q},t)\, k_m \sigma_{m,\tau}}_{B_M^\tau(q,\dot{q},\tau,t)} u_\tau, \end{aligned} \tag{1}$$

where the measurable crank position and velocity are represented by $q : \mathbb{R}_{\geq 0} \to \mathcal{Q}$ and $\dot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively, and the unmeasurable crank acceleration is denoted by $\ddot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The set $\mathcal{Q} \subseteq \mathbb{R}$ contains all potential crank angles. Furthermore, $d : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $b_c \in \mathbb{R}_{>0}$, $P : \mathcal{Q}\times\mathbb{R} \to \mathbb{R}$, $G : \mathcal{Q} \to \mathbb{R}$, $V : \mathcal{Q}\times\mathbb{R} \to \mathbb{R}$, and $M : \mathcal{Q} \to \mathbb{R}_{>0}$ represent the disturbance, viscous damping, passive viscoelastic tissue, gravitational, centripetal-Coriolis, and inertial effects, respectively. The volitional torque contribution about the cycle crank is represented by $\tau_{vol} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. Let $k_e, k_m \in \mathbb{R}_{>0}$, $\forall m \in \mathcal{M}$ represent selectable constants, where the set $\mathcal{M} \triangleq \{LG, LH, LQ, RG, RH, RQ\}$ contains the left (L) and right (R) gluteal (G), hamstrings (H), and quadriceps femoris (Q) muscle groups. The motor's unknown effectiveness, unknown control effectiveness, and control input are represented by $B_e, B_E \in \mathbb{R}_{>0}$ and $u_e : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. The stimulation's unknown control effectiveness, implemented FES control input, and delayed FES control input are represented by $B_M^\tau : \mathcal{Q}\times\mathbb{R}\times\mathbb{S}\times\mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, $u : \mathbb{R}_{\geq 0} \to \mathbb{R}$, and $u_\tau : \mathbb{S}\times\mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. For each $m \in \mathcal{M}$, the unknown muscle effectiveness, designed muscle switching signal, and delayed muscle switching signal are represented by $B_m : \mathcal{Q}\times\mathbb{R}\times\mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$, $\sigma_m : \mathcal{Q}\times\mathbb{R} \to \{0, 1\}$, and $\sigma_{m,\tau}$ respectively. The delayed and implemented FES stimulation inputs $\forall m \in \mathcal{M}$ are defined as $u_{m,\tau} \triangleq k_m\sigma_{m,\tau}u_\tau$ and $u_m \triangleq k_m\sigma_m u$, respectively, and the motor current input is defined as $u_E \triangleq k_e u_e$. The signal σ_m is defined as

$$\sigma_m(q,\dot{q}) \triangleq \begin{cases} 1, & q_\alpha(q,\dot{q}) \in \mathcal{Q}_m \\ 0, & \text{otherwise} \end{cases}, \forall m \in \mathcal{M}, \tag{2}$$

where $q_\alpha : \mathcal{Q}\times\mathbb{R} \to \mathbb{R}$ represents a trigger condition that projects q forward to alter the application of stimulation based on known bounds on the EMD (e.g., refer to Allen et al. (2020b)) to yield a kinematically efficient muscle contribution. For a given muscle $m \in \mathcal{M}$, the set $\mathcal{Q}_m \subset \mathcal{Q}$ contains the angles where a force from muscle m efficiently generates positive crank motion (i.e., forward pedaling) and is defined as (cf., Bellman et al. (2017))

$$\mathcal{Q}_m \triangleq \{q \in \mathcal{Q} \mid T_m(q) > \varepsilon_m\}, \forall m \in \mathcal{M},$$

where $\varepsilon_m \in \mathbb{R}_{>0}$ and $T_m : \mathcal{Q} \to \mathbb{R}$ represent a lower threshold and the torque transfer ratio, respectively. The desired portions of the crank for muscle contractions to occur are defined collectively as $\mathcal{Q}_{FES} \triangleq \cup_{m\in\mathcal{M}}\{\mathcal{Q}_m\}$. The remaining portions of the crank are considered kinematic deadzones (i.e., inefficient regions) and are defined as $\mathcal{Q}_{KDZ} \triangleq \mathcal{Q} \setminus \mathcal{Q}_{FES}$.

Although the parameters in (1) are uncertain, the subsequent control development only assumes known bounds on each parameter, as summarized in the subsequent properties (Bellman et al., 2017). **Property: 1** The cycle–rider parameters can be bounded as $c_m \leq M \leq c_M$, $|V| \leq c_V|\dot{q}|$, $|G| \leq c_G$, $|P| \leq c_{P_1} + c_{P_2}|\dot{q}|$, $b_c\dot{q} \leq c_c|\dot{q}|$, $|d| \leq c_d$, and $|\tau_{vol}| \leq c_{vol}$, respectively, where $c_m, c_M, c_V, c_G, c_{P_1}, c_{P_2}, c_c, c_d, c_{vol} \in \mathbb{R}_{>0}$ are known constants. **Property: 2** $\frac{1}{2}\dot{M} = V$. **Property: 3** The motor and FES (when $\sum_{m\in\mathcal{M}}\sigma_{m,\tau} > 0$) control effectiveness terms are bounded as $c_e \leq B_E \leq c_E$ and $c_b \leq B_M^\tau \leq c_B$, respectively, where $c_b, c_B, c_e, c_E \in \mathbb{R}_{>0}$ are known constants. **Property: 4** The EMD can be bounded as $\underline{\tau} \leq \tau \leq \bar{\tau}$, where $\underline{\tau}, \bar{\tau} \in \mathbb{R}_{>0}$ are known constants.

3. Control development

The objective of this work is to track a desired cadence. The measurable position and cadence tracking errors, represented by $e, \dot{e} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively, are defined as[3]

$$e \triangleq q_d - q, \qquad \dot{e} \triangleq \dot{q}_d - \dot{q}, \tag{3}$$

[2] All explicit dependence on time, t, is suppressed within $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)$ for notational brevity.

