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Global Adaptive Output Feedback Tracking Control of Robot Manipulators

F. Zhang, D. M. Dawson, M. S. de Queiroz, and W. E. Dixon

***Abstract*—This paper presents a solution to the problem of global, output feedback, tracking control of uncertain robot manipulators. Specifically, a desired compensation adaptation law plus a nonlinear feedback term coupled to a dynamic nonlinear filter is designed to produce global asymptotic link position tracking while compensating for parametric uncertainty and requiring only link position measurements.**

***Index Terms*—Adaptive control, output feedback, robot manipulator, tracking control.**

I. Introduction

The problem of output feedback[1] (OFB) link position tracking control of robot manipulators has been a topic of considerable interest over the past several years. In [2], Belanger provided motivation for using control design techniques to eliminate the need for velocity measurements by illustrating how a Kalman filter provided significant improvement over an *ad hoc* method such as numerical integration of the position measurements. A limitation that exists in almost all of the proposed OFB link position *tracking* controllers is the semiglobal nature of the stability results. In contrast, global solutions to the OFB link position *setpoint* control problem have been presented by several researchers. For example, exact model knowledge OFB controllers, composed of a dynamic linear feedback loop plus feedforward gravity compensation, were proposed in [3], [7], and [13] to globally asymptotically stabilize the robot manipulator dynamics. In [1], Arimoto *et al.* also presented an exact model knowledge, global regulating OFB controller; however, the gravity compensation term was dependent only on the desired link position setpoint as opposed to the actual link position. With the intent of overcoming the requirement of exact model knowledge, Ortega *et al.* [21] designed a OFB regulator which compensated for uncertain gravity effects; however, the stability result was semiglobal asymptotic[2] . In [11], Colbaugh *et al.* proposed a global regulating OFB controller that compensates for uncertain gravity effects; however, the control strategy requires the use of two different control laws (i.e., one control law is used to drive the setpoint error to a small value, then another control law is used to drive the setpoint error to zero).

With respect to the more general problem of OFB link position tracking control, semiglobal results have dominated the scenario. For example, in [3] and [17] exact model knowledge, observer-based controllers yielded semiglobal exponential link position tracking while in [19] a semiglobal asymptotic tracking result was achieved. Robust, filter-based control schemes were designed in [4], [22], and [25] to compensate for robot parametric uncertainty while producing semiglobal uniform ultimate bounded link position tracking. In [9] and [10], variable structure OFB controllers were designed to compensate for uncertainty and the lack of link velocity measurements. For other work in this area, the reader is referred to the literature review in [17]. For other tracking control work, the reader is referred to [15]. For work on the OFB problem for rigid-link flexible-joint robots, the reader is referred to [20] and the references therein. Finally, in [5], [6], and [12], adaptive OFB controllers were presented which yielded semi-global asymptotic link position tracking.

To the best of our knowledge, the only previous work which is targeted at the global OFB tracking control problem is given in [18] and [8]. Specifically, in [18], Loria developed an exact model knowledge controller that yields global uniform asymptotic stability of the closed-loop system; however, the control was designed only for a nonlinear, one degree-of-freedom (DOF) system. In [8], Burkov used a singular perturbation analysis to show that an exact model knowledge controller, used in conjunction with a linear observer, can force the link position to asymptotically track a trajectory from any initial condition; however, as pointed out in [18], no explicit bound on the singular perturbation parameter was given.

In this paper, we design a global, adaptive, OFB tracking controller for uncertain robot manipulators. The control law is composed of: i) a desired compensation adaptation law (DCAL) [23] feedforward term to compensate for parametric uncertainty and ii) a nonlinear feedback term coupled to a nonlinear, dynamic filter to compensate for the lack of velocity measurements and the difference between the actual system dynamics and the feedforward term based on the desired trajectory. That is, the proposed controller ensures global asymptotic link position tracking while compensating for parametric uncertainty and lack of link velocity measurements. While the structure of the proposed torque input control law resembles that of [18] in certain aspects, the use of

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[1]We use the term output feedback to denote that only robot's link position measurements are available.

[2]It is interesting to note that the adaptive OFB link position tracking controller proposed in [5] yields as a subresult, a PID regulator that also semiglobally asymptotically stabilizes the robot dynamics with uncertain gravity effects.

a DCAL-based feedforward term, a different filter structure, and a different error system development/analysis allows us to extend the exact model knowledge controller for one DOF systems given in [18] to uncertain, n DOF robot manipulators.

