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Reactive synthesis for relay-explorer consensus with intermittent communication☆

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ABSTRACT

A distributed multi-agent system architecture is explored to reach approximate consensus with intermittent communication. The multi-agent system is cast as a relay-explorer problem, where a relay agent intermittently provides navigational feedback to multiple explorer agents that do not have on-board absolute navigational sensors in a pre-defined sub-region. Within each sub-region, there is one relay agent responsible for servicing the corresponding explorer agents, and the estimated trajectory of an explorer agent can cross the boundary and enter another sub-region. We develop a reactive synthesis approach to formulate the mission specifications, while the state–space system dynamics provide real-time information for state corrections. Specifically, we pre-synthesize a set of planning strategies corresponding to candidate instantiations (i.e., pre-specified representative information scenarios) to dynamically switch among the explorers, and the planning strategies enable transfer of the servicing responsibility between relay agents. To guarantee stability of the switching strategies and the approximate consensus of the explorer agents, we develop maximum dwell-time conditions using a Lyapunov-based analysis to allow the explorer agents to drift for a pre-defined period without requiring servicing from the relay agents. Finally, we include a simulation study to demonstrate the performance of the developed method.

1. Introduction

Motivated by the advantages of intermittent communication versus requiring continuous communication in multi-agent systems (MASs), recent research has focused on developing event-triggered and self-triggered control. In Cheng, Kan, Klotz, Shea, and Dixon (2017), Heemels and Donkers (2013), Li, Liao, Huang, and Zhu (2015), Meng and Chen (2013), Tabuada (2007) and Wang and Lemmon (2009), the control methods only use sampled data for networked agents when desired stability and performance properties trigger the communication conditions. However, these methods typically assume the network is connected to ensure communication when required.

Recently a class of relay-explorer problems has emerged in Chen, Bell, Deptula, and Dixon (2019), Sun, Harris, Bell and Dixon (2020) and Zegers, Chen, Deptula, and Dixon (2019) where a relay agent intermittently provides state feedback to a set of explorer agents. A unique challenge is that the relay agent must maintain a sufficiently small estimation error of the relay agent's trajectory so that it can service the explorer agent when required. To guarantee stability of the resulting switched systems, a set of stabilizing dwell-time conditions are developed using a Lyapunov-based switched systems approach. As in results such as (Chen et al., 2019; Sun, Harris et al., 2020; Zegers et al., 2019), in this paper, maximum dwell-time conditions are developed that determine how long the explorer agents can operate using dead-reckoning from non-absolute sensors such as wheel encoders before requiring state feedback from the relay agent. Minimum dwell-time conditions are developed in Chen et al. (2019) and Sun, Bell, Zegers and Dixon (2020) to ensure the trajectory tracking error converges within a desired neighborhood of the desired trajectory. The objective of investigating the relay-explorer problem is to maximize the time an explorer

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agent can execute a mission objective without absolute sensing information while ensuring the explorer agent can get to the feedback available region. Authors of Zegers et al. (2019) use a relay agent with absolute navigational sensing to switch between multiple explorers lacking absolute positional sensors to provide each explorer navigational information to achieve consensus. Similarly, authors of Sun, Harris et al. (2020) develop a distributed controller to enable formation control and leader tracking for the explorer agents, while a relay agent intermittently provides state feedback to an explorer, enabling an MAS to explore an unknown environment indefinitely. However, the methods in Sun, Harris et al. (2020) and Zegers et al. (2019) rely on one relay agent servicing multiple explorer agents, which requires the relay agent to reach certain explorer agents within specified time periods to guarantee system stability. When the number of explorer agents is increased, the relay agent needs to maneuver to the corresponding explorer agent fast enough to ensure stability, which might be impractical in some applications and limits scalability.

Alternatively, metric temporal logic (MTL) specifications can encode the aforementioned maximum dwell-time conditions as in Ouaknine and Worrell (2005) and Xu, Zegers, Wu, Dixon, and Topcu (2019). MTL specifications in Xu et al. (2019) express the maximum dwell-time condition and practical constraints for the relay agent such as charging its battery and staying in specific regions of interest. Specifically, authors of Xu et al. (2019) design the explorers' controllers to ensure stability of the switched system, and use the MTL specifications to synthesize the relay agent's controller and to encode dwell-time conditions and additional practical constraints. A mixed-integer linear programming (MILP) problem is solved iteratively to obtain the optimal control inputs for the relay agent. Hence, the relay agent is required to iteratively compute the inputs to ensure the explorer agents can be serviced sufficiently often to reach approximate consensus. However, the computation requirements for the relay agent might not be applicable to agents with limited computation power. Another method such as signal temporal logic (STL) can also encode timing constraints. However, it is typically represented as an MILP which can scale exponentially (Raman et al., 2014).

The previous example can be treated as a reactive planning problem, where the MAS has to react to an uncontrolled environment, and guarantee correctness with respect to a given mission specification for all possible behaviors of the environment for all time. Such a planning problem can be solved by using a standard reactive synthesis method such as Bloem, Jobstmann, Piterman, Pnueli, and Sa'ar (2012) and Piterman, Pnueli, and Sa'ar (2006). Particularly, there is a rich literature focused on synthesis for a fragment of linear temporal logic (LTL), i.e., generalized reactivity 1 (GR(1)) in Alonso-Mora, DeCastro, Raman, Rus, and Kress-Gazit (2018), Bharadwaj, Dimitrova, and Topcu (2018), Bharadwaj, Vinod, Dimitrova, and Topcu (2020) and Ehlers and Raman (2016).

We consider a relay-explorer consensus problem where the relay agents have to provide state information to the explorer agents in pre-defined sub-regions, and the number of explorer agents within sub-regions could be time-varying. We use a reactive synthesis method to formulate the mission specifications, which can encode the dwell-time conditions derived from the dynamics to ensure system stability. We pre-synthesize the planning strategy to enable the relay agents to determine the next servicing explorer agent based on the states of real-time execution. Additionally, the planning strategy is flexible to adapt to service a different number of explorer agents, i.e., when an explorer agent leaves a certain region, the relay agents can transfer servicing responsibilities and switch to corresponding strategies. We conduct a simulation study which includes both local maneuvering (i.e., the number of explorer agents within sub-regions is fixed) and global maneuvering (i.e., the number of explorer agents within sub-regions is time-varying) scenarios to demonstrate the performance of the developed technique. The simulation results show the relay-explorer consensus objective can be achieved in both local and global maneuvering cases. Additionally, the developed reactive synthesis planner only requires 49.74% control effort compared to the control effort needed for a round-robin scheduler[1] to achieve the consensus objective.

The contributions of this work include developing a relay-explorer method to enable an MAS to reach consensus with intermittent communication. Unlike typical relay-explorer methods such as Chen et al. (2019), Sun, Bell et al. (2020), Sun, Harris et al. (2020), Xu et al. (2019), Zegers et al. (2019), the result developed in this paper investigates the explorer consensus problem with a network of relay agents. In comparison to previous relay-explorer problems, a pre-synthesized planning strategy enables the relay agents to determine the next explorer agent to service and adapts the servicing sequence to account for a variable number of agents operating within a sub-region. The explorer agents are only serviced when necessary by the relay agents. Unlike previous single relay results such as Chen et al. (2019), Sun, Bell et al. (2020), Sun, Harris et al. (2020), Xu et al. (2019), Zegers et al. (2019), the planning strategy is scalable because additional explorer agents can be serviced by incorporating additional relay agents.

2. Preliminaries

Let $\mathbb{R}$ and $\mathbb{Z}$ denote the set of real numbers and integers, respectively. For $p, q \in \mathbb{Z}_{>0}$, the $p \times q$ zero matrix and the $p \times 1$ zero column vector are denoted by $0_{p\times q}$ and 0_p, respectively. The $p \times p$ identity matrix is denoted by $I_{p\times p}$. The maximum singular value of $(\cdot)$ is denoted as $S_{\max}(\cdot)$. The maximum and minimum eigenvalues of a symmetric matrix $G \in \mathbb{R}^{p\times p}$ are denoted by $\lambda_{\max}(G) \in \mathbb{R}$ and $\lambda_{\min}(G) \in \mathbb{R}$, respectively.

3. Problem formulation

3.1. Problem statement

Consider an MAS consisting of M relay agents indexed by a set $L \triangleq \{1, 2, \ldots, M\}$ and N explorer agents indexed by a set $F \triangleq \{1, 2, \ldots, N\}$ for some $M, N \in \mathbb{Z}_{>0}$, where $M < N$. Given the MAS, the following assumption is made to describe the operating region for the agents.

Assumption 1. The MAS is operating within a region denoted by a compact set $\mathcal{D} \subset \mathbb{R}^z$, where $z \in \mathbb{Z}_{>0}$. The entire operating region $\mathcal{D} \triangleq \bigcup_{i\in L} S_i$ is covered by M number of sub-regions, and each sub-region is defined by a compact set $S_i \subset \mathbb{R}^z$, where i denotes the index of the corresponding sub-region. Additionally, the number of sub-regions equals the number of relay agents.

The following assumption is made to describe the servicing responsibility of the relay agents to the explorer agents.

Assumption 2. Each relay agent $i \in L$ is responsible for servicing[2] the explorer agents $j \in F$ within the sub-region S_i for all $t \in [0, \infty)$.

[1] The round-robin scheduler in this result indicates a scheduling method whereby each relay agent services each of the responsible explorer agents an equal number of times in a circular order.

[2] An explorer agent is serviced by a relay agent when state information is shared when $\left\|y_i^r(t) - y_j^e(t)\right\| \leq R$, where $y_i^r, y_j^e : [0, \infty) \to \mathbb{R}^z$ denote the position of relay agent i and explorer agent j, respectively, and $R \in \mathbb{R}_{\geq 0}$ denotes the communication radius of the relay agents and explorer agents. When a relay agent is communicating with an explorer agent, the state estimate of the explorer agent $\hat{x}_j^e : [0, \infty) \to \mathbb{R}^m$ is updated to the true state $x_j^e : [0, \infty) \to \mathbb{R}^m$ with a known constant error $c_{\text{init}} \in \mathbb{R}_{\geq 0}$, i.e., $\left\|\hat{x}_j^e(t) - x_j^e(t)\right\| \leq c_{\text{init}}$, where $m \in \mathbb{Z}_{>0}$.