[3] For notational brevity, hereafter all functional dependencies are suppressed unless required for clarity of exposition.

where the desired position and cadence of the bicycle crank are sufficiently smooth and represented by $q_d, \dot{q}_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. To aid the stability analysis and compensate for the EMD, measurable auxiliary errors, represented by $r, e_u : \mathbb{R}_{\geq 0} \to \mathbb{R}$, are defined as

$$r \triangleq \dot{e} + \alpha_1 e + \alpha_2 e_u, \tag{4}$$

$$e_u \triangleq - \int_{t-\hat{\tau}(t)}^{t} \sigma_s(\theta)\, u(\theta)\, d\theta, \tag{5}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}_{\geq 0}$ denote selectable constants, $\hat{\tau} : \mathbb{R}_{>0} \to \mathbb{R}$ denotes an estimate of the EMD, and $\sigma_s : \mathbb{R}_{\geq 0} \to \{0,\ 1\}$ denotes a switching signal that indicates when stimulation is being applied to any muscle group and is designed as

$$\sigma_s(t) \triangleq \begin{cases} 1, & \sum_{m\in\mathcal{M}} \sigma_m \geq 1 \\ 0, & \text{otherwise} \end{cases}. \tag{6}$$

A predictor for the estimate of the EMD is designed as

$$\dot{\hat{\tau}} = \operatorname{proj}\left(g\left(t, q, \dot{q}, \hat{\tau}\right)\right), \tag{7}$$

where $\operatorname{proj}(\cdot)$ represents the smooth projection operator from Cai, de Queiroz, and Dawson (2006), which is designed to bound the EMD estimate such that $\underline{\tau} \leq \hat{\tau} \leq \bar{\tau}$ and $\left|\dot{\hat{\tau}}\right| < 1$. In (7), $g : \mathbb{R}_{\geq 0} \times \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{>0} \to \mathbb{R}$ represents a continuous function that updates the EMD estimate. For example, in Allen et al. (2020b), the EMD is modeled during FES-cycling by using $\hat{\tau}(t) = A + Bt + Ct^2, \forall t \in [0, 10]$, where $t \in \mathbb{R}_{\geq 0}$ denotes the cycling run time in minutes, and $A, B, C \in \mathbb{R}$ are constants with statistical information provided in tables. For example, A, B, and C have typical values ranging from 80–100 ms, 1.0–2.8 ms/min, and -0.107 ms/min^2, respectively. Thus, $g(t) = B + 2Ct, \forall t \in [0, 10]$ is used during the subsequent experiments to estimate the EMD; however, g is modular and can be selected by the designer provided that g is continuous.

The open-loop error system is derived by substituting (3) and (5) into (4), taking the time derivative, multiplying by M, adding/subtracting $B_M^{\tau} u_{\hat{\tau}} + e$, and substituting in (1) to yield

$$\begin{aligned} M\dot{r} &= -Vr - e + B_M^{\tau}\left(u_{\hat{\tau}} - u_{\tau}\right) - B_E u_e \\ &\quad - \sigma_s M\alpha_2 u + \left(\sigma_{s,\hat{\tau}} M\alpha_2 - B_M^{\tau}\right) u_{\hat{\tau}} \\ &\quad - \sigma_{s,\hat{\tau}} M\alpha_2 \dot{\hat{\tau}} u_{\hat{\tau}} + \chi, \end{aligned} \tag{8}$$

where $\chi : \mathcal{Q} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ is an auxiliary term defined as $\chi \triangleq M\ddot{q}_d + V\left(\dot{q}_d + \alpha_1 e + \alpha_2 e_u\right) + G + P + b_c\dot{q} + d + M\alpha_1\dot{e} + e - \tau_{vol}$, which, by using Property 1, can be bounded as

$$|\chi| \leq \Phi + \rho\left(\|z\|\right)\|z\|, \tag{9}$$

where $\rho(\cdot)$ is a radially unbounded, positive, and strictly increasing function, $\Phi \in \mathbb{R}_{>0}$ is a known constant, and $z \in \mathbb{R}^3$ is defined as

$$z \triangleq \begin{bmatrix} e & r & e_u \end{bmatrix}^T. \tag{10}$$

Based on (8) and the subsequent analysis, the motor and FES controllers are defined as[4]

$$u_e \triangleq k_1 \operatorname{sgn}(r) + \sigma_e\left(k_2 + k_3\right) r, \tag{11}$$

$$u \triangleq k_s r, \tag{12}$$

respectively, where $k_1, k_2, k_3, k_s \in \mathbb{R}_{>0}$ represent selectable constants, $\operatorname{sgn}(\cdot)$ denotes the signum function, and $\sigma_e : \mathcal{Q} \times \{0,\ 1\} \to \{0,\ 1\}$ represents a switching signal for the motor defined as

$$\sigma_e(q, \sigma_s) \triangleq \begin{cases} 1, & q \in \mathcal{Q}_{KDZ} \\ 1, & q \in \mathcal{Q}_{FES}, \sigma_s = 0 \\ 0, & \text{otherwise} \end{cases}. \tag{13}$$

Substituting (11) and (12) into (8) yields the closed-loop error system

$$\begin{aligned} M\dot{r} &= -B_E\left(k_1 \operatorname{sgn}(r) + \sigma_e\left(k_2 + k_3\right) r\right) \\ &\quad + k_s B_M^{\tau}\left(r_{\hat{\tau}} - r_{\tau}\right) - \sigma_{s,\hat{\tau}} M\alpha_2 k_s \dot{\hat{\tau}} r_{\hat{\tau}} \\ &\quad + \left(\sigma_{s,\hat{\tau}} M\alpha_2 - B_M^{\tau}\right) k_s r_{\hat{\tau}} \\ &\quad - \sigma_s M\alpha_2 k_s r - e - Vr + \chi. \end{aligned} \tag{14}$$