II. Robot Manipulator Model and Properties

The system model for an n-rigid link, revolute, direct-drive robot is assumed to be of the following form [16]:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} = \tau \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ denote the link position, velocity, and acceleration vectors, respectively, $M(q) \in \mathbb{R}^{n\times n}$ represents the link inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ represents the gravity effects, $F_d \in \mathbb{R}^{n\times n}$ is the constant, diagonal, positive-definite, viscous friction coefficient matrix, and $\tau \in \mathbb{R}^n$ represents the torque input vector. We will assume that the left-hand side of (1) is first-order differentiable.

The dynamic equation of (1) has the following properties [16], [20] that will be used in the controller development and analysis.

Property 1: The inertia matrix $M(q)$ is symmetric and positive-definite and satisfies the following inequality:

$$m_1\|\xi\|^2 \le \xi^T M(q)\xi \le m_2\|\xi\|^2, \qquad \forall \xi \in \mathbb{R}^n \tag{2}$$

where m_1, m_2 are known positive constants, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: The inertia and centripetal-Coriolis matrices satisfy the following relationship:

$$\xi^T\left(\tfrac{1}{2}\dot{M}(q) - V_m(q,\dot{q})\right)\xi = 0, \quad \forall \xi \in \mathbb{R}^n \tag{3}$$

where $\dot{M}(q)$ is the time derivative of the inertia matrix.

Property 3: The dynamic equation of (1) can be linear parameterized as

$$Y_d(q_d,\dot{q}_d,\ddot{q}_d)\theta = M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F_d\dot{q}_d \tag{4}$$

where $\theta \in \mathbb{R}^p$ contains the constant system parameters, and the *desired* regression matrix $Y_d(q_d,\dot{q}_d,\ddot{q}_d) \in \mathbb{R}^{n\times p}$ contains known functions of the desired link position, velocity, and acceleration, $q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \in \mathbb{R}^n$, respectively.

Property 4: There exist positive scalar constants ζ_f, ζ_{c1} such that

$$\|F_d\| \le \zeta_f, \quad \|V_m(q,\dot{q})\| \le \zeta_{c1}\|\dot{q}\| \quad \forall q,\dot{q} \in \mathbb{R}^n. \tag{5}$$

Property 5: The centripetal-Coriolis matrix satisfies the following relationship:

$$V_m(q,\xi)\nu = V_m(q,\nu)\xi, \quad \forall \xi,\nu \in \mathbb{R}^n. \tag{6}$$

To aid the subsequent control design and analysis, we define the vector function $\mathrm{Tanh}(\cdot) \in \mathbb{R}^n$ and the matrix function $\mathrm{Cosh}(\cdot) \in \mathbb{R}^{n\times n}$ as follows:

$$\mathrm{Tanh}(\xi) = [\tanh(\xi_1),\cdots,\tanh(\xi_n)]^T \tag{7}$$

and

$$\mathrm{Cosh}(\xi) = \mathrm{diag}\{\cosh(\xi_1),\cdots,\cosh(\xi_n)\} \tag{8}$$

where $\xi = [\xi_1,\cdots,\xi_n]^T \in \mathbb{R}^n$, and $\mathrm{diag}\{\cdot\}$ denotes the operation of forming a matrix with zeros everywhere except for the main diagonal. Based on the definition of (7), it can easily be shown that the following inequalities hold for $\forall \xi, \nu \in \mathbb{R}^n$:

$$\begin{aligned}\tfrac{1}{2}\tanh^2(\|\xi\|) &\le \ln(\cosh(\|\xi\|)) \\ &\le \sum_{i=1}^n \ln(\cosh(\xi_i)) \le \|\xi\|^2 \\ \tanh^2(\|\xi\|) &\le \|\mathrm{Tanh}(\xi)\|^2 = \mathrm{Tanh}^T(\xi)\,\mathrm{Tanh}(\xi)\end{aligned} \tag{9}$$

and

$$\begin{aligned}&\left|\cos\left(\sum_{i=1}^n \xi_i\right) - \cos\left(\sum_{i=1}^n \nu_i\right)\right| \\ &\qquad \le 8\sum_{i=1}^n |\tanh(\xi_i - \nu_i)| \\ &\left|\sin\left(\sum_{i=1}^n \xi_i\right) - \sin\left(\sum_{i=1}^n \nu_i\right)\right| \\ &\qquad \le 8\sum_{i=1}^n |\tanh(\xi_i - \nu_i)|\end{aligned} \tag{10}$$

where ξ_i, ν_i denote the ith elements of the vectors ξ, ν, and $|\cdot|$ denotes the standard absolute value operation.

