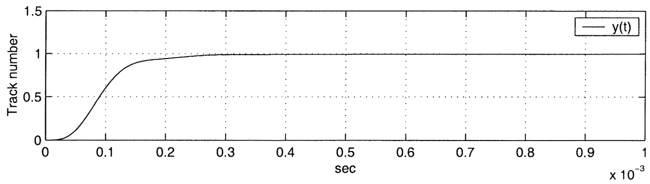

Fig. 15. Track seeking performance of the hard disk servo system.

APPENDIX A
PROOF OF THEOREM 1

By the stability assumption, $|S(j\omega)|$ is continuous and bounded. Given $\epsilon > 0$, choose ω_0 such that

$$||S(j0)|^2 - |S(j\omega)|^2| < \frac{\epsilon}{2} \qquad \forall \omega < \omega_0 \tag{A.1}$$

is satisfied. Using (A.1) and the fact that $\|F(s;\Omega)\|_2 = 1$, the following chain of inequalities holds:

$$\begin{aligned} &|RS(\Omega)^2 - |S(j0)|^2| \\ &\quad = \left|\frac{1}{\pi}\int_0^\infty |F(j\omega;\Omega)|^2(|S(j\omega)|^2 - |S(j0)|^2)\,d\omega\right| \\ &\quad \le \frac{1}{\pi}\int_0^\infty |F(j\omega;\Omega)|^2 ||S(j\omega)|^2 - |S(j0)|^2|\,d\omega \\ &\quad = \frac{\epsilon}{2}\frac{1}{\pi}\int_0^{\omega_0} |F(j\omega;\Omega)|^2 d\omega \\ &\qquad + M_r^2\frac{1}{\pi}\int_{\omega_0}^\infty |F(j\omega;\Omega)|^2 d\omega \\ &\quad < \frac{\epsilon}{2} + M_r^2\frac{1}{\pi}\int_{\omega_0}^\infty |F(j\omega;\Omega)|^2 d\omega. \end{aligned} \tag{A.2}$$

The term $M_r^2(1/\pi)\int_{\omega_0}^\infty |F(j\omega;\Omega)|^2 d\omega$ can be made arbitrarily small if Ω is sufficiently small. Hence, we obtain

$$|RS(\Omega)^2 - |S(j0)|^2| < \epsilon \tag{A.3}$$

for Ω sufficiently small. This proves i).

Part ii) can be proved similarly. Finally, part iii) follows directly from (4). ■

REFERENCES

[1] G. F. Franklin, J. D. Powell, and M. L. Workman, *Digital Control of Dynamic Systems*, 2nd ed. Reading, MA: Addison-Wesley, 1998.

[2] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Upper Saddle River, NJ: Prentice-Hall, 1996.

[3] T. B. Goh, Z. Li, B. M. Chen, T. H. Lee, and T. Huang, "Design and implementation of a hard disk drive servo system using robust and perfect tracking approach," *IEEE Trans. Contr. Syst. Technol.*, vol. 9, pp. 221–233, Feb. 2001.

Comments on "A Composite Energy Function-Based Learning Control Approach for Nonlinear Systems With Time-Varying Parametric Uncertainties"

W. E. Dixon and J. Chen

***Abstract*—In the above paper, a composite energy function learning control approach was proposed to asymptotically eliminate the mismatch between the desired and actual periodic trajectory of a system. Upon review of this result, there appear to be several philosophical and technical issues that invalidate the result including the use of a resetting condition and the lack of boundedness of the learning estimate. The intent of this comment is to highlight these technical errors, especially since the boundedness of the learning estimate has historically been a problematic issue.**