To facilitate the subsequent analysis, Lyapunov–Krasovskii functionals, represented by $Q_1, Q_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$, are defined as

$$Q_1 \triangleq \frac{1}{2} k_s\left(\omega_4\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right) + \varepsilon_3\omega_3\right) \int_{t-\hat{\tau}}^{t} r(\theta)^2\, d\theta, \tag{15}$$

$$Q_2 \triangleq \frac{\omega_2 k_s}{\bar{\tau}} \int_{t-\bar{\tau}}^{t} \int_{s}^{t} r(\theta)^2\, d\theta ds, \tag{16}$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \omega_1, \omega_2, \omega_3, \omega_4 \in \mathbb{R}_{>0}$ represent selectable constants, $\Upsilon \in \mathbb{R}_{>0}$ is a known constant, $\beta_1, \beta_2, \delta_1, \delta_2 \in \mathbb{R}$ are known auxiliary bounding constants, and $\beta_1, \beta_2, \delta_1, \delta_2 \in \mathbb{R}_{>0}$ provided that the following sufficient conditions are satisfied

$$\alpha_1 > \frac{\varepsilon_2\alpha_2^2}{2}, \quad \omega_2 > 3k_s\bar{\tau}^2\left(\frac{1}{2\varepsilon_2} + \frac{k_s\omega_3}{2\varepsilon_3}\left(2 + \varepsilon_4\right)\right), \tag{17}$$

$$\omega_1 \geq \frac{1}{\varepsilon_1}\max\left(\left|c_M\alpha_2 - c_b\right|, \left|c_m\alpha_2 - c_B\right|\right), \tag{18}$$

$$c_m\alpha_2 > \left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right)\left(1 + \omega_4\right) + 2\varepsilon_3\omega_3 + 2\omega_2, \tag{19}$$

$$k_1 \geq \frac{1}{c_e}\left(\Phi + k_s c_B \Upsilon\left(\bar{\tau} - \underline{\tau}\right) + k_s\bar{\tau}\Upsilon \max\left(c_b, c_m\alpha_2\right)\right), \tag{20}$$

$$k_2 > \frac{2k_s}{c_e}\left(\varepsilon_3\omega_3 + \frac{1}{2}\left(1 + \omega_4\right)\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right) + \omega_2\right), \tag{21}$$

$$k_3 \geq \frac{k_s}{c_e}\max\left(c_b, c_m\alpha_2\right), \quad \left|\dot{\hat{\tau}}\right| \leq \varepsilon_4 < 1. \tag{22}$$

4. Stability analysis

To facilitate the subsequent analysis, let switching times be denoted by $\left\{t_n^i\right\}, i \in \{m, e\}, n \in \{0, 1, 2, \ldots\}$, which denote the time instances when σ_e becomes zero ($i = m$) or nonzero ($i = e$). A positive definite and continuously differentiable common Lyapunov functional candidate $V_L : \mathcal{D} \to \mathbb{R}_{\geq 0}$ is defined as

$$V_L \triangleq \frac{1}{2}e^2 + \frac{1}{2}Mr^2 + \frac{1}{2}\omega_3 e_u^2 + Q_1 + Q_2, \tag{23}$$

where $\mathcal{D}, S_{\mathcal{D}} \subseteq \mathbb{R}^5$ denote open connected sets that are defined as $\mathcal{D} \triangleq \left\{y \in \mathbb{R}^5 \mid \|y\| < \gamma\right\}$ and $S_{\mathcal{D}} \triangleq \left\{y \in \mathbb{R}^5 \mid \|y\| < \sqrt{\frac{\lambda_1}{\lambda_2}}\gamma\right\}$, where $\gamma \in \mathbb{R}_{>0}$ represents a known constant defined as[5] $\gamma \leq \inf\left\{\rho^{-1}\left(\left(\sqrt{\min\left(\beta_1 c_m\alpha_2 k_s, \beta_2 c_e k_2\right)}, \infty\right)\right)\right\}$. Based on Property 1, the candidate common Lyapunov functional in (23) can be bounded as

$$\lambda_1\|y\|^2 \leq V_L \leq \lambda_2\|y\|^2, \tag{24}$$

where $\lambda_1, \lambda_2 \in \mathbb{R}_{>0}$ and $y \in \mathbb{R}^5$ are known and defined as $\lambda_1 \triangleq \frac{1}{2}\min\left(1, c_m, \omega_3\right)$, $\lambda_2 \triangleq \max\left(1, \frac{c_M}{2}, \frac{\omega_3}{2}\right)$, and

$$y \triangleq \begin{bmatrix} z^T & \sqrt{Q_1} & \sqrt{Q_2} \end{bmatrix}^T. \tag{25}$$

[4] The first motor term remains on for all time to yield exponential position and cadence tracking and to improve the overall performance. Note that during implementation, a small value is sufficient for k_1; thus, the first motor term would result in a relatively small motor input for all time. However, if it is desired to include the switching signal, σ_e, on the first motor term, refer to the development in Allen et al. (2020c). The cost of including σ_e on the first motor term is that a uniformly ultimately bounded result is obtained, which complicates the analysis and yields a worse control performance.

[5] For a set A, the inverse image is defined as $\rho^{-1}(A) \triangleq \{a \mid \rho(a) \in A\}$.

Theorem 1. *For the switched cycle–rider system in (1), the motor and FES controllers defined in (11) and (12) yield semi-global exponential cadence tracking in the sense that*

$$\|y(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\|y(t_0)\| \exp\left(-\frac{1}{2}\lambda_3(t-t_0)\right), \tag{26}$$

$\forall t \in [t_0, \infty)$, where $\lambda_3 \triangleq \lambda_2^{-1}\min(\delta_1, \delta_2)$, provided that the sufficient conditions in (17)–(22) are satisfied and $y(t_0) \in \mathcal{S}_{\mathcal{D}}$.

