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Tracking Control of Robot Manipulators with Bounded Torque Inputs*

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SUMMARY

This paper examines the problem of link position tracking control of robot manipulators with bounded torque inputs. An adaptive, full-state feedback controller and an exact model knowledge, output feedback controller are designed to produce semi-global asymptotic link position tracking errors. Simulation results are provided to validate the theoretical concepts, and a comparative analysis demonstrates the benefits of the proposed controllers.

KEYWORDS: Tracking control; Robots; Bounded torque input; Feedback controllers.

1. INTRODUCTION

The standard tracking control problem for torque input, rigid-link robot manipulators has been the subject of much research during the last twenty years. A common assumption in most of the previously designed controllers is that the robot's link actuators are able to generate the necessary level of torque inputs. In practice, robotic actuators have physical constraints that limit the amplitude of the available torques. Possible problems that could result from the implementation of controllers based on the unlimited available torque assumption include: (i) degraded link position tracking, and (ii) thermal or mechanical failure. With this conundrum in mind, several researchers have proposed robot controllers which are amplitude limited.[1–6]

Most of the previous work on the design of robot controllers with bounded torque inputs has been targeted at the setpoint control problem. For example, in reference 6, Santibáñez and Kelly proposed a controller composed of a saturated proportional derivative (PD) feedback loop plus feedforward gravity compensation which provided global asymptotic regulation. In reference 5, the same authors explicitly characterized a general class of regulations that solve the same control problem as in reference 6. Loria *et al.*[3] provided an output feedback† (OFB) extension of the full-state feedback (FSFB) control design given in reference 5. In references 1 and 2, Colbaugh *et al*. designed bounded full-state and output feedback controllers that achieve global asymptotic regulation while compensating for robot uncertainties. From a review of the literature, it seems that the more general tracking control problem has received less attention. To the best of our knowledge, the only work that addresses the tracking problem with bounded torque inputs is found in reference 4. In this work, Loria and Nijmeijer designed an exact model knowledge OFB controller which yielded semi-global asymptotic link position tracking.

In this paper, we consider the link position tracking control problem of robot manipulators with bounded torque inputs under either the obstacles of parametric uncertainty or output measurements. Specifically, in Section 3, we develop: (i) an adaptive FSFB controller to compensate for the unknown, constant mechanical parameters and (ii) an exact model knowledge OFB controller that eliminates the need for velocity measurements by incorporating a "saturated" version of the filter proposed in references 7 and 8. Both controllers are shown to produce semi-global asymptotic link position tracking while maintaining the torque inputs at prescribed amplitudes. In addition, we explicitly determine the sufficient conditions on the control gains which ensure semi-global asymptotic link position tracking. In Section 4, we provide simulation results as verification of the proposed controllers. In Section 5, we demonstrate the practical advantages of our proposed controller through comparative simulations. Specifically, we compare the performance of standard robot controllers to the proposed controllers. This comparison illustrates the fact that with equal gain values and initial conditions our controller achieves similar tracking performance with less control energy required. Concluding remarks are presented in Section 6.

2. ROBOT MODEL

The system model for a rigid n-link, serially connected, direct-drive robot is assumed to be of the following form[9]

$$M(q)\ddot{q}+V_m(q,\dot{q})\dot{q}+G(q)+F_d\dot{q}=\tau \qquad (1)$$

where $q(t), \dot{q}(t), \ddot{q}(t) \in \Re^n$ denote the link position, velocity, and acceleration vectors, respectively, $M(q) \in \Re^{n\times n}$ represents the link inertia matrix, $V_m(q, \dot{q}) \in \Re^{n\times n}$ represents centripetal-Coriolis matrix, $G(q) \in \Re^n$ represents gravity effects, F_d is the constant, viscous friction coefficent matrix, and $\tau(t) \in \Re^n$ represents the torque input vector.

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† Here, we use the term output feedback to mean that only link position measurements are available while the term full-state feedback is used to mean that both link position and velocity are available for measurement.

The dynamic equation of (1) has the following properties[9] that will be used later in the controller development and analysis

Property 1: The inertia matrix $M(q)$ is symmetric and positive definite, and satisfies the following inequalities

$$m_1\|\xi\|^2 \le \xi^T M(q)\xi \le m_2\|\xi\|^2 \quad \forall \xi \in \Re^n \tag{2}$$

where m_1, m_2 are known positive constants, and $\|\cdot\|$ is the standard Euclidean norm.

Property 2: The inertia and centripetal-Coriolis matrices satisfy the following skew symmetric relationship

$$\xi^T\left(\frac{1}{2}\dot{M}(q) - V_m(q,\dot{q})\right)\xi = 0 \quad \forall \xi \in \Re^n \tag{3}$$

where $\dot{M}(q)$ is the time derivative of the inertia matrix.

Property 3: The dynamic equation of (1) can be linear parameterized as

$$Y_d(q_d,\dot{q}_d,\ddot{q}_d)\theta = M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F_d\dot{q}_d \tag{4}$$

where $\theta \in \Re^p$ contains the constant system parameters, and the *desired* regression matrix $Y_d(q_d,\dot{q}_d,\ddot{q}_d) \in \Re^{n\times p}$ contains known functions of the desired link position, velocity, and acceleration, $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t) \in \Re^n$, respectively.

Property 4: The centripetal-Coriolis, gravity, and friction terms in (1) can be bounded in the following manner

$$\|V_m(q,\dot{q})\| \le \zeta_c\|\dot{q}\|, \quad \|G(q)\| \le \zeta_g, \quad \|F_d\| \le \zeta_f \tag{5}$$

where ζ_c, ζ_g, and ζ_f are positive bounding constants.