***Index Terms*—Learning systems, Lyapunov methods, periodic systems.**

I. COMMENTARY

In [11], a so-called composite energy function (CEF)-based learning control approach was proposed to asymptotically eliminate the mismatch between the desired and actual periodic trajectory of a system containing nonglobal Lipschitzian functions and unknown, time-varying periodic parameters. This is an important problem that has been examined by various researchers using Lyapunov-based techniques. A few examples of these results are provided in [3]–[6], and [8] (for an in-depth overview of various learning controllers, see [9] and [10]). Although eliminating the mismatch between the desired and actual periodic trajectory of a system containing a general periodic nonlinear function is well motivated, the result in [11] formulates the problem in a manner that yields an impractical controller that lacks robustness. For example, the problem formulated in [11] requires that the parametric uncertainty of the actual system be periodic. It is not clear what actual control problem has naturally occurring time-varying, periodic parametric uncertainty. A more realistic (and previously solved) problem is based on the practical assumption that the desired trajectory is periodic, resulting in a disturbance by a nonlinear function that is composed of parametric uncertainty as a function of the desired trajectory that can be bounded by a known constant. The CEF approach is also predicated on the restrictive resetting condition (i.e., as stated in [11, Remark 3], the assumption that $e_i(0) = 0 \; \forall i \in \mathcal{N}_+$ is crucial for the CEF algorithm). That is, the system is required to return to the same initial configuration after each learning trial. This assumption is similar to the early betterment learning controllers (see [1] and [2]). However, several authors have demonstrated the deficiency and lack of robustness of controllers that are formulated based on this assumption. For example, Heinzinger *et al.* provided several examples in [7] that illustrated the lack of robustness of these controllers to variations in the initial conditions of the system. Motivated by the results from the betterment learning research, several researchers investigated the use of repetitive learning controllers. One of the advantages of the repetitive learning

Manuscript received December 4, 2002. Recommended by Associate Editor W. Kang. This work was supported in part by the United States Department of Energy (DOE) Office of Biological and Environmental Research (OBER) Environmental Management Sciences Program (EMSP) under Project ID 82797 at Oak Ridge National Laboratory, Oak Ridge, TN.

W. E. Dixon is with the Engineering Science and Technology Division, Robotics, Oak Ridge National Laboratory, Oak Ridge, TN 37831-6305 USA.

J. Chen is with the Department of Electrical and Computer Engineering, Clemson University, Clemson, SC 29634-0915 USA (e-mail: dixonwe@ornl.gov).

Digital Object Identifier 10.1109/TAC.2003.817003

0018-9286/03$17.00 © 2003 IEEE

scheme is that the requirement for the system to return to the exact same initial condition after each learning trial is replaced by the less restrictive requirement that the desired trajectory of the system be periodic (the result in [11] requires both assumptions). An example of a Lyapunov-based controller that solves the problem of eliminating the mismatch between the desired and actual periodic trajectory of a system containing a general unknown and bounded, time-varying periodic nonlinear function is provided in [4]. A hybrid learning/adaptive controller is also presented in [4] that illustrates how controllers can be constructed to compensate for periodic disturbances via a learning control element and nonperiodic disturbances via an adaptive control element.

From a review of the CEF approach, there also appear to be several technical issues. The first issue is concerned with the proof that the time derivative of the nonnegative CEF is negative. Specifically, to examine the stability of the developed controller, a CEF denoted by $E_i(t) \in \mathbb{R}$, is defined in [11] as follows:

$$E_i = V(e_i) + \frac{1}{2\beta_v}\int_0^t \mathrm{trace}[(\Psi_i - \Theta)^T(\Psi_i - \Theta)]\,d\tau \qquad (1)$$

$\forall i \in \mathcal{N}_+$ where $V(e_i) \in \mathbb{R}$ is a nonnegative function, $\beta_v \in \mathbb{R}$ denotes a learning gain, $\Theta(t) \in \mathbb{R}^{m\times n_1}$ is an unknown continuous and bounded, time-varying periodic parameter matrix, $\Psi_i(t) \in \mathbb{R}^{m\times n_1}$ is learning estimate for $\Theta(t)$, and $e_i(t) \in \mathbb{R}^n$ denotes the tracking error during the ith learning cycle defined as follows:

$$e_i = x_i - x_d \quad \forall i \in \mathcal{N}_+. \qquad (2)$$

In (2), $x_i(t) \in \mathbb{R}^n$ denotes the output state, and $x_d(t) \in \mathbb{R}^n$ denotes the desired state. Development is then provided to prove that the time derivative of (1) can be upper bounded during the first learning cycle as follows:

$$\dot{E}_1(t) \le -\gamma_3(\|e_1\|) - \frac{\beta_v}{2}\alpha_1\alpha_1^T\xi^T\xi + \frac{1}{2\beta_v}\mathrm{trace}(\Theta^T\Theta). \qquad (3)$$

In (3), $\gamma_3 \in \mathbb{R}$ denotes a positive bounding function, and $\xi(x_i, t) \in \mathbb{R}^{n_1}$ denotes a known vector function that may include global and local Lipschitzian functions as a subset. An assertion is then made that since $\Theta(t)$ is a continuous and bounded function, there will always exist a sufficiently large error $e_1(t)$ such that

$$\gamma_3(\|e_1\|) \ge \frac{1}{2\beta_v}\mathrm{trace}(\Theta^T\Theta) \qquad (4)$$

then

$$\dot{E}_1(t) \le 0. \qquad (5)$$

However, there is no development that proves that $e_1(t)$ will not remain small for large $\Theta(t)$ and, hence, the inequality given in (4) may be invalid, and the time derivative of the CEF may be positive. Thus, the CEF may become unbounded. Moreover, since [11, Part B] is predicated on (5), perfect learning convergence may not be ensured.

