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Global robust output feedback tracking control of robot manipulators*

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SUMMARY
This paper addresses the problem of global output feedback, link position tracking control of robot manipulators. Specifically, a robust, Lyapunov-based controller is designed to ensure that the link position tracking error is globally uniformly ultimately bounded despite the fact that only link position measurements are available in the presence of incomplete model information (i.e., parametric uncertainty and additive bounded disturbances).

KEYWORDS: Tracking control; Robots; Output feedback.

1. INTRODUCTION

The development of controllers that only require position measurements (i.e., output feedback (OFB)) has received considerable interest in robotics literature due to the desire to eliminate the need to incorporate a velocity sensor in the robot design. As an outcome of the research directed at this topic, several global solutions to the OFB link position setpoint control problem have been developed. For example, model-based OFB controllers, composed of a linear feedback loop plus feedforward gravity compensation, were proposed in references [1]–[3] to globally asymptotically regulate the manipulator dynamics. In reference [4], Arimoto et al. also presented a model-based, global regulating OFB controller; however, the gravity compensation term was dependent on the desired link position setpoint as opposed to the actual link position. With the intent of overcoming the requirement of exact model knowledge, Ortega et al.[5] designed a OFB controller that achieved semi-global asymptotic regulation while compensating for uncertain gravity effects. In reference [6], Colbaugh et al. proposed a global regulating OFB controller that compensates for uncertain gravity effects where the control strategy requires the use of two different control laws (i.e., one control law is used to drive the setpoint error to a small value, then another control law is used to drive the setpoint error to zero).

Unfortunately, most of the OFB controllers that have targeted the more general tracking control problem have been limited to semi-global stability results. For example, in references [1, 7], a model-based observer was used to construct a semi-global exponential link position tracking controller. In references [8]–[10], variable structure OFB controllers were designed to compensate for parametric uncertainty and the lack of link velocity measurements. Filter-based schemes were designed in references [11]–[13] to accommodate for the lack of velocity information in the development of robust controllers that produce semiglobal, uniformly ultimately bounded (UUB) link position tracking in the presence of parametric uncertainty. Finally, adaptive OFB controllers were presented in references [14]–[17] that yield semi-global asymptotic link position tracking in the presence of parametric uncertainty.

A factor that limited previous controllers to semi-global link position tracking can be related to quadratic nonlinearities in the unmeasurable velocity states due to the centripetal-Coriolis effects. From a review of literature, only a few controllers address this issue to yield global OFB link position tracking. Specifically, in reference [18], Loria developed a model-based controller that yields global uniform asymptotic tracking. To achieve the result, Loria introduced a unique non-quadratic Lyapunov function candidate that enabled the closed-loop error system to compensate for the quadratic nature of the centrifugal-Coriolis effects. Unfortunately, as stated in reference [18], the technique is only valid for a one degree-of-freedom (DOF) system and could not be extended to the general n-DOF case. To address this issue, Zhang et al.[19] proposed the first global OFB adaptive tracking controller for an n-DOF robot manipulator by using a similar non-quadratic Lyapunov function candidate as in reference [18]. In particular, while the structure of the controller of reference [19] resembles that of reference [18] in certain aspects, differences such as the use of a desired compensation adaptation law[20], a different filter structure, and a different error system development/analysis allowed the extension of the controller of reference [18] to the more general, multivariable case with parametric uncertainty. In essence, the closed-loop error system developed in reference [19] allowed the quadratic nonlinearities of the centripetal-Coriolis effects to be upper bounded by a linear function due to the use of hyperbolic

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trigonometric terms. The result in reference [19] was further refined with a more elegant stability analysis method in reference [21]. An extension to rigid-link flexible-joint robots was also presented in reference [22]. In reference [23], Loria and Melham proposed a new dynamic-kinematic model for Euler-Lagrange systems so that the unmeasurable state appears linearly. Based on this new model formulation and the use of an integral Lyapunov stability criterion, global uniform exponential OFB tracking is proven provided exact knowledge of the system is available. More recently, Besancon et al.[24] cast the tracking problem as a stabilization problem for a linear time-varying error system to achieve a global stability result. Specifically, Besancon et al. developed an exact model knowledge controller that is based on a separation technique that exploits state feedback and output to state stability properties[25] related to an unboundedness observability scheme[26].

