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# Data-Based Learning for Uncertain Robotic Systems

*Anup Parikh*,[1,*] *Rushikesh Kamalapurkar*[2] and *Warren E. Dixon*[1]

## ABSTRACT

An adaptive controller based on the concurrent learning method is developed for uncertain Euler-Lagrange systems. Using a Lyapunov-based analysis, it is shown that this design achieves the tracking objective, as well as identifies the uncertain parameters, without requiring the well-known and restrictive persistence of excitation condition. Simulation results are provided to demonstrate faster convergence compared to gradient based adaptive controllers without concurrent learning.

**Keywords:** Robotic systems, concurrent learning method, adaptive controller

## INTRODUCTION

Robot manipulators have been used for a variety of motion control applications, and therefore high precision control of robot manipulators has been of interest in the control community for a number of decades. The general equations of motion for robot manipulators (i.e., Euler-Lagrange system) can be used to describe the dynamics for a variety of electromechanical systems, and therefore have become a benchmark system for novel control design techniques, e.g., adaptive control, robust control, output feedback, and control with limited actuation (Lewis et al. 2003, Behal et al. 2009). Adaptive control refers to a number of techniques for achieving a tracking or regulation

[1] Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville FL 32611-6250.
[2] School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater OK 74078.
[*] Corresponding author

control objective while compensating for uncertainties in the model by estimating the uncertain parameters online. It is well known that least-squares or gradient based adaptation laws rely on persistence of excitation (PE) to ensure parameter convergence (Ioannou and Sun 1996, Narendra and Annaswamy 1989, Sastry and Bodson 1989), a condition which cannot be guaranteed *a priori* for nonlinear systems, and is difficult to check online, in general.

Motivated by the desire to learn the true parameters, or at least to gain the increased robustness (i.e., bounded solutions in the presence of disturbances) and improved transient performance (i.e., exponential tracking versus asymptotic tracking of many adaptive controllers) that parameter convergence provides, a new adaptive update scheme known as concurrent learning (CL) was recently developed in the pioneering work of (Chowdhary and Johnson 2011, Chowdhary 2010, Chowdhary et al. 2013). The principle idea of CL is to use recorded input and output data of the system dynamics to apply batch-like updates to the parameter estimate dynamics. These updates yield a negative definite, parameter estimation error term in the stability analysis, which allows parameter convergence to be established provided a finite excitation condition is satisfied. The finite excitation condition is a weaker condition than persistent excitation (since excitation is only required for a finite amount of time), and can be checked online by verifying the positivity of the minimum singular value of a function of the regressor matrix.

In this chapter, a concurrent learning based adaptation law for general Euler-Lagrange systems is developed. We also demonstrate faster tracking error and parameter estimation error convergence compared to a gradient based adaptation law through a simulation.

## CONTROL DEVELOPMENT

Consider robot manipulator equations of motion of the form (Lewis et al. 2003, Spong and Vidyasagar 1980)

$$M(q)\,\ddot{q} + V_m(q,\dot{q})\,\dot{q} + F_d\dot{q} + G(q) = \tau \tag{1}$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ represent the measurable link position, velocity and acceleration vectors, respectively, $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ represents the inertial matrix, $V_m : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ represents centripetal-Coriolis effects, $F_d \in \mathbb{R}^{n\times n}$ represents frictional effects, $G : \mathbb{R}^n \to \mathbb{R}^n$ represents gravitational effects and $\tau(t) \in \mathbb{R}^n$ denotes the control input. The system in (1) has the following properties (See Lewis et al. 2003).

**Property 1.** The system in (1) can be linearly parameterized, i.e., (1) can be rewritten as

$$Y_1(q,\dot{q},\ddot{q})\,\theta = M(q)\,\ddot{q} + V_m(q,\dot{q})\,\dot{q} + F_d\dot{q} + G(q) = \tau$$

where $Y_1 : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ denotes the regression matrix, and $\theta \in \mathbb{R}^m$ is a vector of uncertain parameters.

**Property 2.** The inertia matrix is symmetric and positive definite, and satisfies the following inequalities

$$m_1\,\|\xi\|^2 \le \xi^T M(q)\,\xi \le m_2\,\|\xi\|^2, \quad \forall \xi \in \mathbb{R}^n$$

where $m_1$ and $m_2$ are known positive scalar constants, and $\|\cdot\|$ represents the Euclidean norm.