Property 5: The centripetal-Coriolis matrix satisfies the following relationship

$$V_m(q,\xi)\nu = V_m(q,\nu)\xi \quad \forall \xi, \nu \in \Re^n. \tag{6}$$

3. CONTROL FORMULATION

The control objective is to design *amplitude-limited*, link position *tracking* controllers for the aforementioned robot model. Two controllers will be described in the subsequent sections: (i) an adaptive FSFB controller and (ii) an exact model knowledge OFB controller. We quantify the control objective by defining the link position tracking error $e(t) \in \Re^n$ as follows

$$e = q_d - q \tag{7}$$

where $q_d(t)$ was defined in property 3. We assume that $q_d(t)$ and its first two time derivatives are bounded time functions such that

$$\|q_d(t)\| \le \zeta_{dp}, \quad \|\dot{q}_d(t)\| \le \zeta_{dv}, \quad \|\ddot{q}_d(t)\| \le \zeta_{da} \tag{8}$$

where ζ_{dp}, ζ_{dv}, and ζ_{da} are positive bounding constants. We also assume that each of the constant system parameters defined in (4) can be lower and upper bounded as follows

$$\underline{\theta}_i < \theta_i < \overline{\theta}_i \tag{9}$$

where θ_i denotes the i-th component of the vector θ, and $\underline{\theta}_i$, $\overline{\theta}_i$ denote known, constant bounds for each unknown parameter.

Remark 1 *To aid the subsequent control design and analysis, we define the vector* $Tanh(\cdot) \in \Re^n$ *and the matrix* $Cosh(\cdot) \in \Re^{n\times n}$ *as follows*

$$Tanh(\xi) = [\tanh(\xi_1), \ldots, \tanh(\xi_n)]^T \tag{10}$$

and

$$Cosh(\xi) = diag\{\cosh(\xi_1), \ldots, \cosh(\xi_n)\} \tag{11}$$

where $\xi = [\xi_1, \ldots, \xi_n]^T \in \Re^n$. *Based on the definition of (10), it can be shown that the following inequalities hold for* $\forall \xi \in \Re^n$

$$\|\xi\| \ge \|Tanh(\xi)\|,$$

$$\|\xi\|^2 \ge \sum_{i=1}^{n} \ln(\cosh(\xi_i)) \ge \ln(\cosh(\|\xi\|)) \ge \frac{1}{2}\tanh^2(\|\xi\|),$$

$$\xi^T\, Tanh(\xi) \ge Tanh^T(\xi)\, Tanh(\xi) = \|Tanh(\xi)\|^2 \ge \tanh^2(\|\xi\|),$$

$$\|\xi\| + 1 \ge \frac{\|\xi\|}{\tanh(\|\xi\|)}. \tag{12}$$

3.1 Adaptive FSFB controller

To facilitate the design of the adaptive FSFB controller, we define the filtered tracking error $r(t) \in \Re^n$ as follows[10]

$$r = \dot{e} + \alpha Tanh(e) \tag{13}$$

where $\dot{e}(t)$ is the time derivative of the position tracking error defined in (7), $\alpha \in \Re^{n\times n}$ is a constant, diagonal, positive-definite control gain matrix and $Tanh(\cdot)$ was defined in (10). Since the control objective in this subsection is to be met under the constraint of parametric uncertainty, the controller will contain an adaptation law to estimate the unknown system parameters; hence, the difference between the actual and estimated parameters will be denoted as

$$\tilde{\theta} = \theta - \hat{\theta} \tag{14}$$

where $\tilde{\theta}(t) \in \Re^p$ represents the parameter estimation error vector, and $\hat{\theta}(t) \in \Re^p$ is the dynamic estimate of θ defined in (4).

We begin the development of the controller by first ascertaining the open-loop dynamics of $r(t)$. To this end, we take the time derivative of (13), multiply both sides of the equation by $M(q)$, and use (1) in the resulting equation to yield

$$M(q)\dot{r} = -V_m(q,\dot{q})r + Y\theta - \tau \tag{15}$$

where the linear parameterization $Y(e,r,t)\theta$ is defined as

$$Y(e,r,t)\theta = M(q)(\ddot{q}_d + \alpha Cosh^{-2}(e)\dot{e}) + V_m(q,\dot{q})(\dot{q}_d + \alpha Tanh(e)) + G(q) + F_d\dot{q}. \tag{16}$$

To obtain the final expression for the open-loop dynamics of $r(t)$, the term $Y_d\theta$, defined in (4) is added and subtracted to the right-hand side of (15) to produce

$$M(q)\dot{r} = -V_m(q,\dot{q})r + Y_d\theta - \tau + \tilde{Y} \tag{17}$$

where $\tilde{Y}(e,r,t) \in \Re^n$ denotes the mismatch between the actual and desired linear parameterization, and is defined as

$$\tilde{Y} = Y\theta - Y_d\theta. \quad (18)$$

Remark 2 *By exploiting Properties 1, 4, and 5 of the robot dynamics, the first inequality of (12) and the definition of (13), is easy to show by using the results given in reference 11 that*

$$\|\tilde{Y}\| \leq \zeta_0\|e\| + \zeta_1\|r\| \quad (19)$$

where ζ_0 and ζ_1 are some positive bounding constants that depend on the physical properties of the robot (e.g. link masses, link lengths, friction coefficients, etc.), the desired trajectory bounds given by (8), and the control gain matrix α defined in (13).

Based on the form of (17) and on the subsequent stability analysis, we design the torque input, adaptive FSFB controller as follows

$$\tau = Y_d\hat{\theta} + K_p Tanh(e) + K_v Tanh(r) \quad (20)$$

where K_p, $K_v \in \Re^{n\times n}$ are constant, diagonal, positive-definite, control gain matrices. The parameter estimate vector $\hat{\theta}(t)$ is generated on-line according to the following adaptation algorithm

$$\dot{\hat{\theta}}(t) = \mathrm{Proj}(\Gamma Y_d^T r) \quad (21)$$

where $\Gamma \in \Re^{p\times p}$ is a constant, diagonal, positive-definite, adaptation gain matrix, and $\mathrm{Proj}(\cdot)$ denotes a projection algorithm utilized to guarantee $\hat{\theta}(t)$ stays within the known region of θ as prescribed by (9) (e.g., see reference 12 for details). After substituting (20) into (17), we can form the closed-loop dynamics of $r(t)$ as given below

$$M(q)\dot{r} = -V_m(q, \dot{q})r + Y_d\tilde{\theta} - K_p\, Tanh(e) - K_v\, Tanh(r) + \tilde{Y} \quad (22)$$

where $\tilde{\theta}(t)$ was defined in (14).

