Similarly, from (A17), (A20) and $\underline{\beta} \in R^+$, we can get

$$-(\underline{\alpha} u_{k+1} + \underline{\beta} v_{k+1})\sigma - (\underline{\alpha} u_{k+1} + \underline{\beta} v_{k+1}) \leq 0. \tag{A23}$$

By (A18), (A19), (A22) and (A23), it can be ensured that $-1 \leq \sigma \leq 1$. Then, it follows from (A20) that there exists $\tilde{y} = [y_1 \ \cdots \ y_k \ \sigma]^T \in \partial D_1^{(k+1)+}$ satisfying $u\tilde{y} = 0$ and $v\tilde{y} > 0$. This contradicts with (A12). So, we can conclude that if Proposition 1 is true when $n = k$ then Proposition 1 is true also when $n = k + 1$.

From (i)–(iii), it is easy to see that Proposition 1 always holds. This completes the proof of Proposition 1.

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Modular Adaptive Control of Uncertain Euler–Lagrange Systems With Additive Disturbances

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***Abstract*—A novel adaptive nonlinear control design is developed which achieves modularity between the controller and the adaptive update law. Modularity between the controller/update law design provides flexibility in the selection of different update laws that could potentially be easier to implement or used to obtain faster parameter convergence and/or better tracking performance. For a general class of linear-in-the-parameters (LP) uncertain Euler-Lagrange systems subject to additive bounded non-LP disturbances, the developed controller uses a model-based feedforward adaptive term in conjunction with the recently developed robust integral of the sign of the error (RISE) feedback term. Modularity in the adaptive feedforward term is made possible by considering a generic form of the adaptive update law and its corresponding parameter estimate. This generic form of the update law is used to develop a new closed-loop error system and stability analysis that does not depend on nonlinear damping to yield the modular adaptive control result.**

***Index Terms*—Linear-in-the-parameters (LP), robust integral of the sign of the error (RISE).**

I. INTRODUCTION

A variety of adaptive control results have been developed to compensate for linear-in-the-parameters (LP) uncertainty in nonlinear systems. Most of this research has exploited Lyapunov-based techniques (i.e., the controller and the adaptive update law are designed in concert based on a Lyapunov analysis); however, Lyapunov-based methods restrict the design of the adaptive update law. For example, many of the previous adaptive controllers are restricted to utilizing gradient update laws to cancel cross terms in a Lyapunov-based stability analysis. Gradient update laws can potentially exhibit slower parameter convergence which could lead to a degraded transient performance of the tracking error in comparison to other possible adaptive update laws (e.g., least-squares update law). Several results have been developed in literature that aim to augment the typical position/velocity tracking error-based gradient update law including: composite adaptive update laws [1], [2]; prediction error-based update laws [3]–[7], and various least-squares update laws [8]–[10]. The adaptive update law in these results are all still designed to cancel cross terms in the Lyapunov-based stability analysis. In contrast to these results, researchers have also developed a class of modular adaptive controllers (cf. [3], [5]–[7]) where a feedback mechanism is used to stabilize the error dynamics provided certain conditions are satisfied on the adaptive update law. For example, nonlinear damping [4] is typically used to yield an input-to-state stability (ISS) result with respect to the parameter estimation error where

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it is assumed a priori that the update law yields bounded parameter estimates. Often the modular adaptive control development exploits a prediction error in the update law (e.g., see [1], [3]–[6]), where the prediction error is often required to be square integrable (e.g., [3], [5], [6]). A brief survey of modular adaptive control results is provided in [3]

Recently, a new high gain feedback control strategy coined the robust integral of the sign of the error (RISE) in [11] was developed to accommodate for sufficiently smooth bounded disturbances. A significant outcome of this control structure is that asymptotic stability is obtained despite general uncertain disturbances. In fact, the early work in [12]–[15] illustrate how different RISE-based controllers/estimation methods yield an asymptotic result for nonlinear systems with uncertainty and additive bounded disturbances without an adaptive feedforward component. Since the RISE method exploits high gain feedback, results such as [11], [16] were developed with various modifications to the stability analysis to amalgamate the RISE feedback with model-based adaptive or neural network feedforward components. The results in [11] experimentally demonstrate the well accepted paradigm that the inclusion of an adaptive feedforward term can reduce the control effort, improve the transient performance, and reduce the steady-state error over feedback only methods. However, the results in [11], [16] were developed using the typical Lyapunov-based gradient adaptive update law. Since the RISE feedback mechanism alone can yield an asymptotic result without a feedforward component to cancel cross terms in the stability analysis, the research in this technical note is motivated by the following question: *Can the RISE control method be used to yield a new class of modular adaptive controllers?*

