$\mathrm{Re}\ s > 0$. Moreover, we may differentiate past the integral to see by parts that

$$\begin{aligned} F'(s) &= -\int_0^\infty t\sin(t^2/2)e^{-st}\,dt \\ &= e^{-st}\cos(t^2/2)\big|_{t=0}^\infty + s\int_0^\infty \cos(t^2/2)e^{-st}\,dt \\ &= -1 + s\int_0^\infty \cos(t^2/2)e^{-st}\,dt. \end{aligned} \tag{4}$$

Differentiating again by the product rule and integration by parts

$$\begin{aligned} F''(s) &= \int_0^\infty \cos(t^2/2)e^{-st}\,dt \\ &\quad -s\int_0^\infty t\cos(t^2/2)e^{-st}\,dt \\ &= \int_0^\infty \cos(t^2/2)e^{-st}\,dt - s\sin(t^2/2)e^{-st}\big|_{t=0}^\infty \\ &\quad -s^2\int_0^\infty \sin(t^2/2)e^{-st}\,dt \\ &= \int_0^\infty \cos(t^2/2)e^{-st}dt - s^2\int_0^\infty \sin(t^2/2)e^{-st}\,dt. \end{aligned} \tag{5}$$

Combining (4) and (5), we see that $F(s)$ satisfies the second-order differential equation

$$sF''(s) - F'(s) + s^3F(s) = 1 \tag{6}$$

everywhere on the open right half-plane $\mathrm{Re}\ s > 0$.

But the ordinary differential equation

$$sw'' - w' + s^3w = 1 \tag{7}$$

has, as can be seen in the usual way, a full complement of series solutions

$$w = \sum_{n=0}^\infty c_n s^n \tag{8}$$

about $s = 0$, where c_0 and c_2 can be freely chosen, where $c_1 = -1, c_3 = 0$, and where

$$c_{n+4} = \frac{-c_n}{(n+4)(n+2)}. \tag{9}$$

Because the recursion (9) forces factorial growth in the denominators of the coefficients, it is clear that each series solution (8) has an infinite radius of convergence and converges to an entire (everywhere analytic) function. Since all other points $s_0 \neq 0$ are ordinary points of (7), each solution of (7) near $s = s_0$ is the sum of a fixed particular solution plus a linear combination of two fixed independent homogeneous solutions—all three of which can be supplied by the series solutions (8) about $s = 0$. Hence our transfer function $w = F(s)$, itself a solution to (7) on the open right half-plane, has a unique analytic extension to the entire complex plane.

IV. Summary

There exist BIBO-unstable SISO plants whose transfer functions have no singularities in the finite plane.

Acknowledgment

The author would like to thank B. Jacob and J. Partington for bringing Körner's example [4] to his attention.

References

[1] C. R. MacCluer, *Industrial Mathematics*. Upper Saddle River, NJ: Prentice Hall, 2000.
[2] ——, "Stability from an operational viewpoint," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 458–460, May 1988.
[3] M. Miklavčič, *Applied Functional Analysis and Partial Differential Equations*, Singapore: World Scientific , 1998, p. 180.
[4] T. W. Körner, *Fourier Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1988, p. 406.
[5] C. R. MacCluer, *Boundary Value Problems and Orthogonal Expansions*. New York: IEEE Press, 1994, p. 280.

Comments on "Sliding-Mode Motion/Force Control of Constrained Robots"

W. E. Dixon and E. Zergeroglu

***Index Terms*—Constrained robots, motion/force control, sliding-mode.**

This note addresses the proposed sliding-mode solution of the motion/force control problem for constrained robots given in the above-mentioned paper. In the above paper[1] the authors claim their control law ensures the motion and force errors are asymptotically driven to zero and that the actual forces will asymptotically approach the desired forces; however, the authors' claim is invalid due to an error in the sliding-mode control stability analysis. Specific details of the errors[1] are given below with the equation numbers referring to those in the paper.[1]

