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A composite adaptive output feedback tracking controller for robotic manipulators*

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SUMMARY
This paper provides a solution to the composite adaptive output feedback tracking control problem for robotic manipulators. The proposed controller utilizes an update law that is a composite of a gradient update law driven by the link position tracking error and a least squares update law driven by the prediction error. In order to remove the controller's dependence on link velocity measurements, a linear filter and a new prediction error formulation are designed. The controller provides semi-global asymptotic link position tracking performance. Experimental results illustrate that the proposed controller provides improved link position tracking error transient performance and faster parameter estimate convergence in comparsion to the same controller using a gradient update law.

KEYWORDS: Robot control; Output feedback; Tracking performance; Update law.

1. INTRODUCTION

Robot manipulators are nonlinear, multi-input/multi-output mechatronic systems that have well defined dynamic models; however, exact knowledge of the parameters needed to complete the model are often not known precisely. In order to provide for robustness, many researchers have design full-state feedback adaptive controllers that compensate for parametric uncertainty (See reference 1 for a review paper on this topic). Much of the previous full-state feedback adaptive control work focused on the use of a standard gradient update to compensate for parametric uncertainty. In comparison with a least-squares update law, a gradient-type update law often exhibits slower parameter convergence, and hence suffers from slower link position tracking error transient response. In order to incorporate the desirable features of using a least-squares update law, Middleton *et al.*[2] augmented the adaptive controller of reference 3 with additional terms which allowed the closed-loop error system to be written as a stable, linear, strictly-proper, transfer function with the link position tracking error as the output and a prediction error related term as the input. Provided the estimated inertia matrix was forced to be positive-definite (PD), i.e. a projection-type algorithm was required, this novel input-output relationship facilitated the design of a least squares update law driven by the prediction error* while also fostering a bounded-input, bounded-output, global asymptotic link position tracking stability result. Later, the restriction involving the estimated inertia matrix required in reference 2 was removed in references 4, 5 and 6; however, none of these algorithms utilized the traditional least squares update law driven by the prediction error. Specifically, Slotine *et al.*[4] removed the restriction on the estimated inertia matrix required in reference 2 by utilizing a composite update law (i.e. the composite update law was the sum of a least-squares update law driven by the prediction error and a modified† gradient update law driven by the link position/velocity tracking error). Lozano Leal *et al.*[5] utilized the flexibility provided by previous passivity-based adaptive control designs to construct a modified least-squares update law with the link position/velocity tracking error as the input. As time goes to infinity, the parameter estimates in this modified least-squares update law converge to the parameter estimates generated by a standard least squares update law driven by the link position/velocity tracking error. A modified least-squares update law driven by the link position/velocity tracking error was also proposed by Sadegh *et al.*[6] in the design of an exponentially stable desired trajectory-based controller. This exponential tracking result was predicated on the assumption that the *desired* regression matrix satisfies a *semi*-persistency of excitation condition; furthermore, the modified least-squares update law requires the calculation of a matrix dependent on the excitation condition. In Tang *et al.*[7] developed an adaptive controller which included the standard gradient update law, the composite adaptation update law, and an averaging gradient update law as special cases. Recently, de Queiroz *et al.*[8] illustrated how a modification of the controller/update law modularity technique developed in reference 9 could be used to design a least squares update law driven by the prediction error while also fostering a bounded-input, bounded-output stability result; however, the global asymptotic link position

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* The prediction error is defined as the difference between an estimated, filtered version of the the robot dynamics and a filtered version of the input torque.
† The modified gradient update law uses the same time-varying adaptation gain as the least-squares update law.

tracking stability result still mandated that the estimated inertia matrix be positive definite. It should be noted the all of the above robot controllers required link position and link velocity measurements.

In addition to providing robustness for parametric uncertainty, many researchers have also been motivated to reduce system cost and complexity by reducing the number of required sensors. That is, researchers have been motivated to design adaptive or robust output feedback (OFB) controllers which only require measurement of link position. For example, robust, OFB high-gain controllers, based on linear filtering of the link position signal were utilized to achieve semi-global, uniform ultimately bounded stability results in references 10, 11 and 12. Through the use of a linear filtering tool, adaptive OFB controllers that incorporated gradient update laws driven by the link position tracking error were utilized to achieve semi-global asymptotic stability results in references 13 and 14. Zhang *et al.*[15] utilized a nonlinear filtering scheme (derived from a modified version of the controller given in reference 13) to obtain global asymptotic link position tracking.

In this paper, we redesign the full-state feedback, composite adaptive controller given[4] such that link velocity measurements are not required. Similar to the controller given in reference 4, the proposed controller holds the potential of improved link position tracking error transient response since the update law includes a least squared term driven by prediction error. The elimination of link velocity measurements is achieved by: (i) utilizing a linear dynamic filter whose output serves as a surrogate for link velocity measurements, (ii) constructing the prediction error as the difference between the filtered torque control input and a filtered version of the desired regression matrix-parameter estimate formulation, and (iii) using a linear, high-gain feedback term to compensate for the difference between the actual dynamics (i.e. dynamic terms that are functions of link position and link velocity) and the desired dynamics (i.e. dynamic terms with the link position and link velocity being replaced by the desired quantities). Provided the controller and filter gains satisfy some sufficient conditions and the robot manipulator is initially at rest, the proposed control delivers semi-global asymptotic link position tracking. To illustrate the viability and improved performance of the OFB composite adaptive controller, an experimental comparison is performed between the proposed control law and the same controller with a standard gradient update law (i.e. the controller given in reference 13).

The paper is organized as follows. Section 2 provides the foundation for the subsequent control development and analysis. Specifically, we present the mathematical model for the dynamics of a n-link revolute direct drive robot and its associated properties. In Section 3, we utilize the properties of Section 2, for the design and analysis of the OFB composite adaptive controller. Experimental verification of the controller is presented in Section 4, and concluding remarks are given in Section 5.