The results in this work provide the first investigation of the ability to yield controller/update law modularity using the RISE feedback. The class of nonlinear dynamic systems considered in this technical note are modeled by the Euler–Lagrange formulation which describes the behavior of a large class of engineering systems. The development in this technical note focuses on dynamic systems with structured (i.e., LP) and unstructured uncertainties, and a controller is developed with modularity between the controller/update law, where a model-based adaptive feedforward term is used in conjunction with the RISE feedback term [11] to yield asymptotic tracking. The RISE-based approach is different than previous modular adaptive work (cf. [3], [4], [6]) in the sense that it does not rely on nonlinear damping. The use of the RISE method in lieu of nonlinear damping has several advantages that motivate this investigation including: an asymptotic modular adaptive tracking result can be obtained for nonlinear systems with non-LP additive bounded disturbances; the dual objectives of asymptotic tracking and controller/update law modularity are achieved in a single step unlike the two stage analysis required in some results (cf., [3], [6]); the development does not require that the adaptive estimates are *a priori* bounded; and the development does not require a positive definite estimate of the inertia matrix or a square integrable prediction error as in [3], [6]. Modularity in the adaptive feedforward term is made possible by considering a generic form of the adaptive update law and its corresponding parameter estimate. The general form of the adaptive update law includes examples such as gradient, least-squares, and etc. This generic form of the update law is used to develop a new closed-loop error system, and the typical RISE stability analysis is modified to accommodate the generic update law. New sufficient gain conditions are derived to prove an asymptotic tracking result.

The class of RISE-based modular adaptive controllers can be extended to include uncertain dynamic systems that do not satisfy the LP assumption. Neural networks (NNs) have gained popularity as a feedforward adaptive control method that can compensate for non-LP uncertainty in nonlinear systems. Since multilayer NNs are nonlinear in the weights, a challenge is to derive weight tuning laws in closed-loop feedback control systems that yield stability as well as bounded weights. The preliminary development in [17] illustrates how to extend the class of modular adaptive controllers for NNs.

While the current result encompasses a large variety of adaptive update laws, an update law design based on the prediction error is not possible because the formulation of a prediction error requires the system dynamics to be completely LP. Future efforts can focus on developing a RISE-based adaptive controller for a completely LP system that could also use a prediction error/torque filtering approach. Also, one of the shortcomings of current work is that only a semi-global asymptotic stability is achieved, however, the region of attraction can be made arbitrarily large to include any initial conditions by increasing the control gain. Further investigation is needed to achieve a global stability result. Inroads to solve the global tracking problem are provided in [18] under a set of assumptions.

II. Dynamic Model and Properties

The class of nonlinear dynamic systems considered in this technical note can be described by the following Euler–Lagrange formulation:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d(t) = \tau(t). \qquad (1)$$

In (1), $M(q) \in \mathbb{R}^{n\times n}$ denotes the inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes the gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ denotes friction, $\tau_d(t) \in \mathbb{R}^n$ denotes a general nonlinear disturbance (e.g., unmodeled effects), $\tau(t) \in \mathbb{R}^n$ represents the torque input control vector, and $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration vectors, respectively. The subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable and that $M_(q)$, $V_m(q,\dot{q})$, $G(q)$, $F(\dot{q})$ and $\tau_d(t)$ are unknown. Throughout the technical note $|\cdot|$ denotes the absolute value of the scalar argument, $\|\cdot\|$ denotes the standard Euclidean norm for a vector or the induced infinity norm for a matrix, and $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. The following properties and assumptions will be exploited in the subsequent development. **Property 1**: The inertia matrix $M(q)$ is symmetric, positive definite, and satisfies $m_1\|\xi\|^2 \leq \xi^T M(q)\xi \leq \bar{m}(q)\|\xi\|^2 \forall \xi(t) \in \mathbb{R}^n$ where $m_1 \in \mathbb{R}$ is a known positive constant, $\bar{m}(q) \in \mathbb{R}$ is a known positive function. **Property 2**: The functions $M(q), V_m(q,\dot{q}), F(\dot{q})$, and $G(q)$ are second order differentiable such that their second time derivatives are bounded if $q^{(i)}(t) \in \mathcal{L}_\infty$, $i = 0, 1, \ldots, 3$ [14]. **Property 3**: The nonlinear disturbance term and its first two time derivatives, i.e. $\tau_d(t), \dot{\tau}_d(t), \ddot{\tau}_d(t)$ are bounded by known constants. **Property 4**: Part of the dynamics in (1) can be linearly parameterized as