Proof. For $t \in [t_0, \infty)$, let $y(t)$ be a Filippov solution to $\dot{y} \in K[h](y)$, where $h \triangleq \begin{bmatrix} \dot{e} & \dot{r} & \dot{e}_u & \dot{\sqrt{Q_1}} & \dot{\sqrt{Q_2}} \end{bmatrix}^T$ (see Fischer, Kamalapurkar, and Dixon (2013)) and $K[\cdot]$ is defined in Filippov (1964). Since the controllers in (11) and (12) are discontinuous, the time derivative of (23) exists within $t \in [t_0, \infty)$ almost everywhere (a.e.) such that $\dot{V}_L(y) \overset{\text{a.e.}}{\in} \dot{\tilde{V}}_L(y)$, where $\dot{\tilde{V}}_L$ represents the generalized time derivative of (23) along $\dot{y} = h(y)$. Using (4), (12), (14), and the calculus of $K[\cdot]$ from Paden and Sastry (1987), and applying the Leibniz integral rule to (5), (15), and (16) yields

$$\begin{aligned}\dot{\tilde{V}}_L \subseteq\ & e(r - \alpha_1 e - \alpha_2 e_u) + \tfrac{1}{2}\dot{M}r^2 \\ & + r\left(-Vr - e + \chi + k_s K\left[B_M^{\tau}\right](r_{\hat{\tau}} - r_\tau)\right. \\ & - B_E\left(k_1 K[\mathrm{sgn}(r)] + K[\sigma_e](k_2 + k_3)r\right) \\ & - K[\sigma_s]M\alpha_2 k_s r - K\left[\sigma_{s,\hat{\tau}}\right]M\alpha_2 k_s \dot{\hat{\tau}} r_{\hat{\tau}} \\ & \left. + \left(K\left[\sigma_{s,\hat{\tau}}\right]M\alpha_2 - K\left[B_M^{\tau}\right]\right)k_s r_{\hat{\tau}}\right) \\ & + \omega_3 e_u\left(-K[\sigma_s]k_s r + K\left[\sigma_{s,\hat{\tau}}\right]k_s r_{\hat{\tau}}\left(1 - \dot{\hat{\tau}}\right)\right) \\ & + \tfrac{1}{2}k_s\left(\omega_4\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right) + \varepsilon_3\omega_3\right)r^2 \\ & - \tfrac{1}{2}k_s\left(\omega_4\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right) + \varepsilon_3\omega_3\right)\left(1 - \dot{\hat{\tau}}\right)r_{\hat{\tau}}^2 \\ & + \tfrac{\omega_2 k_s}{\bar{\tau}}\left(\bar{\tau}r^2 - \int_{t-\bar{\tau}}^{t} r(\theta)^2\, d\theta\right),\end{aligned} \tag{27}$$

where, $K[\mathrm{sgn}(\cdot)] = SGN(\cdot)$ and $SGN(\cdot) = \{1\}$ if $(\cdot) > 0$, $[-1, 1]$ if $(\cdot) = 0$, and $\{-1\}$ if $(\cdot) < 0$. By examination of (27) and the switching conditions defined in (2), (6), and (13) it can be seen that nine unique cases exist, where Case 1 represents the case when $\sigma_e = 0$ (i.e., $t \in [t_n^m, t_{n+1}^e)$), which by design only occurs when FES-induced muscle forces are present. Cases 2–9 will subsequently be considered simultaneously by using an overall upper bound, since $\sigma_e = 1$ across each case.

During a given case each switching signal is constant; thus, Case 1 can be investigated by setting $K[\sigma_s] = 1$, $K\left[\sigma_{s,\hat{\tau}}\right] = 1$, and $K[\sigma_e] = 0$. By invoking Properties 1–3 (e.g., to bound M, $K\left[B_M^{\tau}\right]$, and B_E), choosing ε_1 and ω_1 such that $\max\left(|c_M\alpha_2 - c_b|, |c_m\alpha_2 - c_B|\right) \leq \varepsilon_1\omega_1$, requiring that $\left|\dot{\hat{\tau}}\right| \leq \varepsilon_4 < 1$, and recalling that $\dot{V}_L(y) \overset{\text{a.e.}}{\in} \dot{\tilde{V}}_L(y)$ then (27) can be upper bounded as

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq}\ & -\alpha_1 e^2 + \alpha_2|ee_u| + |r||\chi| + k_s c_B|r(r_{\hat{\tau}} - r_\tau)| \\ & - k_1 c_e|r| + k_s\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right)|rr_{\hat{\tau}}| \\ & - c_m\alpha_2 k_s r^2 + k_s\omega_3|e_u r| + k_s\omega_3\left(1 - \dot{\hat{\tau}}\right)|e_u r_{\hat{\tau}}| \\ & + \tfrac{1}{2}k_s\left(\omega_4\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right) + \varepsilon_3\omega_3\right)r^2 \\ & - \tfrac{1}{2}k_s\left(\omega_4\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right) + \varepsilon_3\omega_3\right)\left(1 - \dot{\hat{\tau}}\right)r_{\hat{\tau}}^2 \\ & + \tfrac{\omega_2 k_s}{\bar{\tau}}\left(\bar{\tau}r^2 - \int_{t-\bar{\tau}}^{t} r(\theta)^2\, d\theta\right).\end{aligned}$$

Selecting ω_4 such that $\omega_4 = 1/(1 - \varepsilon_4)$, applying Young's Inequality, and simplifying the resulting expression yields

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq}\ & -\left(\alpha_1 - \tfrac{\varepsilon_2\alpha_2^2}{2}\right)e^2 + \left(\tfrac{1}{2\varepsilon_2} + \tfrac{k_s\omega_3}{2\varepsilon_3}(2 + \varepsilon_4)\right)e_u^2 \\ & + |r||\chi| + k_s c_B|r(r_{\hat{\tau}} - r_\tau)| - k_1 c_e|r| \\ & + k_s\left(\tfrac{1}{2}\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right)(1 + \omega_4) + \varepsilon_3\omega_3\right)r^2 \\ & - c_m\alpha_2 k_s r^2 + k_s\omega_2 r^2 - \tfrac{\omega_2 k_s}{\bar{\tau}}\int_{t-\bar{\tau}}^{t} r(\theta)^2\, d\theta.\end{aligned} \tag{28}$$