2. MATHEMATICAL MODEL

The mathematical model for an *n*-DOF revolute direct drive robotic manipulator is assumed to have the following form[16,17]

$$M(q)\ddot{q} + V_m(q,\dot{q})\,\dot{q} + G(q) + F_d\,\dot{q} = \tau \tag{1}$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \Re^n$ denote the link position, velocity, and acceleration, respectively, $M(q) \in \Re^{n\times n}$ represents the positive-definite, symmetric inertia matrix, $V_m(q, \dot{q}) \in \Re^{n\times n}$ represents the centripetal-Coriolis matrix, $G(q) \in \Re^n$ is the gravitational vector, $F_d \in \Re^{n\times n}$ denotes the constant, diagonal, positive-definite viscous friction matrix, and $\tau(t) \in \Re^n$ represents the torque input vector. We will assume that the left-hand side of (1) is first-order differentiable.

The dynamic system given by (1) exhibits the following properties that are utilized in the subsequent control development and stability analysis.

Property 1: The inertia matrix can be upper and lower bounded by the following inequalities[16]

$$m_1\|\xi\|^2 \le \xi^T M(q)\xi \le m_2\|\xi\|^2 \quad \forall\,\xi \in \Re^n \tag{2}$$

where m_1 and m_2 are positive scalar bounding constants, and $\|\cdot\|$ denotes the Euclidean norm.

Property 2: The inertia and the centripetal-Coriolis matrices satisfy the following relationships[18]

$$\dot{M}(q) = V_m(q,\dot{q}) + V_m^T(q,\dot{q}) \tag{3}$$

$$\xi^T\left(\frac{1}{2}\dot{M}(q) - V_m(q,\dot{q})\right)\xi = 0 \quad \forall\,\xi \in \Re^n \tag{4}$$

where $\dot{M}(q)$ represents the time derivative of the inertia matrix.

Property 3: The centripetal-Coriolis matrix satisfies the following relationship[16]

$$V_m(q,\nu)\xi = V_m(q,\xi)\nu \quad \forall\,\xi,\ \nu \in \Re^n. \tag{5}$$

Property 4: The norm of the centripetal-Coriolis matrix, the dynamic friction matrix, and the time derivative of inertia matrix can be upper bounded as follows[16]

$$\|V_m(q,\dot{q})\|_{i\infty} \le \zeta_{c1}\|\dot{q}\|, \quad \|F_d\| \le \zeta_{fd}, \quad \|\dot{M}(q)\|_{i\infty} \le \zeta_{m1}\|\dot{q}\|, \tag{6}$$

where ζ_{c1}, ζ_{fd}, and ζ_{m} are positive scalar bounding constants, and $\|\cdot\|_{i\infty}$ denotes the induced infinity norm of a matrix.

Property 5: The robot dynamics given in (1) can be linearly parameterized as follows[16]

$$Y(q,\dot{q},\ddot{q})\,\theta = M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} \tag{7}$$

where $\theta \in \Re^p$ contains the unknown constant system parameters, and $Y(q,\dot{q},\ddot{q}) \in \Re^{n\times p}$ denotes the known regression matrix that is a function of $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \Re^n$. The regression matrix formulation of (7) is also written in terms of the desired trajectory in the following manner

$$Y_d(q_d,\dot{q}_d,\ddot{q}_d)\,\theta = M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F_d\dot{q}_d \tag{8}$$

where the desired regression matrix is defined by $Y_d(q_d, \dot{q}_d, \ddot{q}_d) \in \Re^{n\times p}$ which is a function of the desired link position, velocity, and acceleratino vectors denoted by $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t) \in \Re^n$, respectively.

Property 6: The inertia, centripetal-Coriolis, and gravity terms of (1) can be upper bounded as follows[19]

$$\|M(\xi)-M(\nu)\|_{i\infty} \le \zeta_{m2}\|(\xi-\nu)\| \quad \forall \xi,\ \nu \in \Re^n$$
$$\|V_m(\xi,\dot{q})-V_m(\nu,\dot{q})\|_{i\infty} \le \zeta_{c2}\|\dot{q}\|\,\|(\xi-\nu)\| \quad \forall \xi,\ \nu \in \Re^n \quad (9)$$
$$\|G(\xi)-G(\nu)\| \le \zeta_g\|(\xi-\nu)\| \quad \forall \xi,\ \nu \in \Re^n$$

where ζ_{m2}, ζ_{c2}, *and* ζ_g are positive bounding constants.

Assumption 1: In the subsequent design/analysis, we must assume that the initial desired velocity of the system is zero (i.e. $\dot{q}_d(0)=0$) and that the system is initially at rest (i.e. $\dot{q}(0)=0$). It is important to note that although we must assume $\dot{q}_d(0)=\dot{q}(0)=0$, which is the case with most practical applications, we do not require the initial link position tracking error to be zero.

3. CONTROL DEVELOPMENT

In order to quantify the control objective, we define the link position tracking error, $e(t)\in\Re^n$ as the difference between the desired link position and the actual link position as shown below

$$e = q_d - \mathrm{q} \quad (10)$$

where we assume the first three time derivatives of the desired link position trajectory, defined in (8), are bounded functions of time. To facilitate the design of a link velocity independent controller, we define a filtered tracking error-like variable $\eta(t)\in\Re^n$ as follows[20]

$$\eta = \dot{e} + \alpha_1 e + \alpha_2 e_f \quad (11)$$

where $\dot{e}(t)\in\Re^n$ denotes the time derivative of $e(t)$ given in (10), the filter variable $e_f(t)\in\Re^n$ is defined by the following dynamic relationship

$$\dot{e}_f = -\alpha_3 e_f + \alpha_2 e - k\eta \qquad e_f(0)=0, \quad (12)$$

and k, α_1, α_2, $\alpha_3 \in \Re^1$ are positive, constant control gains. To quantify the performance of the adaptation algorithm, we define the parameter estimation error signal, denoted by $\tilde{\theta}(t)\in\Re^p$, as follows

$$\tilde{\theta} = \theta - \hat{\theta} \quad (13)$$

where $\hat{\theta}(t)\in\Re^p$ denotes the yet to be defined dynamic parameter estimate of θ.

