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Adaptive set–point control of robotic manipulators with amplitude–limited control inputs*

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SUMMARY
This paper addresses the link position setpoint control problem of *n–link* robotic manipulators with amplitude-limited control inputs. We design a global-asymptotic exact model knowledge controller and a semi-global asymptotic controller which adapts for parametric uncertainty. Explicit bounds for these controllers can be determined; hence, the required input torque can be calculated *a priori* so that actuator saturation can be avoided. We also illustrate how the proposed control algorithm in this paper can be slightly modified to produce a proportional-integral-derivative (PID) controller which contains a saturated integral term. Experimental results are provided to illustrate the improved performance of the proposed control strategy over a standard adaptive controller that has been artificially limited to account for torque saturation.

KEYWORDS: Control inputs; Adaptive control; Robot manipulators; PID controller.

1. INTRODUCTION

Over the past twenty years, a considerable amount of research has targeted the link position control problem for rigid-link robots. Unfortunately, most of the proposed controllers do not take into account the fact that the commanded input may require more torque than is physically possible (*i.e.,* due to large initial condition offsets, an aggressive desired trajectory, or some other disturbance). That is, when the actuator constraints are surpassed, hard nonlinearities, not included in the robot model, are encountered. Once the unmodelled actuator constraints have been breached, degraded control performance in addition to thermal and/or mechanical failure can occur; hence, the need for a control scheme which can ensure that the actuator limits are not breached, is well motivated.

Based on the need for controllers that take actuator constraints into account, several researchers have proposed amplitude limited controllers[1–6]. Specifically, Santibáñez and Kelly[6], proposed a global asymptotic regulating controller that is composed of a saturated proportional derivative (PD) feedback loop plus an exact model knowledge feedforward gravity compensation term. In reference [3], the same authors generalized a class of regulators for the control problem given in reference [6]. Motivated by the research given in references [3] and [6], Loria *et al.*[4] designed an output feedback (OFB) global asymptotic regulating controller; however, exact knowledge of the gravity terms was still required. To provide for robustness, Colbaugh *et al.*[1,2] designed full-state feedback (FSFB) and OFB global asymptotic regulating controllers that compensate for uncertainty; however, the control strategy switches between one controller that is used to drive the setpoint error to a small value, and another controller that is used to drive the setpoint error to zero. To the best of our knowledge, the only researchers to attack the tracking control problem with amplitude-limited torque inputs are given in references [5] and [7]. Specifically in the former Loria *et al.* designed an exact model knowledge OFB semi-global tracking controller. In reference [7], Dixon *et al.* proposed an adaptive FSFB semi-global tracking controller; however, the magnitude of the feedback portion of the control laws could not be arbitrarily small.

In this paper, we design two amplitude-limited torque input, link position setpoint controllers for robot manipulators. The first controller is a global FSFB exact model knowledge controller that is presented in order to facilitate the development of a second controller which provides for robustness. The second controller is a semi-global FSFB adaptive controller that includes an amplitude-limited proportional derivative (PD) feedback loop plus a feedforward term that adapts for gravity and static friction effects. The advantage of the proposed algorithm is that: (i) the controller compensates for unknown parametric effects, (ii) the magnitude of the feedback portion of the controller can be made arbitrarily small provided dynamic friction is not included in the model, (iii) and the maximum required torque can be calculated *a priori*. This paper is organized as follows. Section 2 presents the robot manipulator dynamic model and its associated properties. Sections 3 and 4 present the design and analysis of an exact model knowledge controller and an adaptive controller, respectively. In Section 5, we demonstrate how the proposed controller can be reconfigured as a global link position setpoint PID controller, similar to that given in Kelly[8]. Verification of the control strategy is provided through experimental results given in Section 6.

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2. MATHEMATICAL MODEL

The mathematical model for a rigid n-link, revolute, direct-drive robot as follows[9]

$$M(q)\ddot{q}+V_m(q,\dot{q})\dot{q}+G(q)+F_d\dot{q}+F_s sgn(\dot{q})+\tau \quad (1)$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)\in\Re^n$ denote the link position, velocity, and acceleration vectors, respectively, the symmetric, positive definite inertia matrix is denoted by $M(q)\in\Re^{n\times n}$, the centripetal-Coriolis matrix is denoted by $V_m(q,\dot{q})\in\Re^{n\times n}$, the gravitational vector is denoted by $G(q)\in\Re^n$, the constant, diagonal, viscous friction coefficient matrix is denoted by $F_d\in\Re^{n\times n}$, the constant, diagonal, static friction matrix is denoted by $F_s\in\Re^{n\times n}$, and $\tau(t)\in\Re^n$, represents the torque input vector.

The robot model of (1) has the following beneficial properties[9] that are utilized in the subsequent control design and analysis.

Property 1: The inertia matrix, satisfies the following inequalities

$$m_1\|\xi\|^2\leq\xi^T M(q)\xi\leq m_2\|\xi\|^2, \qquad \forall\xi\in\Re^n \quad (2)$$

where m_1, m_2 are known positive bounding constants, and $\|\cdot\|$ is the standard Euclidean norm.

Property 2: The time derivative of the inertia matrix and the centripetal-Coriolis matrix satisfy the following skew symmetric relationship

$$\xi^T\left(\frac{1}{2}\dot{M}(q)-V_m(q,\dot{q})\right)\xi=0 \qquad \forall\xi\in\Re^n \quad (3)$$

Property 3: The gravitational and static friction terms can be linearly parameterized as follows

$$G(q)+F_s sgn(\dot{q})=Y\phi \quad (4)$$

where $\phi(t)\in\Re^r$ contains mechanical system parameters, and the regression matrix $Y(q,\dot{q})\in\Re^{n\times r}$ contains measurable functions of the link position and link velocity. We will assume that lower and upper bounds for each parameter can be calculated as follows

$$\underline{\phi_i}\leq\phi_i\leq\overline{\phi_i} \quad (5)$$

where $\phi_i\in\Re^1$ denotes the i-th component of ϕ, $\underline{\phi_i}$, $\overline{\phi_i}\in\Re$ denote the i-th component of $\underline{\phi}$, $\overline{\phi}\in\Re^r$, which are defined as follows

$$\underline{\phi}=[\underline{\phi}_1,\underline{\phi}_2,\ldots,\underline{\phi}_r]^T$$
$$\overline{\phi}=[\overline{\phi}_1,\overline{\phi}_2,\ldots,\overline{\phi}_r]^T.$$

Property 4: The gravity vector, static and dynamic friction matrices, centripetal-Coriolis matrix, and the time derivative of the inertia matrix can be upper bounded in the following manner

$$\|G(q)\|\leq\zeta_g,\ \|F_s\|\leq\zeta_{fs},\ \|V_m(q,\dot{q})\|_{i\infty}\leq\zeta_c\|\dot{q}\|,\ \|\dot{M}(q)\|_{i\infty}\leq\zeta_m\|\dot{q}\|. \quad (6)$$

where ζ_g, ζ_{fs}, ζ_c, and ζ_m are positive scalar bounding constants, and $\|\cdot\|_{i\infty}$ denotes the induced infinity norm of a matrix.