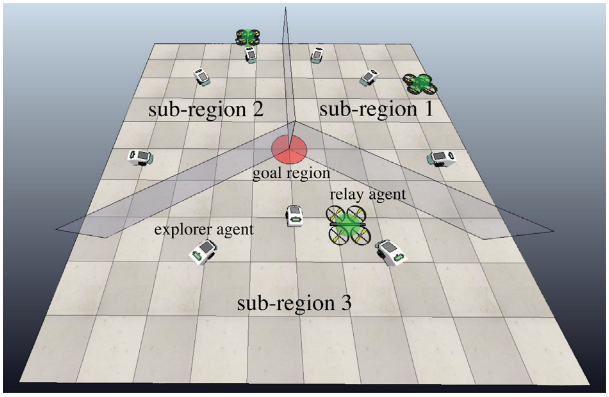

Fig. 1. An illustrative example of an MAS consisting of three relay agents (represented by quadcopters) in three different sub-regions (separated by virtual walls) to regulate nine explorer agents (represented by ground robots) to a goal region (represented by a red circle). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Within the operating region, the explorer agents lack absolute position information, while the relay agents have absolute sensing (e.g., GPS). Let $x_i^r : [0, \infty) \to \mathbb{R}^l$ and x_j^e denote the state of relay agent i and explorer agent j, respectively, where $i \in L$, $j \in F$, and $l \in \mathbb{Z}_{>0}$. Similar to Chen et al. (2019), Sun, Harris et al. (2020), Xu et al. (2019) and Zegers et al. (2019), the objective is to regulate states of the explorer agents (i.e., x_j^e for all $j \in F$) within a goal region centered at $g \in \mathbb{R}^z$ with radius $R_g \in \mathbb{R}_{>0}$. However, unlike Chen et al. (2019), Sun, Harris et al. (2020), Xu et al. (2019), Zegers et al. (2019), we consider the problem where the relay agents are confined to operate within specific sub-regions, and the explorer agents can move between the sub-regions. A unique challenge is that the relay agents have to dynamically adapt to the number of explorer agents within the corresponding sub-regions, and satisfy maximum-dwell time conditions for the explorer agents (see Fig. 1).

3.2. Agent dynamics

The linear time-invariant dynamics of relay agent i and explorer agent j are

$$\dot{x}_i^r(t) = A_i x_i^r(t) + B_i u_i^r(t), \tag{1}$$

$$y_i^r(t) = C_i x_i^r(t), \tag{2}$$

$$\dot{x}_j^e(t) = A x_j^e(t) + B u_j^e(t) + d_j(t), \tag{3}$$

$$y_j^e(t) = C x_j^e(t), \tag{4}$$

where $A_i \in \mathbb{R}^{l\times l}$, $A \in \mathbb{R}^{m\times m}$, $B_i \in \mathbb{R}^{l\times n}$, $B \in \mathbb{R}^{m\times n}$, $C_i \in \mathbb{R}^{z\times l}$, and $C \in \mathbb{R}^{z\times m}$ are known system matrices, and $n \in \mathbb{Z}_{>0}$. In (1) and (3), $u_i^r, u_j^e : [0, \infty) \to \mathbb{R}^n$ denote the control input of relay agent i and explorer agent j, respectively, and $d_j : [0, \infty) \to \mathbb{R}^m$ denotes an exogenous disturbance acting on explorer agent j.

4. Control objective

To quantify the objective, let the regulation error $e_j : [0, \infty) \to \mathbb{R}^m$ of explorer agent j be defined as

$$e_j(t) \triangleq x_g - x_j^e(t), \tag{5}$$

where $x_g \in \mathbb{R}^m$ denotes a predetermined user-selected state, and it is a point in the neighborhood of g with radius R_g. To facilitate the subsequent analysis, state estimation errors $e_{1,j} : [0, \infty) \to \mathbb{R}^m$ and estimated regulation errors $e_{2,j} : [0, \infty) \to \mathbb{R}^m$ are defined as

$$e_{1,j}(t) \triangleq \hat{x}_j^e(t) - x_j^e(t), \tag{6}$$

$$e_{2,j}(t) \triangleq x_g - \hat{x}_j^e(t), \tag{7}$$

respectively. Using (6) and (7), (5) can also be expressed as

$$e_j(t) = e_{1,j}(t) + e_{2,j}(t). \tag{8}$$

To facilitate the stability analysis of the relay agents, we define the relay agent's tracking error $e_{3,j} : [0, \infty) \to \mathbb{R}^z$ as

$$e_{3,j}(t) \triangleq C\hat{x}_j^e(t) - C_i x_i^r(t). \tag{9}$$

Given the system dynamics described in (1)–(4) and the error signals defined in (5)–(9), the following assumptions are made to facilitate the observer and controller development for the relay and explorer agents.

Assumption 3. The state estimate of explorer agent $\hat{x}_j^e(t)$ is initialized as $\left\|\hat{x}_j^e(0) - x_j^e(0)\right\| \le c_{\text{init}}$ for all $j \in F$.

Assumption 4. The initial position of explorer agent $x_j^e(0)$ is known to the corresponding relay agent $i \in L$ for all $j \in F$.

Assumption 5. The exogenous disturbance $d_j(t)$ is bounded, i.e., $\left\|d_j(t)\right\| \le \overline{d}_j$ for all $t \in [0, \infty)$, $j \in F$, where $\overline{d}_j \in \mathbb{R}_{>0}$ is a known constant and $\|\cdot\|$ denotes the Euclidean norm.

Assumption 6. The system matrices B_i and C_i are full-row rank matrices for all $t \in [0, \infty)$, $i \in L$.

The right pseudo inverses of B_i and C_i are denoted by B_i^+ and C_i^+, respectively, where $B_i^+ \triangleq B_i^{\mathrm{T}}\left(B_i B_i^{\mathrm{T}}\right)^{-1}$ and $C_i^+ \triangleq C_i^{\mathrm{T}}\left(C_i C_i^{\mathrm{T}}\right)^{-1}$.

Objective 1. *Given the system dynamics described in (1)–(4) for a sub-region S_i, the control objective is to design controllers u_j^e and observers $\hat{x}_j^e$ for the explorer agents, and design controllers u_i^r for the relay agents to satisfy the following properties. The regulation error e_j is uniformly ultimately bounded (UUB) for all $j \in F$ within the sub-region S_i for all $i \in L$. The explorer state estimates reach approximate consensus when $\limsup_{t\to\infty}\left\|e_j(t)\right\| \le \frac{R}{S_{\max}(C)}$ for all $j \in F$ (Xu et al., 2019).*

4.1. Approach

Objective 1 can be achieved by combining the reactive synthesis planning and control design. To facilitate the subsequent development, an overview is provided.

- Definitions and notations required to formulate the reactive synthesis mission specifications are introduced in this section.
- Controllers and observers for the explorer and relay agents with state–space representation are designed in Section 5.
- The corresponding stability conditions required to reach approximate consensus are derived in Section 6.
- The GR(1) specifications for the relay agents are formulated in (55).
- The required maximum-dwell time conditions are incorporated in the synthesis of correct-by-construction strategy planning in Theorem 5, which shows the explorer agents reach approximate consensus using the developed technique.

We are interested in designing a strategy for the relay agents to service the explorer agents for them to reach approximate consensus. The relay agents cannot control the actions of the explorer agents or the other relay agents. Hence, we represent each relay agent i a reactive system in an uncontrolled environment.

Formally, we define a finite set $I_i \triangleq \{\mu_{i,1}, \ldots, \mu_{i,a}\}$ of atomic propositions or Boolean inputs, controlled by the environment, and a finite set $O_i \triangleq \{\nu_{i,1}, \ldots, \nu_{i,b}\}$ of Boolean outputs, controlled by the relay agent i, where $a, b \in \mathbb{Z}_{>0}$. Together, they define the reactive system's input alphabet $\Sigma_{I,i} \triangleq 2^{I_i}$ and the output alphabet $\Sigma_{O,i} \triangleq 2^{O_i}$. We define $\Sigma_i \triangleq \Sigma_{I,i} \times \Sigma_{O,i}$. Informally, we model the status of the environment as observed as agent i's physical sensors by the valuations of the atomic propositions in set I_i. Similarly, we model the actions and state of relay agent i by the valuations of the atomic propositions in set O_i.

We represent the interaction between relay agent i and the uncontrolled environment as a two-player game. Formally, the game includes a tuple $\mathcal{G}_i = (Q_i, q_0, \Sigma_i, \delta_i)$, where Q_i is a finite set of states, $q_0 \in Q_i$ is the initial state, $\Sigma_i = \Sigma_{I,i} \times \Sigma_{O,i}$ is the alphabet of actions available to the environment and the agent, respectively, and $\delta_i : Q_i \times \Sigma_i \to Q_i$ is a complete transition function, that maps each state, input (environment action) and output (relay agent action) to a successor state.

In every state $q \in Q_i$ (starting with q_0), the environment chooses an input $\sigma_I \in \Sigma_{I,i}$, and then the relay agent chooses some output $\sigma_O \in \Sigma_{O,i}$. These choices define the next state $q' = \delta(q, (\sigma_I, \sigma_O))$, and so on. The resulting (infinite) sequence $\overline{\pi} = (q_0, \sigma_{I,0}, \sigma_{O,0}, q_1)(q_1, \sigma_{I,1}, \sigma_{O,1}, q_2)\ldots$ is called a play.

We are interested in computing a strategy for the relay agent such that every play that may be generated in the game, while the relay agent implementing that strategy, will satisfy a so-called winning condition. Temporal logic has been used to express such winning conditions (Piterman et al., 2006). While we refer the reader to Baier and Katoen (2008) and Manna and Pnueli (2012) for details on temporal logic, we note that a temporal logic specification is interpreted against infinitely long plays in our setting. If there is a strategy for the relay agent that ensures that all plays in the game will satisfy a winning condition expressed in temporal logic, then the relay agent wins the game. Computing such a *winning* strategy had been regarded as computationally intractable (Pnueli & Rosner, 1989). On the other hand, Alonso-Mora et al. (2018) and Ehlers and Raman (2016) showed that the complexity of computing a winning strategy reduces to a polynomial (in the size of the underlying game graph) when the winning conditions are restricted to so-called GR(1) fragment of temporal logic. We assume that the specification is an implication between a set of assumptions and a set of guarantees (Bloem et al., 2012), and the GR(1) specifications are written in the following assume-guarantee form

$$\varphi = \left(\Box\, G_I \wedge \bigwedge_{d=1}^{u} \Box \Diamond D_d\right) \to \left(\Box\, G_O \wedge \bigwedge_{e=1}^{v} \Box \Diamond E_e\right),$$

where $\Box G_I$ and $\Box G_O$ indicate the invariants in the assumption (e.g., membership to a set of states or transitions in the underlying game of interest) and $\Box\Diamond D_d$ and $\Box\Diamond E_e$ refer to the propositions that hold infinitely often. With abuse of notation, we sometimes use G_I, G_O, D_d, E_e to refer to both the sets of states and propositions that indicate membership of the corresponding sets of states.