3.1 Torque Filtering

To facilitate the construction of a composite adaptive controller that utilizes a prediction error-based update law, we filter the torque input signal according to the standard procedure given in references 16 & 21 as follows

$$\tau_f = f * \tau \quad (14)$$

where $\tau_f(t)\in\Re^n$ denotes the measurable filtered torque signal, $*$ denotes the standard convolution operation, $\tau(t)$ was defined in (1), and the filter function, denoted by $f(t)\in\Re^1$, is given by

$$f = \beta \exp(-\beta t) \quad (15)$$

where $\beta\in\Re^1$ denotes a positive filter constant. After substituting the left-hand side of (1) into (14) for $\tau(t)$, we can utilize standard convolution properties and (7) to rewrite (14) in terms of the following linear parameterization

$$\tau_f = Y_f\theta \quad (16)$$

where θ denotes the same unknown parameter vector defined in (7), $Y_f(q,\ \dot{q})\in\Re^{n\times p}$ denotes the known, filtered regression matrix which does not depend on link acceleration measurements and is explicitly given by

$$\begin{aligned} Y_f\theta = {} & \dot{f}(t)*\{M(q(t))\dot{q}(t)\} + f(0)M(q(t))\dot{q}(t) \\ & - f(t)M(q(0))\dot{q}(0) \\ & + f(t)*\{-\dot{M}(q(t))\dot{q}(t) + V_m(q(t),\dot{q}(t))\,\dot{q}(t) \\ & + G(q(t)) + F_d\dot{q}(t)\} \end{aligned} \quad (17)$$

and $\dot{f}(t)\in\Re^1$ is given by

$$\dot{f}(t) = -\beta^2\exp(-\beta t). \quad (18)$$

To facilitate the removal of link velocity measurements from the controller, we utilize the structure of (17) to define the following additional linear parameterization

$$\begin{aligned} Y_{df}\theta = {} & \dot{f}(t)*\{M(q_d(t))\dot{q}_d(t)\} + f(0)M(q_d(t))\,\dot{q}_d(t) \\ & - f(t)M(q_d(0))\dot{q}_d(0) \\ & + f(t)*\{-\dot{M}(q_d(t))\dot{q}_d(t) + V_m(q_d(t),\dot{q}_d(t))\,\dot{q}_d(t) \\ & + G(q_d(t)) + F_d\dot{q}_d(t)\} \end{aligned} \quad (19)$$

where θ denotes the same unknown parameter vector defined in (7), and $Y_{df}(q_d,\ \dot{q}_d)\in\Re^{n\times p}$ denotes the known, desired filtered regression matrix.

We now define the measurable prediction error signal $\varepsilon(t)\in\Re^n$ as shown below

$$\varepsilon = \tau_f - Y_{df}\hat{\theta} \quad (20)$$

where $\hat{\theta}(\mathrm{t})\in\Re^p$ denotes the yet to be defined dynamic parameter estimate of θ, and $Y_{df}(q_d,\ \dot{q}_d)$ was defined in (19). To facilitate the subsequent control design and stability analysis, we substitute the right-hand side of (16) for $\tau_f(t)$ in (20), and then add and subtract $Y_{df}(\cdot)\theta$ to the right-hand side of the resulting expression to obtain

$$\varepsilon = Y_f\theta - Y_{df}\theta + Y_{df}\theta - Y_{df}\hat{\theta} = \Omega + Y_{df}\tilde{\theta} \quad (21)$$

where $\tilde{\theta}(t)$ was defined in (13), and $\Omega(t)\in\Re^n$ denotes the following unmeasurable, auxiliary function

$$\Omega \triangleq Y_f\theta - Y_{df}\theta \quad (22)$$

Based on (15), (17), (18), (19), (22), and *Assumption 1*, it is easy to upper bound $\Omega(t)$ in the following manner

$$\|\Omega\| \le \beta\zeta_2\|x\| + \beta\zeta_3\|x\|^2 \quad (23)$$

where ζ_2 and ζ_3 are positive constant scalar bounding terms, β is the same constant weighting term defined in (15), and $x(t)\in\Re^{3n}$ is defined as follows

$$x = [e^T\ e_f^T\ \eta^T]^T \quad (24)$$

where $e(t)$, $e_f(t)$, and $\eta(t)$ were defined in (10), (12), and (11), respectively.

3.2 Control Formulation

Based on the subsequent open-loop error dynamics, and the corresponding stability analysis, we propose the following control torque input

$$\tau = Y_d \hat{\theta} - k K_s e_f + K_s e \tag{25}$$

where $K_s \in \Re^{n \times n}$ is a diagonal positive definite constant weighting matrix, the desired regression matrix $Y_d(\cdot)$ was defined in (8), and the parameter estimate vector $\hat{\theta}(t)$ is computed on-line according to the following composite update law

$$\dot{\hat{\theta}} = P Y_{df}^T \epsilon + P Y_d^T \eta \tag{26}$$

where $Y_{df}(\cdot)$ was defined in (19), $\varepsilon(t)$ was defined in (20), $\eta(t)$ was defined in (11), and $P(t) \in \Re^{p \times p}$ is a time-varying, gain matrix which is updated according to

$$\dot{P} = -P Y_{df}^T Y_{df} P \tag{27}$$

where $P(0)$ is selected to be a positive definite, symmetric matrix. Based on the definition given in (27), we can use the matrix equality, $\dot{P}^{-1} = -P^{-1}\dot{P}P^{-1}$, to determine the following dynamic expression for $\dot{P}^{-1}(t)$

$$\dot{P}^{-1} = Y_{df}^T Y_{df} \tag{28}$$

where $P^{-1}(0)$ is also a positive definite, symmetric matrix because $P(0)$ has been selected to a positive definite, symmetric matrix. Since $Y_{df}^T(\cdot) Y_{df}(\cdot)$ is a positive semi-definite, symmetric matrix, we can see from (28) that $P^{-1}(t)$ will remain a positive definite, symmetric matrix for all time.