**Property 3.** The inertia and centripetal-Coriolis matrices satisfy the following skew symmetric relation

$$\xi^T \left( \frac{1}{2} \dot{M}(q) - V_m(q, \dot{q}) \right) \xi = 0, \quad \forall \xi \in \mathbb{R}^n$$

where $\dot{M}(q)$ is the time derivative of the inertial matrix.

To quantify the tracking objective, the link position tracking error, $e(t) \in \mathbb{R}^n$, and the filtered tracking error, $r(t) \in \mathbb{R}^n$, are defined as

$$e = q_d - q \tag{2}$$

$$r = \dot{e} + \alpha e \tag{3}$$

where $q_d(t) \in \mathbb{R}^n$ represents the desired trajectory, whose first and second time derivatives exist and are continuous (i.e., $q_d \in \boldsymbol{C}^2$). To quantify the parameter identification objective, the parameter estimation error, $\tilde{\theta}(t) \in \mathbb{R}^m$, is defined as

$$\tilde{\theta} = \theta - \hat{\theta} \tag{4}$$

where $\hat{\theta}(t) \in \mathbb{R}^m$ represents the parameter estimate.

Taking the time derivative of (3), premultiplying by $M(q)$, substituting in from (1), and adding and subtracting $V_m(q, \dot{q})\, r$ results in the following open loop error dynamics

$$M(q)\, \dot{r} = Y_2(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d)\, \theta - V_m(q, \dot{q})\, r - \tau \tag{5}$$

where $Y_2 : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times m}$ is defined based on the relation

$$Y_2(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d)\, \theta \triangleq M(q)\, \ddot{q}_d + V_m(q, \dot{q})\, (\dot{q}_d + \alpha e) + F_d \dot{q} + G(q) + \alpha M(q)\, \dot{e}.$$

To achieve the tracking objective, the controller is designed as

$$\tau(t) = Y_2 \hat{\theta} + e + k_1 r \tag{6}$$

where $k_1 \in \mathbb{R}$ is a positive constant. To achieve the parameter identification objective, the parameter estimate update law is designed as

$$\dot{\hat{\theta}} = \Gamma Y_2^T r + k_2 \Gamma \sum_{i=1}^{N} Y_{1i}^T (\tau_i - Y_{1i} \hat{\theta}) \tag{7}$$

where $k_2 \in \mathbb{R}$ and $\Gamma \in \mathbb{R}^{m \times m}$ are constant positive definite and symmetric control gains, $Y_{1i} \triangleq Y_1(q(t_i), \dot{q}(t_i), \ddot{q}(t_i))$, $\tau_i \triangleq \tau(t_i)$, $t_i$ represent past time points, i.e., $t_i \in [0, t]$, and $N \in \mathbb{N}_0$. Using (1), (7) can be rewritten as

$$\begin{aligned} \dot{\hat{\theta}} &= \Gamma Y_2^T r + k_2 \Gamma \sum_{i=1}^{N} Y_{1i}^T (Y_{1i} \theta - Y_{1i} \hat{\theta}) \\ &= \Gamma Y_2^T r + k_2 \Gamma \left[ \sum_{i=1}^{N} Y_{1i}^T Y_{1i} \right] \tilde{\theta}. \end{aligned} \tag{8}$$

The principal idea behind this design is to use recorded input and trajectory data to identify the uncertain parameter vector $\theta$. The time points $t_i$ and the corresponding $\tau_i$ and $Y_{1i}$ used in the summation in (7) are referred to as the history stack. As shown in the subsequent stability analysis, the parameter estimate learning rate is related to the minimum eigenvalue of $\sum_{i=1}^{N} Y_{1i}^T Y_{1i}$, motivating the use of the singular value maximization algorithm in (Chowdhary 2010) for adding data to the history stack. It is also important to note that although this design uses higher state derivatives which are typically not measured (i.e., $\ddot{q}$), this data is only required for time points in the past, and therefore smoothing techniques can be utilized to minimize noise without inducing a phase shift, e.g., (Mühlegg et al. 2012).