In light of the previous results, the controller developed in this paper achieves global UUB OFB tracking in the sense of references [21] and [24] for a n-DOF robot manipulator with incomplete model information. That is, the robust OFB controller in this paper is designed for a more general model that contains parametric uncertainty and additive bounded disturbances. To develop the controller, a nonlinear feedback term coupled to a nonlinear, dynamic filter is used to indemnify for the loss of link velocity measurements. The paper is organized as follows. Section 2 presents the robot model along with its properties. The control design, error system development, and stability analysis are presented in Section 3. Concluding remarks are provided in Section 4.

2. ROBOT MODEL

The dynamic model for an n-DOF, revolute-joint robot manipulator is assumed to have the following form[27]

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} + T_d = \tau. \tag{1}$$

In (1), $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration, respectively, $M(q) \in \mathbb{R}^{n\times n}$ represents the positive-definite, symmetric inertia matrix, $V_m(q, \dot{q}) \in \mathbb{R}^{n\times n}$ represents the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ is the gravitational vector, $F_d \in \mathbb{R}^{n\times n}$ denotes the constant, diagonal, positive-definite, viscous friction matrix, $T_d \in \mathbb{R}^n$ is a bounded disturbance vector that represents other unmodeled dynamics (e.g., static friction), and $\tau(t) \in \mathbb{R}^n$ represents the torque input control vector. The left-hand side of (1) is assumed to be first-order differentiable. Moreover, the dynamic system given by (1) exhibits the following properties that are utilized in the subsequent control development and stability analysis.

Property 1: The inertia matrix can be upper and lower bounded by the following inequalities[27]

$$m_1\|\xi\|^2 \le \xi^T M(q)\xi \le m_2\|\xi\|^2 \quad \forall \xi \in \mathbb{R}^n \tag{2}$$

where m_1 and m_2 are positive constants, and $\|\cdot\|$ denotes the Euclidean norm.

Property 2: The inertia and the centripetal-Coriolis matrices satisfy the following relationship[27]

$$\xi^T\left(\frac{1}{2}\dot{M}(q) - V_m(q, \dot{q})\right)\xi = 0 \quad \forall \xi \in \mathbb{R}^n \tag{3}$$

where $\dot{M}(q)$ represents the time derivative of the inertia matrix.

Property 3: The centripetal-Coriolis matrix satisfies the following relationship[27]

$$V_m(q, \nu)\xi = V_m(q, \xi)\nu \quad \forall \xi, \nu \in \mathbb{R}^n. \tag{4}$$

Property 4: The robot dynamics given in (1) can be linearly parameterized as follows[27]

$$Y(q, \dot{q}, \ddot{q})\theta = M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} \tag{5}$$

where $\theta \in \mathbb{R}^p$ contains the constant system parameters, and $Y(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n\times p}$ denotes the regression matrix that is a function of $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)$. The formulation of (5) can also be written in terms of the desired trajectory in the following manner

$$Y_d(q_d, \dot{q}_d, \ddot{q}_d)\theta = M(q_d)\ddot{q}_d + V_m(q_d, \dot{q}_d)\dot{q}_d + G(q_d) + F_d\dot{q}_d \tag{6}$$

where the desired regression matrix $Y_d(q_d, \dot{q}_d, \ddot{q}_d) \in \mathbb{R}^{n\times p}$ is a function of the desired link position, velocity, and acceleration vectors denoted by $q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \in \mathbb{R}^n$, respectively, where it is assumed that the desired trajectory is selected so that $q_d(t), \dot{q}_d(t), \ddot{q}_d(t), \dddot{q}_d(t)$ exist and are bounded.