Remark 1. *To facilitate the subsequent control design and stability analysis, we define a vector function Tanh $(\cdot)\in\Re^n$ and a matrix function Cosh $(\cdot)\in\Re^{n\times n}$ as follows*

$$Tanh(\xi)=[\tanh(\xi_1),\tanh(\xi_2),\ldots,\tanh(\xi_n)]^T \quad (7)$$

and

$$Cosh(\xi)=diag\{\cosh(\xi_1),\cosh(\xi_2),\ldots,\cosh(\xi_n)\} \quad (8)$$

where $\xi(t)=[\xi_1,\xi_2,\ldots,\xi_n]^T\in\Re^n$, and diag $\{\cdot\}$ represents the standard diagonal matrix whose off-diagonal elements are zero. Based on the definitions given in (7) and (8), it can easily be shown that the following inequalities hold for all $\xi(t)\in\Re^n$, $\nu(t)\in\Re^n$ and $\Psi\in\Re^{n\times n}$

$$2\sum_{i=1}^{n}\ln(\cosh(\xi_i))\geq\|Tanh(\xi)\|^2\geq\tanh^2(\|\xi\|), \quad (9)$$

$$\xi^T\Psi Tanh(\xi)\geq\lambda_{\min}\{\Psi\}\|Tanh(\xi)\|^2, \quad (10)$$

$$\|\xi\|+1\geq\frac{\|\xi\|}{\tanh(\|\xi\|)}, \quad (11)$$

$$\|\xi\|\ \|\nu\|\leq\|\xi\|^2+\|\nu\|^2, \quad (12)$$

$$\|\xi\|\geq\|Tanh(\xi)\|, \quad (13)$$

where $\lambda_{\min}$ $\{\cdot\}$ denotes the minimum eigenvalue of a matrix.

3. EXACT MODEL KNOWLEDGE CONTROL DEVELOPMENT

Our primary control objective is to regulate each link of a robotic manipulator to a desired link position using an amplitude limited torque input. To quantify the performance of the controller, we define the link position setpoint error $\tilde{q}(t)\in\Re^n$ as follows

$$\tilde{q}=q-q_d \quad (14)$$

where $q_d\in\Re^n$ is the bounded, constant, desired link position. Based on the control objective, and the subsequent stability analysis, we propose the following exact model knowledge controller

$$\tau=Y\phi-K_p Tanh(\tilde{q})-K_v Tanh(\dot{q}) \quad (15)$$

where K_p, $K_v\in\Re^{n\times n}$ are constant, diagonal, positive definite gain matrices, and $Y\phi$ was defined in (4). After substituting (15) into (1), we have the following closed-loop system

$$M(q)\ddot{q}+V_m(q,\dot{q})\dot{q}=-K_p Tanh(\tilde{q})-K_v Tanh(\dot{q})-F_d\dot{q} \quad (16)$$

where (4) has been utilized.

Theorem 1. *Given the robot manipulator dynamic equation defined in (1), the control torque input defined in (15) ensures global asymptotic link position setpoint control in the sense that*

$$\lim_{t\to\infty}\tilde{q}(t)=0. \quad (17)$$

Proof: To prove (17), we define the following non-negative function

$$V(t) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^{n} k_{pi} \ln(\cosh(\tilde{q}_i)) \tag{18}$$

where k_{pi} and $\cosh(\tilde{q}_i)$ represents the i-th diagonal elements of K_p and $Cosh(\tilde{q})$, respectively. After taking the time derivative of (18) and then substituting (16), we have utilized *Property 2* and the facts that q_d is a constant vector and that F_d is positive definite symmetric matrix, we have the following upper bound for $\dot{V}(t)$

$$\dot{V}(t) \le -\dot{q}^T K_v Tanh(\dot{q}) \triangleq -g(t) \tag{19}$$

where $g(t) \in \Re^1$, is a non-negative function. Since $V(t)$ is a radially unbounded, globally positive function (See *Property 1*), and its time derivative is negative semi-definite we can conclude that $V(t) \in \mathcal{L}_\infty$, and hence, $\dot{q}(t)$, $\tilde{q}(t) \in \mathcal{L}_\infty$. Due to the fact that $\tilde{q}(t) \in \mathcal{L}_\infty$, and the assumption that the desired setpoint is bounded, we have from (14) that $q(t) \in \mathcal{L}_\infty$. Utilizing *Property 3, 4*, (15), and the fact that $q(t) \in \mathcal{L}_\infty$ gives that $\tau(t) \in \mathcal{L}_\infty$; hence, from the closed loop dynamics $\ddot{q}(t) \in \mathcal{L}_\infty$. Since both $\dot{q}(t)$, $\ddot{q}(t) \in \mathcal{L}_\infty$, we have that $\dot{g}(t) \in \mathcal{L}_\infty$; hence, $g(t)$ is uniformaly continuous (UC). From a direct application of Barbalat's Lemma [10] we conclude that $\lim_{t\to\infty} g(t) = \lim_{t\to\infty} (\dot{q}^T K_v Tanh(\dot{q})) = 0$ and from the properties of hyperbolic functions $\lim_{t\to\infty} \dot{q}(t) = 0$. Since $\ddot{q}(t)$, $\ddot{q}(t)$, $M(q)$, $\dot{M}(q)$, $V_m(q, \dot{q}) \in \mathcal{L}_\infty$, we can utilize the time deriative of (16), to conclude that $\dddot{q}(t) \in \mathcal{L}_\infty$; thus, $\ddot{q}(t)$ is UC. Based on these arguments, we have that $\lim_{t\to\infty} \left[\int_0^t \frac{d^2}{d\tau^2}(q(\tau))dt\right]$ exists and its finite. Since $\ddot{q}(t)$ is UC, we can apply the integral form of Barbalat's Lemma to conclude that $\lim_{t\to\infty} \ddot{q}(t) = 0$. Finally, by taking the limit, as $t \to \infty$, of both sides of (16) and applying the properties of hyperbolic functions we conclude that (17)□.

Remark 2. *Note that the control torque input given in (15) can be explicitly upper bounded as follows*

$$\|\tau\| \le \zeta_g + \zeta_{fs} + \lambda_{\max}\{K_p\} + \lambda_{\max}\{K_v\} \tag{20}$$

where ζ_g, ζ_{fs} are defined in (6); furthermore, the magnitude of feedback portion of the control law can be made arbitrarily small. That is, the elements of the feedback gain matrices K_p and K_v can be arbitrarily small.

4. ADAPTIVE CONTROL DEVELOPMENT

In the previous subsection, the control required exact knowledge of the vector containing the static friction and gravitational parameters. To provide for a method of quantifying robustness, we define the parameter estimation error $\tilde{\phi}(t) \in \Re^r$ as follows

$$\tilde{\phi} = \phi - \hat{\phi} \tag{21}$$

where $\hat{\phi}(t) \in \Re^r$ represents the parameter estimate for ϕ defined in (4) which is now assumed to be an unknown constant vector. Motivated by the results of the previous section, we design an adaptive torque control input as follows

$$\tau = Y\hat{\phi} - K_p Tanh(\tilde{q}) - K_v Tanh(\dot{q}) \tag{22}$$

with the parameter adaption law designed in the following manner

$$\dot{\hat{\phi}} = proj\{\Omega_o\} \tag{23}$$

where the auxiliary term $\Omega_o \in \Re^r$ is given by

$$\Omega_o = -\Gamma Y^T(\dot{q} + \varepsilon Tanh(\tilde{q})), \tag{24}$$

$\Gamma \in \Re^{r\times r}$ is a constant, diagonal gain matrix, $\varepsilon \in \Re^1$ is a positive, adaption weighting gain, the function $proj\{\Omega_o\}$ is defined as follows

$$proj\{\Omega_{oi}\} \triangleq \begin{cases} \Omega_{oi} & if \;\; \hat{\phi}_i > \underline{\phi_i} \\ \Omega_{oi} & if \;\; \hat{\phi}_i = \underline{\phi_i} \;\; \text{and} \;\; (\Omega_o)_i \ge 0 \\ 0 & if \;\; \hat{\phi}_i = \underline{\phi_i} \;\; \text{and} \;\; (\Omega_o)_i < 0 \\ 0 & if \;\; \hat{\phi}_i = \underline{\phi_i} \;\; \text{and} \;\; (\Omega_o)_i > 0 \\ \Omega_{oi} & if \;\; \hat{\phi}_i = \underline{\phi_i} \;\; \text{and} \;\; (\Omega_o)_i \le 0 \\ \Omega_{oi} & if \;\; \hat{\phi}_i < \phi_i \end{cases} \tag{25}$$

$$\underline{\phi}_i \le \hat{\phi}(0) \le \bar{\phi}_i$$

where $(\Omega_o)_i$ denotes the i-th component of Ω_o, and $\hat{\phi}_i(t)$ denotes the i-th component of $\hat{\phi}(t)$ (Note that the above projection algorithm ensures that $\underline{\phi_i} \le \hat{\phi}(t) \le \bar{\phi}_i$. For further details the reader is referred to references [11] and [12]). After substituting (22) into (1), we have the following closed-loop system

$$\begin{aligned} M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} = & -Y\tilde{\phi} - K_p Tanh(\tilde{q}) \\ & -K_v Tanh(\dot{q}) - F_d\dot{q} \end{aligned} \tag{26}$$

where (4) and (21) have been utilized.