Assumption 1: The positive constants ζ_m, ζ_g, and ζ_{c2} are assumed to exist for all $\xi, \nu \in \mathbb{R}^n$ such that

$$\begin{aligned}\|M(\xi) - M(\nu)\| &\le \zeta_m\|\mathrm{Tanh}(\xi-\nu)\|, \\ \|G(\xi) - G(\nu)\| &\le \zeta_g\|\mathrm{Tanh}(\xi-\nu)\| \\ \|V_m(\xi,\dot{q}) - V_m(\nu,\dot{q})\| &\le \zeta_{c2}\|\dot{q}\|\|\mathrm{Tanh}(\xi-\nu)\|.\end{aligned} \tag{11}$$

In Appendix A, we illustrate how the bounds given in (11) hold for the six DOF Puma robot. In a similar manner to that provided in Appendix A, we can show that the bounds given in (11) hold for other revolute robots; hence, from a practical point of view, Assumption 1 resembles a property more than an assumption.

III. Control Development

The control objective is to design a global link position tracking controller[3] for the robot manipulator model given by (1) under the given constraints that only the link position variable q is available for measurement and that the parameter vector θ defined in (4) is an unknown constant vector. We will quantify the control objective by defining the link position tracking error $e(t) \in \mathbb{R}^n$ as follows:

$$e = q_d - q \tag{12}$$

where we assume that $q_d(t)$, defined in Property 3, and its first three time derivatives are bounded functions of time. In addition, we define the difference between the actual and estimated parameters as follows:

$$\tilde{\theta} = \theta - \hat{\theta} \tag{13}$$

where $\tilde{\theta} \in \mathbb{R}^p$ represents the parameter estimation error vector, and $\hat{\theta} \in \mathbb{R}^p$ represents a dynamic estimate of θ defined in (4).

A. Control Formulation

To facilitate the design of the controller, we define a filtered tracking error-like variable $\eta \in \mathbb{R}^n$ as follows:

$$\eta = \dot{e} + \mathrm{Tanh}(e) + \mathrm{Tanh}(e_f) \tag{14}$$

[3]Here, global link position tracking means that the controller must drive the link position tracking error to zero asymptotically for any finite, initial position, and velocity tracking errors with no conditions on the size of the initial tracking errors.

where e and $\text{Tanh}(\cdot)$ were defined in (12) and (7), respectively, and $e_f \in \mathbb{R}^n$ is an auxiliary filter variable which is defined to have the following dynamics:

$$\dot{e}_f = -\text{Tanh}(e_f) + \text{Tanh}(e) - k\,\text{Cosh}^2(e_f)\eta,\quad e_f(0) = 0 \tag{15}$$

with k being a positive scalar control gain and $\text{Cosh}(\cdot)$ being defined in (8). Based on the subsequent error system development and the corresponding stability analysis, we propose the following torque input:

$$\tau = Y_d\hat{\theta} - k\,\text{Cosh}^2(e_f)\,\text{Tanh}(e_f) + \text{Tanh}(e) \tag{16}$$

where k is the same control gain defined in (15), and the parameter estimate vector $\hat{\theta}$ is generated on-line according to the following update law:

$$\dot{\hat{\theta}} = \Gamma Y_d^T \eta \tag{17}$$

with $\Gamma \in \mathbb{R}^{p\times p}$ being a constant, diagonal, positive-definite, adaptation gain matrix.

Remark 1: Based on the definition of η given in (14), we can see that link velocity measurements are required for control implementation in (15)–(17). However, after the stability proof, we will illustrate how the proposed controller has an equivalent form which only depends on link position measurements.

B. Error System Development

We begin the error system development by first calculating the open-loop filtered tracking error dynamics. To this end, we take the time derivative of (14), multiply both sides of the equation by $M(q)$, and then substitute (1) for $M(q)\ddot{q}$ in the resulting expression to yield

$$\begin{aligned} M(q)\dot{\eta} = {} & M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} - \tau \\ & + M(q)\,\text{Cosh}^{-2}(e)\dot{e} \\ & + M(q)\,\text{Cosh}^{-2}(e_f)\dot{e}_f. \end{aligned} \tag{18}$$

After adding and subtracting $Y_d\theta$ of (4) to the right-hand side of (18), we can utilize (12), (14), and (15) to rewrite the open-loop dynamics for η as follows:

$$M(q)\dot{\eta} = -V_m(q,\dot{q})\eta - kM(q)\eta + Y_d\theta + \chi + \tilde{Y} - \tau \tag{19}$$

where $\chi(e, e_f, \eta, t), \tilde{Y}(e, e_f, \eta, t) \in \mathbb{R}^n$ are defined as