Property 5: The norm of the centripetal-Coriolis and friction matrices and the gravity, disturbance, and unknown parameter vectors can be upper bounded as follows[27]

$$\begin{gathered}\|V_m(q, \dot{q})\| \le \zeta_{c1}\|\dot{q}\|, \quad \|F_d\| \le \zeta_f, \\ \|G(q)\| \le \zeta_g, \quad \|T_d\| \le \zeta_{td}, \quad \|\theta\| \le \zeta_{\theta 1}\end{gathered} \tag{7}$$

where $\zeta_{c1}, \zeta_f, \zeta_g, \zeta_{td}, \zeta_{\theta 1} \in \mathbb{R}$ denote positive constants.

To foster the subsequent control design and analysis, the vector function $Tanh(\cdot) \in \mathbb{R}^n$ and the matrix function $Cosh(\cdot) \in \mathbb{R}^{n\times n}$ are defined as follows

$$Tanh(\xi) \triangleq [\tanh(\xi_1), \ldots, \tanh(\xi_n)]^T \tag{8}$$

and

$$Cosh(\xi) \triangleq diag\{\cosh(\xi_1), \ldots, \cosh(\xi_n)\} \tag{9}$$

where $\xi = [\xi_1, \ldots, \xi_n]^T \in \mathbb{R}^n$, and $diag\{\cdot\}$ denotes a diagonal matrix. Based on the definition of (8), it can easily be shown that the following inequalities hold[19]

$$\begin{gathered}\frac{1}{2}\tanh^2(\|\xi\|) \le \ln(\cosh(\|\xi\|)) \le \sum_{i=1}^{n}\ln(\cosh(\xi_i)) \le \|\xi\|^2, \\ \tanh^2(\|\xi\|) \le \|Tanh(\xi)\|^2 = Tanh^T(\xi)Tanh(\xi).\end{gathered} \tag{10}$$

3. CONTROL DESIGN AND ANALYSIS

The objective in this paper is to design a global UUB link position tracking controller for the robot manipulator model given by (1) under the constraints that only the link position variable $q(t)$ is available for measurement and that the parameter vector θ defined in (5) is an unknown constant vector. To quantify this objective, a link position tracking error, denoted by $e(t) \in \mathbb{R}^n$, is defined as follows

$$e \triangleq q_d - q \tag{11}$$

where $q_d(t)$ and its first three time derivatives are assumed to be bounded functions of time. In addition, the difference between the actual and estimated parameters are defined as follows

$$\tilde{\theta} \triangleq \theta - \hat{\theta} \tag{12}$$

where $\tilde{\theta} \in \mathbb{R}^p$ represents the parameter estimation error, and $\hat{\theta} \in \mathbb{R}^p$ represents the constant best-guess estimates of θ defined in (5). In the subsequent analysis, the following fact will be utilized

$$\|\tilde{\theta}\| \leq \zeta_{\theta 2} \tag{13}$$

where $\zeta_{\theta 2} \in \mathbb{R}$ denotes a known positive bounding constant.

3.1. *Robust output feedback tracking control law*

Based on the subsequent error system development and stability analysis, the following torque input is designed

$$\tau \triangleq Y_d\hat{\theta} - k\Gamma^{-1}y + Tanh(e). \tag{14}$$

In (14), $k(t) \in \mathbb{R}$ is a subsequently designed positive, differentiable time-varying control gain, $\Gamma(y) \in \mathbb{R}^{n\times n}$ is defined as the following diagonal matrix

$$\Gamma \triangleq diag\{(1-y_1^2)^2, (1-y_2^2)^2, \ldots, (1-y_n^2)^2\}, \tag{15}$$

and $y(t) \in \mathbb{R}^n$ denotes a surrogate signal for the link velocity tracking error that is designed as follows

$$y_i \triangleq p_i - ke_i. \tag{16}$$

In (16), $p_i(t) \in \mathbb{R}$ is defined as the solution to the following differential equation

$$\begin{aligned}\dot{p}_i \triangleq & -\left(1-(p_i - ke_i)^2\right)^2(p_i - ke_i - \tanh(e_i)) \\ & - k(\tanh(e_i) + p_i - ke_i) + \dot{k}e_i \end{aligned}\tag{17}$$

where the initial conditions for $p_i(0)$ are selected as follows

$$-\frac{1}{\sqrt{n}} + k(0)e_i(0) < p_i(0) < \frac{1}{\sqrt{n}} + k(0)e_i(0). \tag{18}$$