Provided that $\|y(\cdot)\| < \gamma$, $\forall \cdot \in [t_0, t)$, then (9), (10), (14), (25), and Properties 1 and 3 can be used to conclude that $\dot{r}(\cdot) < c_1 + c_2\gamma + c_3\gamma^2 \leq \Upsilon$, $\forall \cdot \in [t_0, t)$, where $c_1, c_2, c_3 \in \mathbb{R}_{>0}$ are known constants. Hence, by invoking the Mean Value Theorem (MVT) on the $(r_{\hat{\tau}} - r_\tau)$ term in (28), substituting (9) into (28), completing the squares on $-\frac{1}{2}c_m\alpha_2 k_s r^2 + |r|\,\rho(\|z\|)\|z\|$, grouping terms, and imposing (20) yields

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq}\ & -\left(\alpha_1 - \tfrac{\varepsilon_2\alpha_2^2}{2}\right)e^2 + \left(\tfrac{1}{2\varepsilon_2} + \tfrac{k_s\omega_3}{2\varepsilon_3}(2 + \varepsilon_4)\right)e_u^2 \\ & - k_s\left(\tfrac{1}{2}c_m\alpha_2 - \tfrac{1}{2}\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right)(1 + \omega_4)\right. \\ & \left. - \varepsilon_3\omega_3 - \omega_2\right)r^2 + \tfrac{1}{2c_m\alpha_2 k_s}\rho^2(\|z\|)\|z\|^2 \\ & - \tfrac{\omega_2 k_s}{\bar{\tau}}\int_{t-\bar{\tau}}^{t} r(\theta)^2\, d\theta.\end{aligned} \tag{29}$$

To further simplify (29), e_u^2 (via Cauchy–Schwarz inequality), Q_1, and Q_2 are bounded such that

$$\begin{aligned}-\tfrac{\omega_2 k_s}{\bar{\tau}}\int_{t-\bar{\tau}}^{t} r(\theta)^2\, d\theta \leq\ & -\tfrac{2\omega_2}{3\bar{\tau}\left(\omega_4\left(\varepsilon_1\omega_1 + c_M\alpha_2\varepsilon_4\right) + \varepsilon_3\omega_3\right)}Q_1 \\ & - \tfrac{\omega_2}{3k_s\bar{\tau}^2}e_u^2 - \tfrac{1}{3\bar{\tau}}Q_2.\end{aligned} \tag{30}$$

Now using (24), (30), and the fact that $\|y\| \geq \|z\|$ yields

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\frac{\delta_1}{\lambda_2}V_L, \tag{31}$$

$\forall t \in \left[t_n^m, t_{n+1}^e\right)$, provided that $y(t) \in \mathcal{D}, \forall t \in \left[t_n^m, t_{n+1}^e\right)$ and $\|y(\cdot)\| < \gamma$, $\forall \cdot \in [t_0, t)$, where the latter expression is equivalent to $y(\cdot) \in \mathcal{D}$, $\forall \cdot \in [t_0, t)$.

Cases 2–9 represent the cases when $\sigma_e = 1$ (i.e., $t \in \left[t_n^e, t_{n+1}^m\right)$) and all possible switching combinations of σ_s, $\sigma_{s,\hat{\tau}}$, and B_M^{τ}. Notice that an overall upper bound for Cases 2–9 allows each case to be solved simultaneously. Note that the following inequality holds by individually considering each case, selecting ε_1 and ω_1 such that $c_M\alpha_2 - c_b \leq |c_M\alpha_2 - c_b| \leq \varepsilon_1\omega_1$ and $c_B - c_m\alpha_2 \leq |c_m\alpha_2 - c_B| \leq \varepsilon_1\omega_1$, and using Properties 1 and 3:

$$k_s\left|\sigma_{s,\hat{\tau}}M\alpha_2 - B_M^{\tau}\right||rr_{\hat{\tau}}| \leq k_s\varepsilon_1\omega_1|rr_{\hat{\tau}}| + k_s\max\left(c_b, c_m\alpha_2\right)|rr_{\hat{\tau}}|. \tag{32}$$

Using the inequality in (32) and following a similar development as for Case 1, the inequality in (27) can be upper bounded for Cases 2–9 as

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\frac{\delta_2}{\lambda_2}V_L, \tag{33}$$

$\forall t \in \left[t_n^e, t_{n+1}^m\right)$, provided $y(t) \in \mathcal{D}$, $\forall t \in \left[t_n^e, t_{n+1}^m\right)$, and $y(\cdot) \in \mathcal{D}$, $\forall \cdot \in [t_0, t)$.

Upper bounding (31) and (33), and defining $\lambda_3 \triangleq \lambda_2^{-1}\min(\delta_1, \delta_2)$ yields an overall upper bound $\forall t \in [t_0, \infty)$ as

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\lambda_3 V_L, \tag{34}$$

which confirms that (23) is a common Lyapunov-like function for every case. Solving the differential inequality in (34) yields

$$V_L(t) \leq V_L(t_0)\exp\left(-\lambda_3(t - t_0)\right), \ \forall t \in [t_0, \infty), \tag{35}$$

provided that $y(t) \in \mathcal{D}, \forall t \in [t_0, \infty)$. The result in (26) is obtained by using (24) and (35). A sufficient condition for $y(t) \in \mathcal{D}, \forall t \in [t_0, \infty)$ is that the gains k_m, k_e, k_s, α_1, α_2, k_1, k_2, and k_3 are selected so that $y(t_0) \in \mathcal{S}_{\mathcal{D}}$. From (11), (12), (23) and (34), $e, r, e_u, u, u_e \in \mathcal{L}_\infty$ and the remaining signals are bounded.