$$\begin{aligned} \chi = {} & M(q)\text{Cosh}^{-2}(e)\,(\eta - \text{Tanh}(e_f) - \text{Tanh}(e)) \\ & + M(q)\,\text{Cosh}^{-2}(e_f)\,(-\text{Tanh}(e_f) + \text{Tanh}(e)) \\ & + V_m(q, \dot{q}_d + \text{Tanh}(e_f) \\ & + \text{Tanh}(e))(\text{Tanh}(e_f) + \text{Tanh}(e)) \\ & + V_m(q,\dot{q}_d)(\text{Tanh}(e_f) + \text{Tanh}(e)) \\ & - V_m(q,\eta)\,(\dot{q}_d + \text{Tanh}(e_f) + \text{Tanh}(e)). \end{aligned} \tag{20}$$

and

$$\tilde{Y} = M(q)\ddot{q}_d + V_m(q,\dot{q}_d)\dot{q}_d + G(q) + F_d\dot{q} - Y_d\theta \tag{21}$$

where Property 5 given by (6) has been utilized. After substituting (16) into (19), we can form the closed-loop dynamics for η as

$$\begin{aligned} M(q)\dot{\eta} = {} & -V_m(q,\dot{q})\eta - kM(q)\eta + Y_d\tilde{\theta} + \tilde{Y} + \chi \\ & + k\,\text{Cosh}^2(e_f)\,\text{Tanh}(e_f) - \text{Tanh}(e) \end{aligned} \tag{22}$$

where $\tilde{\theta}$ was defined in (13).

Remark 2: By exploiting the fact that the desired trajectory is bounded, Properties 1and 4, and the properties of hyperbolic functions, we note that $\chi(e, e_f, \eta, t)$ of (20) can be upper bounded as follows:

$$\|\chi\| \le \zeta_1\|x\| \tag{23}$$

where ζ_1 is some positive bounding constant that depends on the mechanical parameters and the desired trajectory, and $x \in \mathbb{R}^{3n}$ is defined as

$$x = [\text{Tanh}^T(e) \quad \text{Tanh}^T(e_f) \quad \eta^T]^T. \tag{24}$$

Furthermore, by utilizing the fact that the desired trajectory is bounded and Assumption 1, it can be shown that $\tilde{Y}(e, e_f, \eta, t)$ of (21) can be upper bounded as follows:

$$\|\tilde{Y}\| \le \zeta_2\|x\| \tag{25}$$

where ζ_2 is also some positive bounding constant that depends on the mechanical parameters and the desired trajectory.

C. Stability Analysis

Theorem 1: Given the robot dynamics of (1), the proposed adaptive controller of (15)–(17) ensures global asymptotic link position tracking in the sense that

$$\lim_{t\to\infty} e(t) = 0 \tag{26}$$

provided the control gain k is chosen as follows:

$$k = \frac{1}{m_1}\,(1 + k_n(\zeta_1 + \zeta_2)^2) \tag{27}$$

where m_1, ζ_1, and ζ_2 are constants defined in (2), (23), and (25), respectively, and k_n is a control gain that must satisfy the following sufficient condition:

$$k_n > 1. \tag{28}$$

Proof: We start the proof by defining the following nonnegative function[4]

$$\begin{aligned} V(e, e_f, \eta, \tilde{\theta}) = {} & \sum_{i=1}^{n}\ln(\cosh(e_i)) + \sum_{i=1}^{n}\ln(\cosh(e_{fi}) \\ & + \tfrac{1}{2}\eta^T M(q)\eta + \tfrac{1}{2}\,\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \end{aligned} \tag{29}$$

where e, e_{fi} are the ith elements of the vectors e and e_f defined in (12) and (15), respectively. After taking the time derivative of (29), we obtain the following expression for $\dot{V}(e, e_f, \eta, \tilde{\theta})$:

$$\begin{aligned} \dot{V}(e, e_f, \eta, \tilde{\theta}) = {} & \text{Tanh}^T(e)\dot{e} + \text{Tanh}^T(e_f)\dot{e}_f + \eta^T M(q)\dot{\eta} \\ & + \tfrac{1}{2}\eta^T\dot{M}(q)\eta - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} \end{aligned} \tag{30}$$

where we have used the fact from (13) that $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$. We can now utilize (14), (15), (22), and (17) in (30) to simplify the expression for $\dot{V}(e, e_f, \eta, \tilde{\theta})$ as follows:

$$\begin{aligned} \dot{V} = {} & -\text{Tanh}^T(e)\,\text{Tanh}(e) - \text{Tanh}^T(e_f)\,\text{Tanh}(e_f) \\ & + \eta^T(\tilde{Y} + \chi) - k\eta^T M(q)\eta \end{aligned} \tag{31}$$

where (3) has been employed. After applying (23), (25), (2), and (27) to (31), we obtain the following upper bound for $\dot{V}(e, e_f, \eta, \tilde{\theta})$:

$$\begin{aligned} \dot{V} \le {} & -\|\text{Tanh}(e)\|^2 - \|\text{Tanh}(e_f)\|^2 - \|\eta\|^2 \\ & + [(\zeta_1 + \zeta_2)\|x\|\,\|\eta\| - k_n(\zeta_1 + \zeta_2)^2\|\eta\|^2]. \end{aligned} \tag{32}$$

[4]It should be noted that the first two terms in $V(e, e_f, \eta, \tilde{\theta})$ are motivated by the work given in [18].