Substituting the controller from (6) into the error dynamics in (5) results in the following closed-loop tracking error dynamics

$$M(q)\,\dot{r} = Y_2(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d)\,\tilde{\theta} - e - V_m(q, \dot{q})\,r - k_1 r. \tag{9}$$

Similarly, taking the time derivative of (4) and substituting the parameter estimate update law from (8) results in the following closed-loop parameter estimation error dynamics

$$\dot{\tilde{\theta}} = -\Gamma Y_2^T r - k_2 \Gamma \left[\sum_{i=1}^{N} Y_{1i}^T Y_{1i}\right] \tilde{\theta}. \tag{10}$$

## STABILITY ANALYSIS

To analyze the stability of the closed loop system, two periods of time are considered. During the initial phase, insufficient data has been collected to satisfy a richness condition on the history stack. In Theorem 1 it is shown that the designed controller and adaptive update law are still sufficient for the system to remain bounded for all time despite the lack of data. After a finite period of time, the system transitions to the second phase, where the history stack is sufficiently rich and the controller and adaptive update law are shown, in Theorem 2, to bound the system by an exponentially decaying envelope, therefore achieving the tracking and identification objectives. To guarantee that the transition to the second phase happens in finite time, and therefore the overall system trajectories can be bounded by an exponentially decaying envelope, we require the history stack be sufficiently rich after a finite period of time, i.e.,

$$\exists \underline{\lambda}, T > 0 : \forall t \geq T, \lambda_{\min}\left\{\sum_{i=1}^{N} Y_{1i}^T Y_{1i}\right\} \geq \underline{\lambda}. \tag{11}$$

The condition in (11) requires that the system be sufficiently excited, though is weaker than the persistence of excitation condition since excitation is unnecessary once $\sum_{i=1}^{N} Y_{1i}^T Y_{1i}$ is full rank.

**Theorem 1.** *For the system defined in (1), the controller and adaptive update law defined in (6) and (7) ensure bounded tracking and parameter estimation errors.*

*Proof*: Let $V : \mathbb{R}^{2n+m} \to \mathbb{R}$ be a candidate Lyapunov function defined as

$$V(\eta) = \frac{1}{2} e^T e + \frac{1}{2} r^T M(q)\, r + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{12}$$

where $\eta \triangleq [e^T\, r^T\, \tilde{\theta}^T]^T \in \mathbb{R}^{2n+m}$ is a composite state vector. Taking the time derivative of (12) and substituting (3), (9), and (10) yields

$$\dot{V} = e^T(r - \alpha e) + \frac{1}{2} r^T \dot{M}(q) r + r^T \left(Y_2(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d)\, \tilde{\theta} - e\right.$$
$$\left. - V_m(q, \dot{q})\, r - k_1 r\right) - \tilde{\theta}^T Y_2^T r - k_2 \tilde{\theta}^T \left[\sum_{i=1}^{N} Y_{1i}^T Y_{1i}\right] \tilde{\theta}.$$

Simplifying and noting that $\sum_{i=1}^{N} Y_{1i}^T Y_{1i}$ is always positive semi-definite, $\dot{V}$ can be upper bounded as

$$\dot{V} \le -\alpha e^T e - k_1 r^T r$$

Therefore, $\eta$ is bounded based on (Khalil 2002). Furthermore, since $\dot{V} \le 0$, $V(\eta(T)) \le V(\eta(0))$ and therefore $\|\eta(T)\| \le \sqrt{\frac{\beta_2}{\beta_1}} \|\eta(0)\|$, where $\beta_1 \triangleq \frac{1}{2} \min\{1, m_1, \lambda_{\min}\{\Gamma^{-1}\}\}$ and $\beta_2 \triangleq \frac{1}{2} \max\{1, m_2, \lambda_{\max}\{\Gamma^{-1}\}\}$. ■

**Theorem 2.** *For the system defined in (1), the controller and adaptive update law defined in (6) and (7) ensure globally exponential tracking in the sense that*

$$\|\eta(t)\| \le \left(\frac{\beta_2}{\beta_1}\right) \exp(\lambda_1 T)\, \|\eta(0)\| \exp(-\lambda_1 t), \quad \forall t \in [0, \infty) \tag{13}$$

where $\lambda_1 \triangleq \frac{1}{2\beta_2} \min\{\alpha, k_1, k_2 \underline{\lambda}\}$.

