Distributed Connectivity Preserving Target Tracking With Random Sensing

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***Abstract*—A networked multiagent system is tasked to cooperatively track a mobile target, where agents experience random loss of sensing due to occlusions resulting from the target moving in a complex environment. A directed random graph is used to model the time-varying availability of the target states to agents, where the connection of the directed edge in the graph is assumed to be probabilistic and evolves according to a two-state Markov Model. An almost sure consensus algorithm is developed for all agents to achieve consensus on the target position. Due to limited communication capabilities (i.e., the agents can only communicate within a certain range), agent motion may result in a disconnected communication network, leading to the failure of consensus to the target states. Motivated to preserve the graph connectivity of the position-dependent communication network, an algebraic-connectivity-based distributed motion controller is developed to ensure that the communication network remains connected during cooperative target tracking. Compared to existing results, our approach allows agents to break existing links when necessary as long as the algebraic connectivity remains positive, which provides more motion freedom for agents in mission operation. Moreover, our approach only requires information exchange within two-hop neighbors to preserve the network connectivity, which eliminates the need of iterative estimation of the algebraic connectivity.**

***Index Terms*—Distributed control, network connectivity, random graphs, target tracking.**

I. Introduction

A networked multiagent system is tasked in the current work to cooperatively track a mobile target in a complex environment (e.g., an urban environment). The agents are able to estimate the target position through local sensors (e.g., cameras or Lidar). However, such local sensors can experience occasional failures and sensor measurements can be interrupted due to occlusions, resulting in inconsistent observation of the target. The lack of persistent sensing motivates methods that allow agents to improve their estimates based on information exchange over a wireless communication network. Since communication capabilities are limited (i.e., the agents can only communicate within a certain range), a connected communication network based on the agents' positions must also be available. Since the motion of agents can lead to a partitioned communication network resulting in a permanent loss of communication between agents, designing a cooperative motion controller to preserve network connectivity is also practically motivated.

In consensus problems (cf., [1] and [2] and references therein), agents are generally required to agree upon certain quantities of interest based on local information exchange. In most consensus applications, dynamic agents interact with other team members over a directed or undirected deterministic network, where it is assumed that the agents can consistently share or exchange information. However, sensors could experience random failures (e.g., the target can move out of the field-of-view of the on-board camera) in practice, resulting in inconsistent interaction among agents. Based on these potential issues, extensions of classical consensus results to random networks is desired. In [3], almost sure agreement is achieved over an undirected random network if the links between any pair of agents are activated independently with a common probability. For general directed random graphs, almost sure consensus is developed in [4] and [5] with the assumption that the existence probability of a strongly connected graph is nonzero. Necessary and sufficient conditions for consensus are reported in [6] for graphs that are generated by an ergodic and stationary random process. A leader–follower containment control over directed random graphs is developed in [7], where almost sure convergence of the followers' states to the convex hull spanned by the static leaders' states is established. When considering communication noise, mean-square-robust consensus over a network with random packet loss is considered in the work of [8] and stochastic consensus with Markovian switching topologies is investigated in [9]. However, few existing results consider consensus to a dynamic equilibrium point of interest (e.g., the time-varying target position) in the presence of occasional sensor failures. Hence, consensus to a time-varying target position over random networks is motivated.

A comprehensive review of recent results on maintaining network connectivity of mobile networks is provided in [10]. Potential-field-based controllers are widely used in the works of [11]–[17] to preserve the network connectivity, where every existing link remains connected during mission operation. However, since agents are not allowed to break the existing links, the mobility of agents can be constrained when performing tasks. Based on the fact that a positive Fiedler value $\lambda_2(L)$ (i.e., the second smallest eigenvalue of the graph Laplacian L) implies network connectivity [18], Laplacian eigenvalue spectrum-based approaches are developed in results such as [19]–[24] to ensure the network connectivity by either ensuring $\lambda_2(L)$ remains positive or maximizing $\lambda_2(L)$. However, to obtain $\lambda_2(L)$, global information is generally required to compute the graph Laplacian, which demands significant communication bandwidth especially for the large-scale networks. Motivated to compute $\lambda_2(L)$ in a distributed manner, a decentralized subgradient algorithm and a consensus-based iterative approach are developed to estimate and maximize the algebraic connectivity in [25] and [26], respectively. In [27], distributed estimation

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of Laplacian eigenvalues is developed based on constrained consensus and non-convex optimization. Other representative results focusing on preserving the network connectivity via distributed estimation of the spectrum of graph Laplacian include [28]–[32].