Theorem 2: *Given the system equation defined in* (1), *the control torque input given in* (22), *along with the adaptation law given in* (23), (24), *and* (25), *ensures semi-global asymptotic link position setpoint control in the sense that*

$$\lim_{t\to\infty} \tilde{q}(t) = 0 \tag{27}$$

provided the control gains K_p and K_v introduced in (22), and the adaptation weighting gain ε introduced in (24) are chosen to satisfy the following inequalities

$$\lambda_{\min}\{K_p\} > \lambda_{max}\{K_v\} + \lambda_{\max}\{F_d\} > 0, \tag{28}$$

$$\varepsilon < \min\left\{\frac{1}{2}\frac{m_1}{m_2}, \frac{\lambda_{\min}\{K_p\}}{2m_2}\right\}, \tag{29}$$

and

$$\frac{\lambda_{\min}\{K_v\}}{2\varepsilon(\zeta_\chi + \lambda_{\max}\{K_v\} + \lambda_{\max}\{F_d\})} \ge \left[\sqrt{\frac{\lambda_2(0)}{\frac{1}{2}m_1 - \varepsilon m_2}} + 1\right]^2, \tag{30}$$

where the positive function $\lambda_2(t) \in \Re^1$, and the positive constant ζ_χ are defined as follows

$$\lambda_2(t) = \left(\frac{1}{2} m_2 + \varepsilon m_2 \right) \|\dot{q}(t)\|^2$$

$$+ \sum_{i=1}^{n} (\lambda_{\max}\{K_p\} + 2\varepsilon m_2) \ln(\cosh(\tilde{q}_i(t)))$$

$$+ \frac{1}{2} \lambda_{\max}\{\Gamma^{-1}\} \|\tilde{\phi}(t)\|^2, \quad (31)$$

$$\zeta_\chi = \max\{m_2, \zeta_m, \zeta_c\}, \quad (32)$$

m_1, m_2 *are defined in* (2), *and* ζ_m, ζ_c *are defined in* (6).

Proof: To prove the result given by (27), we define the following non-negative function

$$V(t) = \frac{1}{2} \dot{q}^T M(q)\dot{q} + \varepsilon Tanh^T(\tilde{q}) M(q)\dot{q}$$

$$+ \sum_{i=1}^{n} k_{pi} \ln(\cosh(\tilde{q}_i)) + \frac{1}{2} \tilde{\phi}^T \Gamma^{-1} \tilde{\phi}, \quad (33)$$

where ε was defined in (24), and k_{pi}, $\cosh(\tilde{q}_i)$ are the ***i***-th diagonal components of K_p and $Cosh(\tilde{q}_i)$, respectively. Based on (2), (9), and (12), and the form of (33), we can utilize the Raleigh-Ritz theorem to bound $V(t)$ by the following inequalities

$$\lambda_1(t) \leq V(t) \leq \lambda_2(t) \quad (34)$$

where $\lambda_2(t)$ was defined in (31), and the positive function $\lambda_1(t) \in \Re^1$ is defined as follows

$$\lambda_1(t) = \left(\frac{1}{2} m_1 - \varepsilon m_2 \right) \|\dot{q}(t)\|^2$$

$$+ \sum_{i=0}^{n} \lambda_{\min}\{K_p\} - 2\varepsilon m_2) \ln(\cosh(\tilde{q}_i(t)))$$

$$+ \frac{1}{2} \lambda_{\min}\{\Gamma^{-1}\} \|\tilde{\phi}(t)\|^2. \quad (35)$$

Based on (35), it is straightforward to see that if ε is selected according to (29), we can ensure that $\lambda_1(t) \geq 0$; hence, from (34) we have that $V(t) \geq 0$.

After taking the time derivative of (33), substituting for (26), utilizing *Property 2,* and canceling common terms, we obtain the following expression

$$\dot{V}(t) = -\varepsilon Tanh^T(\tilde{q}) K_p Tanh(\tilde{q}) - \dot{q}^T K_v Tanh(\dot{q})$$
$$+ \varepsilon\chi - \varepsilon Tanh^T(\tilde{q}) K_v Tanh(\dot{q}) - \dot{q}^T F_d \dot{q}$$
$$- \varepsilon Tanh^T(\tilde{q}) F_d \dot{q} + \tilde{\phi}^T \Gamma^{-1}(\Omega_o - \dot{\hat{\phi}}) \quad (36)$$

where (24) has been utilized, and the auxiliary term $\chi(t) \in \Re^n$ is defined as follows

$$\chi = \dot{q}^T Cosh^{-2}(\tilde{q}) M(q)\dot{q}$$

$$+ Tanh^T(\tilde{q}) \left(\dot{M}(q)\dot{q} - V_m(q, \dot{q})\dot{q} \right). \quad (37)$$

Based on the form of (37) and the properties of $Tanh(\cdot)$ and $Cosh(\cdot)$ defined in (7) and (8), respectively, we can use *Properties 1* and *4* to show that

$$\|\chi\| \leq \zeta_\chi \|\dot{q}\|^2 \quad (38)$$

where ζ_χ was defined in (32).

We now utilize (10), (12), (23), (24), (25), and (38) to obtain the following advantageous expression for the upper bound* for $\dot{V}(t)$ given in (36)

$$\dot{V}(t) \leq -\varepsilon(\lambda_{\min}\{K_p\} - \lambda_{\max}\{K_v\}$$

$$- \lambda_{\max}\{F_d\}) \|Tanh(\tilde{q})\|^2$$

$$- \frac{\lambda_{\min}\{K_v\}}{2} \|Tanh(\dot{q})\|^2$$

$$- \frac{\lambda_{\min}\{K_v\}}{2} \|Tanh(\dot{q})\|^2$$

$$+ \varepsilon(\zeta_\chi + \lambda_{\max}\{K_v\} + \lambda_{\max}\{F_d\}) \|\dot{q}\|^2. \quad (39)$$

From (39), we can see that $\dot{V}(t) \leq 0$ provided that the condition given in (28) and the following inequality are both satisfied

$$\varepsilon(\zeta_\chi + \lambda_{\max}\{K_v\} + \lambda_{\max}\{F_d\} \|\dot{q}\|^2$$

$$- \frac{\lambda_{\min}\{K_v\}}{2} \|Tanh(\dot{q})\|^2 \leq 0. \quad (40)$$

In order to facilitate further analysis, we utilize (11), (34), and (35) to obtain the following sufficient condition for (40)

$$\frac{\lambda_{\min}\{K_v\}}{2\varepsilon(\zeta_\chi + \lambda_{\max}\{K_v\} + \lambda_{\max}\{F_d\})} \geq \left(\sqrt{\frac{V(t)}{\frac{1}{2}m_1 - \varepsilon m_2}} + 1 \right)^2 . (41)$$

If the conditions in (28) and (41) are satified, we can utilize (39) to express the upper bound for $\dot{V}(t)$ as follows

$$\dot{V}(t) \leq -\beta \|x\|^2 \quad (42)$$

where β is some positive scalar constant, and $x(t) \in \Re^{2n}$ is given by

$$x = [Tanh^T(\dot{q}) \qquad Tanh^T(\tilde{q})]^T \quad (43)$$

From (42), we have that $\dot{V}(t) \leq 0$; therefore,

$$V(z(t), t) \leq V(z(0), 0)\lambda_2(z(0), 0) \qquad \forall t \geq 0 \quad (44)$$

where $\lambda_2(t)$ was defined in (31), and $z(t) \in \Re^3$ is given by

* For more details on how the projection algorithm allows one to proceed from (36) to (39), the reader is referred to reference [15].

$$z = \left[\; \|\dot{q}\|^2 \quad \sum_{i=1}^{n} \ln(\cosh(\tilde{q}_i)) \quad \left\|\tilde{\phi}\right\|^2 \; \right]^T. \tag{45}$$

Based on (44), we can now express the final sufficient condition for (41) by the inequality given by (30) (For more details on the above semi-global stability argument, the reader is referred to [13], where a similar type of argument was utilized for different problem).