After completing the squares on the bracketed term in (32), we get the following new upper bound on $\dot{V}(e, e_f, \eta, \tilde{\theta})$:

$$\dot{V} \leq -\|x\|^2 + \frac{1}{k_n}\|x\|^2. \tag{33}$$

Finally, if k_n is selected according to (28), we can rewrite (33) as

$$\dot{V} \leq -\beta\|x\|^2 \tag{34}$$

where β is some positive constant.

As discussed in [18], we note that $\ln(\cosh(0)) = 0$ and that $\ln(\cosh(\cdot))$ is a radially unbounded, globally positive-definite function. Hence, due to the structure of $V(e, e_f, \eta, \tilde{\theta})$ given in (29), $V(e, e_f, \eta, \tilde{\theta})$ is a radially unbounded, globally positive-definite function for all $e(t)$, $e_f(t)$, $\eta(t)$, $\tilde{\theta}(t)$, and t. Since $\dot{V}(e, e_f, \eta, \tilde{\theta})$ is negative semidefinite as illustrated by (34), we now know that $V(e(t), e_f(t), \eta(t), \tilde{\theta}(t)) \in \mathcal{L}_\infty$; hence, $e(t), e_f(t), \eta(t), \tilde{\theta}(t) \in \mathcal{L}_\infty$ (and hence, due to the fact that desired trajectory is bounded, $q(t)$, $\dot{q}(t)$, $\hat{\theta}(t)$, $\dot{\hat{\theta}}(t) \in \mathcal{L}_\infty$). We can now utilize (14), (15), and (22) to state that $\dot{e}(t), \dot{e}_f(t), \dot{\eta}(t) \in \mathcal{L}_\infty$. The above boundedness statements together with the fact that the desired trajectory is bounded allow us to conclude that $\ddot{q}(t), \tau(t) \in \mathcal{L}_\infty$.

From the above information, we can state from the definition of (24) that $x(t), \dot{x}(t) \in \mathcal{L}_\infty$ (note that $\dot{x}(t) \in \mathcal{L}_\infty$ is a sufficient condition for $x(t)$ being uniformly continuous). From (34), it is easy to show that $x(t) \in \mathcal{L}_2$. We can now apply Barbalat's lemma [24] to conclude that $\lim_{t\to\infty} x(t) = 0$. From the definition given in (24), we can now see that $\lim_{t\to\infty} \text{Tanh}(e(t)) = 0$; hence, the properties of the hyperbolic tangent function lead to the result given by (26). □

D. Output Feedback Form of the Controller

We now illustrate how the control law proposed in Section III-A has an equivalent form which only requires link position measurements. Note that the torque control input given by (16) and (17) does not actually require the computation of e_f; rather, only of $\text{Tanh}(e_f)$ and $\text{Cosh}^2(e_f)$. Hence, if we define the following relationship:

$$y_i = [\text{Tanh}(e_f)]_i = \tanh(e_{fi}) \tag{35}$$

then according to standard hyperbolic identities

$$\cosh^2(e_{fi}) = \frac{1}{1 - \tanh^2(e_{fi})} = \frac{1}{1 - y_i^2} \tag{36}$$

where y_i is the ith element of the vector $y \in \mathbb{R}^n$.

We will now show that y_i [and hence $\tanh(e_{fi})$ and $\cosh^2(e_{fi})$] can be calculated with only link position measurements. First, note that filter given by (15) can be written in terms of its ith component as follows:

$$\dot{e}_{fi} = -\tanh(e_{fi}) + \tanh(e_i) - k\cosh^2(e_{fi})\eta_i, \quad e_{fi}(0) = 0 \tag{37}$$

where η_i is the ith element of the vector η defined in (14). After taking the time derivative of (35), we can substitute (37) and (36) into the resulting expression to obtain

$$\begin{aligned}\dot{y}_i = \cosh^{-2}(e_{fi})\dot{e}_{fi} &= -(1 - y_i^2)(y_i - \tanh(e_i)) \\ &\quad - k(\dot{e}_i + \tanh(e_i) + y_i), \qquad y_i(0) = 0.\end{aligned} \tag{38}$$

It is now straightforward to utilize (38) to construct the following filter which also computes y_i:

$$\begin{aligned}\dot{p}_i &= -(1 - (p_i - ke_i)^2)(p_i - ke_i - \tanh(e_i)) \\ &\quad - k(\tanh(e_i) + p_i - ke_i), \qquad p_i(0) = ke_i(0) \\ y_i &= p_i - ke_i\end{aligned} \tag{39}$$

where p_i is an auxiliary variable which allows y_i (and hence, $\tanh(e_{fi})$ and $\cosh^2(e_{fi})$) to be calculated with only link position measurements.