Theorem 1. *Given the robot dynamics of (1), the adaptive FSFB controller composed of (20) and (21) ensures semi-global asymptotic link position tracking in the sense that*

$$\lim_{t\to\infty} e(t) = 0 \quad (23)$$

provided the control gain matrix K_v defined in (20) satisfies the following sufficient condition

$$\lambda_{\min}\{K_v\} > \zeta_1(\cosh^{-1}(\exp(\tfrac{\lambda_2}{\lambda_1}\|z(0)\|^2)) + 1)^2 + \frac{\zeta_0^2}{4\lambda_{\min}\{K_p\alpha\}}(\cosh^{-1}(\exp(\tfrac{\lambda_2}{\lambda_1}\|z(0)\|^2)) + 1)^4 \quad (24)$$

where $z(t) \in \Re^{3n}$ is defined as

$$z = [e^T\ r^T\ \tilde{\theta}^T]^T, \quad (25)$$

and λ_1, λ_2 are positive scalar constants given by

$$\lambda_1 = \min\left\{\lambda_{\min}\{K_p\}, \frac{m_1}{2}\right\},$$

$$\lambda_2 = \max\left\{\lambda_{\max}\{K_p\}, \frac{m_2}{2}, \frac{1}{2}\lambda_{\max}\{\Gamma^{-1}\}\right\} \quad (26)$$

with $\lambda_{\min}\{\cdot\}$ and $\lambda_{\max}\{\cdot\}$ denoting the minimum and maximum eigenvalues of a matrix, respectively, and m_1, m_2 being defined in (2).

Proof: To prove Theorem 1, we first define the following non-negative function*

$$V(t) = \frac{1}{2}r^T(t)M(q)r(t) + \sum_{i=1}^{n} k_{pi}\ln(\cosh(e_i(t))) + \frac{1}{2}\tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t) \quad (27)$$

where k_{pi} is the i-th diagonal element of the control gain matrix K_p defined in (20), and $e_i(t)$ is the i-th element of the vector $e(t)$ defined in (7). Based on (12), it is easy to show that $V(t)$ can be bounded as

$$\frac{1}{2}\lambda_1\tanh^2(\|y\|) \leq \lambda_1\ln(\cosh(\|y\|)) \leq V \leq \lambda_2\|z\|^2 \quad (28)$$

where $y(t) \in \Re^{2n}$ is given by

$$y = [e^T\ r^T]^T, \quad (29)$$

and $z(t)$, λ_1, and λ_2 were defined in (25) and (26).

After taking the time derivative of (27), we have

$$\dot{V} = \frac{1}{2}r^T\dot{M}(q)r + r^T M(q)\dot{\Gamma} + Tanh^T(e)K_p\dot{e} - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} \quad (30)$$

where we have used the fact that $\dot{\tilde{\theta}}(t) = -\dot{\hat{\theta}}(t)$. After substituting (22) for $M(q)\dot{r}(t)$, substituting (13) for $\dot{e}(t)$, utilizing Property 2, and cancelling common terms, we obtain

$$\dot{V} = -Tanh^T(e)K_p\alpha Tanh(e) - r^T K_v Tanh(r) + r^T\tilde{Y} + \tilde{\theta}^T\left(Y_d^T r - \Gamma^{-1}\dot{\hat{\theta}}\right). \quad (31)$$

After substituting (21) for $\dot{\hat{\theta}}$, the last term of (31) can be shown to be less than or equal to zero for all time (see reference 12 for details); hence, the inequalities of (12) can be used to formulate the following upper bound for (31)

$$\dot{V} \leq -\lambda_{\min}\{K_p\alpha\}\tanh^2(\|e\|) - \lambda_{\min}\{K_v\}\tanh^2(\|r\|) + \|r\|\left\|\tilde{Y}\right\|. \quad (32)$$

As a sidenote, we can utilize (19) to upper bound the last term of (32) as follows

$$\|r\|\left\|\tilde{Y}\right\| \leq \zeta_0\|e\|\,\|r\| + \zeta_1\|r\|^2 = \zeta_0\frac{\|e\|}{\tanh(\|e\|)}\frac{\|r\|}{\tanh(\|r\|)}\tanh(\|e\|)\tanh(\|r\|) + \zeta_1\frac{\|r\|^2}{\tanh^2(\|r\|)}\tanh^2(\|r\|); \quad (33)$$

hence, $\dot{\mathrm{V}}(\mathrm{t})$ can be further upper bounded as

* Note that the $\ln(\cosh(\cdot))$ part of $V(t)$ in (27) is similar to that used in reference 4.

$$\dot{V} \le -x^T Q x \quad (34)$$

where $x(t) \in \Re^2$ and $Q(e, r) \in \Re^{2\times 2}$ are explicitly defined as follows

$$x = [\tanh(\|e\|) \quad \tanh(\|r\|)]^T \quad (35)$$

and

$$Q = \begin{bmatrix} \lambda_{\min}\{K_p\alpha\} & -\frac{\zeta_0}{2}\frac{\|e\|\,\|r\|}{\tanh(\|e\|)\tanh(\|r\|)} \\ -\frac{\zeta_0}{2}\frac{\|e\|\,\|r\|}{\tanh(\|e\|)\tanh(\|r\|)} & \lambda_{\min}\{K_v\} - \zeta_1\frac{\|r\|^2}{\tanh^2(\|r\|)} \end{bmatrix}. \quad (36)$$