From (42), we now have that $V(t) \in \mathcal{L}_\infty$; hence, $\dot{q}(t)$, $\tilde{q}(t)$, $\tilde{\phi}(t)$, $x(t) \in \mathcal{L}_\infty$. Since $\tilde{q}(t) \in \mathcal{L}_\infty$, and the desired trajectory is assumed to be bounded, we have that $q(t) \in \mathcal{L}_\infty$. From (21), (23), (24), (25), (5), and the preceding arguments we can now conclude that $\dot{\hat{\phi}}(t)$, $\hat{\phi}(t)$, $\tau(t) \in \mathcal{L}_\infty$. Moreover, based on (1) and *Property 1*, and the fact that $\dot{q}(t)$, $\hat{\phi}(t)$, $\tau(t) \in \mathcal{L}_\infty$, we have that $\ddot{q}(t)$, $\dot{x}(t) \in \mathcal{L}_\infty$; hence, $x(t)$ is uniformly continuous (UC). Since $x(t)$ is UC, it follows that $\|x(t)\|$ is UC. Now, since $\|x(t)\|$ is UC, we can directly apply Barbalat's Lemma [10] to (42) to state that $\lim_{t\to\infty} \|x(t)\| = 0$; hence, the properties of the hyperbolic function can be applied to (43) to yield the result given in (27).

Remark 3. *An important advantage of the proposed adaptive FSFB controller given by (22), (23), (24), (25), and (5), is that it can be bounded as follows*

$$\|\tau\| \le \|Y\|_{i\infty} \|\bar{\phi}\| + \lambda_{\max}\{K_p\} + \lambda_{\max}\{K_v\} \tag{46}$$

where $\|\cdot\|_{i\infty}$ denotes the induced infinity norm of a matrix. The conditions given in (28), (29), and (30), can be satisfied by selecting the adaptive weighting gain ε arbitrarily small; hence, the magnitude of feedback portion of the control law can be made arbitrarily small, provided that dynamic friction is excluded from the model. That is, the elements of the feedback gain matrices K_p and K_v can be arbitrarily small (at least theoretically) if dynamic friction is neglected in the model and if one follows a tuning procedure governed by the conditions given in (28), (29), and (30).

5. SATURATED* PID EXTENSION

Recently, Kelly[8] illustrated how a proportional integral derivative (PID) controller with a saturated integral term provides global asymptotic setpoint regulation even though the gravity vector may be uncertain (For background on this problem, the reader is referred to reference [8] and the references therein). In this section of the paper, we illustrate how the proposed control algorithm in this paper can be slightly modified to yield the same type of result given in reference [8] from an adaptive control perspective. That is, the analysis utilized in [8] relied on the use of LaSalle's Theorem and the use of potential energy terms in the Lyapunov function while the subsequent analysis uses Barbalat's Lemma and properties of the robot manipulator dynamics.

As done in reference [8], we begin the development by rewriting (1) without static friction as follows

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} = \tau. \tag{47}$$

In a similar manner as given in (4), we defined the following parameterization in terms of the desired link position

$$Y_d(q_d)\phi = G(q_d) \tag{48}$$

where the desired constant regression matrix $Y_d(q_d) \in \Re^{n\times r}$ contains known constants of the desired setpoint position, and $\phi \in \Re^r$ contains the unknown gravitational parameters. In Reference [14], Zhang *et al*. demonstrated that the following relationship

$$\|G(\xi) - G(\nu)\| \le \zeta_{g2} \|Tanh(\xi - \nu)\| \qquad \forall \xi, \nu \in \Re^n \tag{49}$$

holds for the dynamics of the 6-DOF PUMA robotic manipulator; hence, this relationship resembles a standard robot manipulator property (*i.e.* the above relationship can be shown to be valid for a number of revolute robot manipulators).

Based on the control objective, and the subsequent stability analysis, we propose the following torque controller

$$\tau = Y_d\hat{\phi} - K_p\tilde{q} - K_v\dot{q} \tag{50}$$

and adaptation law

$$\dot{\hat{\phi}} = -\Gamma Y_d^T(\dot{q} + \varepsilon Tanh(\tilde{q})) \qquad \hat{\phi}(0) = \Gamma Y_d^T q_d - \Gamma Y_d^T q(0) \tag{51}$$

where K_p, K_v, Γ, ε are all defined as before. After substituting (50) into (47) and then adding and subtracting the term $Y_d(\cdot)\phi$ defined in (48), we obtain the following closed-loop system

$$\begin{aligned} M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} = &- F_d\dot{q} - G(q) + G(q_d) \\ &- Y_d\tilde{\phi} - K_p\tilde{q} - K_v\dot{q} \end{aligned} \tag{52}$$

where (21) has been utilized.

Theorem 3: *The closed loop system given by (52) and (51) renders $\|Tanh(\tilde{q})\| \in \mathcal{L}_2$ and global asymptotic link position setpoint regulation in the sense that*

$$\lim_{t\to\infty} \tilde{q}(t) = 0 \tag{53}$$

provided the control gains K_p and K_v given in (50), and the weighting constant ε defined in (51) are selected to satisfy the following inequalities

$$\lambda_{\min}\{K_p\} > \lambda_{\max}\{K_v\} + F_d + \left(\frac{\varepsilon + 1}{\varepsilon}\right)\zeta_{g2} \tag{54}$$

$$\lambda_{\min}\{K_v\} > \zeta_{g2} + \varepsilon(\lambda_{\max}\{K_v\} + F_d + \zeta_\chi) \tag{55}$$

and

$$\varepsilon < min\left\{ \frac{1}{2}\,\frac{m_1}{m_2}, \frac{\lambda_{\min}\{K_p\}}{2m_2}, 1 \right\} \tag{56}$$

where F_d and ζ_{g2} were defined in (1) and (49), respectively.