We now illustrate how the torque control input given by (16) and (17) can be computed with only link position measurements. First, we substitute the definition of η given by (14) into (17) and then formulate an equivalent expression as follows:

$$\begin{aligned}\hat{\theta} &= \Gamma Y_d^T e + \Gamma\varrho \\ \dot{\varrho} &= Y_d^T(q_d, \dot{q}_d, \ddot{q}_d)(\text{Tanh}(e) + y) - \dot{Y}_d^T(q_d, \dot{q}_d, \ddot{q}_d)e\end{aligned} \tag{40}$$

where we have utilized the fact from (35) that $y = \text{Tanh}(e_f)$, and $\varrho \in \mathbb{R}^p$ is an auxiliary variable that allows the adaptation law to be calculated with link velocity measurements. By utilizing (39) to compute y, it is now easy to see that $\hat{\theta}$ can be computed with only link position measurements. After substituting (35) and (36) into (16), the ith component of the torque input control can be written as follows:

$$\tau_i = (Y_d\hat{\theta})_i - k\frac{y_i}{1 - y_i^2} + \tanh(e_i) \tag{41}$$

where τ_i, $(Y_d\hat{\theta})_i$ are the ith element of the vectors τ, $Y_d\hat{\theta}$, respectively, $\hat{\theta}$ is computed using (40), and y_i is computed using (39).

Remark 3: It is clear from (35) and (41) that y_i must be restricted such that $|y_i(t)| < 1$ for all time. To illustrate that this does indeed occur, first, note that since $e_f(0) = 0$ [see (15)], we know from (35) that $y(0) = 0$. Secondly, from the proof of Thoerem 1, it follows that $e_{fi}(t) \in \mathcal{L}_\infty$; hence, we can use the definition given by (35) and the properties of the hyperbolic functions to show that $|y_i(t)| < 1$ for $t > 0$.

Remark 4: From the proof of Theorem 1, it also follows that $\hat{\theta}(t) \in \mathcal{L}_\infty$ and $e(t) \in \mathcal{L}_\infty$; hence, it is now easy to see that all of the signals in the OFB form of the control given by (39), (41), and (40) remain bounded for all time (i.e., $p_i(t)$, $\dot{p}_i(t)$, $\hat{\theta}_i(t)$, $\tau_i(t)$, $\varrho_i(t) \in \mathcal{L}_\infty$).

Remark 5: It should be noted that since the proposed controller is a link position tracking controller, a simplified version of the controller can be used for global setpoint control (i.e., q_d = constant, $\dot{q}_d = 0$). Specifically, $Y_d\theta$ defined in (4) now becomes

$$Y_d(q_d)\theta = G(q_d) \tag{42}$$

and hence, the update law given by (40) simplifies as follows:

$$\begin{aligned}\hat{\theta} &= \Gamma Y_d^T e + \Gamma\varrho \\ \dot{\varrho} &= Y_d^T(q_d)(\text{Tanh}(e) + y)\end{aligned} \tag{43}$$

The filter given by (39) and the torque control input given by (41) are utilized to complete the structure of the adaptive setpoint controller.

Remark 6: If we assume exact model knowledge, the torque input can be redesigned as follows:

$$\begin{aligned}\tau = {} & M(q)\ddot{q}_d + V_m(q, \dot{q}_d)\dot{q}_d + G(q) + F_d\dot{q}_d \\ & - k\,\text{Cosh}^2(e_f)\,\text{Tanh}(e_f) + \text{Tanh}(e);\end{aligned} \tag{44}$$

hence, the closed-loop dynamics for η becomes

$$\begin{aligned}M(q)\dot{\eta} = {} & -V_m(q, \dot{q})\eta - kM(q)\eta + \tilde{Y} + \chi \\ & + k\,\text{Cosh}^2(e_f)\,\text{Tanh}(e_f) - \text{Tanh}(e)\end{aligned} \tag{45}$$

where $\chi(e, e_f, \eta, t)$ is still given by (20), but now $\tilde{Y}(e, e_f, \eta, t)$ is defined as