A strategy for the relay agent i is a function $\rho_{O,i} : [0, \infty) \times \Sigma_{I,i} \to \Sigma_{O,i}$ which maps the current time step and an action of the environment to an action of the relay agent. A strategy for the environment is a function $\rho_{I,i} : [0, \infty) \to \Sigma_{I,i}$ that maps the current time step to an action of the environment. We denote the sets of all strategies for the relay agent and for the environment by $\mathcal{M}_{O,i}$ and $\mathcal{M}_{I,i}$, respectively. Every pair of strategies $\rho_{O,i} \in \mathcal{M}_{O,i}$ for the relay agent and $\rho_{I,i} \in \mathcal{M}_{I,i}$ for the environment define a play, denoted by $\Pi(\rho_{O,i}, \rho_{I,i}) = \overline{\pi}$.

Given a game structure $\mathcal{G}$ and a GR(1) winning condition φ for the relay agent, we seek to synthesize a strategy ρ for every relay agent such that for every strategy for the environment it holds that all resulting plays satisfy φ. In such cases we say that ρ satisfies φ, denoted as $\rho \models \varphi$. The strategy synthesis problem for GR(1) winning conditions was solved in Bloem et al. (2012).

In the context of a control synthesis problem, the environment encompasses all variables that cannot be directly set by the controller. From the perspective of a relay agent, the environment variables correspond to the actions of other agents. The game-based formulation is used as, from the perspective of an agent, it sees environment as a second player in a game. The goal of the synthesis then is to construct a strategy of the agent that is *winning* with respect to the specifications for any possible action of the environment. Since our goal is decentralized synthesis, every agent sees the collection of all other agents (the environment) as a second player and the goal of the contracts between agents is to facilitate feasibility of finding winning strategies in the game.

4.2. Approximate consensus

A goal region centered at the position denoted by g with radius R_g is capable of providing state information to each explorer agent $j \in F$ once $\left\|Cx_j^e(t) - Cx_g\right\| \leq R_g$. Without loss of generality, let $R_g = R$ for simplicity of exposition. The task of relay agent i is to service each explorer agent j within S_i intermittently by providing state (i.e., position) information while the explorer agents are navigating to g for all $i \in L, j \in F$.

Given an integer $K \in \mathbb{Z}_{\geq 0}$, an explorer agent j is in the sub-region S_i at time $t + K$ if its estimated position $C\hat{x}_j^e \in S_i$ at time $t + K$. We define the function $\eta_i^K : [0, \infty) \to 2^F$ that outputs the subset of explorer agents that will be within the sub-region S_i in K time steps. Put simply, $\eta_i^K(t)$ will output the set of explorer agents $F_i \subseteq F$ whose estimated state is in sub-region S_i at time $t + K$. If the estimated trajectory of an explorer agent crosses the boundary of a sub-region in less than $t + K$ steps, the relay agent will communicate with the neighboring relay agent to notify the crossing action, hand-over the servicing responsibility, and transfer the last serviced position of the explorer agent. The parameter K is a user-defined time parameter[3] to allow relay agents to conduct the hand-over without violating the dwell-time condition. This forms an assume-guarantee contract between relay agents and we formalize this notion in Section 6. Note that covering the region for optimal distribution of relay and explorer agents (such as minimizing boundary crossings and hand-overs) is an active area of current interest. In this paper, we manually covered the operating region by three and two sub-regions in the subsequent simulations for simplicity.

Let $\zeta_i : [0, \infty) \to F$ be a piece-wise constant switching signal that determines which explorer the relay agent i is to service within the sub-region S_i. At $t = 0$, relay agent i will compute the servicing time of each explorer agent j as denoted by t_s^j, where s indicates the sth servicing instance. Immediately after $t = 0$, relay agent i will maneuver towards explorer agent j. Hence, the $(s + 1)$th servicing time for explorer agent j is defined as $t_{s+1}^j \triangleq \inf\left\{t > t_s^j : \left(\left\|y_i^r(t) - y_j^e(t)\right\| \leq R\right) \wedge (\zeta_i(t) = j)\right\}$, where

[3] The selection of K is dependent on the nature and geometry of the sub-regions. It is unnecessary for K to be the same for all sub-regions, but without loss of generality, we used a common K for ease of notation. If K is too large for a small sub-region, then the relay agent who is responsible for the corresponding sub-region will simply hand off the explorer agents to another relay agent. In this case, the selection of a large K may cause a disproportionate burden. Similarly, if K is too small, then there may be too little time for a relay agent to take over responsibility and service the explorer agent without violating the dwell-time condition. Generally speaking, similar sized regions should require similar K values, however it cannot be generalized completely as it depends on the geometry of the space.

$\wedge$ denotes the conjunction logical connective.[4] Let $\left\{t_s^j\right\}_{s=0}^{\infty} \subset \mathbb{R}$ be an increasing sequence of servicing times determined by the subsequently defined maximum dwell-time condition (see Theorem 1) for explorer agent j. The servicing time in t_{s+1}^j defines the necessary conditions to enable communication between the relay agent i and explorer agent j. Nonetheless, the maximum dwell-time condition provides an upper bound on the servicing time based on the need to ensure stability.

At time t_s^j, $\left\|y_i^r\left(t_s^j\right) - y_j^e\left(t_s^j\right)\right\| \leq R$, where the relay agent i will service explorer agent j and compute the future servicing time t_{s+1}^j. Immediately after t_s^j, the relay agent i will leave explorer agent j to go service other explorers. Let t_r^j denote the time the relay agent i begins maneuvering towards explorer agent j, where $t_r^j \triangleq \inf\left\{t > t_s^j : \left(\left\|y_i^r(t) - y_j^e(t)\right\| > R\right) \wedge \left(\zeta_i(t) = j\right)\right\}$. Proper design of $\zeta_i(t)$ requires $t_r^j < t_{s+1}^j$ for the relay agent i to satisfy the maximum dwell-time condition. Let $\left\{t_r^j\right\}_{r=0}^{\infty} \subset \mathbb{R}$ be an increasing sequence of return times for explorer agent j. One of the contributions of this work is to provide a scalable and provably correct method to compute $\zeta_i(t)$ for all relay agents $i \in L$. We detail this process in Section 6.

5. Observer and controller development

The state estimate of explorer agent $j \in F$ is obtained from the following model-based observer

$$\dot{\hat{x}}_j^e(t) \triangleq -Ae_{2,j}(t) + Bu_j^e(t), t \in \left[t_s^j, t_{s+1}^j\right), \tag{10}$$

$$\hat{x}_j^e\left(t_s^j\right) \triangleq x_j^e\left(t_s^j\right) + c_{\text{init}}, \tag{11}$$

where the position estimate $\hat{y}_j^e : [0,\infty) \to \mathbb{R}^z$ of explorer agent j can be modeled as

$$\hat{y}_j^e(t) \triangleq C\hat{x}_j^e(t). \tag{12}$$

The control input of explorer agent j is designed as

$$u_j^e(t) \triangleq B^{\mathrm{T}}Pe_{2,j}(t), \tag{13}$$

where $P \in \mathbb{R}^{m\times m}$ is the positive definite solution to the Algebraic Riccati Equation (ARE) given by

$$A^{\mathrm{T}}P + PA - 2PBB^{\mathrm{T}}P + k_{ARE}I_{m\times m} = 0_{m\times m}, \tag{14}$$

and $k_{ARE} \in \mathbb{R}_{>0}$ is a user-defined parameter. The control input of relay agent i is designed as

$$\begin{aligned} u_i^r(t) \triangleq\ & B_i^+C_i^+\left(-C_iA_ix_i^r(t) + k_i(t)e_{3,j}(t)\right) \\ & + B_i^+C_i^+C\left(-Ae_{2,j}(t) + Bu_j^e(t)\right), \end{aligned} \tag{15}$$

where $k_i : [0,\infty) \to \mathbb{R}_{>0}$ is a subsequently defined piece-wise constant parameter. Substituting (3), (6), (7), (10), and (11) into the time derivative of (6) yields

$$\dot{e}_{1,j}(t) = Ae_{1,j}(t) - Ax_g - d_j(t), \quad t \in \left[t_s^j, t_{s+1}^j\right), \tag{16}$$

$$e_{1,j}\left(t_s^j\right) = c_{\text{init}}. \tag{17}$$

Substituting (10), (11), and (13) into the time derivative of (7) yields

$$\dot{e}_{2,j}(t) = \left(A - BB^{\mathrm{T}}P\right)e_{2,j}(t), \quad t \in \left[t_s^j, t_{s+1}^j\right), \tag{18}$$

$$e_{2,j}\left(t_s^j\right) = x_g - x_j^e\left(t_s^j\right) - c_{\text{init}}. \tag{19}$$

Substituting (3), (8), and (13) into the time derivative of (5) yields

$$\begin{aligned} \dot{e}_j(t) =\ & \left(A - BB^{\mathrm{T}}P\right)e_j(t) + BB^{\mathrm{T}}Pe_{1,j}(t) \\ & - Ax_g - d_j(t). \end{aligned} \tag{20}$$

Substituting (1), (10), and (15) into the time derivative of (9) yields

$$\begin{aligned} \dot{e}_{3,j}(t) =\ & C\left(-Ae_{2,j}(t) + Bu_j^e(t)\right) \\ & - C_i\left(A_ix_i^r(t) + B_iu_i^r(t)\right), t \in \left[t_s^j, t_r^j\right) \end{aligned} \tag{21}$$

$$e_{3,j}\left(t_s^j\right) = C\left(x_j^e\left(t_s^j\right) + c_{\text{init}}\right) - C_ix_i^r\left(t_s^j\right) \tag{22}$$

and

$$\dot{e}_{3,j}(t) = -k_i(t)e_{3,j}(t), \quad t \in \left[t_r^j, t_{s+1}^j\right) \tag{23}$$

$$e_{3,j}\left(t_r^j\right) = C\hat{x}_j^e\left(t_r^j\right) - C_ix_i^r\left(t_r^j\right). \tag{24}$$

Remark 1. A regulation control objective is investigated in this result to highlight the novelties with the relay-explorer consensus problem with intermittent communication and because of the additional complexities involved with the resulting nonautonomous system resulting from a tracking control problem. One possible approach to extend the current approach for a tracking objective is to adapt the method presented in Kamalapurkar, Dinh, Bhasin, and Dixon (2015). In Kamalapurkar et al. (2015), an optimal tracking problem is solved by including the partial derivative of the value function with respect to the desired trajectory in the Hamilton–Jacobi–Bellman (HJB) equation. By using a system transformation, the problem is converted to a time-invariant optimal control problem such that the resulting value function is a time-invariant function of the transformed states. To apply this strategy for the current result, further investigation is required to examine the ability of the switching signal $\zeta_i(t)$ for all relay agents $i \in L$ to satisfy the maximum dwell-time conditions in Section 6.