Proof: To prove (53), we define the following non-negative, scalar function

* The term "saturated" is used here to point out that the integral portion of the control is saturated via the $Tanh(\cdot)$ function. It does not imply that the control torque input can be explicitly bounded *a priori* as done for the previous controllers.

$$V(t) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_p \tilde{q} + \varepsilon Tanh^T(\tilde{q}) M(q)\dot{q}$$

$$+\frac{1}{2}\tilde{\phi}^T \Gamma^{-1}\tilde{\phi} \tag{57}$$

where ε is the same gain as defined in (51). Based on (2), (12), and the Raleigh-Ritz theorem, we can lower bound $V(t)$ by the following inequality

$$\lambda_1 \left[\|\tilde{q}\|^2 \quad \|\dot{q}\|^2 \quad \|\tilde{\phi}\|^2 \right]^T \leq V(t) \tag{58}$$

where $\lambda_1 \in \Re^1$ is defined as follows

$$\lambda_1 = \min\left\{ \left(\frac{1}{2} m_1 - \varepsilon m_2 \right), \right.$$

$$\left. \left(\frac{\lambda_{\min}\{K_p\}}{2} - \varepsilon m_2 \right), \frac{1}{2}\lambda_{\min}\{\Gamma^{-1}\} \right\}. \tag{59}$$

Based on (59), it is straightforward to see that if ε is selected according to (56), we can ensure that λ_1 is positive, hence, from (58), we have that $V(t) \geq 0$. After taking the time derivative of (57), we can utilize *Property 2*, (52), (51), and the time derivative of (14) to obtain the following expression

$$\begin{aligned}\dot{V}(t) = & -\dot{q}^T [K_v \dot{q} + G(q) - G(q_d) + F_d \dot{q}] \\ & - \varepsilon Tanh^T(\tilde{q})[G(q) - G(q_d) + K_p \tilde{q} + K_v \dot{q} \\ & + F_d \dot{q}] + \varepsilon\chi \end{aligned} \tag{60}$$

where $\chi(t)$ was defined in (37). After utilizing (10), (12), (38), and (49), we can upper bound $\dot{V}(t)$ of (60) as follows

$$\dot{V}(t) \leq -(\lambda_{\min}\{K_v\} - \zeta_{g2} - \varepsilon\zeta_\chi - \varepsilon\lambda_{\max}\{K_v\} - \varepsilon F_d)\|\dot{q}\|^2$$

$$-\varepsilon\left(\lambda_{\min}\{K_p\} - \lambda_{\max}\{K_v\} - F_d - \left(\frac{\varepsilon+1}{\varepsilon} \right)\zeta_{g2} \right)\|Tanh(\tilde{q})\|^2$$

$$\tag{61}$$

where F_d, ζ_{g2}, and ζ_χ were defined in (1), (49), and (38), respectively. Provided the conditions given in (54), (55), and (56) are satisfied, $\dot{V}(t)$ can upper bounded as follows

$$\dot{V}(t) \leq -\beta\|z\|^2 \tag{62}$$

where β is some positive bounding constant, and $z(t) \in \Re^{2n}$ is defined as

$$z = [\dot{q}^T \quad Tanh(\tilde{q})^T]^T. \tag{63}$$

From (62), we now have that $V(t) \in \mathcal{L}_\infty$; hence, from (58), we have that $\dot{q}(t)$, $\tilde{q}(t)$, $\tilde{\phi}(t)$, $z(t) \in \mathcal{L}_\infty$. Since $\tilde{q}(t) \in \mathcal{L}_\infty$, we can utilize (14) to obtain $q(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can now be employed to show that all signals remain bounded during closed loop operation; hence, it is easy to show from (52) that $\dot{z}(t) \in \mathcal{L}_\infty$ (*i.e.* $z(t)$ is uniformly continuous). It also follows directly from (62), that $z(t) \in \mathcal{L}_2$ (and hence $\|Tanh(\tilde{q})\| \in \mathcal{L}_2$). Since $\dot{z}(t) \in \mathcal{L}_\infty$ and $z(t) \in \mathcal{L}_2$, Barbalat's Lemma[15] can be used to state that $\lim_{t\to\infty} \|z(t)\| = 0$;

hence, the properties of hyperbolic functions can be applied to (63) to yield the result given in (53).□

Remark 4. *We note that the adaptive controller given by (50) and (51) can be rewritten in form very similar to that given in reference [8]. That is, we note that the adaptation law given in (51) can be written in the following integral form*

$$\hat{\phi} = -\Gamma Y_d^T \tilde{q} - \varepsilon\Gamma Y_d^T \int_0^t Tanh(\tilde{q})d\sigma \tag{64}$$

where $\hat{\phi}(0)$ defined in (51) has been utilized. After substituting (64) into (50) and then grouping common terms, we obtain a PID controller with a saturated integral term as follows

$$\tau = -\bar{K}_p \tilde{q} - \bar{K}_v \dot{q} - \bar{K}_I \int_0^t Tanh(\tilde{q})d\sigma \tag{65}$$

where $\bar{K}_p = K_p + Y_d \Gamma Y_d^T$, $\bar{K}_I = \varepsilon Y_d \Gamma Y_d^T$, and $\bar{K}_v = K_v$

Remark 5. *In reference [14], Zhang et.al. proposed a global adaptive output feedback tracking controller that can also be written as a setpoint controller in a form that is somewhat similar to that given by (65). Specifically, the i-th component of the control torque input $\tau(t) \in \Re^n$ proposed in reference [14] can be rewritten as a nonlinear controller with a saturated integral term as follows*

$$\tau_i = -(\bar{K}_p \tilde{q})_i - \tanh(\tilde{q}_i) - \bar{K}_v \left(\frac{y_i}{1 - y_i^2} \right)$$

$$-\bar{K}_I \int_0^t (y_i - \tanh(\tilde{q}_i))\, d\sigma \tag{66}$$

where $\bar{K}_p = Y_d \Gamma Y_d^T$, $\bar{K}_v = k$, $\bar{K}_I = Y_d \Gamma Y_d^T$, $(\cdot)_i$ denotes the i-th component of a vector in general, and $y(t) \in \Re^n$ is computed by the following filter

$$\begin{cases} \dot{p}_i = -(1 - (p_i + k\tilde{q}_i)^2)(p_i + k\tilde{q}_i + \tanh(\tilde{q}_i)) \\ \qquad + k(\tanh(\tilde{q}_i) - p_i - k\tilde{q}_i), \quad p_i(0) = -k\tilde{q}_i(0) \\ y_i = p_i + k\tilde{q}_i \end{cases}$$

while $p(t) \in \Re^n$ is an auxiliary that allows y(t) to be computed without link velocity measurements.

6. EXPERIMENTAL VERIFICATION

The proposed adaptive link position setpoint controller was implemented on an Integrated Motion Inc. 2-link, revolute, direct-drive robot manipulator with the following dynamics[16]

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} p_1 + 2p_3 cos(q_2) & p_2 + p_3 cos(q_2) \\ p_2 + p_3 cos(q_2) & p_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} -p_3 \sin(q_2)\dot{q}_2 & -p_3 \sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ p_3 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} f_{s1} & 0 \\ 0 & f_{s2} \end{bmatrix} \begin{bmatrix} sgn(\dot{q}_1) \\ sgn(\dot{q}_2) \end{bmatrix} \tag{67}$$

where $p_1 = 3.31$ kg.m^2, $p_2 = 0.116$ kg.m^2, $p_3 = 0.16$ kgm^2, $f_{d1} = 1.0$ Nm.sec, $f_{d2} = 0.6$ Nm.sec, $f_{s1} = 8.45$ Nm, and $f_{s2} = 2.35$ Nm. For these dynamics the unknown parameter vector given in (5) is defined as follows

$$[\, \phi_1 \;\; \phi_2 \,]^T = [\, f_{s1} \;\; f_{s2} \,]^T. \tag{68}$$

The links of the robotic manipulator are directly actuated by switched-reluctance motors which are controlled through NSK torque controlled amplifiers. A Pentium 266 MHz PC operating under QNX (a real-time micro-kernal based operating system) hosts the control algorithm. The control algorithm was implemented via Qmotor 2.0, an in-house graphical user-interface that facilitates realtime graphing, data logging, and the ability to vary control gains without recompiling the program. Data acquisition and control implementation were performed using the MultiQ I/O board at a frequency of 3.0 kHz.