For the matrix $Q(e, r)$ above to be positive-definite, the following conditions must be satisfied

$$\lambda_{\min}\{K_p\alpha\} > 0, \quad (37)$$

$$\lambda_{\min}\{K_v\} > \zeta_1 \frac{\|r\|^2}{\tanh^2(\|r\|)}, \quad (38)$$

and

$$\frac{\zeta_0^2}{4}\frac{\|e\|^2\|r\|^2}{\tanh^2(\|e\|)\tanh^2(\|r\|)} < \lambda_{\min}\{K_p\alpha\}\,\lambda_{\min}\{K_v\} - \lambda_{\min}\{K_p\alpha\}\,\zeta_1\,\frac{\|r\|^2}{\tanh^2(\|r\|)}. \quad (39)$$

After using (25) and the last inequality of (12), we can easily develop the following sufficient condition for (38) and (39)

$$\lambda_{\min}\{K_v\} > \zeta_1(\|y\|+1)^2 + \frac{\zeta_0^2}{4\lambda_{\min}\{K_p\alpha\}}(\|y\|+1)^4 \quad (40)$$

where $y(t)$ was defined in (29). We can now utilize the first lower bound on $V(t)$ from (28) to state that

$$\|y\| \le \cosh^{-1}\left(\exp\left(\frac{V}{\lambda_1}\right)\right); \quad (41)$$

hence, a sufficient condition for (40) can be obtained as follows

$$\lambda_{\min}\{K_v\} > \zeta_1\left(\cosh^{-1}\left(\exp\left(\frac{V}{\lambda_1}\right)\right)+1\right)^2 + \frac{\zeta_0^2}{4\lambda_{\min}\{K_p\alpha\}}\left(\cosh^{-1}\left(\exp\left(\frac{V}{\lambda_1}\right)\right)+1\right)^4. \quad (42)$$

If condition* (42) is satisfied, then we can state from (34) and (12) that

$$\dot{V} \le -\beta\|x\|^2 \le -\beta\tanh^2(\|y\|) \quad (43)$$

where β is some positive constant. From (43), we know $\dot{V}(t) \le 0$, and hence,

$$V(z(t), t) \le V(z(0), 0) \le \lambda_2\|z(0)\|^2 \quad \forall t \ge 0 \quad (44)$$

where the upper bound from (28) has been utilized. Therefore, by the use of (44), a final sufficient condition for (42) can be written as in (24).

If (24) is satisfied, a direct implication of (27), (28), and (43) is that $V(t)$ is bounded, and thus, $r(t)$, $e(t)$, and $\tilde{\theta}(t)$ are bounded. Since $e(t)$ and $r(t)$ are bounded, it follows from (13) that $\dot{e}(t)$ is bounded. A consequence of $e(t)$ and $\dot{e}(t)$ being bounded is that $q(t)$ and $\dot{q}(t)$ are bounded due to the boundedness of the desired link trajectory. From the above boundedness statements, we know from (22) that $\dot{r}(t)$ is also bounded. It is now obvious from (29) that $\dot{y}(t)$ is bounded; hence $y(t)$ is uniformly continuous (UC). Since $y(t)$ is UC, we know $\|y(t)\|$ is UC which implies $\tanh(\|y(t)\|)$ is also UC. Now, from the boundedness of $V(t)$ and the form of (43), we can state that $\tanh(\|y(t)\|)$ is square integrable; hence, application of Barbalat's Lemma[10] yields that $\lim_{t\to\infty}\tanh(\|y(t)\|)=0$ which implies that $\lim_{t\to\infty}\|y(t)\|=0$. This directly leads to the result of Theorem 1 given in (23) due to the definition of (29). □

Remark 3 *An important feature of the adaptive FSFB controller given by (20) and (21) is its applicability to the case where constraints exist on the available actuator torques. Note that the control law (20) is bounded since we can explicitly obtain the following upper bound on the norm of the required torque input*

$$\|\tau\| \le \zeta_y\zeta_p + \lambda_{\max}\{K_p\} + \lambda_{\max}\{K_v\} \quad (45)$$

where ζ_y, ζ_p are positive constants defined as follows

$$\zeta_y = \sup_t\|Y_d(t)\|, \quad \zeta_p = \sup_t\|\hat{\theta}(t)\| \quad (46)$$

which exist due to the boundedness of $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$ and the use of a projection algorithm to generate $\hat{\theta}(t)$.

3.2 Exact model knowledge OFB controller

In this subsection, we will design a controller for the dynamics of (1) under the constraint that link velocity measurements are not available for measurement; however, we now assume that exact knowledge of the system parameters is available. Since link velocity is assumed to be unmeasureable, we first introduce a filter that generates a suitable surrogate signal for the link velocity tracking error. The filter is given by the following dynamic relationship

$$\begin{cases} \dot{p} = -(k+1)Tanh(-ke+p) - (k-1)\tanh(\mathrm{e}) \\ e_f = -ke+p \end{cases} \quad (47)$$

where $e_f(t) \in \Re^n$ is the output of the filter, $p(t) \in \Re^n$ is an auxiliary variable, k is a positive scalar control gain, and $Tanh(\cdot)$ was defined in (10).

We start the controller development by obtaining the

* Note that condition (37) is automatically satisfied due to the definitions of K_p and α in (20) and (13), respectively.

dynamics of the filter output $e_f(t)$. Specifically, we take the time derivative of the bottom equation (47) and then substitute the right-hand side of the top equation of (47) for $\dot{p}(t)$ to produce

$$\dot{e}_f = -Tanh(e_f) + Tanh(e) - k\eta \tag{48}$$

where $\eta(t) \in \Re^n$ represents a filtered tracking error-like variable defined as follows

$$\eta = Tanh(e) + Tanh(e_f) + \dot{e}. \tag{49}$$

It should be noted that (49) can be rearranged as follows

$$\dot{e} = -Tanh(e) - Tanh(e_f) + \eta \tag{50}$$

to provide the dynamics of $e(t)$.

