
Intermittent Image-Based Estimation



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Abstract

Image feedback can be used to estimate Euclidean distances between features in an image and/or relative motion between an image feature and the camera. Typical current methods assume that the image feature is continuously observed; yet, in most practical scenarios, the image feature can be occluded. Image occlusion/intermittency segregates the image dynamics into two subsystems: when the feature is visible, feedback is available and the estimator/observer is stabilizable, otherwise its unstable. This entry discusses the use of switched/hybrid systems methods including the development of sufficient dwell-time conditions to ensure the stability of such estimators/observers.

Keywords

Nonlinear estimation · Image estimation · Hybrid system · Lyapunov methods · Intermittent sensing

Introduction

Numerous advances have led to widespread adoption of image-based sensing. One of the technical challenges of image-based feedback for navigation and visual servo control for autonomous systems is the lack of depth information (i.e., the distance from the camera to an image feature along the focal axis), resulting in scale ambiguity. Typical solutions to recover the depth information use multiple views of a feature. Multiple views can be obtained using multiple physical cameras (e.g., a stereo pair) with calibrated relative position and orientation (pose) to determine the properly scaled Euclidean coordinates of image features (i.e., structure estimation) and/or the relative motion between the camera and the feature(s) (i.e., motion estimation) (Faugeras 1993; Hartley and Zisserman 2000; Chaumette and Hutchinson 2006a). Alternatively, monocular solutions use multiple images taken by a moving camera to determine the structure from known motion of the camera (structure from motion (SfM)), or motion from known structure, either using image geometry, provided a sufficient number of features on an object are tracked, or using an estimator/observer (cf. Chaumette et al. 1996; Chaumette and Hutchinson 2006b; Dani et al. 2012; Hu et al. 2008; Dixon et al. 2003). A common assumption among these strategies is that the image features are continuously available.

In many realistic scenarios, image feedback may be intermittent due to failures in feature

tracking due to feature occlusions or features leaving the camera field-of-view (FoV) due to motion constraints of the camera or even purposeful motion by the camera. Various path planning and control methods have been motivated by the need to ensure uninterrupted feedback, where the desired trajectory or behavior of such systems is inherently constrained. For example, systems with nonholonomic constraints may experience non-ideal, sharp-angled, or non-smooth trajectories to keep a navigational landmark in the FoV. Moreover, such approaches are still not robust to arbitrary occlusions of the image feature that result from environmental conditions. Typically when features are temporarily not visible, the estimator/observer is reinitialized with the previous state estimate when the feature is reacquired; however, as shown in Liberzon (2003), such switching is insufficient: the estimates may not converge.

In contrast to the strategy of altering the trajectory of the camera to keep the image feature in the FoV, this entry examines recent and emerging efforts that use switched/hybrid systems methods to examine the stability of image-based estimates in the presence of feature intermittency. These methods include the development of dwell-time conditions that provide sufficient (and potentially conservative) timing conditions which indicate how long features can be occluded, relative to the amount of time the features are visible. A challenge is that image occlusion/intermittency segregates the image dynamics into two subsystems: when the feature is visible, feedback is available and the estimator/observer is stabilizable, otherwise it is unstable. Despite this challenge, tolerance to image feedback intermittency provides inherent robustness to the observer and enables new dwell-time-aware path-planning strategies that ease trajectory generation constraints that require the image feature to remain visible.

Image-Based Estimation Dynamics

The unknown Euclidean coordinates, X , Y , $Z \in \mathbb{R}$, of a tracked feature point can be related

to the states of an image-based observer by the normalization $x = [x_1, x_2, x_3]^T \triangleq [\frac{X}{Z}, \frac{Y}{Z}, \frac{1}{Z}]^T \in \mathbb{R}^3$. Using projective geometry, the measurable image coordinates, $p \in \mathbb{R}^3$, of the feature point can be related to the normalized Euclidean coordinates as $p = A [\frac{X}{Z} \frac{Y}{Z} 1]$, where $A \in \mathbb{R}^{3 \times 3}$ is a known, invertible camera intrinsic parameter matrix (Faugeras 1993; Hartley and Zisserman 2000; Chaumette and Hutchinson 2006a). The perspective state dynamics can be written as $\dot{x} = g(t, x)$, where $g(t, x) : [0, \infty) \times \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a nonlinear function that depends on the partially measurable states (i.e., x_3 is not measurable).