*Proof*: Let $V: \mathbb{R}^{2n+m} \to \mathbb{R}$ be a candidate Lyapunov function defined as in (12). Taking the time derivative of (12), substituting (3), (9), (10) and simplifying yields

$$\dot{V} = -\alpha e^T e - k_1 r^T r - k_2 \tilde{\theta}^T \left[\sum_{i=1}^{N} Y_{1i}^T Y_{1i}\right] \tilde{\theta}. \tag{14}$$

From the finite excitation condition, $\lambda_{\min}\left\{\sum_{i=1}^{N} Y_{1i}^T Y_{1i}\right\} > 0, \forall t \in [T, \infty)$, which implies that $\sum_{i=1}^{N} Y_{1i}^T Y_{1i}$ is positive definite, and therefore $\dot{V}$ can be upper bounded as

$$\dot{V} \le -\alpha e^T e - k_1 r^T r - k_2 \underline{\lambda} \|\tilde{\theta}\|^2, \quad \forall t \in [T, \infty).$$

Invoking (Khalil 2002), $\eta$ is globally exponentially stable, i.e., $\forall t \in [T, \infty)$,

$$\|\eta(t)\| \le \sqrt{\frac{\beta_2}{\beta_1}} \|\eta(T)\| \exp(-\lambda_1 (t - T)).$$

The composite state vector can be further upper bounded using the results of Theorem 1, yielding (13). ■

**Remark 1.** Although the analysis only explicitly considers two periods, i.e., before and after the history stack is sufficiently rich, additional data may be added into the history stack after $T$ as long as the data increases the minimum eigenvalue of $\sum_{i=1}^{N} Y_{1i}^T Y_{1i}$. By using the data selection algorithm in (Chowdhary 2010), the minimum eigenvalue of $\sum_{i=1}^{N} Y_{1i}^T Y_{1i}$ is always increasing, and therefore the Lyapunov function derivative upper bound in (14), is valid for all time after $T$. Hence (12) is a common Lyapunov function (Liberzon 2003).

## SIMULATION

A simulation of this control design applied to a two-link planar robot was performed. The dynamics of the two-link robot are given as

$$\underbrace{\begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix}}_{M(q)} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} -p_3 s_2 \dot{q}_2 & -p_3 s_2 (\dot{q}_1 + \dot{q}_2) \\ p_3 s_2 \dot{q}_1 & 0 \end{bmatrix}}_{V_m(q,\dot{q})} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix}}_{F_d} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

where $c_2$ denotes cos ($q_2$) and $s_2$ denotes sin ($q_2$). The nominal parameters are given by

$$p_1 = 3.473 \qquad f_{d1} = 5.3$$
$$p_2 = 0.196 \qquad f_{d2} = 1.1$$
$$p_3 = 0.242$$

and the controller gains were selected as

$$\alpha = 1.0 \qquad \Gamma = 0.1 I_5$$
$$k_1 = 0.1 \qquad k_2 = 0.1.$$

The desired trajectory was selected as

$$q_{d1} = (1 + 10\exp(-2t))\sin(t),$$
$$q_{d2} = (1 + 10\exp(-t))\cos(3t),$$

and a history stack of up to 20 data points was used in the adaptive update. The tracking and parameter estimation error trajectories are show in Figs. 1 and 2. From Fig. 2, it is clear that the system parameters have been identified. A comparison simulation was also performed without concurrent learning (i.e., setting $k_2 = 0$), representing a typical control design that asymptotically tracks the desired trajectory based on Theorem 1, with error trajectories shown in Figs. 3 and 4. In comparison to a typical gradient based adaptive controller that yields the trajectories in Figs. 3 and 4, the contribution of the concurrent learning method is evident by the exponential trajectories in Figs. 1 and 2. It is also important to note that a number techniques have been developed for improving transient performance of adaptive control architectures such as the gradient based adaptive update law simulated here (e.g. (Duarte and Narendra 1989, Krstić et al. 1993, Yucelen and Haddad 2013, Pappu et al. 2014, Yucelen et al. 2014)),