6. Stability conditions

In this section, we provide conditions that generate a stable switched system for each sub-region, and then prove approximate consensus for the corresponding explorer agents. When explorer agents cross boundaries, the synthesized strategies are changed for the relay agents to adapt to the different number of explorer agents within sub-regions. Eventually, all the explorer agents in the entire operating region reach approximate consensus within the goal region centered at g with radius R_g. Specifically, Theorem 1 presents the maximum dwell-time condition the relay agent i has to satisfy to ensure the state estimation error $e_{1,j}(t)$ is bounded for all $t \in \left[t_s^j, t_{s+1}^j\right]$. Theorem 2 shows the observer in (10) and controller in (13) ensure the estimated regulation error $e_{2,j}(t)$ is exponentially regulated for all $t \in \left[t_s^j, t_{s+1}^j\right)$ provided the ARE in (14) is satisfied. Theorem 3 indicates the observer in (10) and controller in (13) ensure the regulation error $e_j(t)$ is UUB provided the relay agent i satisfies the maximum dwell-time condition in (25) and $e_{1,j}\left(t_0^j\right) \leq c_{\text{init}}$. Theorem 4 provides a sufficient gain condition to enable timely servicing by the relay agent i, and shows the relay agent's tracking error $e_{3,j}(t)$ is bounded for all $t \in \left[t_s^j, t_{s+1}^j\right]$. Theorem 5 shows when the GR(1) specifications for relay agents described in (55) are satisfied, the observer in (10), the controllers in (13) and (15) enable the explorer agents reach approximate consensus within the goal region.

[4] For $s = 0$, t_0^j is taken to be the initial time, e.g., $t_0^j = 0$.

6.1. Explorer agent analysis

To demonstrate the regulation error e_j is bounded for the explorer agent j, we provide three theorems. The following theorem provides a condition on the relay agent such that the state estimation error $e_{1,j}(t)$ is bounded for all $t \in \left[t_s^j, t_{s+1}^j\right]$.

Theorem 1. *When the relay agent i satisfies the maximum dwell-time condition given by*

$$T_j \triangleq t_{s+1}^j - t_s^j \leq \frac{1}{S_{\max}(A)} \ln\left(\frac{V_T + \varepsilon_1}{c_{init} + \varepsilon_1}\right), \tag{25}$$

where $T_j \in \mathbb{R}_{>0}$ denotes the maximum dwell-time for explorer agent j, $V_T \in \left(0, \frac{R}{S_{\max}(C)}\right)$ is a user-defined parameter, $\varepsilon_1 \triangleq \frac{\kappa_j}{S_{\max}(A)} \in \mathbb{R}_{>0}$, $\kappa_j \triangleq S_{\max}(A)\,\bar{x}_g + \bar{d}_j \in \mathbb{R}_{>0}$, $\bar{x}_g \in \mathbb{R}_{>0}$ is a bounding constant such that $\|x_g\| \leq \bar{x}_g$, then $\|e_{1,j}(t)\| \leq V_T$ for all $t \in \left[t_s^j, t_{s+1}^j\right]$.

Proof. Let $t \geq t_s^j$, and suppose $\left\|e_{1,j}\left(t_s^j\right)\right\| = c_{\text{init}}$.[5] Consider the common Lyapunov-like functional candidate $V_{1,j} : \mathbb{R}^m \to \mathbb{R}_{\geq 0}$ defined as

$$V_{1,j}\left(e_{1,j}(t)\right) \triangleq \frac{1}{2} e_{1,j}^{\mathrm{T}}(t)\, e_{1,j}(t). \tag{26}$$

Substituting the closed-loop error system (16) into the time derivative of (26) yields

$$\dot{V}_{1,j}\left(e_{1,j}(t)\right) = e_{1,j}^{\mathrm{T}}(t)\left(Ae_{1,j}(t) - Ax_g - d_j(t)\right). \tag{27}$$

Using the definition of κ_j in (25), (27) can be upper bounded by

$$\dot{V}_{1,j}\left(e_{1,j}(t)\right) \leq S_{\max}(A)\left\|e_{1,j}(t)\right\|^2 + \kappa_j\left\|e_{1,j}(t)\right\|. \tag{28}$$

Substituting (26) into (28) yields

$$\begin{aligned}\dot{V}_{1,j}\left(e_{1,j}(t)\right) &\leq 2S_{\max}(A)\, V_{1,j}\left(e_{1,j}(t)\right) \\ &\quad + \kappa_j\sqrt{2V_{1,j}\left(e_{1,j}(t)\right)}.\end{aligned} \tag{29}$$

Invoking the Comparison Lemma in Khalil (2002, Lemma 3.4) on (29) over $\left[t_s^j, t_{s+1}^j\right)$ yields

$$\begin{aligned}V_{1,j}\left(e_{1,j}(t)\right) \leq &\left(\left(\frac{\sqrt{2}}{2}(c_{\text{init}} + \varepsilon_1)\exp(S_{\max}(A)(t - t_s^j)\right)\right. \\ &\left. - \frac{\sqrt{2}}{2}\varepsilon_1\right)^2.\end{aligned} \tag{30}$$

Substituting (26) into (30) yields

$$\left\|e_{1,j}(t)\right\| \leq (c_{\text{init}} + \varepsilon_1)\exp\left(S_{\max}(A)\left(t - t_s^j\right)\right) - \varepsilon_1. \tag{31}$$

Define $\Phi_j : \left[t_s^j, t_{s+1}^j\right) \to \mathbb{R}$ as

$$\Phi_j(t) \triangleq (c_{\text{init}} + \varepsilon_1)\exp\left(S_{\max}(A)\left(t - t_s^j\right)\right) - \varepsilon_1. \tag{32}$$

Since $\left\|e_{1,j}(t)\right\| \leq (c_{\text{init}} + \varepsilon_1)\exp\left(S_{\max}(A)\left(t - t_s^j\right)\right) - \varepsilon_1$ for all $t \in \left[t_s^j, t_{s+1}^j\right)$ and $\left\|e_{1,j}\left(t_{s+1}^j\right)\right\| = c_{\text{init}}$, where $t_{s+1}^j > t_s^j$ and $\Phi_j\left(t_{s+1}^j\right) > 0$, therefore $\left\|e_{1,j}(t)\right\| \leq \Phi_j(t)$ for all $t \in \left[t_s^j, t_{s+1}^j\right]$. If $\Phi_j\left(t_{s+1}^j\right) \leq V_T$, then $\left\|e_{1,j}(t)\right\| \leq V_T$ for all $t \in \left[t_s^j, t_{s+1}^j\right]$. In addition, $\Phi_j\left(t_{s+1}^j\right) \leq V_T$ yields the maximum dwell-time condition in (25). Therefore, $\left\|e_{1,j}(t)\right\| \leq V_T$ for all $t \in \left[t_s^j, t_{s+1}^j\right]$ provided $\left\|e_{1,j}\left(t_s^j\right)\right\| = c_{\text{init}}$ and (25) hold.

The value of the maximum dwell-time T_j is dictated by the selection of the user-defined maximum upper bound V_T for the state estimation error $e_{1,j}$, the selection of goal location $\|x_g\| \leq \bar{x}_g$, the exogenous disturbance $\|d_j(t)\| \leq \bar{d}_j$, and the system parameters A, C, c_{init}

Remark 2. Zeno behavior occurs when the difference between $t_{s+1}^j - t_s^j$ is zero. It is essential to show that the difference between consecutive servicing times, i.e., $t_{s+1}^j - t_s^j$ is lower bounded by a finite positive constant. While explorer agent j is not serviced by a relay agent, let $t_{\text{travel}} \in \left(t_s^j, t_{s+1}^j\right)$ represent the minimum time it would take the relay agent to travel between the previous and the current explorer agents. Therefore, the maximum dwell-time condition has a lower constant bound, i.e., $t_{\text{travel}} \leq T_j$, where $t_{\text{travel}} = \frac{y}{\bar{v}_{\max}}$, $y, \bar{v}_{\max} \in \mathbb{R}_{>0}$ denotes the actual distance and the maximum velocity the relay agent travels, respectively. Since $t_{\text{travel}} \leq T_j$, Zeno behavior is excluded.

Next, we show the estimated regulation error $e_{2,j}(t)$ is exponentially regulated for all $t \in \left[t_s^j, t_{s+1}^j\right)$.

Theorem 2. *If the ARE in (14) is satisfied, then the observer in (10) and controller in (13) ensure the estimated regulation error in (7) is exponentially regulated in the sense that*

$$\begin{aligned}&\left\|e_{2,j}(t)\right\| \leq \\ &\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\left\|e_{2,j}\left(t_s^j\right)\right\|\exp\left(-\frac{k_{ARE}}{2\lambda_{\max}(P)}\left(t - t_s^j\right)\right)\end{aligned} \tag{33}$$

for all $t \in \left[t_s^j, t_{s+1}^j\right)$ and each servicing instance $s \in \mathbb{Z}$.

Proof. Consider the common Lyapunov functional $V_{2,j} : \mathbb{R}^m \to \mathbb{R}_{\geq 0}$ defined as

$$V_{2,j}\left(e_{2,j}(t)\right) \triangleq e_{2,j}^{\mathrm{T}}(t)\, Pe_{2,j}(t). \tag{34}$$

By the Rayleigh quotient, (34) can be bounded as

$$\begin{aligned}\lambda_{\min}(P)\left\|e_{2,j}(t)\right\|^2 &\leq V_{2,j}(e_{2,j}(t)) \\ &\leq \lambda_{\max}(P)\left\|e_{2,j}(t)\right\|^2.\end{aligned} \tag{35}$$

Substituting the closed-loop error system (18) into the time derivative of (34) yields

$$\dot{V}_{2,j}(e_{2,j}(t)) = e_{2,j}^{\mathrm{T}}(t)\left(A^{\mathrm{T}}P + PA - 2PBB^{\mathrm{T}}P\right)e_{2,j}(t). \tag{36}$$

Using (14), (36) can be rewritten as

$$\dot{V}_{2,j}\left(e_{2,j}(t)\right) = -k_{ARE}\left\|e_{2,j}(t)\right\|^2. \tag{37}$$

Substituting (35) in (37) yields

$$\dot{V}_{2,j}\left(e_{2,j}(t)\right) \leq -\frac{k_{ARE}}{\lambda_{\max}(P)} V_{2,j}\left(e_{2,j}(t)\right). \tag{38}$$

Invoking the Comparison Lemma in Khalil (2002, Lemma 3.4) on (38) over $\left[t_s^j, t_{s+1}^j\right)$ and substituting (35) yields (33).