The boundedness of the learning estimate and the control is a second technical issue. Specifically, [11] asserts that

$$\int_0^T \mathrm{trace}[(\Psi_i - \Theta)^T(\Psi_i - \Theta)]\,d\tau \le 2\beta_v E_1(T). \qquad (6)$$

However, as stated previously, $E_1(T)$ may not be upper bounded due to the possible fallacy of (4). Moreover, assuming (4) could somehow be proven to be valid, the result in [11] only asserts that the control input is a member of $\mathcal{L}_2$ over an interval. However, the fact that the control input is a member of $\mathcal{L}_2$, does not prove that the control (or the learning estimate) is bounded (i.e., a member of $\mathcal{L}_\infty$). The lack of boundedness of the learning estimate is a significant problem that has historically been an issue for learning controllers. An in-depth discussion of this issue is provided in [4].

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. Robot. Syst.*, vol. 1, no. 2, pp. 123–140, 1984.

[2] ——, "Realization of robot motion based on a learning method," *IEEE Trans. Syst., Man, Cybern.*, vol. 18, pp. 126–134, Feb. 1988.

[3] B. Costic, M. S. de Queiroz, and D. M. Dawson, "A new learning control approach to the active magnetic bearing benchmark system," in *Proc. Amer. Control Conf.*, Chicago, IL, 2000, pp. 2639–2643.

[4] W. E. Dixon, E. Zergeroglu, D. M. Dawson, and B. T. Costic, "Repetitive learning control: A Lyapunov-based approach," *IEEE Trans. Syst., Man, Cybern. B*, vol. 32, pp. 538–545, Aug. 2002.

[5] C. Ham, Z. Qu, and R. Johnson, "A nonlinear iterative learning control for robot manipulators in the presence of actuator dynamics," *Int. J. Robot. Automat.*, vol. 15, no. 3, pp. 119–130, 2000.

[6] C. Ham, Z. Qu, and J. Kaloust, "Nonlinear learning control for a class of nonlinear systems," *Automatica*, vol. 37, no. 3, pp. 419–428, 2001.

[7] D. Heinzinger, B. Fenwick, B. Paden, and F. Miyazaki, "Robot learning control," in *Proc. IEEE Conf. Decision Control*, 1989, pp. 436–440.

[8] W. Messner, R. Horowitz, W.-W. Kao, and M. Boals, "A new adaptive learning rule," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 188–197, Feb. 1991.

[9] K. L. Moore, "Iterative learning control—An expository overview," *Appl.Comput. Controls, Signal Processing, Circuits*, vol. 1, pp. 151–214, 1999.

[10] ——, "Iterative learning control for deterministic systems," in *Advances in Industrial Control*, London, U.K.: Springer-Verlag, 1993.

[11] J.-X. Xu and Y. Tan, "A composite energy function-based learning control approach for nonlinear systems with time-varying parametric uncertainty," *IEEE Trans. Automat. Contr.*, vol. 47, pp. 1940–1945, Nov. 2002.

Authors' Reply

Jian-Xin Xu and Ying Tan

I. COMMENTARY

First of all, we would like to thank Drs. Dixon and Chen for their comments on our paper [1], which presents a new iterative learning control (ILC) scheme based on the composite energy function (CEF) approach, and handles systems with local Lipschitz nonlinearities and time-varying parametric uncertainties.

According to Drs. Dixon and Chen's comments [2], there exist four problems in our paper. Let us briefly summarize them as follows.

1) Our control problem, formulated with the periodic time-varying parametric uncertainty, is not practical.
2) Our assumption on identical initialization condition (i.i.c.), which assumes a restrict resetting condition, lacks robustness.

Manuscript received May 19, 2003. Recommended by Associate Editor W. Kang.

J.-X. Xu is with the Department of Electrical and Computer Engineering, National University of Singapore (e-mail: elexujx@nus.edu.sg).

Y. Tan is with the Department of Chemical Engineering, Mcmaster University, Hamilton, ON L8S 4L7, Canada.

Digital Object Identifier 10.1109/TAC.2003.817004

0018-9286/03$17.00 © 2003 IEEE