Remark 1. *Based on the structure of $\eta(t)$ defined in (11), it is clear that link velocity measurements are required for the implementation of (11), (12), (25), and (26); however, we can construct a link velocity independent, implementable form of the control algorithm. Specifically, based on (11) and (12), we can construct the following linear link-velocity independent filter*

$$\begin{aligned} \dot{w} &= -(w - ke)(\alpha_3 + k\alpha_2) - e(k\alpha_1 - \alpha_2) \qquad w(0) = ke(0) \\ e_f &= w - ke \end{aligned} \tag{29}$$

where $w(t) \in \Re^n$ is an auxiliary filter variable. In addition, the adaptive update law given by (26) and (27) can be integrated by parts to yield the following link velocity-independent form for the update law

$$\begin{aligned} \hat{\theta} = P Y_d^T e &+ \int_0^t \big(P(\sigma) Y_{df}^T(\sigma) \varepsilon(\sigma) \\ &+ P(\sigma) Y_d^T(\sigma)(\alpha_1 e(\sigma) + \alpha_2 e_f(\sigma))\big)\, d\sigma \\ &- \int_0^t \Big(-P(\sigma) Y_{df}^T(\sigma) Y_{df}(\sigma) P(\sigma) Y_d^T(\sigma) e(\sigma) \\ &+ P(\sigma) \dot{Y}_d^T(\sigma) e(\sigma) \Big)\, d\sigma \end{aligned} \tag{30}$$

where (27) has been utilized, and $\dot{Y}_d(\cdot)$ can be calculated by taking the time derivative of the regression matrix formulation given by (8). Hence, the control law can be calculated using only link position measurements via the use of (20), (25), (27), (29), and (30).

3.3 Error System Development

After taking the time derivative of (11), pre-multiplying the result by the inertia matrix $M(q)$, making substitutions for (1), (10), (11) and (12), adding and subtracting the term $Y_d(\cdot)\theta$, and making use of *Property 3*, the dynamics for $\eta(t)$ can be written as follows

$$M(q)\dot{\eta} = -V_m(q, \dot{q})\eta - \alpha_2 k M(q)\eta + Y_d \theta + \chi - \tau \tag{31}$$

where the auxiliary function $\chi(e, e_f, \eta, t) \in \Re^n$ is defined as

$$\begin{aligned} \chi = {} & M(q)\ddot{q}_d + F_d \dot{q} + G(q) - Y_d \theta + \alpha_1 M(q)(\eta - \alpha_1 e - \alpha_2 e_f) \\ & + \alpha_2 M(q)(-\alpha_3 e_f + \alpha_2 e) \\ & - V_m(q, \eta)(\dot{q}_d + \alpha_1 e + \alpha_2 e_f) + V_m(q, \dot{q}_d)\dot{q}_d \\ & + V_m(q, \dot{q}_d)(\alpha_1 e + \alpha_2 e_f) \\ & + V_m(q, \dot{q}_d + \alpha_1 e + \alpha_2 e_f)(\alpha_1 e + \alpha_2 e_f). \end{aligned} \tag{32}$$

The closed-loop dynamics for $\eta(t)$ can be obtained by substituting the control torque input given in (25) and (31) as shown

$$\begin{aligned} M(q)\dot{\eta} = {} & -V_m(q, \dot{q})\eta - \alpha_2 k M(q)\eta + Y_d \tilde{\theta} + \chi \\ & + k K_s e_f - K_s e. \end{aligned} \tag{33}$$

Remark 2. *Based on the expression given in (32), we can utilize Properties 4 thru 6, (10), and (11) to upper bound $\chi(t)$ as follows*

$$\|\chi\| \le \zeta_0 \|x\| + \zeta_1 \|x\|^2 \tag{34}$$

where ζ_0, ζ_1 are positive scalar bounding constants, and $x(t)$ was defined in (24). Based on the bound given by (34), the control gain k introduced in (12) and (25) is selected to facilitate the stability analysis as follows

$$K = \frac{1}{m_1}(1 + k_n(\zeta_0^2 + \zeta_1^2)) \tag{35}$$

where m_1 was defined in (2), and $k_n \in \Re^1$ denotes a positive nonlinear damping gain.

3.4 Stability Analysis

Theorem 1. *The proposed controller yields semi-global asymptomatic link position tracking control in the sense that*

$$\lim_{t \to \infty} e(t) = 0$$

provided the robot manipulator is initially at reset (i.e. $\dot{q}(0) = 0$), the initial desired velocity is selected as $\dot{q}_d(0) = 0$) the control gain k of (12) and (25), and the filter constant β of (15) are selected sufficiently large and sufficiently small, respectively, to satisfy the following conditions

$$\frac{\lambda_1\left(\min\{\alpha_1 \lambda_{\min}\{K_s\}, \alpha_3 \lambda_{\min}\{K_s\}, \alpha_2\} - \frac{1}{\alpha_2 k_n} - \beta\zeta_2\right)}{\frac{1}{\alpha_2 k_n} + \beta\zeta_3} > \gamma_0 \tag{36}$$

$$\beta < \frac{1}{2(\zeta_2 + \zeta_3)} \tag{37}$$

where $\gamma_0 \in \Re^1$ *is defined as*

$$\gamma_0 = \frac{1}{2}\left(e(0)^T K_s e(0) + e_f^T(0) K_s e_f(0) + \eta^T(0) M(q(0))\eta(0) + \tilde{\theta}^T(0) P^{-1}(0)\,\tilde{\theta}(0) \right),$$

$\lambda_1 \in \Re^1$ *is defined as*

$$\lambda_1 = \frac{1}{2}\min\left[\lambda_{\min}\{K_s\},\, m_1,\, \lambda_{\min}\{P^{-1}(0)\}\right], \tag{38}$$

and $\lambda_{\min}\{\cdot\}$ *denotes the minimum eigenvalue of a matrix.*

Proof: In order to prove Theorem 1, we define the following non-negative function

$$V = \frac{1}{2}e^T K_s e + \frac{1}{2}e_f^T K_s e_f + \frac{1}{2}\eta^T M(q)\eta + \frac{1}{2}\tilde{\theta}^T P^{-1}\tilde{\theta}. \tag{39}$$