In the paper[1] the authors develop the following closed-loop error system:

$$\overline{M}\dot{s} = Y(\cdot)\theta - \overline{C}s - K\,\mathrm{sgn}(s) - Y(\cdot)\varphi \tag{7}$$

where $\mathrm{sgn}(\cdot)$ denotes the standard signum function, the switching function $\varphi(t) \in \Re^r$ is designed as follows:

$$\varphi = \overline{\theta}_i\,\mathrm{sgn}\left(\sum_{j=1}^n s_jY_{ji}\right), \qquad i = 1, 2, \cdots, r$$

and $\overline{M} \in \Re^{n\times n}$, $\overline{C} \in \Re^{n\times n}$, $K \in \Re^{n\times n}$, $Y(\cdot) \in \Re^{n\times r}$, $\theta \in \Re^r$, $s(t) \in \Re^n$ are explicitly defined in the paper.[1] Through a Lyapunov-based stability analysis, the authors assert that

$$\lim_{t\to\infty} e_p, \dot{e}_p, e_f = 0$$

where $e_p(t) \in \Re^n$ represents the motion error, and $e_f(t) \in \Re^n$ is defined as the accumulated force tracking error as follows:

$$e_f = A^T\int_0^t (\lambda_d - \lambda)\,dt$$

Manuscript received August 13, 1998; revised July 30, 1999. Recommended by Associate Editor, L. S. Wang.
The authors are with the Department of Electrical and Computer Engineering, Clemson University, Clemson, SC 29634-0915 USA (e-mail: wdixon@ces.clemson.edu).
Publisher Item Identifier S 0018-9286(00)06751-9.

[1]Kuang-Yow Lian and Chia-Ru Lin, "Sliding mode motion/force control of constrained robots," *IEEE Trans. on Automatic Control*, vol. 43, no. 8, pp. 1101–1103, August 1998.

while $\lambda_d(t) \in \Re^n$ represents the desired force, $\lambda(t) \in \Re^n$ represents the actual force, and $A \in \Re^{n\times n}$ is defined in the paper.[1] To show that the actual force tracks the desired force, the authors claim that $\dot{e}_f(t)$, $\ddot{e}_f(t) \in \mathcal{L}_\infty$ [which is a sufficient condition for $\dot{e}_f(t)$ to be uniformly continuous] in order to invoke Barbalat's lemma to conclude that

$$\lim_{t\to\infty} \dot{e}_f = 0.$$

However, the claim that $\ddot{e}_f(t) \in \mathcal{L}_\infty$ is predicated on the fact that $\ddot{s}(t) \in \mathcal{L}_\infty$, which due to the discontinuous nature of the closed-loop error system given in (7) is not true.

In *Remark 1*[1] the authors state that an additional control law can be designed as follows:

$$\overline{\tau} = Ks - T^T A^T \lambda + Y(\cdot)\varphi$$

where $\overline{\tau}(t)$ is defined[1] to prove that $s(t)$ and $e_p(t)$ approach zero exponentially fast; hence, the actual forces approach the desired forces exponentially fast. Unfortunately, this additional control law also results in a discontinuous closed-loop error system due to the discontinuous nature of $\varphi(t)$; hence, the additional control law still does not ensure that $\ddot{s}(t) \in \mathcal{L}_\infty$. Since $\ddot{s}(t) \in \mathcal{L}_\infty$ has not been proven, it is not clear how the additional control law ensures that $\lim_{t\to\infty} \lambda = \lambda_d$ unless the authors intended to craft an exact model knowledge control torque as

$$\overline{\tau} = Ks - T^T A^T \lambda + Y(\cdot)\theta.$$

It should be noted that McClamroch [3] provided an exact model knowledge controller which yielded global exponential position and force tracking.

In *Remark 2*[1] the authors assert that the chattering problem resulting from the discontinuous control law can be eliminated by utilizing a boundary layer technique or replacing the signum function with a continuous function. Unfortunately, the application of these techniques will sacrifice the asymptotic result for a uniformly ultimately bounded stability result. It should also be noted that full-state feedback [1], [4], and [5] and partial state feedback [2] adaptive controllers have been previously designed to yield asymptotic position/force tracking for constrained robot manipulators.

REFERENCES

[1] S. Arimoto, T. Naniwa, and Y. H. Liu, "Model-based adaptive hybrid control for manipulators with geometric endpoint constraint," *Adv. Robot.*, vol. 9, no. 1, pp. 67–80, 1995.

[2] M. S. de Queiroz, J. Hu, D. M. Dawson, T. Burg, and S. R. Donepudi, "Adaptive position/force control of robot manipulators without velocity measurements: Theory and experimentation," *IEEE Trans. Syst., Man, Cyber.—Part B: Cyber.*, vol. 27, no. 5, pp. 796–809, Oct. 1997.