A common objective in image-based navigation and control research is the SfM problem, where the goal of estimating the Euclidean coordinates of the target position can be quantified by the state estimate error

$$e \triangleq x - \hat{x}, \quad (1)$$

where $\hat{x} \in \mathbb{R}^3$ denotes the state estimate provided by an observer. If the image feature is intermittently tracked, then the evolution of e is defined by the family of systems

$$\dot{e} = f_p(t, x, \hat{x}) \quad (2)$$

where $f_p : [0, \infty) \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}^3$ and $p \in \{s, u\}$, where s is an index for the resulting subsystem when the target is visible and u is an index for the subsystem when the target is not visible. When the target is in view, the first two elements of the state vector x (i.e., x_1 and x_2) are measurable, and the closed-loop error dynamics are

$$f_s = g(t, x) - \dot{\hat{x}}, \quad (3)$$

where $\dot{\hat{x}}$ is defined by an observer. For the error dynamics in (3), numerous observers have been developed that achieve a variety of stability results. However, when the feature is not visible, the state estimates cannot depend on x , and the resulting error system is unstable. For example, a zero order hold (ZOH) could be used (i.e.,

$\hat{x} = 0$), and the unstable error dynamics would be given by

$$f_u = g(t, x). \quad (4)$$

Switched Systems Analysis

Switched systems methods (cf. Liberzon 2003; Goebel et al. 2012) provide an analysis framework to evaluate the stability of state estimation error dynamics in (2) after a series of instances when the image feature is visible or not. The underlying concept is that if the image feature is visible long enough and the instances without feedback are short enough, then the state estimate over the entire trajectory could remain stable. To understand the relative timing for the estimate to be stable, switched systems analysis includes the development of a dwell-time. The dwell-time is typically expressed as a ratio of the time spent in each subsystem, along with the convergence rate in the stable subsystem and the divergence rate in the unstable subsystem. The intermittent image-based estimation problem is challenging because it inherently involves switching between stabilizable and unstable subsystems, and if the image feature is not visible long enough then stability may never be achieved. Therefore, the concept of an average dwell-time condition (a weaker dwell-time condition where the time in the stable subsystem is longer than the unstable region on average) is difficult to develop. Moreover, when switching between stable and unstable subsystems, two different dwell times are often required, to determine a minimum amount of time to dwell in the stable subsystem and the maximum amount of time in the unstable that can be tolerated. Typically, such dwell-time conditions provide a sufficient rule for a system to follow to ensure stability; however, often for image-based estimation, the ratios of time in each subsystem are arbitrary. In these cases, the dwell-time conditions provide a sufficient condition for a decision-maker to check as a means to trust or not trust the estimated states.

Because the dwell-time conditions are inherently linked to the ratios of the rates of conver-

gence and divergence, exponential stability of the observer is desired. While various exponentially convergent observers have been developed (cf. Chen and Kano 2002; Dani et al. 2012), open questions remain for developing dwell-time conditions for switching between stable and unstable subsystems for asymptotic convergence rates that are more common for the inherently nonlinear image dynamics. Moreover, exponential divergence can also be difficult to determine. For example, in Parikh et al. (2018), the ZOH scenario yielded a faster than exponential divergence rate, which resulted in complex trigonometric dwell-time conditions. However, results such as Parikh et al. (2017) illustrate that by replacing the ZOH observer with a model-based state-predictor during the periods when feedback is unavailable, the divergence rate can be bounded by an exponential envelope, facilitating the development of traditional (and less restrictive) dwell-time conditions. That is, the dwell-time conditions result in the intuitive conclusion that strategies with better state prediction allow the image feature to be occluded for longer periods; however, such predictors often require more stringent assumptions on knowledge about the camera and image feature relative motion.