Provided the initial condition $p_i(0)$ is selected according to (18), then (16) can be used to conclude that

$$|y_i(0)| < \frac{1}{\sqrt{n}}$$

independently of the magnitude of $e_i(0)$. Based on the subsequent development, the time-varying control gain $k(t)$ of (14) and (16)–(18) is designed as follows

$$\begin{aligned} k \triangleq \frac{1}{m_1}\Bigg(& 1 + k_{n1}np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2\zeta_{\theta 1}^2 + k_{n2}np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2\zeta_{\theta 2}^2 \\ & + k_{n3}\zeta_k^2 + k_{n4}\zeta_1^2 + 16\zeta_2^2 + 8\zeta_3^2 + 4\zeta_4^2 + 16\zeta_5^2 + \zeta_6\Bigg). \end{aligned} \tag{19}$$

In (19), $\zeta_k(t) \in \mathbb{R}$ denotes a known positive function defined as follows

$$\zeta_k \triangleq m_2\|\ddot{q}_d\| + \zeta_{c1}\|\dot{q}_d\|^2 + \zeta_f\|\dot{q}_d\| + \zeta_g + \zeta_{td} \tag{20}$$

where m_1 and m_2 were defined in (2), $k_{n1}, k_{n2}, k_{n3}, k_{n4} \in \mathbb{R}$ denote positive constant control gains, $\zeta_{\theta 1}$ was defined in (7), $\zeta_{\theta 2}$ was defined in (13), and $\zeta_i, i = 1, \ldots, 6$ denote some positive constants used to bound the mechanical parameters of the system dynamics and the desired trajectory. To facilitate the subsequent stability analysis, the control gains k_{n1}, k_{n2}, k_{n3}, and k_{n4} are selected to satisfy the following sufficient conditions

$$\varepsilon < \frac{1}{2} \quad k_{n4} > \frac{1}{4\left(\frac{1}{2} - \varepsilon\right)} \tag{21}$$

where $\varepsilon \in \mathbb{R}$ is a positive constant defined as follows

$$\varepsilon = \frac{1}{4k_{n1}} + \frac{1}{4k_{n2}} + \frac{1}{4k_{n3}}. \tag{22}$$

3.2. *Error system development*

After taking the time derivative of (16), the open-loop error system for the link velocity tracking error surrogate can be determined as follows

$$\dot{y}_i = -(1-y_i^2)^2(y_i - \tanh(e_i)) - k\eta_i \tag{23}$$

where (17) and (18) were utilized, and $\eta(t) \in \mathbb{R}^n$ denotes the following auxiliary filtered tracking error signal

$$\eta \triangleq \dot{e} + Tanh(e) + y. \tag{24}$$

To obtain the open-loop error system for $\eta(t)$, we differentiate (24) and pre-multiply both sides of the resulting equation by $M(q)$ to yield the following expression

$$\begin{aligned} M(q)\dot{\eta} = & M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + T_d - \tau \\ & + M(q)Cosh^{-2}(e)\dot{e} + M(q)\dot{y} \end{aligned} \tag{25}$$

where (1) was utilized. After adding and subtracting $Y_d\theta$ of (6) to (25), we can utilize (4), (11), (23), and (24) to rewrite the open-loop dynamics for $\eta(t)$ as follows

$$M(q)\dot{\eta} = -V_m(q,\dot{q})\eta + Y_d\theta - \tau - kM(q)\eta + \chi \qquad (26)$$