[3] N. H. McClamroch and D. Wang, "Feedback stabilization and tracking of constrained robots," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 419–426, 1998.

[4] B. Yao and M. Tomizuka, "Adaptive control of robot manipulators in constrained motion-controller design," *ASME J. Dynam. Syst., Meas., Contr.*, vol. 117, pp. 320–328, Sept. 1995.

[5] R. R. Y. Zhen and A. A. Goldenburg, "An adaptive approach to constrained robot motion control," in *Proc. IEEE Conf. Robotics Automation*, Nagoya, Japan, May 1995, pp. 1833–1838.

Comments on "Robust Tracking Control for Rigid Robotic Manipulators"

Wen-Hong Zhu

***Abstract*—The incorrectness of a main assumption in note [1] was pointed out by [2]. This note proposes a correction.**

***Index Terms*—Robust control, sliding mode control, robot control, tracking control.**

The incorrectness of *Assumption 2.3* with Eq. (2.7) in the authors' previous publication [1] was pointed out by [2]. It indicates that the uncertainty $\rho(t)$ must include either acceleration $\ddot{q}$ or control $u(t)$. In an authors' late publication [3], this issue was addressed. Theorem 3.1 in [3] states that if the control input $u(t)$ does not contain the acceleration signal, then the system uncertainty $\rho(t)$ can be bounded by a positive function of position and velocity only, i.e. (20) in [3] holds when the control $u(t)$ is subject to (21) in [3]. This leads to Eq. (22) in [3] which imposes restriction on the coefficients of control bound λ_0, λ_1, and λ_2 and the coefficients of uncertainty bound b_0, b_1, and b_2. Unfortunately, this restriction was not considered in the control design (48) and (49) in [3]. Equation (52) in [3] indicates that b_0, b_1, and b_2 are constants. Therefore, λ_0, λ_1, and λ_2 must have upper bounds according to Eq. (22) in [3]. However, the authors in [3] were failed to prove that the control laws (48) and (49), where $\hat{b}_0, \hat{b}_1$, and $\hat{b}_2$ are nondecreasing positive functions, still satisfy (21) in [3]. The algebraic loop problem related to the control $u(t)$, the uncertainty $\rho(t)$, and the acceleration $\ddot{q}$ remains unsolved. On the other hand, in order to guarantee $\alpha_1 < +\infty$ for (22) in [3], $\Delta M(q)$ in (3.a) must be limited according to (13.a) in [3] to avoid zero singular value for $I + \Delta M(q) \cdot M_0(q)^{-1}$. However, no guideline on choosing $M_0(q)$ is provided to guarantee $\alpha_1 < +\infty$. Meanwhile, Remark 4.7 in [3] states that the control space is not compact.

Consider that [2] didn't present correction, this note proposes a correction to the previous publication [1]. A guideline on choosing $M_0(q)$ is proposed and the algebraic loop issue related to the control $u(t)$, the uncertainty $\rho(t)$, and the acceleration $\ddot{q}$ is solved with a compact control space.

From (2.1) and (2.2) in [1], it follows that

$$\Delta M(q)\ddot{q} = E \cdot (u(t) - h(q, \dot{q})), \tag{1}$$

where

$$E \stackrel{\Delta}{=} I_n - M_0(q) M(q)^{-1}. \tag{2}$$

Note that $M(q)^{-1}$ is symmetrically positive-definite. We have $M(q)^{-1} = T(q)^T \cdot \underline{M}(q)^{-1} \cdot T(q)$, where $\underline{M}(q)^{-1} \in R^{n\times n}$ is a diagonal positive-definite matrix and $T(q) \in R^{n\times n}$ is an orthogonal rotational matrix subject to $T(q)^T = T(q)^{-1}$. If $M_0(q)$ is chosen as (page 229 of [4])

$$M_0 = \frac{2}{c_1 + c_2} \cdot I_n, \tag{3}$$

Manuscript received September 7, 1998; revised August 18, 1999. Recommended by Associate Editor, L. S. Wang.

W.-H. Zhu is with the Department of Electrical Engineering, University of British Columbia, 2356 Main Mall, Vancouver, B.C., Canada V6T 1Z4.

Publisher Item Identifier S 0018-9286(00)06746-5.