$$\tilde{Y} = -F_d\dot{e}. \tag{46}$$

Note that $\tilde{Y}(\cdot)$ of (46) can now be bounded as in (25) but without the need for Assumption 1. By slightly modifying the stability analysis given in Theorem 1, it is easy to show that the controller given by (44) yields global asymptotic link position tracking. In a similar manner as done for the adaptive controller, it is also easy to show that the controller given by (44) can be computed with only link position measurements. Furthermore, since the parameter error term is not required in the nonnegative function given by (29) for the exact model knowledge case, we can employ Property 5 to bound the nonadaptive version of (29) as follows:

$$\lambda_1 \ln(\cosh(\|z\|)) \le V \le \lambda_2 \|z\|^2 \tag{47}$$

where $z \in \mathbb{R}^{3n}$ is defined as

$$z = [e^T \quad e_f^T \quad \eta^T]^T \tag{48}$$

and $\lambda_1, \lambda_2 \in \mathbb{R}^1$ are some positive constants. We can also use Property 5, (48), and (34) to bound the time derivative of the nonadaptive version of (29) as follows:

$$\dot{V} \le -\lambda_3 \tanh^2(\|z\|) \tag{49}$$

where $\lambda_3 \in \mathbb{R}^1$ is some positive constant. Based on (47)–(49), it easy to show the requirements of [14], Th. A.5 are satisfied, and hence, we can conclude that the equilibrium $z = 0$ is globally uniformly asymptotically stable.

IV. Conclusion

In this paper, we presented a solution to the problem of global, output feedback, tracking control of uncertain robot manipulators. The solution involves the use of a dynamic filter which is nonlinear in a way fundamentally different from the linear, high-gain filters used in previous work (e.g., see linear filter structures used in [4], [25], and [5]). Furthermore, the solution involves the use of a nonquadratic Lyapunov function that is "softer"[5] than the standard quadratic Lyapunov function. In addition, the control development/analysis and the method in which the control is implemented exploits several properties inherent to the robot manipulator equation. Hence, the filter structure, softness of the Lyapunov function, and the exploitation of the robot manipulator equation are all instrumental in the design of a controller that compensates for parametric uncertainty, achieves global asymptotic link position tracking, and only requires link position measurements. Simulation results for the proposed controller can be found in [26].

Appendix A
Bounds for the Puma Robot (Assumption 1)

To prove the existence of the bounds given in (11), we will make use of the inequalities given in (10). First, note that the matrices in (11) contain mismatches *only* in the link position variable. For revolute joint robots, these mismatches can be arranged to have one of the following general forms:

$$\left|\cos\left(\sum_{i=j}^{k}\xi_i\right) - \cos\left(\sum_{i=j}^{k}\nu_i\right)\right|, \quad \left|\sin\left(\sum_{i=j}^{k}\xi_i\right) - \sin\left(\sum_{i=j}^{k}\nu_i\right)\right| \tag{50}$$

where ξ_i, ν_i are the ith elements of $\forall \xi, \nu \in \mathbb{R}^n$, $j \in \{1, 2, \cdots, n\}$, $k \in \{j, j+1, \cdots, n\}$, and n represents the robot's total DOF's. That is, all the elements of the matrices in (11) will contain terms similar to those given by (50). For example, the most complicated elements of the centripetal-Coriolis matrix [i.e., $V_{m53}(q, \dot{q})$] for the six DOF Puma robot manipulator has the following form [16]:

$$V_{m53} = [0.0025\cos(q_2+q_3)\sin(q_4)\sin(q_5) + 0.000\,64\cos(q_2+q_3)\sin(q_4)]\dot{q}_1 \tag{51}$$

where q_i denotes the ith element of the link position vector $q \in \mathbb{R}^6$. If we define

$$\tilde{V} \triangleq V_m(\xi, \dot{q}) - V_m(\nu, \dot{q}) \tag{52}$$

then, according to (51), the mismatch for element $V_{m53}(q, \dot{q})$ becomes

$$\tilde{V}_{53} = \Omega_1(\xi, \nu, \dot{q}) + \Omega_2(\xi, \nu, \dot{q}) \tag{53}$$

where $\Omega_1(\xi, \nu, \dot{q})$ and $\Omega_2(\xi, \nu, \dot{q})$ are scalar quantities defined by

$$\Omega_1 = 0.000\,64\dot{q}_1[\cos(\xi_2+\xi_3)\sin(\xi_4) - \cos(\nu_2+\nu_3)\sin(\nu_4)] \tag{54}$$

and

$$\Omega_2 = 0.0025\dot{q}_1[\cos(\xi_2+\xi_3)\sin(\xi_4)\sin(\xi_5) - \cos(\nu_2+\nu_3)\sin(\nu_4)\sin(\nu_5)]. \tag{55}$$