To design the exact model knowledge OFB controller, we require the dynamics of $\eta(t)$. To this end, we take the time derivative of (49) and substitute (1) for $\ddot{q}(t)$ to yield.

$$\dot{\eta} = \Omega(e, e_f, t) + Cosh^{-2}(e)\eta - kCosh^{-2}(e_f)\eta + M^{-1}(q)(V_m(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} - \tau) \tag{51}$$

where $\Omega(\cdot) \in \Re^n$ is a *measurable* auxiliary variable given by

$$\Omega = \ddot{q}_d + Cosh^{-2}(e)(-Tanh(e_f) - Tanh(e)) + Cosh^{-2}(e_f)(-Tanh(e_f) + Tanh(e)), \tag{52}$$

and $Cosh(\cdot)$ was defined in (11). Based on the form of (51) and on the subsequent stability analysis, we propose the following torque input

$$\tau = M(q)(\Omega(e, e_f, t) - k\alpha_0 Tanh(e_f) + \alpha_0 Tanh(e)) + V_m(q, \dot{q}_d)\dot{q}_d + G(q) + F_d\dot{q}_d \tag{53}$$

where k is the same control gain defined in (47), and α_0 is a positive, scalar control gain. Substituting (53) into (51) yields the following closed-loop dynamics for $\eta(t)$

$$\dot{\eta} = -kCosh^{-2}(e_f)\eta + \chi(e, e_f, \eta, t) + k\alpha_0\, Tanh(e_f) - \alpha_0\, Tanh(e) \tag{54}$$

where $\chi(\cdot) \in \Re^n$ is an *unmeasurable* auxiliary variable defined as

$$\chi = Cosh^{-2}(e)\eta + M^{-1}(V_m(q, \dot{q})\dot{q} - V_m(q, \dot{q}_d)\dot{q}_d - F_d\dot{e}). \tag{55}$$

Remark 4. *In a similar fashion described in Remark 2, it is easy to show that*

$$\|\chi\| \leq \zeta_0\|x\| + \zeta_1\|x\|^2 \tag{56}$$

where ζ_0, ζ_1 are some positive bounding constants that depended on the robot parameters and the desired motion trajectory, and $x(t) \in \Re^{3n}$ is defined as

$$x = [Tanh^T(e)\ Tanh^T(e_f)\ \eta^T]^T. \tag{57}$$

Theorem 2. *Given the robot dynamics of (1), the exact model knowledge OFB controller composed of (47) and (53) ensures a semi-global asymptotic link position tracking error in the sense that*

$$\lim_{t\to\infty} e(t) = 0 \tag{58}$$

provided the control gains α_0 and k defined in (53) and (47) satisfy the following sufficient conditions

$$\alpha_0 > \max\{1,\ \zeta_0 + \zeta_1\sqrt{4\alpha_0}\|y(0)\|\} \tag{59}$$

$$k > \alpha_0 \exp(4\alpha_0\|y(0)\|^2) \tag{60}$$

where $y(t) \in \Re^{3n}$ is now defined as follows

$$y = [e^T\ e_f^T\ \eta^T]^T, \tag{61}$$

and ζ_0, ζ_1 were defined in (56).

Proof: To begin the proof of Theorem 2, we define the following non-negative function

$$V(t) = \alpha_0 \sum_{i=1}^{n} \ln(\cosh(e_i(+))) + \alpha_0 \sum_{i=1}^{n} \ln(\cosh(e_{f_i}(t)) + \frac{1}{2}\eta^T(t)\eta(t) \tag{62}$$

where α_0 is the same control gain defined in (53), and $e_i(t)$, $e_{f_i}(t)$ are the *i*-th elements of the vectors $e(t)$ and $e_f(t)$ defined in (7) and (47), respectively. Upon utilizing the second inequality of (12), it is evident that (62) can be bounded as follows

$$\lambda_1\|x\|^2 \leq \lambda_2(\ln(\cosh(\|e\|)) + \ln(\cosh(\|e_f\|)) + \|\eta\|^2) \leq V \leq \lambda_3\|y\|^2 \tag{63}$$

where $x(t)$ and $y(t)$ were defined in (57) and (61), respectively, and

$$\lambda_1 = \frac{1}{2}\min\left\{\alpha_0, \frac{1}{2}\right\},\ \lambda_2 = \min\left\{\alpha_0, \frac{1}{2}\right\},\ \lambda_3 = \max\left\{\alpha_0, \frac{1}{2}\right\}, \tag{64}$$

After taking the time derivative of (62), substituting (48), (50), and (54) into the resulting expression, and cancelling common terms, we obtain the following upper bound on $\dot{V}(t)$

$$\dot{V} \leq -\alpha_0\|Tanh(e)\|^2 - \alpha_0\|Tanh(e_f)\|^2 - k\min\{\cosh^{-2}(|e_{f_i}|)\}\|\eta\|^2 + \|\eta\|(\zeta_0\|x\| + \zeta_1\|x\|^2) \tag{65}$$

where (56) has been used. Now, note that if

$$k\min\{\cosh^{-2}(|e_{f_i}|)\} > \alpha_0, \tag{66}$$

then an upper bound can be placed on (65) as follows

$$\dot{V} \leq -(\alpha_0 - \zeta_0 - \zeta_1\|x\|)\|x\|^2 \tag{67}$$

Hence, if α_0 is chosen such that

$$\alpha_0 > \zeta_0 + \zeta_1\|x\| \tag{68}$$

then (67) can be expressed as

$$\dot{V} \leq -\beta\|x\|^2 \tag{69}$$

where β is some positive constant.