Using the relationship described in (8), and results from Theorems 1 and 2, the following theorem indicates the regulation error $e_j(t)$ is UUB.

Theorem 3. *If the relay agent i satisfies the maximum dwell-time condition in (25) for each $s \in \mathbb{Z}$ and $e_{1,j}\left(t_0^j\right) = c_{init}$, then the*

[5] $\left\|e_{1,j}\left(t_s^j\right)\right\| = c_{\text{init}}$ because relay agent i serviced explorer agent j at time t_s^j with a maximum error of c_{init}.

observer in (10) and controller in (13) ensure the regulation error in (5) is UUB in the sense that

$$\begin{aligned}\left\|e_j(t)\right\| \leq\ & \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\left\|e_j(0)\right\| \exp\left(-\frac{k_{ARE}}{2\lambda_{\max}(P)}t\right) \\ & +\frac{c}{k_{ARE}}\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\left(1-\exp\left(-\frac{k_{ARE}}{2\lambda_{\max}(P)}t\right)\right), \end{aligned} \tag{39}$$

where $c \triangleq 2V_T S_{\max}\left(PBB^{\mathrm{T}}P\right)+2\bar{x}_g S_{\max}(PA)+2\bar{d}_j S_{\max}(P) \in \mathbb{R}_{>0}$ is a known constant.

Proof. Suppose the relay agent i satisfies the maximum dwell-time condition in (25) for each $s \in \mathbb{Z}$ and $e_{1,j}\left(t_0^j\right)=c_{\text{init}}$. Consider the common Lyapunov functional $V_j : \mathbb{R}^m \to \mathbb{R}_{\geq 0}$ defined as

$$V_j\left(e_j(t)\right) \triangleq e_j^{\mathrm{T}}(t)\, P e_j(t). \tag{40}$$

By the Rayleigh quotient, (40) can be bounded as

$$\lambda_{\min}(P)\left\|e_j(t)\right\|^2 \leq V_j\left(e_j(t)\right) \leq \lambda_{\max}(P)\left\|e_j(t)\right\|^2. \tag{41}$$

Substituting the closed-loop error system (20) into the time derivative of (40) yields

$$\begin{aligned}\dot{V}_j\left(e_j(t)\right) =\ & 2e_j^{\mathrm{T}}(t)P\big(BB^{\mathrm{T}}Pe_{1,j}(t)-Ax_g-d_j(t)\big) \\ & +e_j^{\mathrm{T}}(t)\left(A^{\mathrm{T}}P+PA-2PBB^{\mathrm{T}}P\right)e_j(t). \end{aligned}\tag{42}$$

Using (14), (42) can be upper bounded as

$$\begin{aligned}\dot{V}_j\left(e_j(t)\right) \leq\ & -k_{ARE}\left\|e_j(t)\right\|^2+2S_{\max}(P)\left\|e_j(t)\right\|\bar{d}_j \\ & +2S_{\max}\left(PBB^{\mathrm{T}}P\right)\left\|e_j(t)\right\|\left\|e_{1,j}(t)\right\| \\ & +2S_{\max}(PA)\left\|e_j(t)\right\|\bar{x}_g. \end{aligned}\tag{43}$$

Since the relay agent i satisfies the maximum dwell-time condition in (25) for each $s \in \mathbb{Z}$, $\left\|e_{1,j}(t)\right\| \leq V_T$ for all $t \in [0,\infty)$ by Theorem 1. Using the definition for c, (43) can be upper bounded as

$$\dot{V}_j\left(e_j(t)\right) \leq -k_{ARE}\left\|e_j(t)\right\|^2+c\left\|e_j(t)\right\|. \tag{44}$$

Invoking the Comparison Lemma in Khalil (2002, Lemma 3.4) on (44) over $[0,\infty)$ and substituting (41) yields (39). The inequality in (39) implies $e_j \in \mathcal{L}_\infty$. Since $e_j \in \mathcal{L}_\infty$ and $e_{1,j} \in \mathcal{L}_\infty$ given the relay agent i satisfies the maximum dwell-time condition in (25) for each $s \in \mathbb{Z}$, (5) and (8) imply $x_j^e, e_{2,j} \in \mathcal{L}_\infty$. Since $x_j^e, e_{1,j}, e_{2,j} \in \mathcal{L}_\infty$, (6) and (13) imply $\hat{x}_j^e, u_j^e \in \mathcal{L}_\infty$ provided B and P are constant matrices. Hence, $\dot{x}_j^e, y_j^e \in \mathcal{L}_\infty$ by (3) and (4). Since $e_{1,j}, e_{2,j}, e_j, u_j^e, \hat{x}_j^e \in \mathcal{L}_\infty$, (10), (12), (16), (18), and (20) imply $\dot{\hat{x}}_j^e, \hat{y}_j^e, \dot{e}_{1,j}, \dot{e}_{2,j}, \dot{e}_j \in \mathcal{L}_\infty$.

Remark 3. From Theorem 3, note that

$$\limsup_{t\to\infty}\left\|e_j(t)\right\| \leq \frac{c}{k_{ARE}}\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \triangleq \gamma(c), \tag{45}$$

where $\gamma(c)$ can be made arbitrarily small by selecting a small c, i.e., selecting a small V_T and setting the desired state as the origin.

6.2. Relay agent analysis

To prove the relay agent's tracking error $e_{3,j}(t)$ is bounded for all $t \in \left[t_s^j, t_{s+1}^j\right]$, we provide the following theorem.

Theorem 4. *If $\left\|y_i^r\left(t_r^j\right)-y_j^e\left(t_r^j\right)\right\| > R$, then the controller of the relay agent i in (15) can satisfy the maximum dwell-time condition in (25) for explorer agent j provided*

$$k_i(t) \geq \frac{1}{\left(t_{s+1}^j-t_r^j\right)}\ln\left(\frac{\left\|e_{3,j}\left(t_r^j\right)\right\|}{R-S_{\max}(C)V_T}\right) \tag{46}$$

for all $t \in \left[t_r^j, t_{s+1}^j\right)$, where $k_i(t)$ is a piece-wise constant. In addition, the relay agent's tracking error in (9) is bounded for all $t \in \left[t_s^j, t_{s+1}^j\right]$.

Proof. Consider the common Lyapunov functional candidate $V_{3,j} : \mathbb{R}^z \to \mathbb{R}_{\geq 0}$ defined as

$$V_{3,j}\left(e_{3,j}(t)\right) \triangleq \frac{1}{2}e_{3,j}^{\mathrm{T}}(t)\, e_{3,j}(t). \tag{47}$$

Substituting the closed-loop error system (23) when $t \in \left[t_r^j, t_{s+1}^j\right)$ into the time derivative of (47) yields

$$\dot{V}_{3,j}\left(e_{3,j}(t)\right) = -k_i(t)\, e_{3,j}^{\mathrm{T}}(t)\, e_{3,j}(t), \tag{48}$$

where $k_i(t)$ is constant over $\left[t_r^j, t_{s+1}^j\right)$. Substituting (47) into (48) yields

$$\dot{V}_{3,j}\left(e_{3,j}(t)\right) = -2k_i(t)\, V_{3,j}\left(e_{3,j}(t)\right). \tag{49}$$

Invoking the Comparison Lemma in Khalil (2002, Lemma 3.4) on (49) over $\left[t_r^j, t_{s+1}^j\right)$ and substituting in (47) yields

$$\left\|e_{3,j}(t)\right\| = \left\|e_{3,j}\left(t_r^j\right)\right\|\exp\left(-k_i(t)\left(t-t_r^j\right)\right). \tag{50}$$

Consider $t \in \left[t_r^j, t_{s+1}^j\right)$, the jump discontinuity of $e_{3,j}(t)$ at t_{s+1}^j is given by $\Psi_j\left(t_{s+1}^j\right) \triangleq e_{3,j}\left(t_{s+1}^j\right) - \lim_{t\to\left(t_{s+1}^j\right)^-} e_{3,j}(t) \in \mathbb{R}^z$, where $e_{3,j}\left(t_{s+1}^j\right)$ is defined by (22) and $\lim_{t\to\left(t_{s+1}^j\right)^-} e_{3,j}(t)$ denotes the limit of $e_{3,j}(t)$ as $t \to t_{s+1}^j$ from the left. Since $\Psi_j\left(t_{s+1}^j\right) = -\lim_{t\to\left(t_{s+1}^j\right)^-} Ce_{1,j}(t) + Cc_{\text{init}}$, then by Theorem 1, $\left\|\Psi_j\left(t_{s+1}^j\right)\right\| \leq S_{\max}(C)\left(V_T+c_{\text{init}}\right)$. Therefore, the magnitude of the jump discontinuity is bounded by

$$\left|\left\|e_{3,j}\left(t_{s+1}^j\right)\right\| - \left\|\lim_{t\to\left(t_{s+1}^j\right)^-} e_{3,j}(t)\right\|\right| \leq S_{\max}(C)\left(V_T+c_{\text{init}}\right). \tag{51}$$