where the disturbance term $\chi(e, y, \eta, t) \in \mathbb{R}^n$ is defined as

$$\begin{aligned}\chi \triangleq\ & M(q)Cosh^{-2}(e)(\eta - Tanh(e) - y) \\ & - M(q)\Gamma(y - Tanh(e)) + V_m(q, \dot{q}_d + Tanh(e) + y) \\ & \times (Tanh(e) + y) + V_m(q, \dot{q}_d)(Tanh(e) + y) \\ & - V_m(q, \eta)(\dot{q}_d + Tanh(e) + y) + M(q)\ddot{q}_d \\ & + V_m(q, \dot{q}_d)\dot{q}_d + F_d\dot{q} + G(q) + T_d - Y_d\theta. \qquad (27)\end{aligned}$$

As illustrated in Appendix A, we can exploit the boundedness properties of the desired trajectory, (2), (7), and the properties of hyperbolic functions to show that $\chi(\cdot)$ of (27) can be upper bounded as follows

$$\begin{aligned}\|\chi\| \le\ & \zeta_1\|x\| + \zeta_2\|y\|^2 + \zeta_3\|y\|^3 + \zeta_4\|y\|^4 + \zeta_5\|y\|^5 \\ & + \zeta_6\|\eta\|\|y\| + \zeta_k + \left(\sqrt{np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2}\right)\zeta_{\theta 1} \qquad (28)\end{aligned}$$

where the composite state vector $x(t) \in \mathbb{R}^{3n}$ is defined as

$$x \triangleq [Tanh^T(e)\ \eta^T\ y^T]^T. \qquad (29)$$

The closed-loop error system for $\eta(t)$ can now be obtained by substituting (14) into (26) as follows

$$\begin{aligned}M(q)\dot{\eta} =\ & -V_m(q,\dot{q})\eta + Y_d\tilde{\theta} + k\Gamma^{-1}y - Tanh(e) \\ & - kM(q)\eta + \chi \qquad (30)\end{aligned}$$

where (12) was used.

3.3. Stability analysis

Theorem 1
Given the robot dynamics of (1), the robust controller of (14)–(19) with the control gains selected as described in (19) and (21) ensures that the link position tracking error is UUB in the sense that

$$\|e(t)\| \le \|z(t)\| < \bar{d} \quad \forall t \ge T(\bar{d}, \|z(0)\|) \qquad (31)$$

in the following global region for the link position tracking error

$$\left\{(e, \eta, y) \in \mathbb{R}^n \times \mathbb{R}^n \times \left(-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)^n\right\} \qquad (32)$$

where the composite state vector $z(t) \in \mathbb{R}^{3n}$ is defined as follows

$$z \triangleq [e^T\ \eta^T\ y^T]^T. \qquad (33)$$

In (31), $\bar{d} \in \mathbb{R}$ is a positive constant that defines the radius of the ball containing the link position tracking errors as follows

$$\bar{d} > \left(\gamma_1^{-1} \circ \gamma_2\right)\left(\gamma_3^{-1}(\varepsilon)\right), \qquad (34)$$

and $T(\bar{d}, \|z(0)\|) \in \mathbb{R}$ denotes the following positive constant that denotes the ultimate time to reach the ball

$$T(\bar{d}, \|z(0)\|) = \begin{cases} 0 & \text{if } \|z(0)\| \le \left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d}) \\ \dfrac{\gamma_2(\|z(0)\|) - \gamma_1\left(\left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d})\right)}{\gamma_3\left(\left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d})\right) - \varepsilon} & \text{if } \|z(0)\| > \left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d}) \end{cases} \qquad (35)$$

where ε is introduced in (22), and $\circ$ denotes the composition operator. In (34) and (35), the strictly increasing nonnegative functions $\gamma_1(\|z\|), \gamma_2(\|z\|), \gamma_3(\|z\|) \in \mathbb{R}$ are defined as follows