$$Y_d\theta \triangleq M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F(\dot{q}_d) \qquad (2)$$

where $\theta \in \mathbb{R}^p$ contains the constant unknown system parameters, and $Y_d(q_d,\dot{q}_d,\ddot{q}_d) \in \mathbb{R}^{n\times p}$ is the desired regression matrix that contains known nonlinear functions of the desired link position, velocity, and acceleration, $q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \in \mathbb{R}^n$, respectively. **Property 5**: The desired trajectory is assumed to be designed such that $q_d^{(i)}(t) \in \mathbb{R}^n$ $(i = 0, 1, \ldots, 4)$ exist and are bounded.

III. Control Objective

The objective is to design a continuous modular adaptive controller which ensures that the system tracks a desired time-varying trajectory $q_d(t)$ despite uncertainties and bounded disturbances in the dynamic model. To quantify this objective, a position tracking error, denoted by $e_1(t) \in \mathbb{R}^n$, is defined as

$$e_1 \triangleq q_d - q. \tag{3}$$

To facilitate the subsequent analysis, filtered tracking errors, denoted by $e_2(t), r(t) \in \mathbb{R}^n$, are also defined as

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1 \qquad r \triangleq \dot{e}_2 + \alpha_2 e_2 \tag{4}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}$ denote positive constants. The filtered tracking error $r(t)$ is not measurable since the expression in (4) depends on $\ddot{q}(t)$.

IV. Control Development

The open-loop tracking error system can be developed as

$$M(q)r = Y_d\theta + S + \tau_d - \tau \tag{5}$$

where the auxiliary function $Y_d(t)\theta \in \mathbb{R}^n$ was defined in (2), and the auxiliary function $S(q, \dot{q}, t) \in \mathbb{R}^n$ is defined as

$$\begin{aligned} S \triangleq\, & M(q)(\alpha_1\dot{e}_1 + \alpha_2 e_2) + M(q)\ddot{q}_d - M(q_d)\ddot{q}_d \\ & + V_m(q,\dot{q})\dot{q} - V_m(q_d,\dot{q}_d)\dot{q}_d + G(q) - G(q_d) + F(\dot{q}) \\ & - F(\dot{q}_d). \end{aligned} \tag{6}$$

Based on the open-loop error system in (5), the control torque input is composed of an adaptive feedforward term plus the RISE feedback term as

$$\tau \triangleq Y_d\hat{\theta} + \mu. \tag{7}$$

In (7), $\mu(t) \in \mathbb{R}^n$ denotes the RISE feedback term defined as [11], [13]

$$\mu(t) \triangleq (k_s + 1)e_2(t) - (k_s + 1)e_2(0) + v(t) \tag{8}$$

where $v(t) \in \mathbb{R}^n$ is the generalized solution to

$$\dot{v}(t) = (k_s + 1)\alpha_2 e_2 + \beta_1 sgn(e_2), \quad v(0) = 0$$

where $k_s, \beta_1 \in \mathbb{R}$ are positive constant control gains, $Y_d(t)$ was introduced in (2), and $\hat{\theta}(t) \in \mathbb{R}^p$ denotes a subsequently designed parameter estimate vector. The subsequent development exploits the fact that the continuous RISE feedback in (8) has the following time derivative:

$$\dot{\mu}(t) = (k_s + 1)r + \beta_1 sgn(e_2). \tag{9}$$

The closed-loop tracking error system can be developed by substituting (7) into (5) as

$$M(q)r = Y_d(\theta - \hat{\theta}) + S + \tau_d - \mu. \tag{10}$$

To facilitate the subsequent modular adaptive control development and stability analysis, the time derivative of (10) is expressed as

$$M(q)\dot{r} = -\frac{1}{2}\dot{M}(q)r + \tilde{N} + N_B - (k_s+1)r - \beta_1 sgn(e_2) - e_2 \tag{11}$$

where (9) was utilized. In (11), the unmeasurable/unknown auxiliary terms $\tilde{N}(e_1, e_2, r, t), N_B(t) \in \mathbb{R}^n$ are defined as

$$\tilde{N}(t) \triangleq -\frac{1}{2}\dot{M}(q)r + \dot{S} + e_2 + \tilde{N}_0$$

$$N_B(t) \triangleq N_{B_1}(t) + N_{B_2}(t) \tag{12}$$

where $N_{B_1}(t) \in \mathbb{R}^n$ is given by

$$N_{B_1} \triangleq \dot{Y}_d\theta + \dot{\tau}_d \tag{13}$$

and the sum of the auxiliary terms $\tilde{N}_0(t), N_{B_2}(t) \in \mathbb{R}^n$ is given by

$$N_{B_2}(t) + \tilde{N}_0 = -\dot{Y}_d\hat{\theta} - Y_d\dot{\hat{\theta}}. \tag{14}$$