Communication between the relay agent i and explorer agent j occurs when $\left\|y_i^r(t)-y_j^e(t)\right\| \leq R$ where $\left\|y_i^r(t)-y_j^e(t)\right\| \leq S_{\max}(C)\left\|e_{1,j}(t)\right\|+\left\|e_{3,j}(t)\right\|$. Therefore, $\left\|y_i^r\left(t_{s+1}^j\right)-y_j^e\left(t_{s+1}^j\right)\right\| \leq R$ can be ensured provided $S_{\max}(C)\left\|e_{1,j}\left(t_{s+1}^j\right)\right\|+\left\|e_{3,j}\left(t_{s+1}^j\right)\right\| \leq R$. From Theorem 1, $\left\|e_{1,j}\left(t_{s+1}^j\right)\right\| \leq V_T$. Using (50) and (51), it follows that $S_{\max}(C)\left\|e_{1,j}\left(t_{s+1}^j\right)\right\|+\left\|e_{3,j}\left(t_{s+1}^j\right)\right\| \leq S_{\max}(C)V_T + \left\|e_{3,j}\left(t_r^j\right)\right\|\exp\left(-k_i(t)\left(t_{s+1}^j-t_r^j\right)\right) \leq R$ provided (46) holds. To ensure $k_i(t)$ for $t \in \left[t_r^j, t_{s+1}^j\right)$ is well-defined, V_T must be selected such that $\left\|e_{3,j}\left(t_r^j\right)\right\| > R-S_{\max}(C)V_T > 0$. Note that if $0 < \left\|e_{3,j}\left(t_r^j\right)\right\| \leq R-S_{\max}(C)V_T$, then $S_{\max}(C)\left\|e_{1,j}\left(t_r^j\right)\right\| + \left\|e_{3,j}\left(t_r^j\right)\right\| \leq S_{\max}(C)V_T+R-S_{\max}(C)V_T \leq R$ provided $V_T \in \left(0, \frac{R}{S_{\max}(C)}\right)$ and communication between the relay agent i and explorer agent j is possible without the need to maneuver the relay agent i towards explorer agent j. By (47) and (50), the relay agent's tracking error in (9) is bounded. Since $e_{3,j} \in \mathcal{L}_\infty$ and $\hat{x}_j^e \in \mathcal{L}_\infty$ by Theorem 3, then $x_i^r \in \mathcal{L}_\infty$. Since $x_i^r, e_{3,j} \in \mathcal{L}_\infty$ and $e_{2,j}, u_j^e \in \mathcal{L}_\infty$ by Theorem 3, the controller $u_i^r \in \mathcal{L}_\infty$ by (15). Substituting (21) when $t \in \left[t_s^j, t_r^j\right)$ into the time derivative of (47) yields

$$\dot{V}_{3,j}\left(e_{3,j}(t)\right) = e_{3,j}^{\mathrm{T}}(t)\Big(C\big(Bu_j^e(t) - Ae_{2,j}(t)\big) - C_i\big(A_i x_i^r(t) + B_i u_i^r(t)\big)\Big). \tag{52}$$

From Theorem 2, $e_{2,j}(t) \in \mathcal{L}_\infty$ for $t \in \left[t_s^j, t_{s+1}^j\right)$. Since $t_r^j < t_{s+1}^j$ by design, $e_{2,j} \in \mathcal{L}_\infty$, i.e., $\left\|e_{2,j}(t)\right\| \le \bar{e}_{2,j}$ for $t \in \left[t_s^j, t_r^j\right)$, where $\bar{e}_{2,j} \in \mathbb{R}_{>0}$. Using (7), since $\left\|x_g\right\| \le \bar{x}_g$ and $e_{2,j} \in \mathcal{L}_\infty$, $\hat{x}_j^e \in \mathcal{L}_\infty$, i.e., $\left\|\hat{x}_j^e(t)\right\| \le \bar{\hat{x}}_j^e$ for $t \in \left[t_s^j, t_r^j\right)$, where $\bar{\hat{x}}_j^e \in \mathbb{R}_{>0}$. Since $u_i^r, u_j^e \in \mathcal{L}_\infty$, then there exist $\overline{U}_i, \overline{U}_j \in \mathbb{R}_{>0}$ such that $\left\|u_i^r(t)\right\| \le \overline{U}_i$ and $\left\|u_j^e(t)\right\| \le \overline{U}_j$ for all t.[6] Therefore, $\dot{V}_{3,j}\left(e_{3,j}(t)\right)$ in (52) can be upper bounded as

$$\dot{V}_{3,j}\left(e_{3,j}(t)\right) \le S_{\max}(A_i)\left\|e_{3,j}(t)\right\|^2 + \epsilon\left\|e_{3,j}(t)\right\|, \tag{53}$$

where $\epsilon \triangleq S_{\max}(CA)\,\bar{e}_{2,j} + S_{\max}(CB)\,\overline{U}_j + S_{\max}(C_iB_i)\,\overline{U}_i + S_{\max}(A_i)\,S_{\max}(C)\,\bar{\hat{x}}_j^e \in \mathbb{R}_{>0}$ is a bounding constant. Invoking the Comparison Lemma in Khalil (2002, Lemma 3.4) on (53) over $\left[t_s^j, t_r^j\right)$ and substituting in (47) yields

$$\begin{aligned}\left\|e_{3,j}(t)\right\| \le\ & \frac{\epsilon}{S_{\max}(A_i)}\left(\exp\left(S_{\max}(A_i)\left(t - t_s^j\right)\right) - 1\right) \\ & + \left\|e_{3,j}\left(t_s^j\right)\right\|\exp\left(S_{\max}(A_i)\left(t - t_s^j\right)\right).\end{aligned} \tag{54}$$

By (51) and (54), $e_{3,j}(t) \in \mathcal{L}_\infty$ for $t \in \left[t_s^j, t_r^j\right)$. Since $e_{3,j}(t) \in \mathcal{L}_\infty$ for $t \in \left[t_r^j, t_{s+1}^j\right)$, the relay agent's tracking error in (9) is bounded for all $t \in \left[t_s^j, t_{s+1}^j\right]$.

6.3. Strategy synthesis

Recall that the goal of the synthesized strategy is to compute switching signal $\zeta_i(t)$ for all relay agents $i \in L$. We approach the problem using reactive synthesis as it is a natural formulation to capture any potential unknowns in the environment (such as travel time between explorer agents) as environmental inputs and still provide theoretical guarantees of correctness that the maximum dwell-time condition given in Theorem 1 for all explorer agents is satisfied. In this subsection, we highlight how we can use contract-based synthesis to decentralize the reactive synthesis problem amongst the relay agents. In other words, our method enables each relay agent to compute their own $\zeta_i(t)$ independently and in parallel.

We decentralize the problem by enforcing each relay agent to only be responsible for servicing explorer agents in its region. Each relay agent thus needs to keep track of which explorer agents it is responsible for, as well as how much time has elapsed since that agent had last been serviced. To this end, we introduce two sets of atomic propositions. First, for a relay agent i, we define a set of service propositions $Y_i = \{y_i^1, \ldots, y_i^N\}$ that corresponds to the explorer agents that relay agent i is currently responsible for servicing, i.e., $y_i^j = \top$ if explorer agent j is in S_i. We additionally define $service_i : [0, \infty) \to 2^{Y_i}$ which maps the current time step to the set of explorer agents in the corresponding sub-region S_i. In practice, the function $\eta_i^K(t)$ outputs the set of explorer agents $F_i \subseteq F$, and $service_i$ converts F_i into valuations of the service propositions Y_i.

Second, we define the discrete time set $\mathbb{T}_d \triangleq \{t[0], t[1], \ldots\}$, where $t[h] = hT_s$ for $h \in \mathbb{I}$, $\mathbb{I} \triangleq \{0, 1, \ldots\}$ is the time index set, and $T_s \in \mathbb{R}_{>0}$ is the sampling period. Then we define the set of timing propositions $\mathcal{T}_i^j = \{\tau_0, \tau_1, \ldots, \tau_{T_j}\}$, where T_j denotes the maximum dwell-time defined in Theorem 1, and $\mathcal{T}_i^j$ encodes how much time explorer agent j has to be serviced before violating the dwell-time condition, i.e., $\tau_h = \top$ if explorer j has to be serviced in at most $t[h]$ time steps for the maximum dwell-time condition to be satisfied.

Formally, each relay agent i will have environment atomic propositions $E_i = Y_i \cup \left(\bigcup_{j=1}^N \mathcal{T}_i^j\right)$. The GR(1) requirements that each relay agent i must satisfy are $\varphi_i = \bigwedge_{j=1}^N \left(\Box\left(y_j \to \neg\tau_0\right)\right)$, where the valuation y_j is set by $service_i$. Informally, φ_i states that if explorer agent j is in S_i, then it must be serviced by relay agent i before the time left to service reaches 0 as denoted by $\tau_0 = \top$.

Each relay agent is unaware of the specification and implementation details of the other relay agents. To ensure that relay agents coordinate to satisfy their specifications, every controller must additionally satisfy contract specifications. These contract specifications take the form of assume-guarantee contracts. Informally, a relay agent gives a guarantee of satisfying a contract specification with all other relay agents. This guarantee is used as an assumption for the synthesis of the other relay agents' controllers and vice-versa. We focus on providing a framework to conduct the assume-guarantee synthesis. However, in practice, the contract specifications are domain and environment-specific. We provide an example of a contract specification used to coordinate hand-offs used in the implementation in Section 7. Since the explorer agents can enter and leave sub-regions, the currently responsible relay agent must ensure there is sufficient time for the next relay agent to service the incoming explorer agent. We denote this contract specification as ϕ_i and define it as $\phi_i = \bigwedge_{j=1}^N \left(\Box\left(\left(y_j \wedge \neg \bigcirc y_j\right) \to \neg\left(\bigwedge_{h=0}^K \tau_h\right)\right)\right)$ for some user-provided integer $K \le T_j$. This contract specification states that if explorer agent j is leaving region S_i in the next time step, it must have at least K time steps before it needs to be serviced again. This contract gives the next relay agent some buffer time to service explorer agent j when it enters the next region.

The full GR(1) specifications for relay agent i to satisfy are

$$\Phi_i = \Box\Diamond\left(\bigwedge_{\alpha=1,\alpha\neq i}^{M} \phi_\alpha\right) \to \bigwedge_{j=1}^{N}\left(\Box\left(y_j \to \neg\tau_0\right) \wedge \phi_i\right). \tag{55}$$

By construction, if $\rho_i \models \Phi_i$ for all $i \in L$ then the maximum dwell-time condition for all explorer agents is satisfied and approximate consensus is achieved. Last, we present Theorem 5, which provides theoretical guarantees for achieving stability and approximate consensus (in Objective 1) by satisfying the full GR(1) specifications described in (55).

Theorem 5. *With the observer in (10), controllers in (13) for explorer agents, controllers in (15) for relay agents, the parameters are selected such that* $k_i(t) \ge \frac{1}{\left(t_{s+1}^j - t_r^j\right)} \ln\left(\frac{\left\|e_{3,j}\left(t_r^j\right)\right\|}{R - S_{\max}(C)V_T}\right)$, $V_T \in \left(0, \frac{R}{S_{\max}(C)}\right)$, $\gamma(c)\,S_{\max}(C) \le R$, *Assumptions 1–6 and the GR(1) specifications for relay agents described in (55) are satisfied, then the explorer agents reach approximate consensus within the goal region in the sense that*

$$\limsup_{t\to\infty}\left\|e_j(t)\right\| \le \gamma\left(c^*\right), \tag{56}$$

[6] The relay agent i executes (15) by cycling through all $j \in F$ for all t, which was shown to be bounded for each $j \in F$.