Provided that the conditions (68) and (66) are satisfied, we can see from (69) that $\dot{V}(t)$ is non-positive; hence, we know $V(t)$ is decreasing or constant. This fact leads to the following inequality

$$V(y(t), t) \leq V(y(0), 0) \leq \lambda_3 \|y(0)\|^2 \quad \forall t \geq 0 \tag{70}$$

where the upper bound on $V(t)$ from (63) has been utilized. If

$$\alpha_0 > 1, \tag{71}$$

then we have from (63) and (70) that

$$\|x(t)\| \leq \sqrt{4V(t)} \leq \sqrt{4\alpha_0}\|y(0)\|, \tag{72}$$

and, as a consequence, a sufficient condition for (68) and (71) is given by (59). Due to the properties of the cosh $(\cdot)$ function, it is easy to see that (66) can be rewritten as

$$k > \alpha_0 \min\{\cosh^2|e_{f_i}|\} = \alpha_0 \cosh^2(\max|e_{f_i}|); \tag{73}$$

hence, a sufficient condition for (73) is as follows

$$k > \alpha_0 \cosh^2(\|e_f\|). \tag{74}$$

From (63) and (71), we can state that

$$\ln(\cosh(\|e_f\|)) \leq 2V \tag{75}$$

or, after solving for $\|e_f\|$,we have

$$\|e_f\| \leq \cosh^{-1}(\exp(2V)) \leq \cosh^{-1}(\exp(2\alpha_0\|y(0)\|^2)) \tag{76}$$

where (72) has been utilized. After utilizing (76), we can develop a sufficient condition for (74) as given in (60).

Finally, we use (12) and the definition of $x(t)$ given by (57) to formulate the final upper bound for (69) as follows

$$\dot{V} \leq -\beta\|Tanh(y)\|^2 \leq -\beta \tanh^2(\|y\|) \tag{77}$$

where $y(t)$ was defined in (61). From (77), similar arguments as those used in the proof of Theorem 1 can be followed to prove the boundedness of all closed-loop signals and the result given by (58).

Remark 5. *The exact model knowledge OFB controller proposed in this subsection also possesses the characteristic of being applicable to robot manipulator systems with limited actuator torques. From (53) and (52), an explicit bound on the required torque can be obtained as follows*

$$\|\tau\| \leq m_2(\zeta_{da} + 4 + k\alpha_0 + \alpha_0) + \zeta_c\zeta_{dv}^2 + \zeta_g + \zeta_f\zeta_{dv} \tag{78}$$

where m_2, ζ_{da}, ζ_{dv}, ζ_c, ζ_g, and ζ_f were defined in (2), (5), and (8).

4 SIMULATION RESULTS

The two controllers developed in this paper were simulated based on the 2-link, Integrated Motion Inc. (IMI) direct-drive robot manipulator which has the following dynamics[13]

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -p_3s_2\dot{q}_2 & -p_3s_2(\dot{q}_1 + \dot{q}_2) \\ p_3s_2\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \tag{79}$$

where $p_1 = 3.473$ kg · m^2, $p_2 = 0.193$ kg · m^2, $p_3 = 0.242$ kg · m^2, $f_{d1} = 5.3$ Nm.sec, $f_{d2} = 1.1$ Nm · sec, c_2 represents $\cos(q_2)$, and s_2 represents $\sin(q_2)$. Based on (79) and (4), the parameter vector θ (required in the adaptive FSFB controller) was constructed as

$$\theta = [p_1\ p_2\ p_3\ f_{d1}\ f_{d2}]^T.$$

The desired position trajectories for links 1 and 2 for all simulations in this section were selected as follows

$$q_{d1}(t) = 1.57 \sin(2t)(1 - e^{-0.05t^3}) \text{ rad}, \qquad q_{d2}(t) = 1.2 \sin(3t)(1 - e^{-0.05t^3}) \text{ rad}. \tag{80}$$

For the adaptive FSFB controller simulation, the actual link positions and velocities were initialized to zero, and the control and adaptation gains were chosen as

$$\alpha = diag\{1.0, 1.0\}, K_p = diag\{1.0, 1.0\}, K_v = diag\{1.0, 1.0\}, \Gamma = diag\{0.5, 5.0 \times 10^{-3}, 1.8 \times 10^{-2}, 8.0, 1.0\}. \tag{81}$$

The position tracking errors for links 1 and 2 are shown in Figure 1. The control torque inputs and the feedback terms of the torque inputs (i.e. the second and third terms of (20)) are depicted in Figure 2. Figure 3 illustrates the parameter estimates. For the exact model knowledge OFB controller, we set $q(0) = [2.0, -2.0]^T$ and $\dot{q}(0) = [0, 0]^T$ while the control gains were selected as

$$\alpha_0 = 1.0, k = 1.0.$$

Figure 4 shows the link position tracking errors. The control torque inputs and the feedback terms of the torque inputs (i.e. the second and third terms of (53)) are shown in Figure 5.

5 COMPARATIVE ANALYSIS

In order to demonstrate the performance of our controller over recently proposed controllers, we now compare the simulation results from the proposed adaptive FSFB controller with a standard unsaturated version given by Slotine.[10] In addition, we compare the proposed exact model

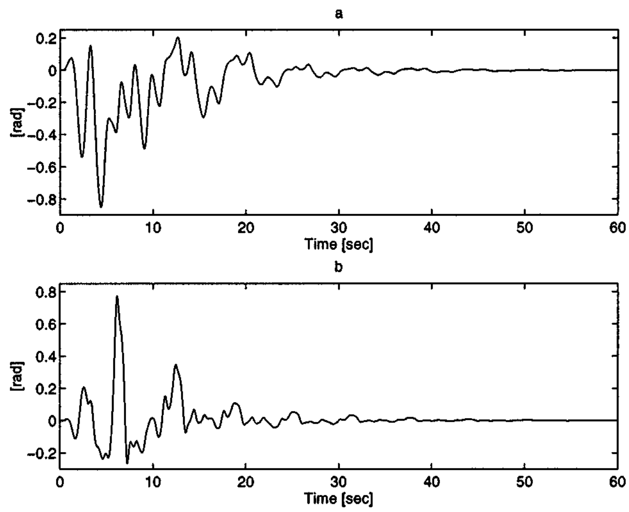

Fig. 1. Adaptive FSFB controller: (a) link 1 tracking error and (b) link 2 tracking error.