Specific definitions for $\tilde{N}_0(t), N_{B_2}(t)$ are provided subsequently based on the definition of the adaptive update law for $\hat{\theta}(t)$. The structure of (11) and the introduction of the auxiliary terms in (12)–(14) is motivated by the desire to segregate terms that can be upper bounded by state-dependent terms and terms that can be upper bounded by constants. Specifically, depending on how the adaptive update law is designed, analysis is provided in the next section to upper bound $\tilde{N}(t)$ by state-dependent terms and $N_B(t)$ by a constant. The need to further segregate $N_B(t)$, is that some terms in $N_B(t)$ have time derivatives that are upper bounded by a constant, while other terms have time-derivatives that are upper-bounded by state dependent terms. The segregation of these terms based on the structure of the adaptive update law (see (14)), is key for the development of a stability analysis for the modular RISE-based adaptive update law/controller.

V. Modular Adaptive Update Law Development

A key difference between the traditional modular adaptive controllers that use nonlinear damping and the current RISE-based approach is that the RISE-based method does not exploit the ISS property with respect to the parameter estimation error. The current approach does not rely on nonlinear damping, but instead uses the implicit learning characteristic of the RISE technique to compensate for smooth bounded disturbances. In general, previous nonlinear damping-based modular adaptive controllers first prove an ISS stability result provided the adaptive update law yields bounded parameter estimates (e.g., $\hat{\theta}(t) \in \mathcal{L}_\infty$ via a projection algorithm), and then use additional analysis along with assumptions (PD estimate of the inertia matrix, and square integrable prediction error, etc.) to conclude asymptotic convergence. In contrast, since the RISE-based modular adaptive control approach in this technical note does not exploit an ISS analysis, the assumptions regarding the parameter estimate are modified. The following development requires some general bounds (i.e., design criteria) on the structure of the adaptive update law $\dot{\hat{\theta}}(t)$ and the corresponding parameter estimate $\hat{\theta}(t)$ to segregate the components of the auxiliary terms introduced in (12)–(14). Specifically, instead of assuming that $\hat{\theta}(t) \in \mathcal{L}_\infty$, the subsequent development is based on the less restrictive assumption that the parameter estimate $\hat{\theta}(t)$ and the update law $\dot{\hat{\theta}}(t)$ can be described by the following design criteria:

$$\hat{\theta}(t) = f_1(t) + \Phi(q, \dot{q}, e_1, e_2, t) \tag{15}$$

$$\dot{\hat{\theta}}(t) = g_1(t) + \Omega(q, \dot{q}, e_1, e_2, r, t). \tag{16}$$

In (15), $f_1(t) \in \mathbb{R}^p$ is a known function such that

$$\|f_1(t)\| \le \gamma_1 \quad \left\|\dot{f}_1(t)\right\| \le \gamma_2 + \gamma_3\|e_1\| + \gamma_4\|e_2\| + \gamma_5\|r\| \tag{17}$$

where $\gamma_i \in \mathbb{R}$, $(i = 1, 2, \ldots, 5)$ are known non-negative constants (i.e., the constants can be set to zero for different update laws), and

$\Phi(q, \dot{q}, e_1, e_2, t) \in \mathbb{R}^p$ is a known function that satisfies the following bound:

$$\|\Phi(t)\| \leq \rho_1(\|\bar{e}\|)\|\bar{e}\| \tag{18}$$

where the bounding function $\rho_1(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function, and $\bar{e}(t) \in \mathbb{R}^{2n}$ is defined as

$$\bar{e}(t) \triangleq \begin{bmatrix} e_1^T & e_2^T \end{bmatrix}^T. \tag{19}$$

In (16), $g_1(t) \in \mathbb{R}^p$ is a known function such that

$$\|g_1(t)\| \leq \delta_1 \qquad \|\dot{g}_1(t)\| \leq \delta_2 + \delta_3\|e_1\| + \delta_4\|e_2\| + \delta_5\|r\| \tag{20}$$

where $\delta_i \in \mathbb{R}$, $(i = 1, 2, \ldots, 5)$ are known non-negative constants, and $\Omega(q, \dot{q}, e_1, e_2, r, t) \in \mathbb{R}^p$ satisfies the following bound:

$$\|\Omega(t)\| \leq \rho_2(\|z\|)\|z\| \tag{21}$$

where the bounding function $\rho_2(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function, and $z(t) \in \mathbb{R}^{3n}$ is defined as

$$z(t) \triangleq \begin{bmatrix} e_1^T & e_2^T & r^T \end{bmatrix}^T. \tag{22}$$