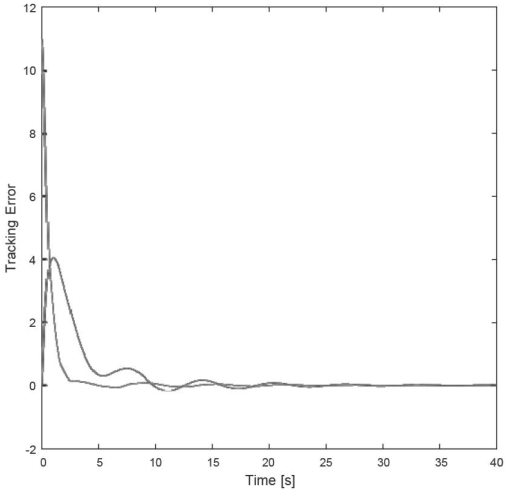


**Figure 1.** Tracking error trajectory using concurrent learning based control design.

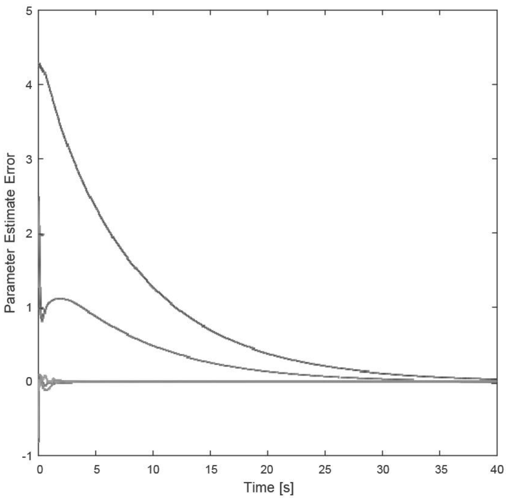


**Figure 2.** Parameter estimation error trajectory using concurrent learning based control design.

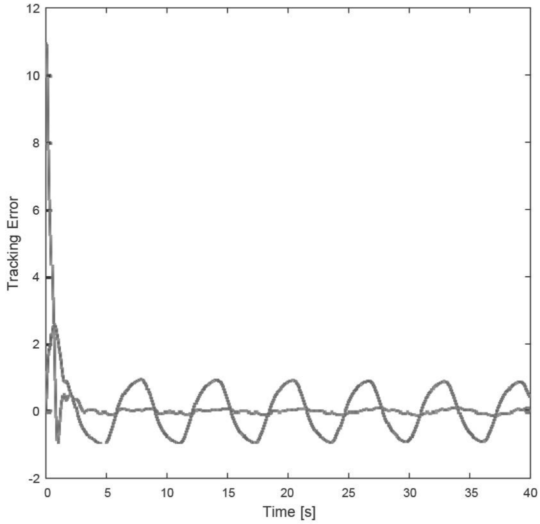


**Figure 3.** Tracking error trajectory using traditional gradient based control design.

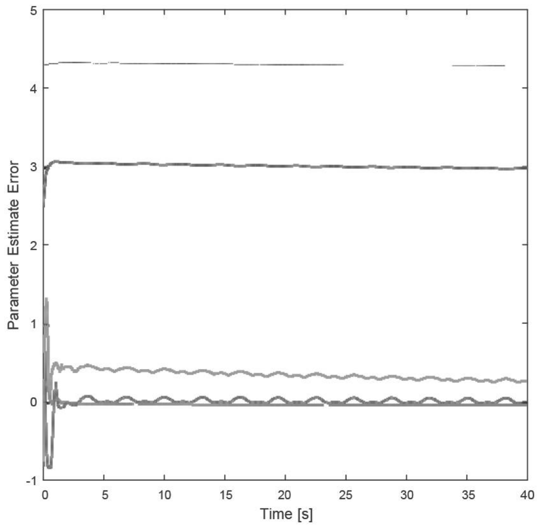


**Figure 4.** Parameter estimation error trajectory using traditional gradient based control design.

though cannot guarantee parameter identification, and hence exponential trajectories, without persistence of excitation.

## CONCLUSION

In this chapter a novel adaptive controller is developed for Euler-Lagrange systems. The concurrent learning based design incorporates recorded data into the adaptive update law, resulting in exponential convergence of the tracking and parameter estimation errors. The excitation condition required for convergence is weaker than persistent excitation, and is easier to check online. The provided simulation results demonstrate the increased convergence rate of this design compared to traditional adaptive controllers with gradient based update laws.

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