Summary and Future Directions

Various strategies are available to analyze image-based estimators in the presence of intermittent visibility of image features. An alternative to the aforementioned switched systems methods is to treat the availability of feedback as a stochastic system with a known distribution and the goal of proving the observer converges in expectation. Another alternative is to develop relationships between multiple visible features, which are assumed to have a constant relative pose, so that when a subset of the features are occluded, the remaining visible features can be used to approximate the trajectory of the hidden features (cf. Mehta et al. 2010). Opportunities also exist to develop such methods within a switched systems framework.

Switched systems methods provide dwell-time conditions that can be checked to determine the stability of image-based observers, with arbitrary loss of features. These conditions also provide design guidelines for potential new advances in image-guided autonomy. Specifically, new image-based path planners could be developed that are informed by the dwell-time conditions to allow the image features to purposefully leave the FoV for controlled periods of time as a means to yield more favorable trajectories. New state-prediction results to minimize the error growth when features are not visible also present new opportunities. With continued development and maturity of switched systems methods (e.g., stochastic switched systems methods), more practical and robust image-based estimation strategies are likely to result.

Cross-References

- ▶ [2.5D Vision Based Estimation and Control](#)
- ▶ [Camera Motion Estimation from Images: A Quaternion-Based Approach](#)
- ▶ [Control Based Image Processing](#)
- ▶ [Image-Based Estimation for Robotics and Autonomous Systems](#)
- ▶ [Image-Based Formation Control of Mobile Robots](#)
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- ▶ [Linear Systems: Continuous-Time, Time-Invariant State Variable Descriptions](#)
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- ▶ [Lyapunov's Stability Theory](#)
- ▶ [Neural Control and Approximate Dynamic Programming](#)
- ▶ [Quantitative Feedback Theory](#)
- ▶ [Sampled-Data Systems](#)
- ▶ [Switching Adaptive Control](#)

Bibliography

- Chaumette F, Hutchinson S (2006a) Visual servo control part I: basic approaches. *IEEE Rob Autom Mag* 13(4):82–90
- Chaumette F, Hutchinson S (2006b) Visual servo control part II: advanced approaches. *IEEE Rob Autom Mag* 14(1):109–118
- Chaumette F, Boukir S, Bouthemy P, Juvin D (1996) Structure from controlled motion. *IEEE Trans Pattern Anal Mach Intell* 18(5):492–504
- Chen X, Kano H (2002) A new state observer for perspective systems. *IEEE Trans Autom Control* 47(4):658–663
- Dani A, Fischer N, Dixon WE (2012) Single camera structure and motion. *IEEE Trans Autom Control* 57(1):241–246
- Dani A, Fischer N, Kan Z, Dixon WE (2012) Globally exponentially stable observer for vision-based range estimation. *Mechatron* 22(4):381–389. Special issue on visual servoing
- Dixon WE, Fang Y, Dawson DM, Flynn TJ (2003) Range identification for perspective vision systems. *IEEE Trans Autom Control* 48:2232–2238
- Faugeras O (1993) *Three-dimensional computer vision: a geometric viewpoint*. MIT Press, Cambridge
- Goebel RG, Sanfelice R, Teel AR (2012) *Hybrid dynamical systems*. Princeton University Press, Princeton
- Hartley R, Zisserman A (2000) *Multiple view geometry in computer vision*. Cambridge University Press, New York
- Hu G, Aiken D, Gupta S, Dixon W (2008) Lyapunov-based range identification for a paracatadioptric system. *IEEE Trans Autom Control* 53(7):1775–1781
- Liberzon D (2003) *Switching in systems and control*. Birkhauser, Boston
- Mehta S, Hu G, Gans N, Dixon WE (2010) Robot localization and map building. InTech, ch. A Daisy-chaining visual servoing approach with applications in tracking, localization, and mapping, pp 383–408. [Online]. Available: <http://ncr.mae.ufl.edu/papers/sid-chap.pdf>
- Parikh A, Cheng T-H, Chen H-Y, Dixon WE (2017) A switched systems framework for guaranteed convergence of image-based observers with intermittent measurements. *IEEE Trans Robot* 33(2):266–280
- Parikh A, Cheng T-H, Licitra R, Dixon WE (2018) A switched systems approach to image-based localization of targets that temporarily leave the camera field of view. *IEEE Trans Control Syst Technol* 26(6):2149–2156