5. Experiment

Henceforth, we will label the motor and FES controllers defined in (11) and (12), the subsequently defined "delay-free" version of (11) and (12), and $u_e = 0$ and $u = 0$ (i.e., no motor or FES assistance) as Controllers A, B, and C, respectively. Controller B represents the motor and FES controllers that compensate for the switched system in (1) if the EMD was assumed to be negligible. Controller B is generated by redefining (4) as $r \triangleq \dot{e} + \alpha_1 e$, redefining (2) and (13) as in Bellman et al. (2017)

$$\sigma_m(q) \triangleq \begin{cases} 1, & q \in \mathcal{Q}_m \\ 0, & \text{otherwise} \end{cases}, \quad \sigma_e(q) \triangleq \begin{cases} 1, & q \in \mathcal{Q}_{KDZ} \\ 0, & \text{otherwise} \end{cases},$$

and using (11) and (12) with these modified signals.

5.1. Experimental testbed

For the testbed, a recumbent tricycle (TerraTrike Rover) was modified to be stationary as detailed in Bellman et al. (2017) and Cousin et al. (2019). The stimulator (Hasomed Rehastim), motor (Unite Motor Co.), and encoder (US Digital H1) were interfaced at 500 Hz using a DAQ (Quanser Q-PIDe) and MATLAB/Simulink/Quarc on a desktop computer. As detailed in Cousin et al. (2019), stimulation pulse width (PW) was controlled by (12), whereas the frequency (60 Hz) and amplitude (70 mA, 80 mA, and 90 mA for the gluteals, hamstrings, and quadriceps, respectively) were fixed.

5.2. Experimental methods

One male and four female able-bodied participants (ages 21.8 ± 0.4 years) in addition to two male and two female participants (ages 44.3 ± 13.5 years) with NCs ranging from cerebral palsy, multiple sclerosis, and spina bifida participated in the study. Written informed consent was provided by each participant as approved by the University of Florida Institutional Review Board (IRB201600881). To allow for equal comparison and to represent the clinical case when a participant is unable to contribute volitionally, passive therapy experiments were performed on the able-bodied participants, which consisted of the rider being instructed to remain passive (i.e., provide no volitional effort) and the rider being blind to the tracking performance. To investigate various clinical conditions, active therapy experiments were performed on the participants with NCs, which consisted of the rider being shown a real-time plot of the actual versus desired cadence, and each rider was asked to contribute to the tracking objective to the best of their ability.

In preparation for the experiments, the participant sat on the cycle's seat, which was adjusted to ensure comfort for the rider. The participant was then secured using orthotic boots (Össur Rebound Air Tall) connected at the pedals and electrodes (Axelgaard ValuTrode CF7515) were placed on the lower limb muscles (i.e., gluteals, hamstrings, quadriceps) and connected to the stimulator. Measurements (i.e., seat position, limb lengths, etc.) were then obtained as detailed in Bellman et al. (2017) to calculate the desired regions of the crank for a contraction of each muscle group (i.e., $\mathcal{Q}_m$). The cycle speed was then continuously increased to 50 RPM via the motor and stimulation was applied in an open-loop manner to determine a comfort limit on the stimulation for each muscle group. During the subsequent experiments, the FES inputs were saturated at a comfort limit indicated by the participant for each muscle group.

A preliminary trial was performed before each experiment that used Controller A to obtain an initial estimate of the EMD, $\hat{\tau}(t_0)$. Using the procedure detailed in Allen et al. (2020b), $\hat{\tau}(t_0)$ was obtained by fixing the crank at an efficient angle and then stimulating the quadriceps of the dominant leg for 0.25 s. The stimulation and torque data were then examined to calculate the CD25[6] measurement of the EMD, which was used as $\hat{\tau}(t_0)$. To update the EMD, recall that $g(t) = B + 2Ct$ was used, where the terms B and C are obtained by using Table V in Allen et al. (2020b).

After the cycle speed was increased to 50 RPM during the first 20 s, Controller A, B, or C was implemented for the remaining 120 s, called the steady-state period, to track a constant desired cadence of 50 RPM. Experiments using Controllers A and B for the able-bodied participants were implemented in a random order. Controller C was implemented before Controller A for the participants with NCs since Controller C provides no FES inputs, which would yield minimal fatigue and provide an unassisted and unfatigued baseline performance for each participant. No practice was allowed for the able-bodied participants; however, a single practice trial was permitted for each controller for the participants with NCs since they provided volition. Between each experiment, rest periods of five minutes were provided.

6. Results

To compare each controller, descriptive statistics of the experimental results (i.e., the peak and root mean square (RMS) cadence errors, FES effort, and motor effort) are included in Table 1 for each participant. On average across the able-bodied participants, Controllers A and B had a cadence tracking error of −0.01 ± 1.35 RPM and −0.01 ± 2.84 RPM, respectively, and FES was applied (to a minimum of one muscle group) 62.3% and 61.2% of the time, respectively. On average across the participants with NCs, FES was applied 64.8% of the time for Controller A, and the average cadence tracking error was −0.05 ± 1.38 RPM and 0.53 ± 3.37 RPM for Controllers A and C, respectively.

6.1. Statistical analysis and discussion

In Allen, Cousin et al. (2022), FES/motor controllers with a constant estimate of the EMD yielded an average cadence tracking error of 0.01 ± 2.00 RPM and 0.01 ± 2.72 RPM across six able-bodied participants and four participants with NCs, respectively. Compared to the controller developed in Allen, Cousin et al. (2022), the standard deviation of the cadence error was 32.5% and 49.3% smaller for Controller A across the able-bodied participants and participants with NCs, respectively. Interestingly, the average cadence error across able-bodied participants produced by Controller B was 0.04 ± 2.85 RPM in Allen, Cousin et al. (2022) (called Controller C in Allen, Cousin et al. (2022)) and −0.01 ± 2.84 RPM in this work.