Table 1
Simulation parameters.

Local maneuvering			Round-robin			Global maneuvering		
$d_1^* = 1$	$d_2^* = 0.45$	$d_3^* = 0.15$	$d_1^* = 1$	$d_2^* = 0.45$	$d_3^* = 0.15$	$d_1^* = 0.75$	$d_2^* = 0.15$	$d_3^* = 0.35$
$d_4^* = 1$	$d_5^* = 0.45$	$d_6^* = 0.15$	$d_4^* = 1$	$d_5^* = 0.45$	$d_6^* = 0.15$	$d_4^* = 0.15$		
$d_7^* = 1$	$d_8^* = 0.45$	$d_9^* = 0.15$	$d_7^* = 1$	$d_8^* = 0.45$	$d_9^* = 0.15$			
$R_g, R = 5$	$V_T = 3$	$k_{ARE} = 0.005$	$R_g, R = 5$	$V_T = 3$	$k_{ARE} = 0.005$	$R_g, R = 5$	$V_T = 3$	$k_{ARE} = 0.005$
$k_1(0) = 2.8$	$k_2(0) = 2.1$	$k_3(0) = 2.8$	$k_1(0) = 4$	$k_2(0) = 3.8$	$k_3(0) = 4$	$k_1(0) = 4$	$k_2(0) = 3$	

where $c^* = 2c_{init}S_{\max}\left(PBB^TP\right) + 2\bar{x}_gS_{\max}(PA) + 2\bar{d}_jS_{\max}(P)$.

Proof. From results of Theorems 1–3, the regulation error $e_j(t)$ is UUB provided the relay agent i satisfies the maximum dwell-time condition described in (25) for all $t \in \left[t_s^j, t_{s+1}^j\right]$. By satisfying the GR(1) specifications for relay agent i described in (55) for all $i \in L$, then the maximum dwell-time condition for all the explorer agents is satisfied. According to (45), $\left\|e_j(t)\right\| \le \gamma(c)$. By satisfying $\gamma(c)S_{\max}(C) \le R$, then $e_{1,j}(t) = c_{\text{init}}$, and $\gamma(c)$ can be reduced to $\gamma(c^*)$. Therefore, we obtain (56).

7. Simulation

Two simulation examples demonstrate that the developed technique of combining the reactive synthesis strategy planning with the control yields approximate consensus by the explorer agents. Specifically, Section 7.1 shows nine explorer agents originated in three different pre-defined sub-regions (divided by functions $X = 0$, $\sqrt{3}X - 3Y = 0$ and $\sqrt{3}X + 3Y = 0$ in the Cartesian coordinate system) that are serviced by three relay agents for state corrections. Each of the three relay agents is responsible for servicing the corresponding three explorer agents within its sub-region, and the nine explorer agents reach a goal region centered at $g \triangleq [0, 0] \in \mathbb{R}^2$ with radius R. To demonstrate the developed method requires less control effort and can be used in a distributed manner, we provide the following two baseline methods for comparison. We use the round-robin scheduler for relay agents to service certain explorer agents while satisfying maximum dwell-time conditions. We also conduct a centralized reactive synthesis planning to compare to the developed distributed strategy planning.

To further demonstrate the applicability of the developed method, Section 7.2 provides an example where four explorer agents reach approximate consensus even when explorer agents' trajectories cross sub-regions. The servicing responsibilities among relay agents can be transferred to account for boundary crossing between sub-regions, and the corresponding planning strategies can accommodate the changing number of explorer agents within a sub-region.

7.1. Local maneuvering

We adopt the dynamics of the relay and explorer agents in (1)–(4), where $A_i = B_i = C_i = A = B = C \triangleq I_{2\times 2}$, and $i = 1, 2, 3$. The disturbances for the explorer agents are modeled as $d_j(t) \triangleq d_j^*[\sin(t), \cos(t)]^T$, where $j = 1, 2, 3, \ldots, 9$.[7] The initial positions of explorer agents 1–9 and relay agents 1–3 are shown in Fig. 5, and the simulation parameters are selected as shown in Table 1. We use the tool Slugs (Ehlers & Raman, 2016) for the strategy synthesis.

Fig. 2 depicts the norm of the state estimation error $e_{1,j}(t)$ throughout the simulation, showing the errors are bounded. Fig. 3 depicts the norm of the estimated regulation error $e_{2,j}(t)$ is regulated to zero. Fig. 4 shows the relay agent's tracking error

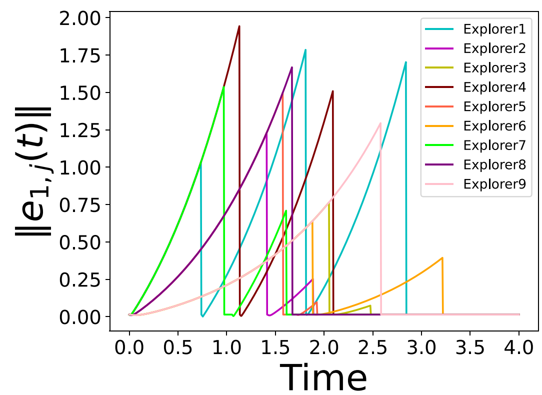

Fig. 2. Norm for state estimation error for the nine explorer agents without crossing the sub-region boundaries, i.e., local maneuvering.

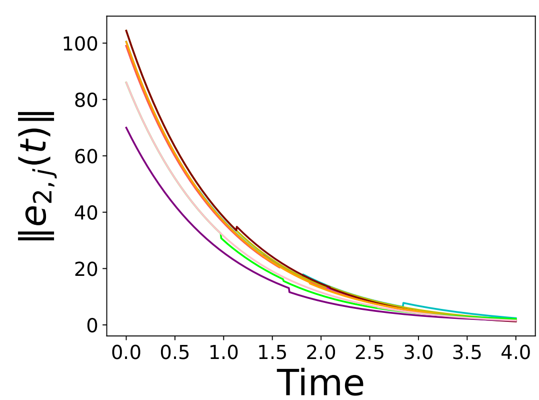

Fig. 3. Norm for estimated tracking error for the nine explorer agents without crossing the sub-region boundaries, i.e., local maneuvering.

$e_{3,j}(t)$ for each explorer agent with respect to its corresponding servicing relay agent. Fig. 5 depicts the true and estimated trajectories for the explorer agents, and the trajectories for the relay agents. As shown in Figs. 2–5, the errors are bounded and the states of nine explorer agents are regulated towards the origin.

To illustrate the developed method requires less control effort than the other standard scheduler methods, we provide a comparison using a round-robin scheduler. Specifically, we set the target servicing sequence to be 1-2-3 in a loop for the relay agent within the sub-region while the round-robin scheduler also satisfies the corresponding maximum dwell-time conditions. Since the round-robin scheduler cannot achieve the objective while using the same initial control gains for the relay agents and exogenous disturbances for the explorer agents, we select the initial gains for the relay agents to be $k_1(0) = 4$, $k_2(0) = 3.8$, and $k_3(0) = 4$ as shown in Table 1. As shown in Figs. 7 and

[7] For the specific values used in the simulation, we refer the reader to Table 1.

Table 2
Computation time for generating the synthesized strategies.

	$M = 2, N = 3$	$M = 2, N = 5$	$M = 2, N = 7$	$M = 3, N = 6$	$M = 3, N = 9$	$M = 3, N = 12$
Distributed	0.028 s	3.05 s	6.91 s	0.084 s	9.06 s	11.67 s
Centralized	109.43 s	TO	TO	TO	TO	TO

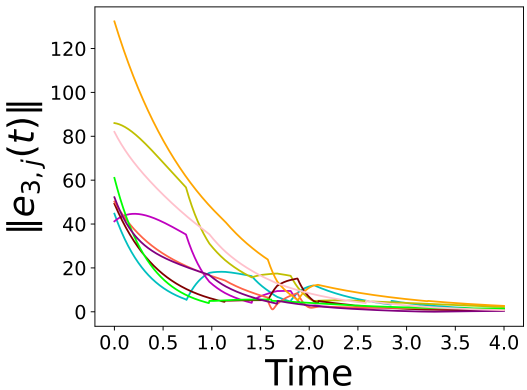

Fig. 4. Norm for relay agent's tracking error for the nine explorer agents without crossing the sub-region boundaries, i.e., local maneuvering.

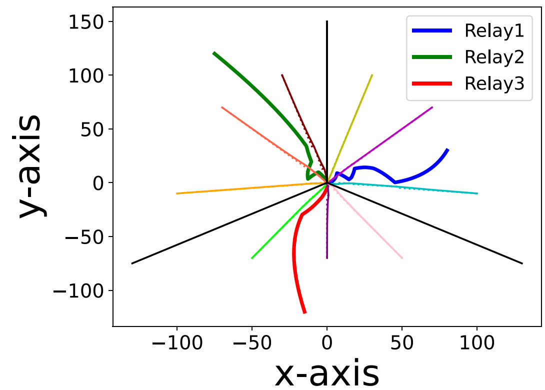

Fig. 5. Agent trajectories for the nine explorer agents without crossing the sub-region boundaries, i.e., local maneuvering. The blue, green and red lines denote the three relay agents, and the other lines denote the nine explorer agents. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

8, the round-robin scheduler requires 98.97% more control effort to complete the objective compared to the control effort needed for the developed method. The synthesized strategies enable the relay agents to service the explorer agents who need the state corrections the most, based on their previous servicing times and the corresponding maximum dwell-time conditions before the state estimation errors exceed the user-defined threshold, i.e., $\left\|e_{1,j}(t)\right\| \leq V_T$. As shown in Fig. 5, the relay agent in the top-right sub-region services Explorer Agents 1 (initialized at $[100, -10]^{\mathrm{T}}$) and 2 (initialized at $[70, 70]^{\mathrm{T}}$) more often than servicing Explorer Agent 3 (initialized at $[30, 100]^{\mathrm{T}}$). Because the explorer agents experienced different exogenous disturbances with the same user-defined state estimation error bound, the corresponding maximum dwell-time conditions are different, i.e., (25), which leads to some explorer agents needing more service than others. Because the round-robin scheduler sets a specific servicing sequence, some explorer agents got redundant services while ensuring the maximum dwell-time condition for each explorer agent is satisfied. Therefore, the developed method requires less control effort to achieve the objective.