For one familiar with the robotics literature, it is easy to see that if the hyperbolic tangent functions and the projection algorithm are removed from the proposed adaptive controller that we recover the following setpoint version of Slotine's robot controller[10]

$$\begin{aligned} \tau^s &= Y\hat{\phi} - K_p\tilde{q} - K_v\dot{q} \\ \dot{\hat{\phi}} &= -\Gamma Y^T(\dot{q} + \varepsilon\tilde{q}). \end{aligned} \tag{69}$$

It is common practice to saturate the control torque input at a level just below the maximum threshold of the actuator in order to prevent potential mechanical/thermal damage. For example, the setpoint version of the controller given by (69) might be implemented in the following manner

$$\tau_i = sat(\tau_i^s;\, \delta_i) \tag{70}$$

where the $\delta_i' s$ denote a positive constant that represents the torque saturation constraint of the actuators, $sat(\cdot)$ denotes the standard, linear piecewise saturation function[6] that saturates at δ_i, τ_i^s denotes the *i*-th component of τ^s, and τ_i denotes the *i*-th component of the control torque that is commanded at each link. However, this *ad hoc* implementation of the control given by (70) lacks a stability proof; hence, there is a potential for instability and degraded control performance. Since the proposed controller was motivated by this conundrum, we performed experiments to compare the controller given by (69) and (70), and the proposed controller given by (22), (23), (25), (24), and (5). The desired setpoints for both experiments were chosen as follows

$$q_{d1} = 70\,\text{deg} \qquad q_{d2} = -70\,\text{deg} \tag{71}$$

with the link position and link velocity being initialized to zero. In order to ensure a fair comparison between the two controllers, we initialized the parameter estimates for both controllers to the lower bound defined in (5) as follows

$$\hat{\phi}_1(0) = \underline{\phi_1} = 3.0 \qquad \hat{\phi}_2(0) = \underline{\phi_2} = 0.80 \tag{72}$$

where the lower bounds were selected to be 35.5%, and 34% of the values given in (67) for f_{s1}, and f_{s2}, respectively. For the projection algorithm of (25), the upper bounds were selected to be 177%, and 191% of the values given in (67) for f_{s1}, and f_{s2}, respectively, as follows

$$\overline{\phi_1} = 15.0 \qquad \overline{\phi_2} = 4.5.$$

For the controller given by (69) and (70), we limited the control torque input to approximately 95% of the maximum available torque by setting the saturation constants as follows

$$\delta_1 = 231 \text{ Nm} \qquad \delta_2 = 37.5 \text{ Nm} \tag{73}$$

In order to achieve the best transient response and good steady-state error performance (see Table I), the control/adaptation gains for the controller given in (69) and (70) were selected as follows

$$K_p = \begin{bmatrix} 1875 & 0 \\ 0 & 700 \end{bmatrix}, \quad K_v = \begin{bmatrix} 175 & 0 \\ 0 & 31.2 \end{bmatrix},$$

$$\Gamma = diag[0.010, 0.022], \quad \varepsilon = 450. \tag{74}$$

Table 1: Comparison of Link Position Setpoint Control Performance

	Proposed controller	Standard controller
Maximum Steady-State Tracking Error (deg.)	Link 1: 0.013 Link 2: 0.004	Link 1: 0.013 Link 2: 0.004
Time required to reach a 1% envelope of the desired position (sec)	Link 1: 0.43 Link 2: 0.40	Link 1: 0.49 Link 2: 0.39
Maximum Computed Torque (Nm)	Link 1: 231.0 Link 2: 37.5	Link 1: 2290.6 Link 2: 855.2
Maximum Delivered Torque (Nm)	Link 1: 231.0 Link 2: 37.5	Link 1: 231.0 Link 2: 37.5

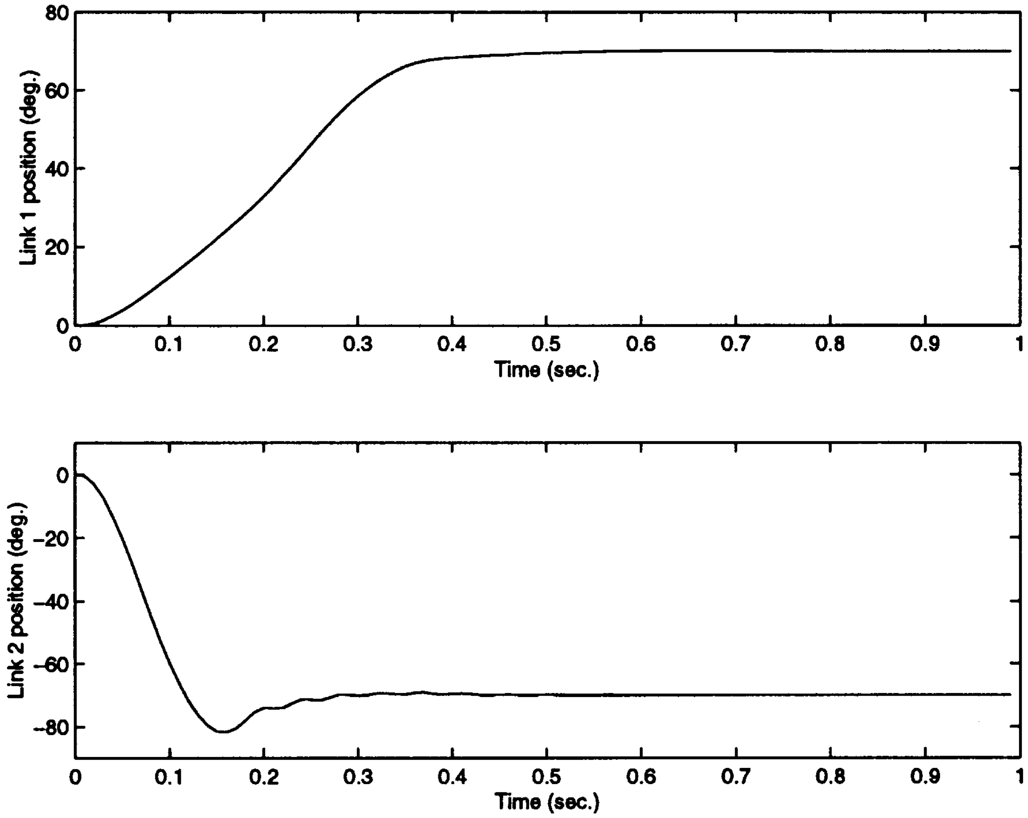

Fig. 1. Link position.