Note that (54) can be rewritten as

$$\Omega_1 = 0.000\,64\dot{q}_1[\cos(\xi_2+\xi_3) - \cos(\nu_2+\nu_3)]\sin(\xi_4) + 0.000\,64\dot{q}_1\cos(\nu_2+\nu_3)\,[\sin(\xi_4) - \sin(\nu_4)]; \tag{56}$$

hence, an upper bound can be placed on $|\Omega_1|$ as follows:

$$|\Omega_1| \le 0.000\,64|\dot{q}_1|\,(|\cos(\xi_2+\xi_3) - \cos(\nu_2+\nu_3)|. + |\sin(\xi_4) - \sin(\nu_4)|). \tag{57}$$

Upon the application of (10) to (57), we have the following new upper bound:

$$|\Omega_1| \le 0.005\,12|\dot{q}_1|[|\tanh(\xi_2-\nu_2)| + |\tanh(\xi_3-\nu_3)| + |\tanh(\xi_4-\nu_4)|]. \tag{58}$$

Similar arguments can be applied to (55) to show that

$$|\Omega_2| \le 0.02|\dot{q}_1|\,[|\tanh(\xi_2-\nu_2)| + |\tanh(\xi_3-\nu_3)| + |\tanh(\xi_4-\nu_4)| + |\tanh(\xi_5-\nu_5)|]. \tag{59}$$

From (58) and (59), it is clear that $\tilde{V}_{53}$ of (53) can be upper bounded as follows:

$$|\tilde{V}_{53}| \le \zeta_{53}|\dot{q}_1|\,[|\tanh(\xi_2-\nu_2)| + |\tanh(\xi_3-\nu_3)| + |\tanh(\xi_4-\nu_4)| + |\tanh(\xi_5-\nu_5)|] \tag{60}$$

where ζ_{53} is some positive bounding constant. We can now use the following inequality:

$$|a| + |b| \le \sqrt{2}\,\sqrt{|a|^2 + |b|^2}, \quad \forall a, b \in \mathbb{R} \tag{61}$$

to show that

$$|\tilde{V}_{53}| \le \overline{\zeta}_{53}\|\dot{q}\|\,\|\mathrm{Tanh}(\xi - \nu)\| \tag{62}$$

where $\overline{\zeta}_{53}$ is some positive bounding constant.

Since all of the elements of $\tilde{V}$, defined in (52), can be upper bounded in a similar fashion as shown in (62), it is now easy to see that $\|\tilde{V}\|$ can

[5]The word softer is used to illustrate the fact that $\ln(\cosh(x)) \le x^2$.

be upper bounded as in the third inequality of (11). Similar arguments can be followed to prove the other two inequalities given in (11).

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Almost Disturbance Decoupling for a Class of High-Order Nonlinear Systems

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***Abstract*—The problem of almost disturbance decoupling with internal stability (ADD) is formulated, in terms of an L_2–L_{2p} (instead of an L_2) gain, for a class of *high-order* nonlinear systems which consist of a chain of *power* integrators perturbed by a lower-triangular vector field. A significant feature of the systems considered in the paper is that they are neither *feedback linearizable* nor *affine* in the control input, which have been two basic assumptions made in all the existing ADD nonlinear control schemes. Using the so-called *adding a power integrator technique* developed recently in [15], we solve the ADD problem via static smooth state feedback, under a set of growth conditions that can be viewed as a *high-order* version of the feedback linearization conditions. We also show how to explicitly construct a smooth state feedback controller that attenuates the disturbance's effect on the output to an arbitrary degree of accuracy, with internal stability.**

***Index Terms*—Almost disturbance decoupling, high-order nonlinear systems, internal stability, smooth state feedback, uncontrollable linearization.**

I. INTRODUCTION

In this paper we investigate the problem of almost disturbance decoupling with internal stability (ADD), for a class of *high-order* nonaffine systems whose Jacobian linearization at the equilibrium is not *controllable*. The ADD problem was originally formulated for linear systems by Willems [26] at the beginning of 1980's. Since then, the problem has attracted considerable attention and many important results have been obtained for both linear and nonlinear systems. For reasons of space, it will not be possible to review here all the developments in linear systems, so we refer the reader to the papers (e.g., [26] and [23]) and the references therein for details.

For affine nonlinear systems, the problem of disturbance decoupling was one of the main subjects in nonlinear control theory. During the period of 1980–1990, the problem was extensively studied from a differential geometry point of view. A complete solution to the problem can be found now, for instance, in the textbooks [8] and [21], where a necessary and sufficient condition is given for the problem of *exact disturbance decoupling* to be solvable by static smooth state feedback. On the contrary, the investigation of the so-called problem of *almost disturbance decoupling* started relatively late. In fact, the first paper on the ADD problem appeared in the literature only about a decade ago

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