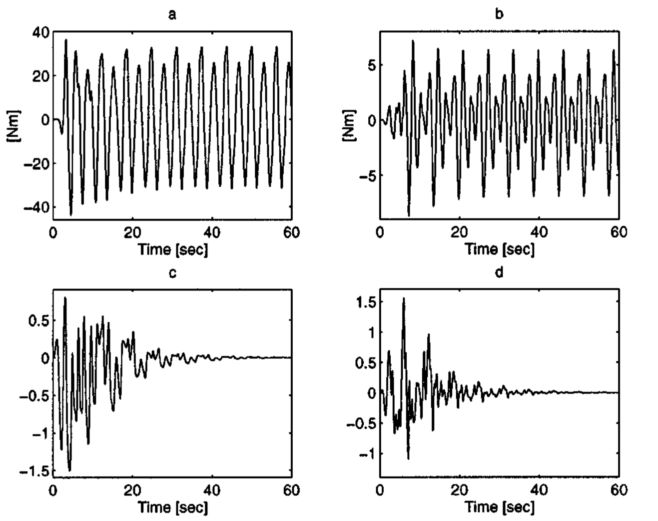

Fig. 2. Adaptive FSFB controller: (a) link 1 control torque input, (b) link 2 control torque input, (c) link 1 feedback terms, and (d) link 2 feedback terms.

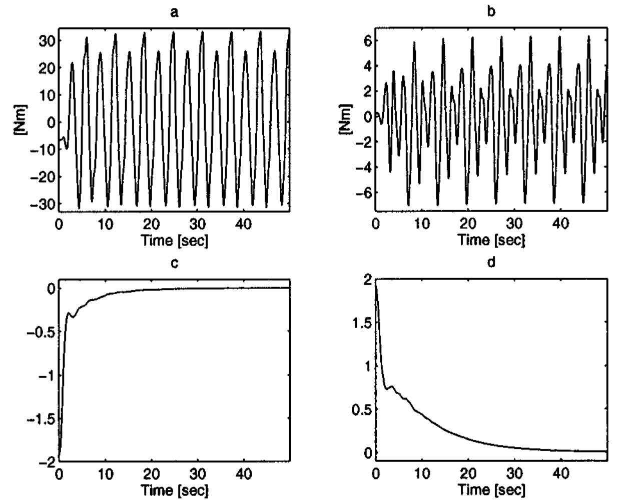

Fig. 5. Exact model knowledge OFB controller: (a) link 1 control torque input, (b) link 2 control torque input, (c) link 1 feedback terms, and (d) link 2 feedback terms.

knowledge controller with a version of the controller that does not include the saturation functions. In order to give an accurate comparison between the controllers, all gains, and initial conditions were chosen exactly the same. Specifically

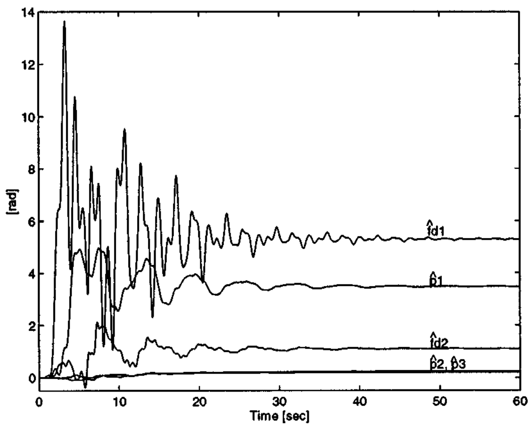

Fig. 3. Adaptive FSFB controller: parameter estimates.

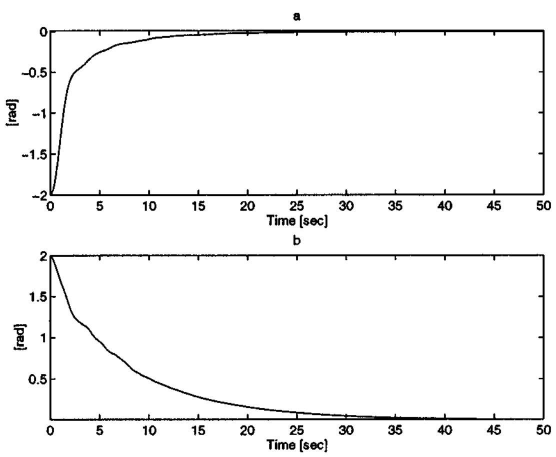

Fig. 4. Exact model knowledge OFB controller: (a) link 1 tracking error and (b) link 2 tracking error.

the gains for the adaptive and exact model knowledge controllers were selected as follows.

$$\alpha = diag\{1.0,\ 1.0\},\ K_p = \mathrm{diag}\{0.5,\ 0.5\},\ K_v = \mathrm{diag}\{0.5,\ 0.5\},$$
$$\Gamma = diag\{0.1,\ 3.0\times 10^{-3},\ 1.0\times 10^{-2},\ 0.5,\ 0.5\} \quad (82)$$

and

$$\alpha_0 = 1.0,\ k = 5.0 \quad (83)$$

respectively, with the following desired trajectory

$$q_{d1}(t) = 1.57\sin(t)\ \mathrm{rad},$$
$$q_{d2}(t) = 1.57\ \sin(t)\ \mathrm{rad}. \quad (84)$$

and initial conditions

$$q(0) = [-1.0,\ 1.0]^T \text{ and } \dot{q}(0) = [0,\ 0]^T. \quad (85)$$

From Figure 6, it is straightforward to see that the steady-state error is practically the same for both controllers. (Note that the proposed controller exhibits superior transient

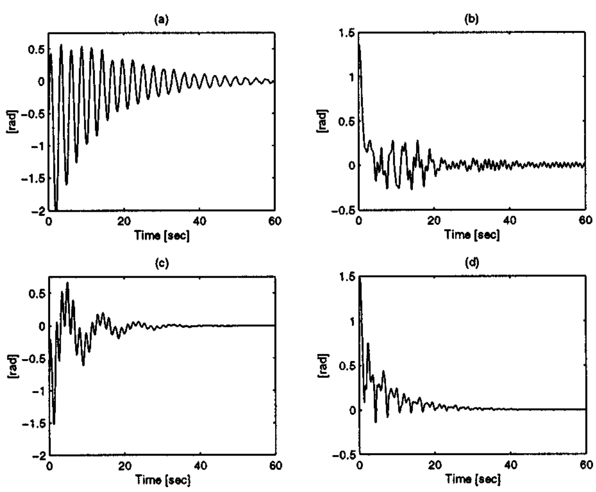

Fig. 6. Adaptive FSFB link position tracking error comparison for Slotine's controller: (a) link 1, (b) link 2, and proposed adaptive controller (c) link 1, and (d) link 2.

