# Comments on "Adaptive Variable Structure Set-Point Control of Underactuated Robots"

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Recently, an adaptive variable structure controller was proposed in the above note[1] to solve the set-point problem for underactuated robots. Specifically, a time-varying gain, denoted by $k(t) \in \mathbb{R}$, was utilized in the control development and stability analysis which was generated via a discontinuous differential equation. Unfortunately, due to the discontinuous nature of the time derivative of $k(t)$, erroneous arguments were utilized in the stability analysis which yield unexpected stability results.

To illustrate that the stability arguments are invalid, we propose the following counter example

$$\dot{x}_1(t) = x_1(t)$$
$$\dot{x}_2(t) = u_a(t) \tag{1}$$

where $x_1(t),\ x_2(t) \in \mathbb{R}$ are states of the system, and $u_a(t) \in \mathbb{R}$ represents the control input. It is clear from (1) that $x_1(t)$ is *uncontrollable* and *unstable*. Based on the controller developed in the note,[1] the control input for the above system can be designed as follows:

$$u_a = -k_a x_2 - u_c \tag{2}$$

where the auxiliary control input, denoted by $u_c(t) \in \mathbb{R}$, is designed as follows:

$$u_c = \frac{(1+k)x_2}{x_2^2 + \delta}\,(k_u + 1)x_1^2 \tag{3}$$

where $k(t) \in \mathbb{R}$ is a positive, time-varying control gain that is generated via the following discontinuous differential equation:

$$\dot{k} = \begin{cases} \dfrac{\eta}{k}\left(\dfrac{kx_2^2 - \delta_1}{x_2^2 + \delta}\right)(k_u + 1)x_1^2, & \text{if } k \neq 0 \\ \delta, & \text{if } k = 0 \end{cases} \tag{4}$$

$k_a,\ k_u,\ \eta \in \mathbb{R}$ are positive, constant design parameters, and $\delta,\ \delta_1 \in \mathbb{R}$ are small positive constants, selected according to the following inequality:

$$\delta < \delta_1. \tag{5}$$

Based on the stability analysis presented in the note,[1] we define a nonnegative function, denoted by $V(t) \in \mathbb{R}$, to examine the stability of the system given in (1) as follows:

$$V = \frac{1}{2}\,x_1^2 + \frac{1}{2}\,x_2^2 + \frac{1}{2}\,\frac{k^2}{\eta}. \tag{6}$$

After taking the time derivative of (6) and substituting (1) for $x_1(t)$ and $x_2(t)$, we obtain the following expression:

$$\dot{V} = x_1^2 + x_2 u_a + \frac{k\dot{k}}{\eta}. \tag{7}$$

Then, we can substitute (2) for $u_a(t)$ and add/subtract the product $k_u x_1^2$ to the right side of (7) to conclude that

$$\dot{V} = -k_u x_1^2 - k_a x_2^2 - x_2 u_c + \frac{k\dot{k}}{\eta} + (k_u + 1)x_1^2. \tag{8}$$

After substituting (3) and (4) into (8) for $u_c(t)$ and $\dot{k}(t)$, respectively, and cancelling common terms, we have

$$\dot{V} = -k_u x_1^2 - k_a x_2^2 + \frac{1}{x_2^2 + \delta}\,(\delta - \delta_1)(k_u + 1)x_1^2, \qquad \text{for } k \neq 0. \tag{9}$$

Based on (5), it is clear that

$$\dot{V} \le -k_u x_1^2 - k_a x_2^2 \le 0, \qquad \text{for } k \neq 0 \tag{10}$$

hence, $x_1(t),\ x_2(t) \in \mathcal{L}_2$. Furthermore, since standard signal chasing arguments can be utilized to prove that $\dot{x}_1(t),\ \dot{x}_2(t) \in \mathcal{L}_\infty$ (which is a sufficient condition for $x_1(t),\ x_2(t)$ to be uniformly continuous), we can use Barbalat's Lemma to prove that

$$\lim_{t \to 0} x_1(t),\ x_2(t) = 0. \tag{11}$$

From (1), it is obvious that the result given in (11) is impossible since $x_1(t)$ is *uncontrollable* and *unstable*. Note that the stability analysis presented in the note[1] does not examine the case when $k(t) = 0$, and, hence, it is not clear if the result given in the note[1] is valid for this case. Thus, based on this counter example, it appears that the piecewise continuous nature of the control development and stability analysis presented in the note[1] is erroneous.

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[1]C.-Y. Su and Y. Stepanenko, *IEEE Trans. Automat. Contr.*, Vol. 44, pp. 2090–2093, Nov. 1999.

## Author's Reply

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We would like to thank Dr. De Luca and Dr. Oriolo, Dr. Zhang, and Dr. Dixon and Dr. Zergeroglu for their inputs and interest with regards to the above note.[1] Through the use of counter-examples, they came across demonstrated results, which differed from those expected.

From the comments, we feel that there are important issues that were not duly emphasized in our note.

3) The constrained condition on the unactuated dynamic equation (3). The basic requirement is that the constrained equation (3) must be nonintegrable. If this is not the case, the second-order

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[1]C.-Y. Su and Y. Stepanenko, *IEEE Trans. Automat. Contr.*, Vol. 44, pp. 2090–2093, Nov. 1999.