$$\begin{aligned}\gamma_1(\|z\|) &\triangleq \lambda_1 \ln\cosh(\|z\|) \\ \gamma_2(\|z\|) &\triangleq \lambda_2\|e\ \eta\|^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{y_i^2}{1-y_i^2} \qquad (36) \\ \gamma_3(\|z\|) &\triangleq \left(\frac{1}{2} - \frac{1}{4k_{n4}}\right)\tanh^2(\|z\|)\end{aligned}$$

where the positive constants $\lambda_1, \lambda_2 \in \mathbb{R}$ are defined as follows

$$\lambda_1 \triangleq \min\left\{\frac{1}{2}, \frac{m_1}{2}\right\} \quad \lambda_2 \triangleq \max\left\{1, \frac{m_2}{2}\right\}. \qquad (37)$$

Proof. To prove Theorem 1, a nonnegative function $V(z, t) \in \mathbb{R}$ is defined as follows

$$V \triangleq \sum_{i=1}^{n}\ln(\cosh(e_i)) + \frac{1}{2}\eta^T M(q)\eta + \frac{1}{2}\sum_{i=1}^{n}\frac{y_i^2}{1-y_i^2}. \qquad (38)$$

With regard to the structure of (38), the first and second terms are positive-definite and radially unbounded, and the last term is positive-definite and radially unbounded on the interval $[-1, 1]$; hence, $V(z, t)$ of (38) is a positive-definite, radially unbounded function in the set

$$S \triangleq \{(e, \eta, y) \in \mathbb{R}^n \times \mathbb{R}^n \times [-1, 1]^n\}. \qquad (39)$$

Based on the inequalities given in (10), the following upper and lower bounds for (38) can be determined

$$\gamma_1(\|z\|) \le V(z, t) \le \gamma_2(\|z\|) \qquad (40)$$

where $z(t)$ was defined in (33), and $\gamma_1(\|z\|)$ and $\gamma_2(\|z\|)$ were introduced in (36).

After taking the time derivative of (38) and utilizing (23), (24), and (30) the following expression can be obtained

$$\dot{V} = -\sum_{i=1}^{m} \tanh^2(e_i) + \eta^T(-kM(q)\eta + \chi + Y_d\tilde{\theta}) - \sum_{i=1}^{n} y_i^2 \tag{41}$$

where (3) was utilized. After utilizing (2), (19), and (28), the expression in (41) can be upper bounded as follows

$$\begin{aligned}\dot{V} \le & -\|\eta\|^2 - \|y\|^2 - \|Tanh(e)\|^2 \\ & + \left[\zeta_{\theta 1}\sqrt{np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2}\|\eta\| - k_{n1}(\zeta_{\theta 1})^2\left(np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2\right)\|\eta\|^2\right] \\ & + \left[\zeta_{\theta 2}\sqrt{np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2}\|\eta\| - k_{n2}(\zeta_{\theta 2})^2\left(np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2\right)\|\eta\|^2\right] \\ & + [\zeta_k\|\eta\| - k_{n3}(\zeta_k)^2\|\eta\|^2] + [\zeta_1\|x\|\|\eta\| - k_{n4}\zeta_1^2\|\eta\|^2] \\ & + [\zeta_2\|y\|^2\|\eta\| - 16\zeta_2^2\|\eta\|^2] + [\zeta_3\|y\|^3\|\eta\| - 8\zeta_3^2\|\eta\|^2] \\ & + [\zeta_4\|y\|^4\|\eta\| - 4\zeta_4^2\|\eta\|^2] + [\zeta_5\|y\|^5\|\eta\| - 16\zeta_5^2\|\eta\|^2] \\ & + \zeta_6(\|y\| - 1)\|\eta\|^2.\end{aligned} \tag{42}$$