Using the data for participants S1–S5 provided in Table 1, paired difference statistical tests (i.e., two-sided paired t-test and Shapiro–Wilk's test) were performed for each measurement to conclude normality of the data and that Controllers A and B had no significant effect on the percent of FES application time (P-Value = 0.352), the FES standard deviation (P-Value = 0.054), or the average motor (P-Value = 0.754) and FES (P-Value = 0.218) efforts. Subsequently, one-sided paired t-tests were used to conclude that the peak (P-Value = 0.001) and RMS (P-Value < 0.001) cadence errors, and the motor (P-Value < 0.001) standard

[6] Note that although the EMD may vary between each muscle group and each leg, the problem was simplified by using the measured EMD from the quadriceps of the dominant leg as $\hat{\tau}(t_0)$. From experience, the quadriceps muscle from the dominant leg tends to produce the highest torques. Furthermore, although the EMD is measured six different ways in Allen et al. (2020b), this paper uses the CD25 measurement to represent the EMD, which represents the delay between the onset of stimulation to the instant that the output torque reached 25% of the maximum torque level.

Table 1
Experimental results for the able-bodied participants (S1–S5) and the participants with NCs (N1–N4) during steady state.

Controller	Participant	RMS cadence error (RPM)	Peak cadence error (RPM)[a]	Motor effort (A)[b]	FES effort (μs)[c]
A	S1	1.69	7.81	1.46 ± 1.09	34.52 ± 1.47
	S2	1.72	4.81	1.73 ± 1.13	36.70 ± 1.83
	S3	1.24	4.10	1.70 ± 0.82	39.47 ± 1.93
	S4	1.02	5.63	1.78 ± 0.80	25.56 ± 1.14
	S5	1.11	3.47	1.53 ± 0.83	18.22 ± 0.38
	Average	**1.35**	**5.17**	**1.64 ± 0.93**	**30.89 ± 1.35**
	N1	1.53	4.70	1.25 ± 0.88	18.11 ± 2.17
	N2	1.75	4.88	1.74 ± 1.12	26.11 ± 0.75
	N3	1.26	4.93	1.76 ± 1.07	44.42 ± 1.82
	N4	1.01	5.37	1.34 ± 0.77	32.98 ± 1.51
	Average	**1.39**	**4.97**	**1.52 ± 0.96**	**30.40 ± 1.56**
B	S1	2.95	14.18	1.42 ± 1.98	39.49 ± 2.14
	S2	3.67	12.14	1.75 ± 2.33	38.06 ± 2.06
	S3	2.71	6.87	1.63 ± 1.61	38.06 ± 3.02
	S4	2.45	10.19	1.89 ± 1.78	27.22 ± 1.27
	S5	2.46	9.38	1.53 ± 1.58	19.10 ± 0.65
	Average	**2.85**	**10.55**	**1.65 ± 1.86**	**32.39 ± 1.83**
C	N1	3.62	14.08	0.00 ± 0.00	0.00 ± 0.00
	N2	3.39	24.36	0.00 ± 0.00	0.00 ± 0.00
	N3	2.62	14.18	0.00 ± 0.00	0.00 ± 0.00
	N4	4.53	27.94	0.00 ± 0.00	0.00 ± 0.00
	Average	**3.54**	**20.14**	**0.00 ± 0.00**	**0.00 ± 0.00**

[a]The maximum value of $|\dot{e}|$.
[b]The average ± standard deviation of $|u_E|$.
[c]The average ± standard deviation of the maximum stimulation delivered to each muscle group within each FES region.

deviation were significantly smaller for Controller A than for Controller B. Therefore, compensating for the EMD (i.e., Controller A) improved the cadence tracking performance while simultaneously reducing the variance of the motor control input, relative to no EMD compensation (i.e., Controller B).

Likewise, the same paired statistical tests were performed using the cadence data in Table 1, for participants N1–N4, to conclude normality of the data and that peak (P-Value = 0.011) and RMS (P-Value = 0.010) cadence errors were significantly smaller for Controller A than for Controller C. Therefore, Controller A improved the cadence tracking performance beyond what the participant could achieve on their own volition (recall Controller C provides no assistance).

Overall, the results for Controller A in Table 1 demonstrate the ability of Controller A to achieve cadence tracking despite uncertain volitional contributions from each participant with a NC, uncertainties and nonlinearities in the dynamics, a time-varying and unknown EMD, unknown disturbances, and a range of capabilities of each participant (e.g., due to some participants being able-bodied and others having a variety of NCs). Thus, Controller A has proven to be a safe and effective cadence tracking controller for individuals with varied capabilities during both active and passive therapy exercises.

7. Conclusion

In this work, FES/motor controllers and a time-varying estimate of the EMD are developed to compensate for a switched, delayed, nonlinear, and uncertain FES cycle system with uncertain volitional effort and disturbances. A switched Lyapunov stability analysis was performed to conclude exponential cadence tracking. Control authority was maintained and efficient muscle contractions were produced through the development of EMD and state dependent switching signals. Using the developed controllers, passive therapy experiments were conducted on five able-bodied participants and active therapy experiments were conducted on four participants with NCs, which produced average cadence errors of 0.01 ± 1.35 RPM and −0.05 ± 1.38 RPM, respectively. Ongoing efforts include the design of additional estimates of the EMD, such as adaptive estimates in real time, in an effort to further improve the tracking performance. Furthermore, the clinical impact and robustness of the proposed control system can be further validated through clinical trials.