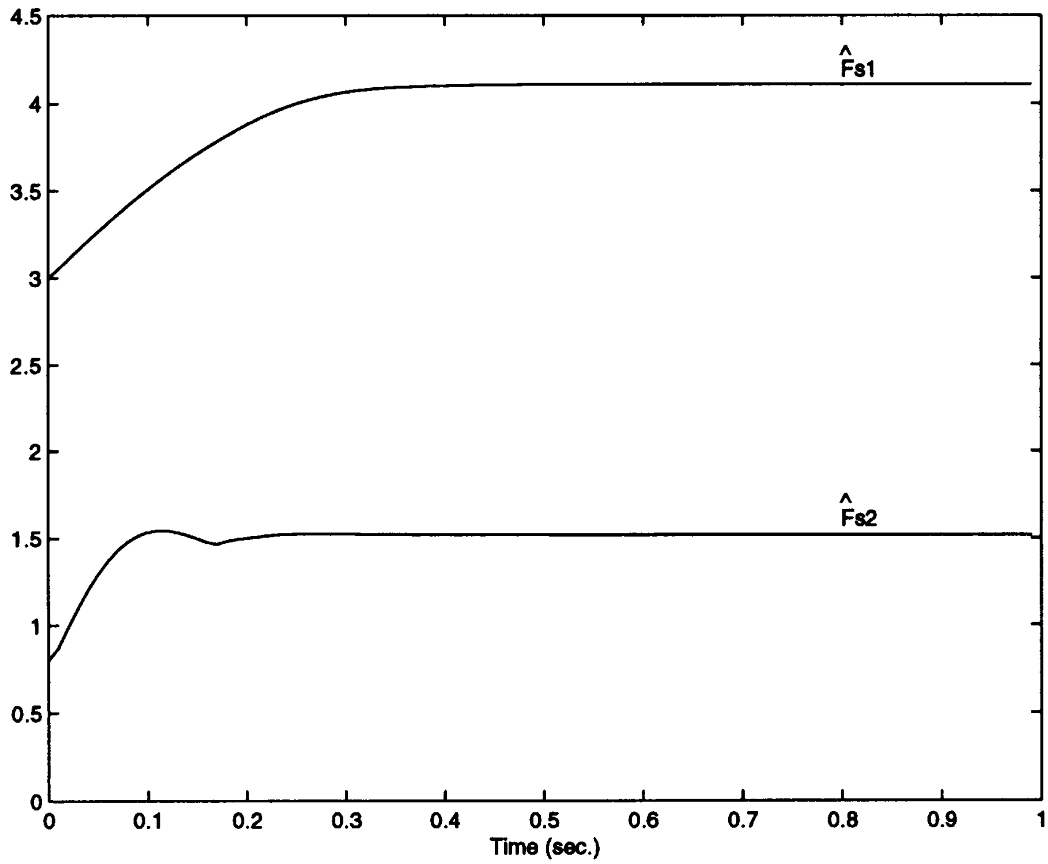

Fig. 2. Parameter estimates.

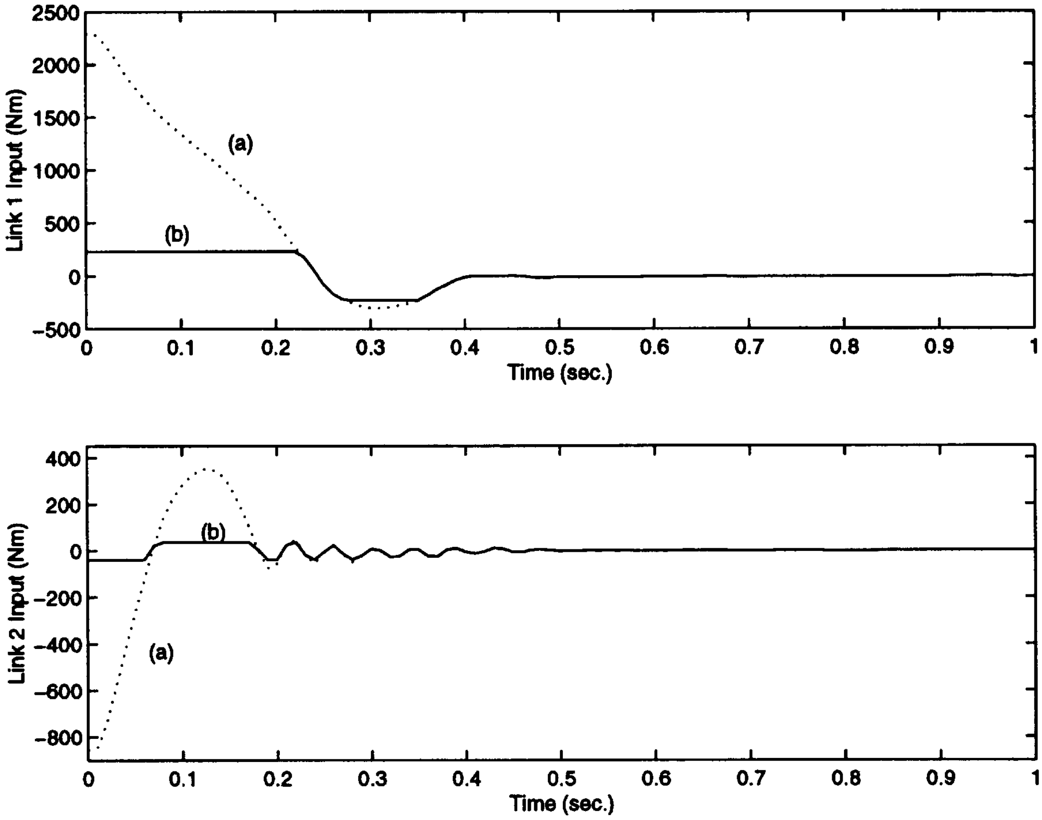

Fig. 3. Control torque inputs (a) computed, (b) actual.

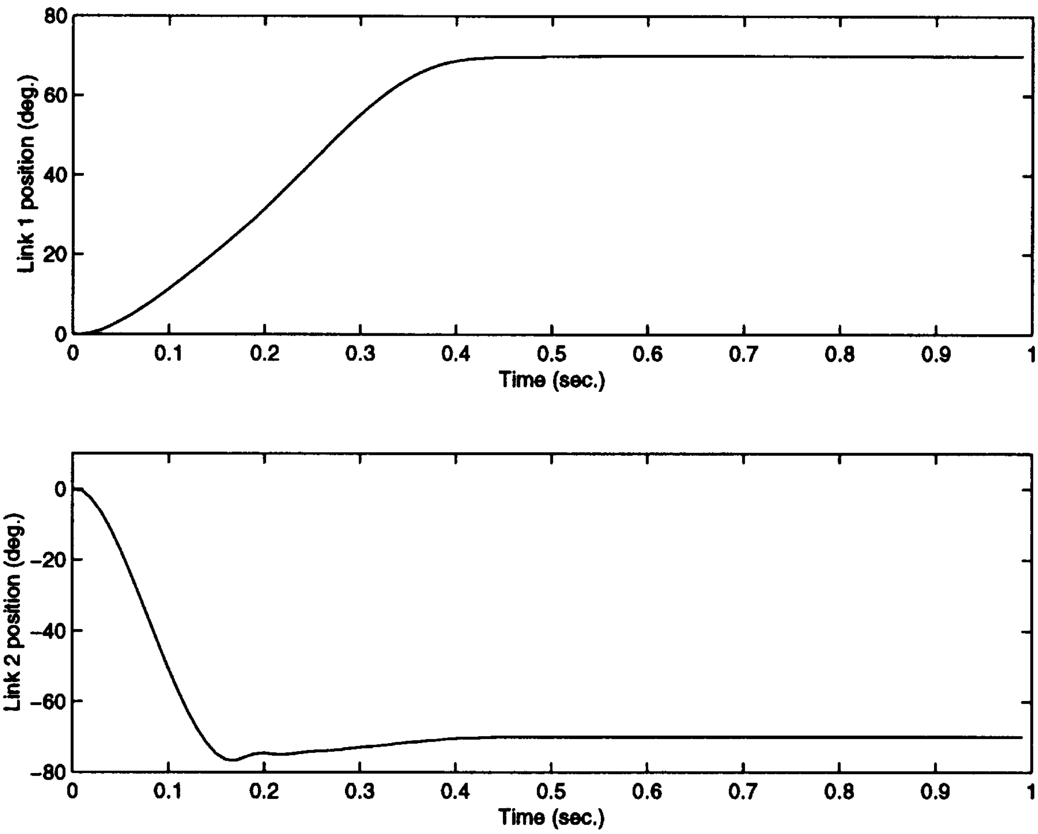

Fig. 4. Link position.