The design criteria for the adaptive estimate and the adaptive update law is flexible in the sense that any of the terms in (15) and (16) can be removed for any specific update law and estimate. For example if all the error-dependent terms in (15) are removed, then the condition on $\hat{\theta}(t)$ is the same as in the standard nonlinear damping-based modular adaptive methods (i.e., $\hat{\theta}(t) \in \mathcal{L}_\infty$). In this sense, the ISS property with respect to the parameter estimation error is automatically proven by considering this special case of $\hat{\theta}(t)$. The results in this technical note are not proven for estimates or update laws with additional terms that are not included in the generic structure in (15) and (16). For example, a standard gradient-based update law is of the form (16), but the corresponding estimate (obtained via integration by parts) is not of the form (15) due to the presence of some terms that are bounded by the integral of the error instead of being bounded by the error. However, the same gradient-based update law and its corresponding estimate can be used in (7) if a smooth projection algorithm is used that keeps the estimates bounded. As shown in [11], the standard gradient-based update law can be used in (7) without a projection algorithm, yet including this structure in the modular adaptive analysis is problematic because the integral of the error could be unbounded (so this update law could not be used in traditional nonlinear damping based-modular adaptive laws without a projection either). Since the goal in this technical note is to develop a modular update law, a specific update law cannot be used to inject terms in the stability analysis to cancel the terms containing the parameter mismatch error. Instead, the terms containing the parameter mismatch error are segregated depending on whether they are state-dependent or bounded by constant (see (14)).

Based on the development given in (15)–(20), the terms $\tilde{N}_0(t)$ and $N_{B_2}(t)$ introduced in (12)–(14) are defined as

$$\tilde{N}_0(t) \triangleq -\dot{Y}_d\Phi - Y_d\Omega \qquad N_{B_2}(t) \triangleq -\dot{Y}_d f_1 - Y_d g_1. \tag{23}$$

In a similar manner as in [13], the Mean Value Theorem can be used along with the inequalities in (18) and (21) to develop the following upper bound for the expression in (12):

$$\left\|\tilde{N}(t)\right\| \leq \rho(\|z\|)\|z\| \tag{24}$$

where the bounding function $\rho(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function, and $z(t) \in \mathbb{R}^{3n}$ is defined in (22). The following inequalities can be developed based on the expressions in (12), (13), their time derivatives, and the inequalities in (17) and (20):

$$\begin{gathered} \|N_B(t)\| \leq \zeta_1 \qquad \left\|\dot{N}_{B_1}(t)\right\| \leq \zeta_2 \\ \left\|\dot{N}_{B_2}(t)\right\| \leq \zeta_3 + \zeta_4\|e_1\| + \zeta_5\|e_2\| + \zeta_6\|r\| \end{gathered} \tag{25}$$

where $\zeta_i \in \mathbb{R}$, $(i = 1, 2, \ldots, 6)$ are known positive constants.

VI. Stability Analysis

Theorem 1: The controller given in (7), (8), (15) and (16) ensures that all system signals are bounded under closed-loop operation and that the position tracking error is regulated in the sense that $\|e_1(t)\| \to 0$ as $t \to \infty$ provided the control gain k_s introduced in (8) is selected sufficiently large (see the subsequent proof), α_1 and α_2 are selected according to the following sufficient conditions:

$$\alpha_1 > \frac{\beta_2}{4} + \frac{1}{2} \qquad \alpha_2 > \frac{\beta_2}{2} + \beta_3 + \frac{\beta_4}{2} + 1 \tag{26}$$

and β_i $(i = 1, 2, 3, 4)$ are selected according to the following sufficient conditions:

$$\begin{aligned} &\beta_1 > \zeta_1 + \frac{1}{\alpha_2}\zeta_2 + \frac{1}{\alpha_2}\zeta_3 \quad \beta_2 > \zeta_4, \\ &\beta_3 > \zeta_5, \qquad \beta_4 > \zeta_6 \end{aligned} \tag{27}$$

where β_1 was introduced in (8), and $\beta_2 - \beta_4$ are introduced in (30).