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References

Allen, B. C., Cousin, C. A., Rouse, C. A., & Dixon, W. E. (2022). Robust cadence tracking for switched FES-cycling with an unknown time-varying input delay. *IEEE Transactions on Control Systems Technology*, *30*(2), 827–834.
Allen, B. C., Stubbs, K., & Dixon, W. E. (2020a). Robust power and cadence tracking on a motorized FES cycle with an unknown time-varying input delay. In *Proc. IEEE conf. decis. control* (pp. 3407–3412).
Allen, B. C., Stubbs, K. J., & Dixon, W. E. (2020b). Characterization of the time-varying nature of electromechanical delay during FES-cycling. *IEEE Transactions on Neural Systems Rehabilitation Engineering*, *28*(10), 2236–2245.
Allen, B. C., Stubbs, K., & Dixon, W. E. (2020c). Robust cadence tracking for switched FES-cycling with an unknown time-varying input delay using a time-varying estimate. In *IFAC world congr.*.
Allen, B. C., Stubbs, K. J., & Dixon, W. E. (2020d). Saturated control of a switched FES-cycle with an unknown time-varying input delay. In *IFAC conf. cyber-phys. hum.-syst.*.
Allen, B. C., Stubbs, K. J., & Dixon, W. E. (2021). Electromechanical delay during functional electrical stimulation induced cycling is a function of lower limb position. *Disability and Rehabilitation: Assistive Technology*, 1–6.
Allen, B. C., Stubbs, K., & Dixon, W. E. (2022). Position and cadence tracking of a motorized FES-cycle with an unknown time-varying input delay using saturated FES control. *Automatica*, *139*.
Anaya, F., Thangavel, P., & Yu, H. (2018). Hybrid FES–robotic gait rehabilitation technologies: a review on mechanical design, actuation, and control strategies. *International Journal of Intelligent Robotics Applications*, 1–28.
Bagheri, M., Naseradinmousavi, P., & Krstic, M. (2019). Feedback linearization based predictor for time delay control of a high-DOF robot manipulator. *Automatica*, *108*.

Bellman, M. J., Downey, R. J., Parikh, A., & Dixon, W. E. (2017). Automatic control of cycling induced by functional electrical stimulation with electric motor assistance. *IEEE Transactons on Automation Science and Engineering*, *14*(2), 1225–1234. http://dx.doi.org/10.1109/TASE.2016.2527716.

Benjamin, E. J., Blaha, M. J., Chiuve, S. E., Cushman, M., Das, S. R., Deo, R., et al. (2017). Heart disease and stroke statistics - 2017 update. *Circulation*, *135*(10), 146–603.

Cai, Z., de Queiroz, M. S., & Dawson, D. M. (2006). A sufficiently smooth projection operator. *IEEE Transactions on Automatic Control*, *51*(1), 135–139. http://dx.doi.org/10.1109/TAC.2005.861704.

Cousin, C. A., Rouse, C. A., Duenas, V. H., & Dixon, W. E. (2019). Controlling the cadence and admittance of a functional electrical stimulation cycle. *IEEE Transactions on Neural Systems Rehabilitation Engineering*, *27*(6), 1181–1192.

Downey, R., Merad, M., Gonzalez, E., & Dixon, W. E. (2017). The time-varying nature of electromechanical delay and muscle control effectiveness in response to stimulation-induced fatigue. *IEEE Transactions on Neural Systems Rehabilitation Engineering*, *25*(9), 1397–1408.

Filippov, A. F. (1964). Differential equations with discontinuous right-hand side. In *American mathematical society translations - series 2*: *Vol. 42*, *Fifteen papers on differential equations* (pp. 199–231). American Mathematical Society.

Fischer, N., Kamalapurkar, R., & Dixon, W. E. (2013). Lasalle-yoshizawa corollaries for nonsmooth systems. *IEEE Transactions on Automatic Control*, *58*(9), 2333–2338.

Karafyllis, L., & Krstic, M. (2017). *Predictor feedback for delay systems: implementations and approximations*. Springer.

Karafyllis, I., Malisoff, M., de Queiroz, M., Krstic, M., & Yang, R. (2015). Predictor-based tracking for neuromuscular electrical stimulation. *International Journal of the Robust Nonlinear*, *25*(14), 2391–2419. http://dx.doi.org/10.1002/rnc.3211.

Krstic, M. (2009). *Delay compensation for nonlinear, adaptive, and PDE systems*. Springer.

Krstic, M. (2010). Input delay compensation for forward complete and strict-feedforward nonlinear systems. *IEEE Transactions on Automatic Control*, *55*, 287–303.

Mazenc, F., & Malisoff, M. (2020). Continuous discrete sequential observers for time-varying systems under sampling and input delays. *IEEE Transactions on Automatic Control*, *65*(4), 1704–1709.

Mazenc, F., Malisoff, M., & Ozbay, H. (2018). Stability and robustness analysis for switched systems with time-varying delays. *SIAM Journal of the Control Optimization*, *56*, 158–182.

Mazenc, F., Mondie, S., Francisco, R., Conge, P., Lorraine, I., & Metz, F. (2004). Global asymptotic stabilization of feedforward systems with delay in the input. *IEEE Transactions on Automatic Control*, *49, (5)*, 844–850.

Nordez, A., Gallot, T., Catheline, S., Guèvel, A., Cornu, C., & Hug, F. (2009). Electromechanical delay revisited using very high frame rate ultrasound. *Journal of the Applications and Physiology*, *106*, 1970–1975. http://dx.doi.org/10.1152/japplphysiol.00221.2009.

Obuz, S., Duenas, V. H., Downey, R. J., Klotz, J. R., & Dixon, W. E. (2020). Closed-loop neuromuscular electrical stimulation method provides robustness to unknown time-varying input delay in muscle dynamics. *IEEE Transactions on Control Systems Technology*, *28*(6), 2482–2489.

Paden, B. E., & Sastry, S. S. (1987). A calculus for computing filippov's differential inclusion with application to the variable structure control of robot manipulators. *IEEE Transactions on Circuits Systems*, *34*(1), 73–82.

Rimmer, J. H., & Rowland, J. L. (2008). Health promotion for people with disabilities: implications for empowering the person and promoting disability-friendly environments. *American Journal of Lifestyle Medicine*, *2*(5), 409–420. http://dx.doi.org/10.1177/1559827608317397.

Wang, Y., Niu, B., Wu, B., & Xie, X. (2018). Asynchronous switching for switched nonlinear input delay systems with unstable subsystems. *Journal of the Franklin Institute*, *355*, 2912–2931.

Wang, Y., Sun, X., & Mazenc, F. (2016). Stability of switched nonlinear systems with delay and disturbance. *Automatica*, *69*.

Yang, D., Li, X., & Qiu, J. (2019). Output tracking control of delayed switched systems via state-dependent switching and dynamic output feedback. *Nonlinear Analysis Hybrid Systems*, *32*, 294–305.

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