A centralized strategy planning approach is also compared to our distributed method. The centralized strategy refers to a method where more than one relay agent is pre-synthesized in the planning to service all the explorer agents at the same time. For example, a distributed strategy can incorporate two relay agents, and each relay agent is responsible for servicing three explorer agents. While the centralized strategy will have these two relay agents servicing all six explorer agents together. As shown in Table 2, the centralized strategies scale badly in computation time as the number of relay and explorer agents increased, which impedes applicability.[8]

7.2. Global maneuvering

To further demonstrate the applicability of the developed method, we now consider four explorer agents and two relay agents initialized in two different pre-defined sub-regions.

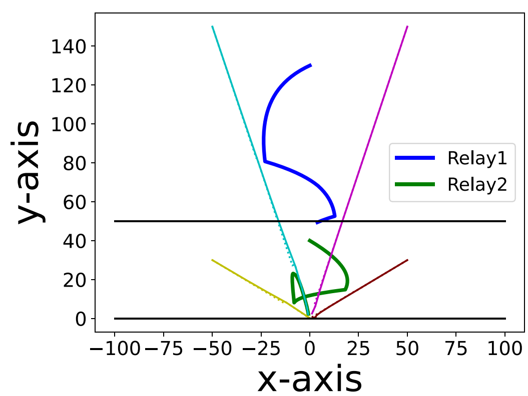

Fig. 6. Agent trajectories for the four explorer agents with two explorer agents crossing the sub-region boundaries, i.e., global maneuvering. The blue and green lines denote the two relay agents, and the other lines denote the four explorer agents. The relay agent in the top sub-region transfers the servicing responsibility to the relay agent in the bottom sub-region after crossing the sub-region boundaries. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Throughout the simulation, two explorer agents (i.e., Explorer Agents 1 and 2 initialized at $[-50, 150]^{\mathrm{T}}$ and $[50, 150]^{\mathrm{T}}$, respectively) leave the top sub-region and enters the bottom sub-region as depicted in Fig. 6. While the trajectories of Explorer Agents 1 and 2 cross the boundaries, servicing responsibilities between the relay agents in the top and bottom sub-regions are transferred, and the relay agents only need to service the explorer agents in their own sub-regions. The dynamics and system matrices used in this simulation example are the same as those in Section 7.1, and the disturbances for the explorer agents are modeled as $d_j(t) \triangleq d_j^* [\sin(t), \cos(t)]^{\mathrm{T}}$, where $j = 1, 2, 3, 4$. The initial positions of Explorer Agents 1–4 and Relay Agents 1–2 are shown in Fig. 6, and the simulation parameters are selected as shown in Table 1.

[8] When generating the synthesized strategies, the maximum dwell-time for each explorer agent is selected as 5 time units. The times listed in Table 2 are generated using a Linux Ubuntu 20.04 operating system, Intel i7-4820K CPU @ 3.70 GHz × 8 processor, and 32 GB memory computer.

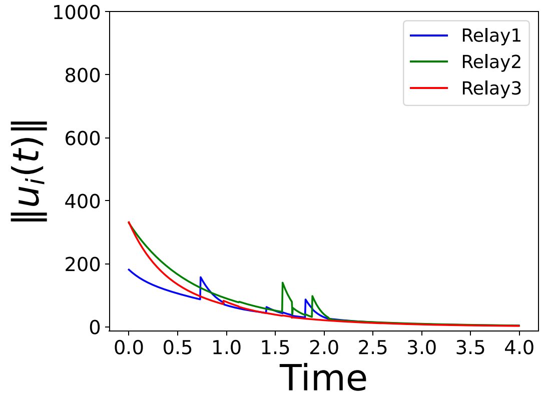

Fig. 7. Control effort of the relay agents using the developed approach.

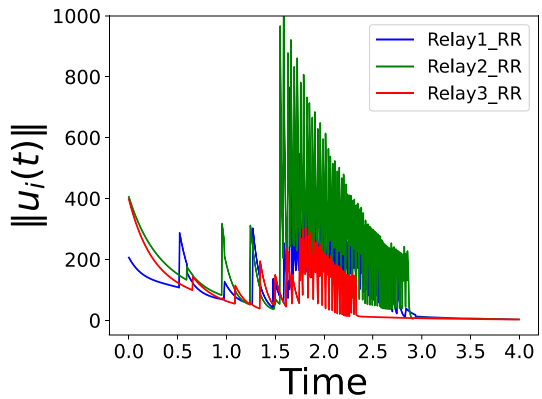

Fig. 8. Control effort of the relay agents using the round-robin scheduler.

Similar to Section 7.1, Fig. 6 shows that the states of the explorer agents reach approximate consensus at the origin. Explorer Agents 1 and 2 leave the top sub-region and enter the bottom sub-region during the simulation, the relay agent in the bottom sub-region needs to start servicing Explorer Agents 1 and 2 after crossing, and therefore, the relay agent in the top sub-region does not need to service Explorer Agents 1 and 2 after crossing. This simulation example shows the developed method can accommodate for transferring of servicing responsibilities between relay agents.

The two provided simulation examples in Sections 7.1 and 7.2 demonstrate that the developed method enables the explorers to reach approximate consensus. Specifically, the formulated reactive synthesis mission specification incorporates the required maximum-dwell time conditions for the relay agents to satisfy. As shown in the results of previous examples, the developed method requires only half of the control effort compared to the round-robin scheduler. The synthesized strategy also has better scalability in terms of incorporating additional explorer agents than the centralized planning strategy.

8. Conclusion

By using the reactive synthesis approach to formulate the mission specifications and control design to provide performance guarantees, we show the distributed MAS can reach approximate consensus while relay agents switch among explorer agents to provide state information. The developed approach requires the specifications for each relay agent to be feasible for the conjunction of specifications to be globally feasible. Future work will focus on extending the current approach to satisfy more complicated mission specifications, such as enabling the relay agents to intermittently provide state information to explorer agents while avoiding inter-agent collision. Such a potential extension would require new control inputs for explorer agents (i.e., (13)) and for relay agents (i.e., (15)), and further formulation of GR(1) specifications (i.e., (55)).

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References

Alonso-Mora, J., DeCastro, J. A., Raman, V., Rus, D., & Kress-Gazit, H. (2018). Reactive mission and motion planning with deadlock resolution avoiding dynamic obstacles. *Autonomous Robots*, *42*, 801–824.
Baier, C., & Katoen, J.-P. (2008). *Principles of model checking*. MIT Press.
Bharadwaj, S., Dimitrova, R., & Topcu, U. (2018). Synthesis of surveillance strategies via belief abstraction. In *Proc. IEEE conf. decis. control* (pp. 4159–4166).
Bharadwaj, S., Vinod, A. P., Dimitrova, R., & Topcu, U. (2020). Near-optimal reactive synthesis incorporating runtime information. In *Proc. IEEE int. conf. robot. autom.* (pp. 10342–10348).
Bloem, R., Jobstmann, B., Piterman, N., Pnueli, A., & Sa'ar, Y. (2012). Synthesis of reactive(1) designs. *Journal of Computer and System Sciences*, *78*, 911–938.
Chen, H.-Y., Bell, Z., Deptula, P., & Dixon, W. E. (2019). A switched systems approach to path following with intermittent state feedback. *IEEE Transactions on Robotics*, *35*(3), 725–733.
Cheng, T. H., Kan, Z., Klotz, J. R., Shea, J. M., & Dixon, W. E. (2017). Event-triggered control of multiagent systems for fixed and time-varying network topologies. *IEEE Transactions on Automatic Control*, *62*(10), 5365–5371.
Ehlers, R., & Raman, V. (2016). Slugs: Extensible GR(1) synthesis. In *Int. conf. on comput. aided verification* (pp. 333–339). Springer.
Heemels, W., & Donkers, M. (2013). Model-based periodic event-triggered control for linear systems. *Automatica*, *49*(3), 698–711.
Kamalapurkar, R., Dinh, H., Bhasin, S., & Dixon, W. E. (2015). Approximate optimal trajectory tracking for continuous-time nonlinear systems. *Automatica*, *51*, 40–48.
Khalil, H. K. (2002). *Nonlinear systems* (3rd ed.). Upper Saddle River, NJ: Prentice Hall.
Li, H., Liao, X., Huang, T., & Zhu, W. (2015). Event-triggering sampling based leader-following consensus in second-order multi-agent systems. *IEEE Transactions on Automatic Control*, *60*(7), 1998–2003.
Manna, Z., & Pnueli, A. (2012). *The temporal logic of reactive and concurrent systems: Specification*. Springer.
Meng, X., & Chen, T. (2013). Event based agreement protocols for multi-agent networks. *Automatica*, *49*, 2125–2132.
Ouaknine, J., & Worrell, J. (2005). On the decidability of metric temporal logic. In *20th annual IEEE symposium on log. in comput. sci. (LICS'05)* (pp. 188–197).
Piterman, N., Pnueli, A., & Sa'ar, Y. (2006). Synthesis of reactive(1) designs. In *Int. workshop on verification, model checking, and abstr. interpret.* (pp. 364–380). Springer.
Pnueli, A., & Rosner, R. (1989). On the synthesis of an asynchronous reactive module. In *Int. colloq. on autom., lang., and program.* (pp. 652–671). Springer.
Raman, V., Donzé, A., Maasoumy, M., Murray, R., Sangiovanni-Vincentelli, A., & Seshia, S. (2014). Model predictive control with signal temporal logic specifications. In *Proc. IEEE conf. decis. control* (pp. 81–87).
Sun, R., Bell, Z., Zegers, F., & Dixon, W. E. (2020). A switched systems approach to unknown environment exploration with intermittent state feedback for nonholonomic systems. In *Proc. Am. control conf.* (pp. 5275–5280).
Sun, R., Harris, C., Bell, Z., & Dixon, W. E. (2020). Relay-explorer approach for multi-agent exploration of an unknown environment with intermittent communication. In *Proc. IEEE conf. decis. control* (pp. 5218–5223).

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, *52*(9), 1680–1685.

Wang, X., & Lemmon, M. (2009). Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability. *IEEE Transactions on Automatic Control*, *54*, 452–467.

Xu, Z., Zegers, F. M., Wu, B., Dixon, W. E., & Topcu, U. (2019). Controller synthesis for multi-agent systems with intermittent communication: A metric temporal logic approach. In *Proc. Allerton conf. on commun., control, and compu.*.

Zegers, F., Chen, H.-Y., Deptula, P., & Dixon, W. E. (2019). A switched systems approach to consensus of a distributed multi-agent system with intermittent communication. In *Proc. Am. control conf.* (pp. 2372–2377).

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