Based on the structure of (39), we can utilize *Property 1* and (28) to lower bound $V(t)$ *as follows*

$$\lambda_1 \|z\|^2 \le V(z(t), t) \tag{40}$$

where λ_1 was defined in (38), and $z(t) \in \Re^{3n+p}$ is defined as

$$z(t) = \begin{bmatrix} e^T & e_f^T & \eta^T & \tilde{\theta}^T \end{bmatrix}^T. \tag{41}$$

After taking the time derivative of (39), utilizing *Property 2*, and making substitutions for (11), (12), (21), (26), (28) and (33), we have

$$\dot{V} = -\alpha_1 e^T K_s e - \alpha_3 e_f^T K_s e_f - \alpha_2 k \eta^T M(q)\eta + \eta^T \chi - \frac{1}{2}\|Y_{df}\tilde{\theta}\|^2 - \tilde{\theta}^T Y_{df}^T \Omega \tag{42}$$

We can now utilize *Property 1*, the triangular inequality (23), (34), and (35) to upper bound (42) as follows

$$\begin{aligned}\dot{V} \le &-\alpha_1 \lambda_{\min}\{K_s\}\|e\|^2 - \alpha_3 \lambda_{\min}\{K_s\}\|e_f\|^2 - \alpha_2\|\eta\|^2 \\ &- \frac{1}{2}\left\|Y_{df}\tilde{\theta}\right\|^2 + \beta\zeta_2\left(\left\|Y_{df}\tilde{\theta}\right\|^2 + \|x\|^2\right) \\ &+ \beta\zeta_3\left(\left\|Y_{df}\tilde{\theta}\right\|^2 + \|x\|^4\right) \\ &+ [\zeta_0\|\eta\|\,\|x\| - k_n\alpha_2\zeta_0^2\|\eta\|^2 + \zeta_1\|\eta\|\,\|x\|^2 \\ &- k_n\alpha_2\zeta_1^2\|\eta\|^2].\end{aligned} \tag{43}$$

After applying the non-linear damping tool[22] to the bracketed terms of (43), utilizing (40), and grouping common terms, we have the following new upper bound for $\dot{V}(t)$

$$\begin{aligned}\dot{V} \le -&\left\{ \min\{\alpha_1\lambda_{\min}\{K_s\}, \alpha_3\lambda_{\min}\{K_s\}, \alpha_2\} - \frac{1}{\alpha_2 k_n} - \beta\zeta_2 - \left(\frac{1}{\alpha_2 k_n} + \beta\zeta_3\right)\frac{V(z(t),t)}{\lambda_1}\right\}\|x\|^2 \\ &- \left(\tfrac{1}{2} - \beta\zeta_2 - \beta\zeta_3\right)\left\|Y_{df}\tilde{\theta}\right\|^2.\end{aligned} \tag{44}$$

If the conditions given in (37) is satisfied along with the

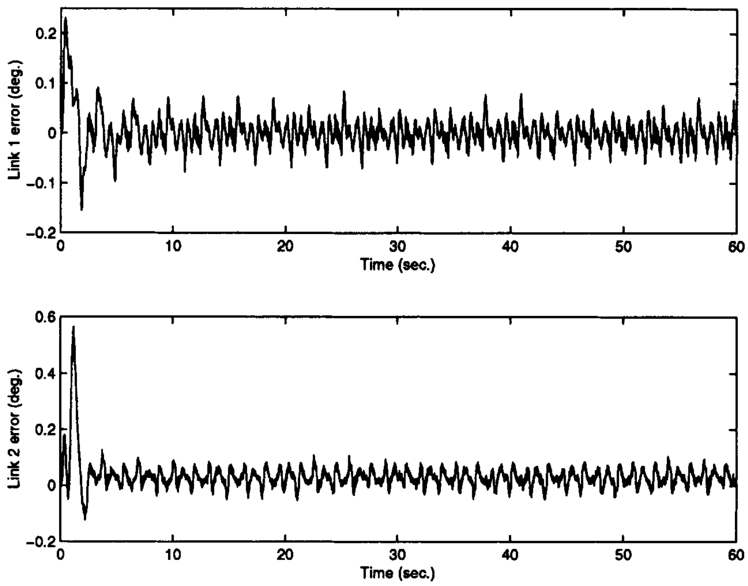

Fig. 1. Link position tracking error with the composite update based controller (CL)

following condition

$$\min\{\alpha_1\lambda_{\min}\{K_s\},\,\alpha_3\lambda_{\min}\{K_s\},\,\alpha_2\}\geq\frac{1}{\alpha_2 k_n}+\beta\zeta_2$$

$$+\left(\frac{1}{\alpha_2 k_n}+\beta\zeta_3\right)\frac{V(z(t),t)}{\lambda_1}, \tag{45}$$

then we can use (44) to upper bound $\dot{V}(t)$ as follows

$$\dot{V}\leq-\lambda_3\|x\|^2 \tag{46}$$

where $\lambda_3\in\Re^1$ is some positive bounding constant, and $x(t)$ was defined in (24). From (46), we have that $\dot{V}(t)\leq 0$; therefore,

$$V(z(t),t)\leq V(z(0),0)\quad\forall t\geq 0 \tag{47}$$

where $z(t)$ was defined in (41). We can now use (47), (46), (45), and (39) to obtain the sufficient condition given by (36) (for more details on the above semi-global proof, the reader is referred to references 13 or 20).