Proof: Let $\mathcal{D} \subset \mathbb{R}^{3n+1}$ be a domain containing $y(t) = 0$, where $y(t) \in \mathbb{R}^{3n+1}$ is defined as

$$y(t) \triangleq \begin{bmatrix} z^T(t) & \sqrt{P(t)} \end{bmatrix}^T. \tag{28}$$

In (28), the auxiliary function $P(t) \in \mathbb{R}$ is the generalized solution to the differential equation

$$\dot{P}(t) = -L(t), \quad P(0) = \beta_1 \sum_{i=1}^{n} |e_{2i}(0)| - e_2(0)^T N_B(0) \tag{29}$$

where $e_{2i}(0)$ denotes the ith element of the vector $e_2(0)$, and the auxiliary function $L(t) \in \mathbb{R}$ is

$$\begin{aligned} L(t) \triangleq r^T\left(N_B(t) - \beta_1 sgn(e_2)\right) - \beta_2\|e_1(t)\|\|e_2(t)\| \\ - \beta_3\|e_2(t)\|^2 - \beta_4\|e_2(t)\|\|r(t)\| \end{aligned} \tag{30}$$

where $\beta_i \in \mathbb{R}$ $(i = 1, 2, 3, 4)$ are positive constants chosen according to the sufficient conditions in (27). Provided the sufficient conditions introduced in (27) are satisfied, then $P(t) \geq 0$.

Let $V_L(y, t) : \mathcal{D} \times [0, \infty) \to \mathbb{R}$ be a Lipschitz continuous regular positive definite function defined as

$$V_L(y, t) \triangleq e_1^T e_1 + \frac{1}{2}e_2^T e_2 + \frac{1}{2}r^T M(q) r + P \tag{31}$$

which satisfies $U_1(y) \leq V_L(y, t) \leq U_2(y)$ provided the sufficient conditions introduced in (26), (27) are satisfied. The continuous positive definite functions $U_1(y)$, and $U_2(y) \in \mathbb{R}$ are defined as $U_1(y) \triangleq \lambda_1\|y\|^2$, and $U_2(y) \triangleq \lambda_2(q)\|y\|^2$, where $\lambda_1, \lambda_2(q) \in \mathbb{R}$ are defined as $\lambda_1 \triangleq (1/2)\min\{1, m_1\}$ and $\lambda_2(q) \triangleq \max(1/2)\bar{m}(q), 1$, where m_1, $\bar{m}(q)$ are introduced in Property 1. After taking the time derivative of (31), $\dot{V}_L(y, t)$ is expressed as

$$\dot{V}_L(y, t) = r^T M(q)\dot{r} + \frac{1}{2}r^T \dot{M}(q) r + e_2^T \dot{e}_2 + 2e_1^T \dot{e}_1 + \dot{P}.$$

From (4), (11), (29) and (30), some of the differential equations describing the closed-loop system for which the stability analysis is being performed have discontinuous right-hand sides as

$$\dot{e}_1 = e_2 - \alpha_1 e_1 \qquad \dot{e}_2 = r - \alpha_2 e_2 \tag{32}$$

$$M\dot{r} = -\frac{1}{2}\dot{M}(q)r + \tilde{N} + N_B - (k_s+1)r - \beta_1 sgn(e_2) - e_2 \tag{33}$$

$$\dot{P}(t) = -r^T\left(N_B - \beta_1 sgn(e_2)\right) + \beta_2\|e_1\|\|e_2\| + \beta_3\|e_2\|^2 + \beta_4\|e_2\|\|r\|. \tag{34}$$

Let $f(y,t) \in \mathbb{R}^{3n+1}$ denote the right-hand side of (32)–(34). As described in [19]–[21], the existence of Filippov's generalized solution can be established for (32)–(34). First, note that $f(y,t)$ is continuous except in the set $\{(y,t)|e_2 = 0\}$. From [19]–[21], an absolute continuous Filippov solution $y(t)$ exists almost everywhere (a.e.) so that $\dot{y} \in K[f](y,t)\ a.e.$ Except the points on the discontinuous surface $\{(y,t)|e_2 = 0\}$, the Filippov set-valued map includes unique solutions. Under Filippov's framework, a generalized Lyapunov stability theory can be used (see [22]–[24] for further details) to establish strong stability of the closed-loop system. The generalized time derivative of (31) exists a.e., and $\dot{V}_L(y,t) \in^{a.e.} \dot{\tilde{V}}_L(y,t)$ where