After completing the squares on the bracketed terms of (42), the following inequality can be obtained

$$\begin{aligned}\dot{V} \le & -\frac{\|\eta\|^2}{2} - \|y\|^2 - \frac{\|Tanh(e)\|^2}{2} \\ & + \frac{1}{4k_{n1}} + \frac{1}{4k_{n2}} + \frac{1}{4k_{n3}} + \frac{1}{4k_{n4}}\|x\|^2 \\ & + \frac{1}{2}\|y\|^2\left[\frac{1}{8}\|y\|^2 + \frac{1}{4}\|y\|^4 + \frac{1}{2}\|y\|^6 + \frac{1}{8}\|y\|^8\right] \\ & + \zeta_6[\|y\| - 1]\|\eta\|^2.\end{aligned} \tag{43}$$

The expression in (43) can be further upper bounded by utilizing (29) as follows

$$\begin{aligned}\dot{V} \le & -\frac{1}{2}\|x\|^2 + \frac{1}{4k_{n1}} + \frac{1}{4k_{n2}} + \frac{1}{4k_{n3}} + \frac{1}{4k_{n4}}\|x\|^2 \\ & + \frac{1}{2}\|y\|^2\left[-1 + \frac{1}{8}\|y\|^2 + \frac{1}{4}\|y\|^4 + \frac{1}{2}\|y\|^6 + \frac{1}{8}\|y\|^8\right] \\ & + \zeta_6[\|y\| - 1]\|\eta\|^2.\end{aligned}$$

Provided the control gains are selected according to (21), then the following expression can be obtained

$$\dot{V} \le -\left(\frac{1}{2} - \frac{1}{4k_{n4}}\right)\|x\|^2 + \varepsilon \quad \text{if} \quad \|y(t)\| < 1 \quad \forall t \ge 0$$

where ε was defined in (22). Utilizing the inequalities given in (10), the following new upper bound can be determined

$$\dot{V} \le -\gamma_3(\|z\|) + \varepsilon \quad \text{if} \quad \|y(t)\| < 1 \quad \forall t \ge 0 \tag{44}$$

where $\gamma_3(\cdot)$ is introduced in (36), and $z(t)$ was defined in (33). After noting that $\|y\|^2 \le n\max_i |y_i|^2$, the set S_1 can be defined as follows

$$S_1 \triangleq \left\{(e, \eta, y) \in \mathbb{R}^n \times \mathbb{R}^n \times \left(-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)^n\right\}. \tag{45}$$

It now follows from (44) and (45) that

$$\dot{V} \le -\gamma_3(\|z\|) + \varepsilon \quad \text{if } (e, \eta, y) \in S_1. \tag{46}$$

From (21) and (36), it is clear that the following conditions hold

$$\gamma_i(0) = 0 \quad i = 1, 2, 3 \quad \lim_{z\to\infty} \gamma_i(z) = \infty \quad i = 1, 2 \tag{47}$$

$$\lim_{z\to\infty} \gamma_3(z) = \left(\frac{1}{2} - \frac{1}{4k_{n4}}\right) \quad \varepsilon < \left(\frac{1}{2} - \frac{1}{4k_{n4}}\right). \tag{48}$$

Hence, for initial conditions inside S_1, Lemma 1 given in Appendix B can applied to (40) and (46) to obtain the result given by (31) where the ultimate bound is given in (34). □

Remark 1

Based on (34) and (36), it can be determined that the size of the ultimate bound $\bar{d}$ can be made arbitrarily small by selecting the control gains $k_{ni} \forall i = 1, 2, \ldots 4$ arbitrarily large.

Remark 2

Despite the fact that the initial conditions for the auxiliary signal $y(t)$ are constrained (i.e., see (17) and (23)), the stability result is still global for the link position tracking error signal $e(t)$, since no restrictions are placed on the size of $\|e(0)\|$. Moreover, no restrictions are placed on the size of $\|\eta(0)\|$, and $\|\dot{e}(0)\|$.