Based on (47), (40), and (41), we can conclude that $V(t)\in\mathfrak{L}_\infty$ and that $e(t)$, $e_f(t)$, $\eta(t)\in\mathfrak{L}_\infty^n$, and $\tilde{\theta}(t)\in\mathfrak{L}_\infty^p$; hence, from (8), (13), (11), (12), (33), (29), (17), (19) and the fact that desired trajectory is assumed to be bounded, $q(t)$, $\dot{q}(t)$, $\dot{e}(t)$, $\dot{e}_f(t)$, $\dot{\eta}(t)$, $w(t)\in\mathfrak{L}_\infty^n$, $\hat{\theta}(t)\in\mathfrak{L}_\infty^p$, and $Y_f(q,\dot{q})$, $Y_{df}(q_d,\dot{q}_d)\in\mathfrak{L}_\infty^{n\times p}$. Next, from (16) and (20), we have that $\tau_f(t)$, $\varepsilon(t)\in\mathfrak{L}_\infty^n$. Due to the fact that $P^{-1}(t)$ is positive definite for all time, we can conclude that $P(t)$ is positive definite for all time. Since $P(t)$ is positive definite and $\dot{P}(t)\leq 0$, as given by (27), we have that $P(t)\in\mathfrak{L}_\infty^{p\times p}$.

Standard signal chasing arguments can now be used to show that all signals in the controller and the robot manipulator dynamics remain bounded during closed-loop operation. Utilizing the above information, we can state from the definition of (24) that $x(t)$, $\dot{x}(t)\in\mathfrak{L}_\infty^{3n}$, which is a sufficient condition for $x(t)$ to be uniformly continuous. Furthermore, from (46), we have that $x(t)$ is square integrable. We can now apply Barbalat's Lemma[21] to conclude that $\lim_{t\to\infty}x(t)=0$; therefore, from (24), we have that $\lim_{t\to\infty}e(t)=0$.□

Table I. Comparison of Link Position Tracking Control Performance

	CR	*GR*
$\lvert e_{1ss}\rvert_{\max}$ (deg.)	0.08	0.1
$\lvert e_{2ss}\rvert_{\max}$ (deg.)	0.1	0.1
$\int_0^{40} e_1^2(t)$	15.2503	63.1422
$\int_0^{40} e_2^2(t)$	27.3991	41.0284
$\lvert\tau_1\rvert_{\max}$ (Nm)	50	42
$\lvert\tau_2\rvert_{\max}$ (Nm)	11	8.4

$\bullet_{ss}$ and $\lvert\bullet\rvert_{\max}$ are used to denote the steady state and maximum values of the parameters, respectively

4 EXPERIMENTAL VERIFICATION

The proposed compositive adaptative link position tracking controller was implemented on an Integrated Motion Inc. 2-link, revolute, direct-drive robot manipulator with the following dynamics[23]

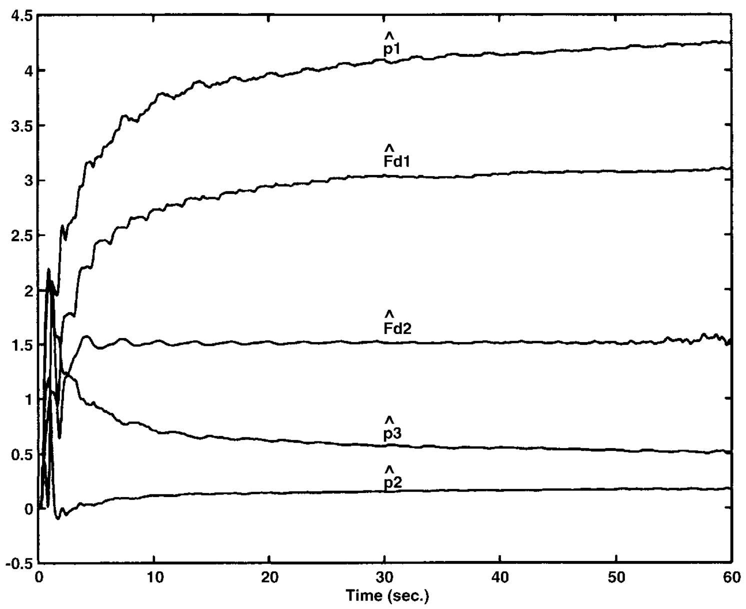

Fig. 2. Parameter estimates with the composite update based controller (CL), $\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$ in *kg.m*^{2}, F_{d1} and F_{d2} in *Nm.sec*

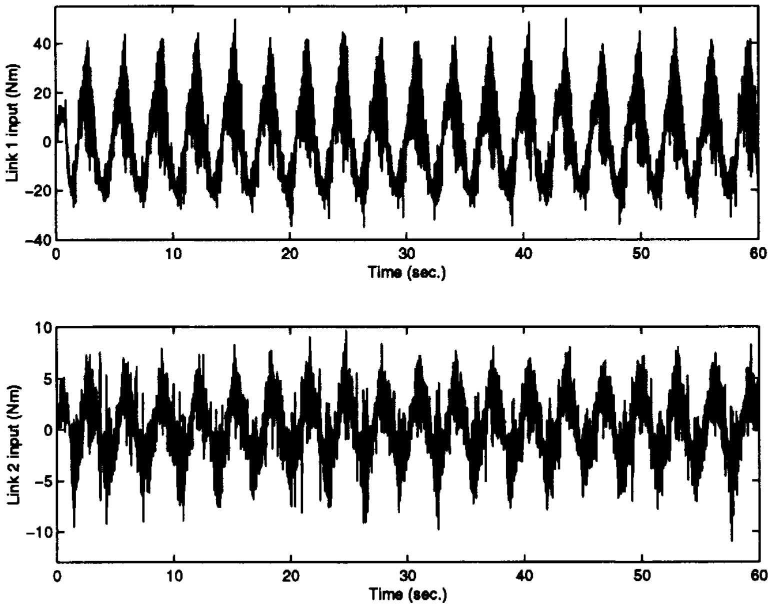

Fig. 3. Control torque inputs with the composite update based controller (CL)

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}p_1+2p_3cos(q_2) & p_2+p_3cos(q_2)\\ p_2+p_3cos(q_2) & p_2\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}-p_3\sin(q_2)\dot{q}_2 & -p_3\sin(q_2)(\dot{q}_1+\dot{q}_2)\\ p_3\sin(q_2)\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}+\begin{bmatrix}F_{d1} & 0\\ 0 & F_{d2}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} \quad (48)$$

where $p_1=3.31$ kg·m^2, $p_2=0.116$ kg·m^2, $p_3=0.16$ kg·m^2, $F_{d1}=5.3$ Nm·sec, and $F_{d2}=1.1$ Nm·sec. For this dynamic model the unknown parameter vector given in (8) is defined as follows

$$\theta=[p_1, p_2, p_3, F_{d1}, F_{d2}]^T \quad (49)$$

The links of the manipulator are actuated by switched-reluctance motors which are controlled through NSK torque controlled amplifiers. A Pentium 266 MHz PC operating under QNX hosts the control algorithm, which was implemented via Qmotor 2.0, an in-house graphical user-interface, to facilitate real-time graphing, data logging, and the ability to adjust control gains without recompiling the program (for further information on Qmotor 2.0 the reader is referred to reference 24). Data acquisition and control implementation were performed at a frequency of 2.0kHz using the Quanser MultiQ I/O board.