$$\begin{aligned}\dot{\tilde{V}}_L &= \bigcap_{\xi\in\partial V_L(y,t)} \xi^T K\begin{bmatrix}\dot{e}_1^T & \dot{e}_2^T & \dot{r}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & 1\end{bmatrix}^T\\ &= \nabla V_L^T K\begin{bmatrix}\dot{e}_1^T & \dot{e}_2^T & \dot{r}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & 1\end{bmatrix}^T\\ &\subset \begin{bmatrix}2e_1^T & e_2^T & r^T M & 2P^{\frac{1}{2}} & \frac{1}{2}r^T\dot{M}r\end{bmatrix}\\ &\quad\times K\begin{bmatrix}\dot{e}_1^T & \dot{e}_2^T & \dot{r}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & 1\end{bmatrix}^T.\end{aligned}$$

.

After utilizing (4), (11), (9), (16), and (34)

$$\dot{\tilde{V}}_L \subset r^T\tilde{N} - (k_s+1)\|r\|^2 - \alpha_2\|e_2\|^2 - 2\alpha_1\|e_1\|^2 + 2e_2^T e_1 + \beta_2\|e_1\|\|e_2\| + \beta_3\|e_2\|^2 + \beta_4\|e_2\|\|r\|. \tag{35}$$

where the fact that $(r^T - r^T)_i SGN(e_{2i}) = 0$ is used (the subscript i denotes the i^{th} element), where $K[sgn(e_2)] = SGN(e_2)$ [24] such that $SGN(e_{2_i}) = 1$ if $e_{2_i} > 0$, $[-1,1]$ if $e_{2_i} = 0$, and -1 if $e_{2_i} < 0$. Based on the fact that $2e_2^T e_1 \leq \|e_1\|^2 + \|e_2\|^2$, the expression in (24) can be used to upper bound $\dot{\tilde{V}}_L(t)$ using the squares of the components of $z(t)$ as

$$\dot{\tilde{V}}_L \subset -\lambda_3\|z\|^2 - \left[\left(k_s - \frac{\beta_4}{2}\right)\|r\|^2 - \rho\left(\|z\|\right)\|r\|\|z\|\right] \tag{36}$$

where $\lambda_3 \triangleq \min\{2\alpha_1 - (\beta_2/2) - 1, \alpha_2 - (\beta_2/2) - \beta_3 - (\beta_4/2) - 1, 1\}$; hence, α_1, and α_2 must be chosen according to the sufficient condition in (26). After completing the squares for the terms inside the brackets in (36), the following expression is obtained:

$$\dot{\tilde{V}}_L \subset -\lambda_3\|z\|^2 + \frac{\rho^2\left(\|z\|\right)\|z\|^2}{4\left(k_s - \frac{\beta_4}{2}\right)}. \tag{37}$$

The expression in (37) can be further upper bounded as

$$\dot{V}_L(y,t) \subset -U(y) = -c\|z\|^2 \qquad \forall y \in \mathcal{D} \tag{38}$$

for some positive constant c, where

$$\mathcal{D} \triangleq \left\{y \in \mathbb{R}^{3n+1} | \|y\| \leq \rho^{-1}\left(2\sqrt{\lambda_3\left(k_s - \frac{\beta_4}{2}\right)}\right)\right\}$$

where k_s is selected as $k_s > (\beta_4/2)$. Larger values of k_s will expand the size of the domain $\mathcal{D}$. The result in (38) indicates that $\dot{V}_L(y,t) \leq -U(y) \forall \dot{V}_L(y,t) \in \dot{\tilde{V}}_L(y,t)$. The inequality in (38) can be used to show that $V_L(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, standard analysis methods can be used to prove that the control input and all closed-loop signals are bounded, and that $U(y)$ is uniformly continuous in $\mathcal{D}$. Let $\mathcal{S} \subset D$ denote the set defined as $\mathcal{S} \triangleq \{y(t) \in \mathcal{D}|U_2(y(t)) < \lambda_1(\rho^{-1}(2\sqrt{\lambda_3(k_s - (\beta_4/2))}))^2\}$. The region of attraction can be made arbitrarily large to include any initial conditions by increasing the control gain k_s (i.e., a semi-global type of stability result), and hence $c\|z(t)\|^2 \to 0$ and $\|e_1(t)\| \to 0$ as $t \to \infty \forall y(0) \in \mathcal{S}$.

VII. Conclusion

A RISE-based approach was presented to achieve modularity in the controller/update law for a general class of Euler-Lagrange systems. Specifically, for systems with structured and unstructured uncertainties, a controller was employed that uses a model-based feedforward adaptive term in conjunction with the RISE feedback term (see [11]). The adaptive feedforward term was made modular by considering a generic form of the adaptive update law and its corresponding parameter estimate. New sufficient gain conditions were derived to show asymptotic tracking of the desired link position.