4. CONCLUSION

In this paper, a novel global output feedback tracking controller for rigid-link robot manipulators is presented. Through the new design, the proposed robust controller overcomes some potential drawbacks associated with the stability analysis used in reference [19]. That is, the closed-loop error system development and Lyapunov stability analysis originate from a velocity-independent version of the control strategy. As in references [19, 24], the proposed control scheme guarantees asymptotic link position tracking with no restrictions on the size of the initial position/velocity tracking error.

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APPENDIX A

In this appendix, the upper bound given in (28) is developed. To this end, the expression given in (27) can be upper bounded as follows by utilizing (2) and (7)

$$\begin{aligned}\|\chi\| \leq\ & m_2(\|\eta\| + \|Tanh(e)\| + \|y\|) \\ & + m_2\|T\|(\|y\| + \|Tanh(e)\|) \\ & + \zeta_{c1}(\|\dot{q}_d\| + \|Tanh(e)\| + \|y\|)(\|y\| + \|Tanh(e)\|) \\ & + \zeta_{c1}\|\dot{q}_d\|(\|y\| + \|Tanh(e)\|) \\ & + \zeta_{c1}\|\eta\|(\|\dot{q}_d\| + \|Tanh(e)\| + \|y\|) \\ & + m_2\|\ddot{q}_d\| + \zeta_{c1}\|\dot{q}_d\|^2 + \zeta_f\|\dot{q}\| + \zeta_g + \zeta_{td} \\ & + \left(\sqrt{np\sum_{i=1}^{n}\sum_{j=1}^{p} Y_{dij}^2}\right)\zeta_{\theta 1}. \end{aligned} \tag{49}$$

From the definition of $\Gamma(y)$ given in (15), the following inequality can be developed

$$\|\Gamma\| \leq (1 + \|y\|^2)^2. \tag{50}$$

After substituting (50) into (49), using the fact that $|\tanh(x)| \leq 1, \forall x \in \mathbb{R}$, and making use of (29), the result given in (28) can be easily obtained.

APPENDIX B

Lemma 1: [28] Given the system

$$\dot{x} = f(x(t), t) \tag{51}$$

where $f(\cdot) \in \mathbb{R}^{n\times 1}$ satisfies $f(0, t) = 0$, if there exists a non-negative function $V(\cdot)$ such that for all x and t

$$\begin{aligned}\gamma_1(\|x\|) \leq V(x, t) \leq \gamma_2(\|x\|) \\ \dot{V}(x, t) \leq -\gamma_3(\|x\|) + \varepsilon\end{aligned} \tag{52}$$

where $\gamma_i(\cdot)$ are scalar, strictly increasing functions, ε is a positive scalar constant, and the following conditions hold

$$\gamma_i(0) = 0 \quad i = 1, 2, 3 \tag{53}$$

$$\lim_{p\to\infty}\gamma_i(p) = \infty \quad i = 1, 2 \quad \lim_{p\to\infty}\gamma_3(p) \triangleq l < \infty \quad \varepsilon < l \tag{54}$$

where l is a positive scalar constant, then $x(t)$ is globally uniformly ultimately bounded in the following sense

$$\|x(t)\| < \bar{d} \quad \forall t \geq T(\bar{d}, \|x(0)\|) \tag{55}$$

where $\bar{d}$ is a positive constant that defines the radius of the ball and is selected according to

$$\bar{d} > \left(\gamma_1^{-1} \circ \gamma_2\right)\left(\gamma_3^{-1}(\varepsilon)\right),$$

and $T(\bar{d}, \|x(0)\|)$ is a positive constant that denotes the ultimate time and is given by

$$T(\bar{d}, \|x(0)\|) = \begin{cases} 0 & \text{if } \|x(0)\| \leq \left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d}) \\ \dfrac{\gamma_2(\|x(0)\|) - \gamma_1\left(\left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d})\right)}{\gamma_3\left(\left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d})\right) - \varepsilon} & \text{if } \|x(0)\| > \left(\gamma_2^{-1} \circ \gamma_1\right)(\bar{d}). \end{cases} \tag{56}$$