In order to illustrate the viability and improved performance of the composite adaptive control design, we

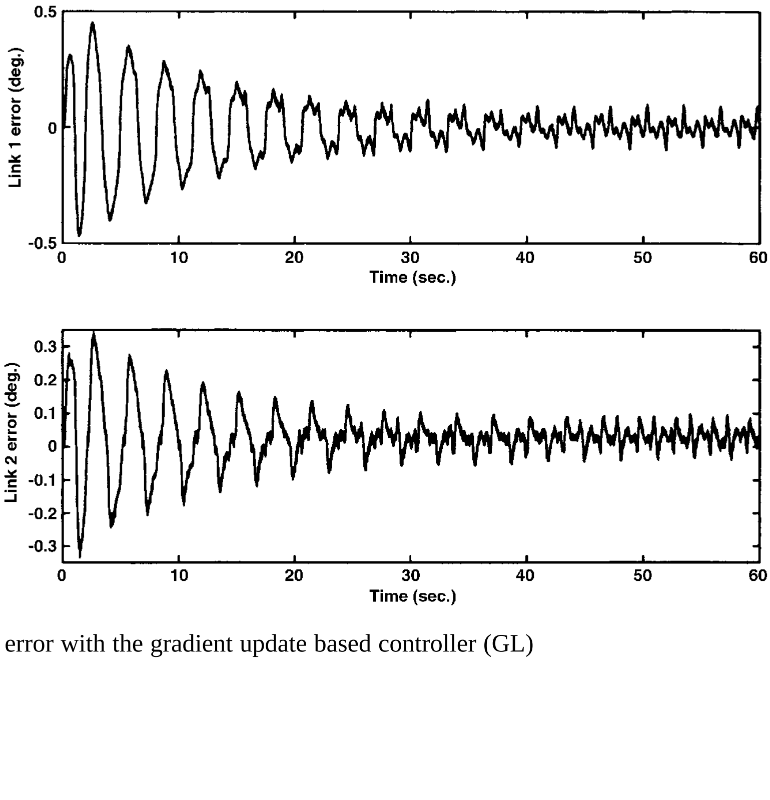

Fig. 4. Link position tracking error with the gradient update based controller (GL)

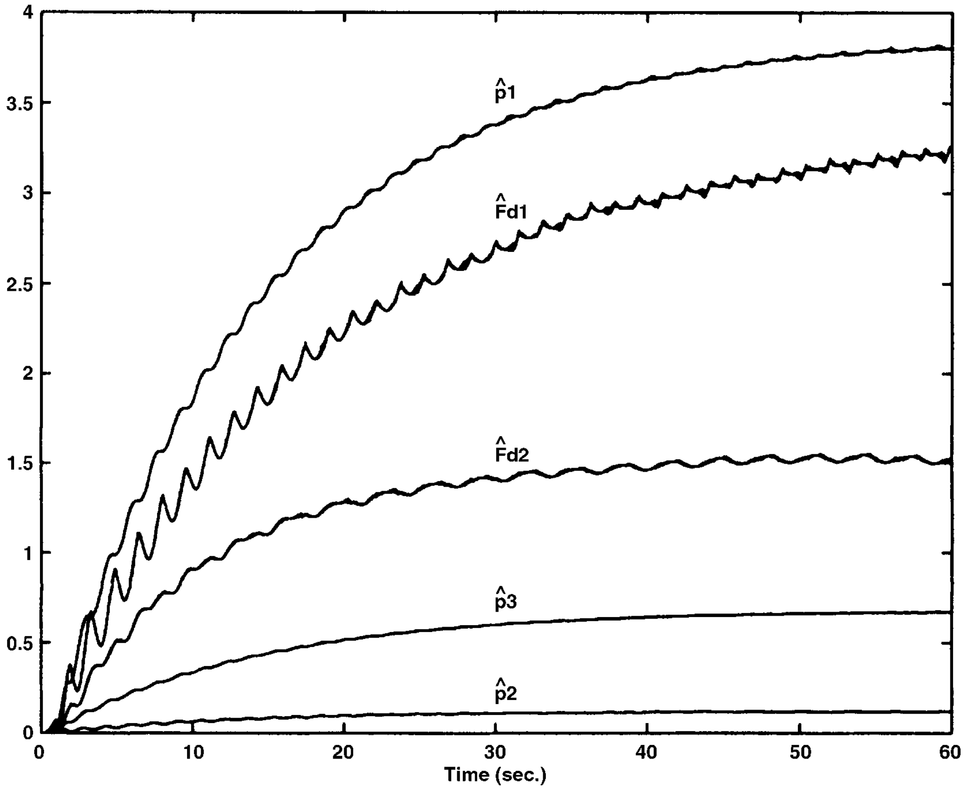

Fig. 5. Parameter estimates with the gradient update based controller (GL), $\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$ in *kg.m*^{2}, F_{d1} and F_{d2} in *Nm.sec*

implemented the proposed control torque input given by (25) and (29) using: (i) the proposed composite update law given in (26) and (27) (hereinafter denoted as *CR*), and (ii) a standard gradient update law (hereinafter denoted as *GR*) as given by