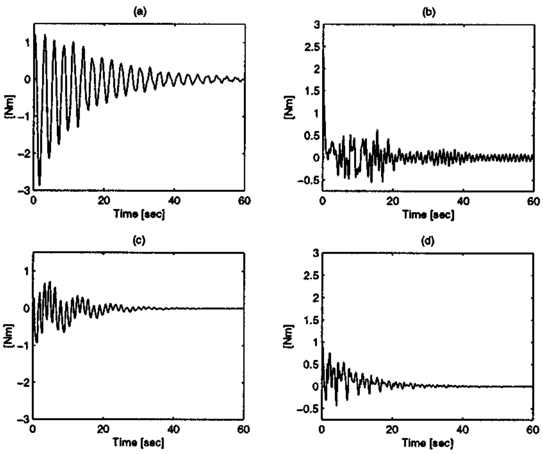

Fig. 7. Adaptive FSFB feedback terms comparison for Slotine's controller: (a) link 1, (b) link 2, and the proposed adaptive controller (c) link 1, and (d) link 2.

performance in comparison to Slotine's[10] controller; however, it may be possible to tune the controllers to achieve different transient results.) Likewise, similar transient/ steady-estate error results are apparent for the exact model knowledge comparison given in Figure 8. In addition to the improved tracking results, it is evident from the plots of the feedback portion of the input control torque given in Figure 7 and Figure 8, that our controllers require less control energy than equivalent unsaturated counterparts. Hence, the steady-state error is on the same order of magnitude, however the proposed controllers take into account the possibility of actuator torque limits.

Remark 6. *Note that the feedbacks terms in Figure 2, Figure 5, Figure 7 and Figure 9 do not exhibit "hard" saturation effects since the* $\tanh(\cdot)$ *is a "slow" saturation function. That is "hard" saturation effects would only be apparent if the tracking error became very large for a substantial interval of time.*

Remark 7. *It is important to note that the conditions given in (24), (59), and (60) are sufficient conditions. Thus it is not surprising that asymptotic tracking is obtained even though the controller gains do not satisfy the aforementioned conditions. In fact, gains as low as 0.01 were chosen and the error was observed to still equal zero after some time.*

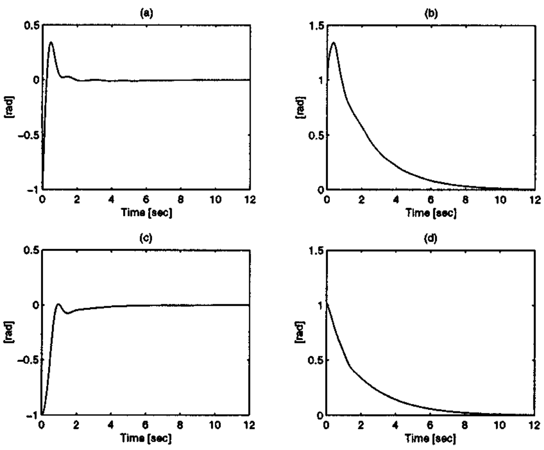

Fig. 8. Exact model knowledge OFB link position tracking error comparison for the unsaturated controller: (a) link 1, (b) link 2 and the proposed exact model knowledge controller (c) link 1, and (d) link 2.

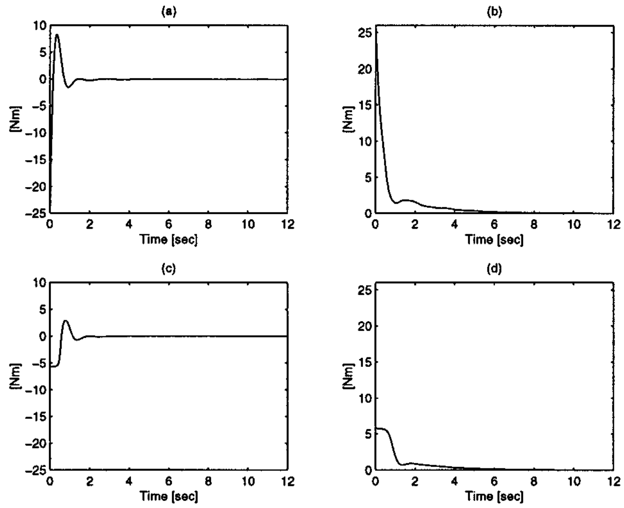

Fig. 9. Exact model knowledge OFB feedback terms comparison for the unsaturated controller: (a) link 1, (b) link 2, and the proposed exact model knowledge controller (c) link 1, and (d) link 2.

6 CONCLUSION

This paper presented the development of two bounded, torque input, tracking controllers for robot manipulators with constraints on the available actuator torques. Specifically, an adaptive, full-state feedback controller and an exact model knowledge, output feedback controller were designed and shown to produce semi-global asymptotic link position tracking. Simulation results were utilized to demonstrate the controllers' tracking performance. Furthermore, a simulation-based comparison demonstrated the practical advantages of the proposed controllers over standard robot controllers. Future work will target the investigation of the proposed control techniques for Rigid-Link Flexible-Joint Robots, and other mechanical systems.

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