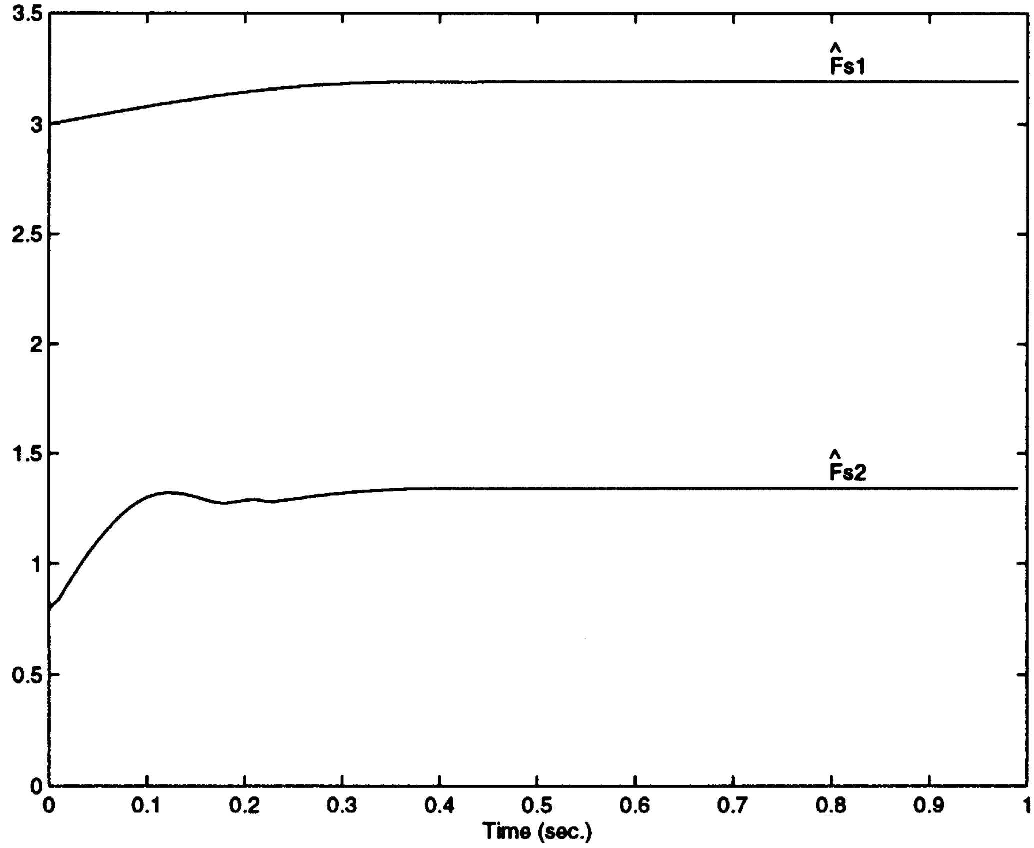

Fig. 5. Parameter estimates.

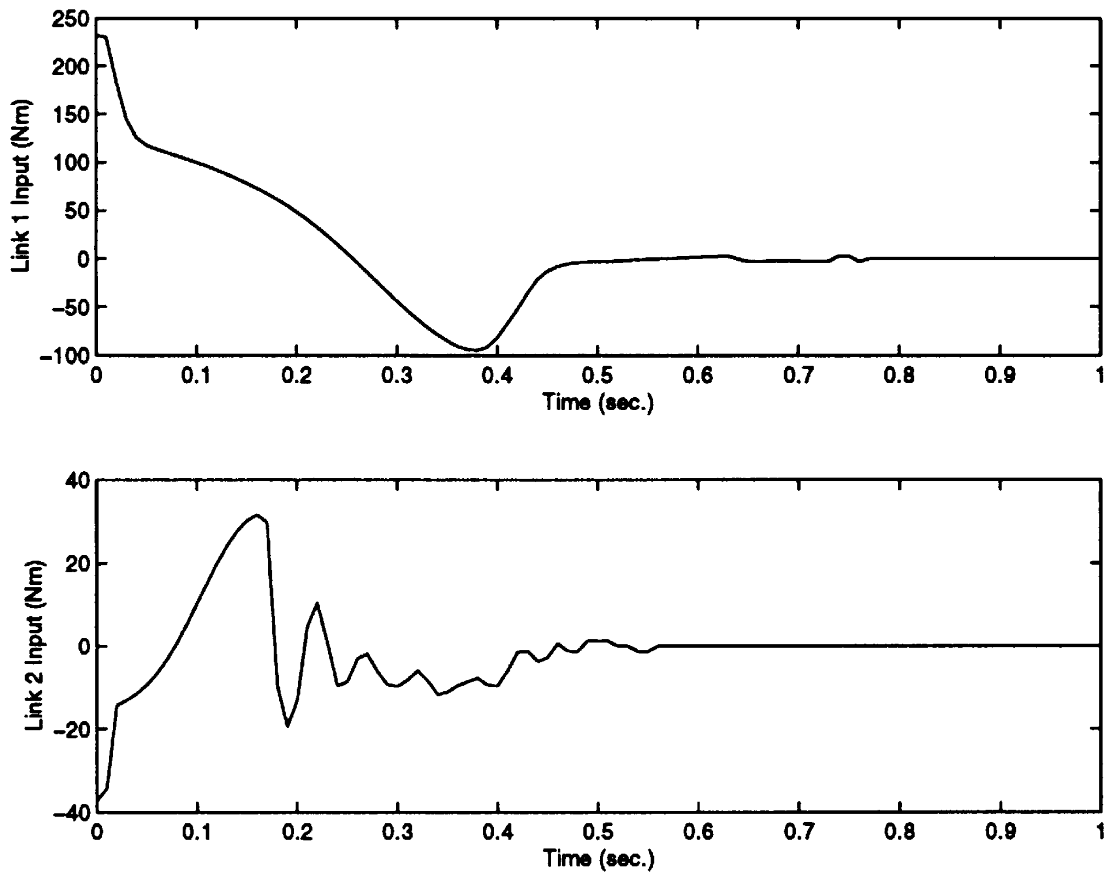

Fig. 6. Control torque inputs.

The performance of the controller is illustrated in Figure 1, the parameter estimates are given in Figure 2, and the associated control torque inputs are given in Figure 3. Likewise, the gains for the proposed adaptive amplitude-limited controller were tuned to the following values

$$K_p = \begin{bmatrix} 276.5 & 0 \\ 0 & 49 \end{bmatrix}, \quad K_v = \begin{bmatrix} 117 & 0 \\ 0 & 27.4 \end{bmatrix},$$

$$\Gamma = diag[0.009, 0.08], \quad \varepsilon = 110. \tag{75}$$

The performance of the controller is illustrated in Figure 4. The parameter estimates and the associated control torque inputs are given in Figure 5 and Figure 6 respectively.

Remark 6. *The control gain values used in (74) and (75) were determined as a result of "tuning" the controller until the transient response and steady-state error improved. It is clearly evident that these gains do not satisfy (29) and (30), however, it should be noted that these conditions are sufficient conditions spawned from a conservative Lyapunov based stability analysis.*

Remark 7. *It is evident from a comparison of the experimental results (see Table I) that the proposed adaptive controller and the controller given in (69) and (70) demonstrate similar staedy-state and transient responses. Based on our experimental results, we concur with Satibáñez and Kelly[17] that we cannot definitively say that the proposed controller is better than the standard counter-part given in (69) and (70). However, due to the fact that the stability/performance of the standard saturated controller with uncertain parameters remains an open problem, we have more confidence in the performance of the proposed controller with its accompanying proof of stability.*

7. CONCLUSION

Through the use of a Lyapunov based design, we have presented two amplitude limited controllers that achieved link position setpoint regulation for robot manipulators. First, a FSFB exact model knowledge controller was designed to achieve global setpoint control. Then, a FSFB adaptive controller was proposed for the semi-global setpoint control problem. As demonstrated in the control development, an advantage of the proposed controllers is that the magnitude of feedback portion of the control law can be made arbitrarily small, provided dynamic friction is excluded; furthermore, an upper bound on the maximum torque required can be calculated *a priori*. We also illustrated how the proposed control algorithm in this paper can be slightly modified to yield a PID controller which yield global adaptive setpoint regulation. Experimental results present a comparison between the proposed adaptive, amplitude limited control scheme with a standard counter-part (*i.e.* an adaptive controller which has been artificially limited to account for torque saturation).

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