$$\hat{\theta} = \Gamma \int_0^t Y_d^T (e+e_f)dt + \Gamma Y_d^T e - \Gamma \int_0^t \frac{d}{dt}\{Y_d^T\} e\, dt \quad (50)$$

where $\Gamma \in \Re^{p \times p}$ is a constant, positive-definite, diagonal,

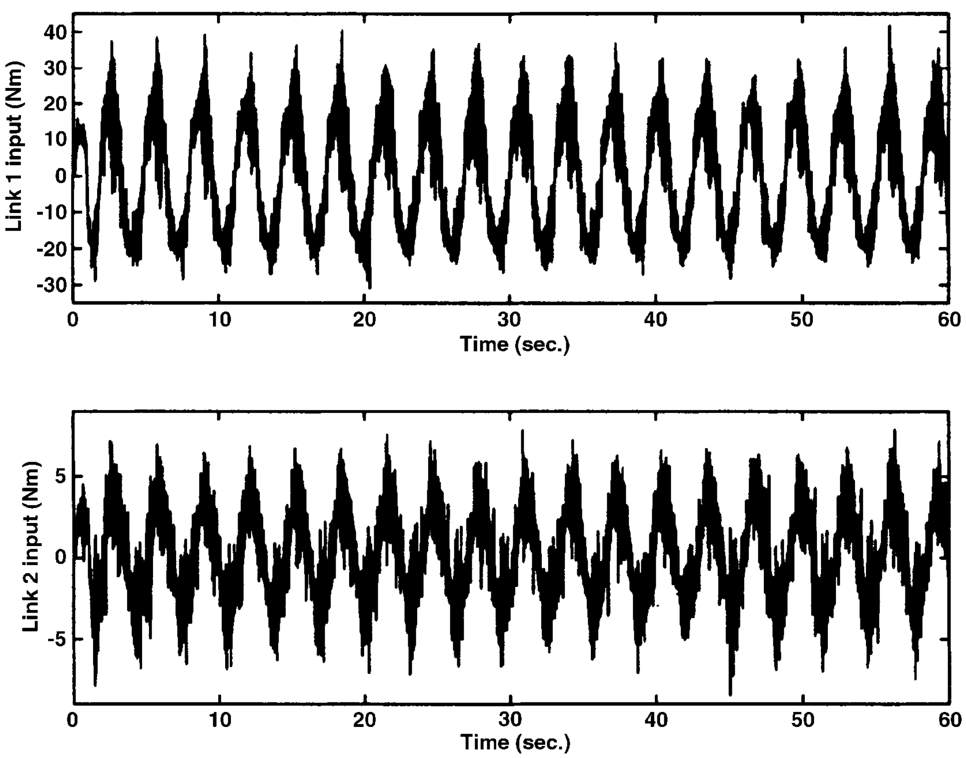

Fig. 6. Control torque input with the gradient update based controller (GL)

adaptation gain matrix. Note that the integral form of both the *CR* and *GR* adaptation laws were computed via a standard trapezoidal algorithm.

For both experiments, the desired position trajectories for links 1 and 2 were selected as follows

$$q_{d1} + 0.8\sin(2t)\left(1 - e^{-0.3t^3}\right)\text{ rad}$$
$$q_{d2} = 0.8\sin(2t)\left(1 - e^{-0.3t^3}\right)\text{ rad} \tag{51}$$

while the actual link positions, velocities, and parameter estimates were initialized to zero. The controller was tuned with the adaptation gains and all of the initial adaptive estimates set to zero. The feedback gains and the nonlinear filter gains were adjusted until the link position tracking error could not be further decreased. We then adjusted the filter gain β and the initial conditions of the time varying control gain matrix $P(t)$ to allow the parameter estimation to reduce the link position tracking error (see Figure 1 and Table I). After the best tracking performance for the *CR* controller was obtained, the final gain values were recorded as follows

$$k = diag\ \{36.25, 30.0\},\ K_s = diag\ \{85.5, 25.53\},$$
$$\beta = 10, \quad \alpha_1 = 10.25, \quad \alpha_2 = 12.5, \quad \alpha_3 = 22.25,$$
$$P(0) = \begin{bmatrix} 1.1034 & 0.0952 & 0.0165 & 0.21 & 0.01 \\ 0.0952 & 0.5517 & 0.085 & 0.015 & 0.315 \\ 0.0165 & 0.085 & 0.66136 & 0.085 & 0.003 \\ 0.21 & 0.015 & 0.085 & 0.60658 & 0.085 \\ 0.01 & 0.315 & 0.003 & 0.085 & 7.54 \end{bmatrix}.$$

After replacing the composite update rule with (50) and tuning the adaptation gains of the *GR* controller until the best tracking control performance was obtained, the control and adaptation gains were recorded as shown below

$$k = diag\{32.25, 28.0\}, \quad K_s = diag\{85.5, 25.53\},$$
$$\beta = 10, \quad \alpha_1 = 10.25, \quad \alpha_2 = 12.5, \quad \alpha_3 = 22.25,$$
$$\Gamma = diag\{22.2, 0.8, 1.255, 100.6, 40.2\}.$$

Table I provides a comparison of the absolute value of the maximum steady state error and maximum control torque input, and the integral of the square of the link position tracking errors. Figures 1, 2, and 3 illustrate the link position tracking, parameter updates, and computed control torques for the *CR* controller, respectively, and Figure 4, 5, and 6 illustrate the link position tracking, parameter updates, and computed control torques for the *GR* controller, respectively. From Table I and Figures 1–6 it is evident that at the expense of slightly larger control torque inputs, the proposed *CR* controller yields faster parameter adaptation and improved transient performance results.

5 CONCLUSION

In this paper, we have provided a solution to the composite adaptive output feedback tracking control problem for robotic manipulators. The controller provided semi-global asymptotic link position tracking performance. Experimental results illustrated that the proposed controller provides improved link position tracking error transient performance and faster parameter estimate convergence in comparison to the same controller using a gradient update law. In the hopes of achieving even faster transient response, future work will concentrate on the development of an OFB adaptive controller with an update law which is solely driven by the prediction error as previously done for the full-state feedback case in references 2 and 8.

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