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A Structurally Stable Globally Adaptive Internal Model Regulator for MIMO Linear Systems

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***Abstract*—The problem of compensating an uncertain disturbance and/or tracking some reference signals for a general linear MIMO system is studied in this work using the robust regulation theory frame. The disturbances are assumed to be composed by a known number of distinct sinusoidal signals with unknown phases, amplitude and frequencies. Under suitable assumptions, an exponentially convergent estimator of the unknown disturbance parameters is proposed and introduced into the classical robust regulator design to obtain an adaptive controller. This controller guarantees that the closed-loop robust regulation is attained in some neighborhood of the nominal values of the parameters of system. A simulated example shows the validity of the proposed approach.**

***Index Terms*—Frequency estimation, nonlinear systems, regulation theory.**

I. Introduction

In many industrial and defense applications, the noise and vibration compensation is an important problem. Periodic and/or quasi-periodic noises like engine noise in turboprop aircraft [1] and automobiles [2], ventilation noise in HVAC systems [3] and sea wave noise in landing systems [4] represent interesting examples of such signals. The problem of rejecting an unknown sinusoidal signal was first addressed in [5]–[7] and recently in [8], [9] and [10] using an adaptive observer scheme developed in [11]. On the other hand, in [12] the output stabilization problem with disturbance rejection was considered for a class of SISO minimum phase nonlinear systems which can be transformed into an output feedback form. A local solution for a stable SISO system with a single frequency sinusoidal signal satisfying the matching condition, was proposed in [5], while a global solution was given in [8]. This solution was extended in [9] to non-minimum phase systems and in [10] to the case of k frequencies giving a $((n+1)(k+1)+2k(k+1))$-th order controller, being n the dimension of the state space vector. The singularity problem presented in [9] was then fixed in [10] including a stable signal into the transformation determinant to avoid zero crossings. Also, in [7] a supervisory control scheme was proposed for the case of k frequencies and a SISO linear system, considering that the number of frequencies is known and all frequency values lie in a predefined set. Using a high gain feedback technique combined with regulator theory, the problem was treated in [13], considering that the values of the frequencies belong to some pre-specified compact set. If however the values of the frequencies leave this set, the gains of the proposed regulator must be adjusted in order to keep the stability property. Recently, in [14] an universal adaptive controller was proposed for minimum phase systems using K-filters and backstepping technique. For MIMO linear systems, a locally exponentially stable adaptive control law was proposed in [6], using the Youla parameterization. Along the same lines, in this work we propose an adaptive control scheme for the case of MIMO linear systems when the number of the frequencies is known but having their values not necessarily belonging to a pre-defined finite set, relaxing as well the minimum phase and matching conditions. Moreover, the proposed scheme is globally stable and robust with respect to plant parameter variations in some neighborhood of the nominal values. The order of the proposed controller is $(n+k+m(2k+1))$ where m is the number of inputs and k is the number of frequencies.

II. Problem Statement

Consider a linear system described by

$$\dot{x}(t) = Ax(t) + Bu(t) + Dd(t) \quad (1)$$
$$e(t) = Cx(t) + \tilde{Q}d(t)$$
$$d(t) = [d_0, d_1\sin(\alpha_1 t+\phi_1), \cdots, d_k\sin(\alpha_k t+\phi_k)]^T \quad (2)$$

where $x \in \Re^n$ is the state, $u \in \Re^m$ is the input, $d \in \Re^{(k+1)}$ is a disturbance and/or reference signal consisting of a constant signal with unknown magnitude d_0 and k sinusoidal signals with unknown magnitudes d_i, frequencies α_i and phases ϕ_i for $i = 1,\ldots,k$. $e \in \Re^m$ represents the tracking error between the plant output $y = Cx(t)$ and a reference signal $r = -\tilde{Q}d(t)$ and $A, B, C, D, \tilde{Q}$ are matrices of appropriate dimensions, whose parameters may possibly vary in some neighborhood of the nominal values A_0, C_0, B_0, D_0 and $\tilde{Q}_0$. Let $d(t)$ be generated by an exosystem [15], thus the system (1), (2) can be rewritten as

$$\dot{x}(t) = Ax(t) + Bu(t) + Pw(t) \quad (3a)$$
$$e(t) = Cx(t) + Qw(t)$$
$$\dot{w}(t) = Sw(t) \quad (3b)$$

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