In this study, motivated by the fact that agents could experience random loss of target sensing, a directed random graph is used to model the time-varying availability of the target states to agents, where the connection of the directed edge in the graph is assumed to be probabilistic and evolves according to a two-state Markov Model. Based on the graph theory and the probability theory, almost sure consensus on the target states is established, which allows each agent to dynamically estimate the target states by only communicating with immediate neighboring agents over a random network. In contrast to classical consensus results for deterministic networks, this study is applicable to cases where only a subset of the agents is able to sense the target and such subsets vary due to target motion and occlusions in the complex dynamic environment. The distributed nature of the result also provides robustness in the sense that occasional failure of any sensor will not lead to failure of the consensus algorithm. Due to position-dependent limited communication capabilities, a connected communication graph enables information exchange among agents in cooperative target tracking. To maintain connectivity of the communication graph, a gradient-based distributed controller is developed to preserve the global network connectivity by ensuring $\lambda_2(L)$ remains positive during target tracking. Compared to most graph Laplacian spectrum-based approaches where global information is required to compute the graph Laplacian, our approach is decentralized in the sense that only local information from two-hop neighbors is used. Moreover, the iterative estimation of $\lambda_2(L)$ adopted in results such as [25]–[32] is eliminated in this paper. Instead, our approach does not rely on estimation of $\lambda_2(L)$ and can ensure $\lambda_2(L)$ will remain positive based on local two-hop information. Moreover, in contrast to results such as [11]–[15] where agent motion is constrained by preserving every existing edge to ensure the global network connectivity, our algebraic-connectivity-based approach relaxes motion constraints by allowing the existing communication links to break when necessary, thereby, reducing agent motion constraints.

II. Problem Formulation

A network of N agents is tasked to cooperatively track a mobile target moving along an unknown trajectory. Let $k \in \mathbb{Z}^+$ represent the discrete time index and $t_\Delta \in \mathbb{R}^+$ be a sampling period. Each agent moves according to the kinematics

$$x_i(k+1) = x_i(k) + t_\Delta u_i(k), i = 1, \ldots, N \tag{1}$$

where $x_i(k) \in \mathbb{R}^2$ denotes the position of agent i at the discrete time instant k, and $u_i(k) \in \mathbb{R}^2$ represents its control input.

Agents exchange information to estimate the target position and perform cooperative tracking over a wireless communication network, which is modeled by an undirected time-varying graph $\mathcal{G}(k) = (\mathcal{V}, \mathcal{E}(k))$, where the node set $\mathcal{V} = \{1, \ldots, N\}$ represents the agents and the edge set $\mathcal{E}(k) \subset \mathcal{V} \times \mathcal{V}$ indicates the communication links between agents. Each agent is assumed to have a limited communication capability that two agents are only able to communicate and exchange information within a distance of R. Specifically, an edge $(i,j) \in \mathcal{E}$ exists if $d_{ij} \leq R$ where $d_{ij} \triangleq \|x_i - x_j\| \in \mathbb{R}^+$. Note that the edge set $\mathcal{E}(k)$ in the communication graph $\mathcal{G}(k)$ is time varying, since existing edges could break when two agents move apart from each other (i.e., $d_{ij} \geq R$) and new edges could be established if two agents move close (i.e., $d_{ij} < R$).

Let the target be indexed as 0, and $x_0(k) \in \mathbb{R}^2$ denotes the target position at k. When considering the target, the time-varying graph $\mathcal{G}(k)$ is augmented as a directed graph $\bar{\mathcal{G}}(k) = (\bar{\mathcal{V}}, \bar{\mathcal{E}}(k))$, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{0\}$ and $\bar{\mathcal{E}} = \mathcal{E}(k) \cup \{(0,i)\}$, where the directed edge $(0,i) \in \bar{\mathcal{E}}$ with $i \in \mathcal{V}$ in $\bar{\mathcal{G}}$ indicates the availability of x_0 to agent i. Since agents experience random loss of target sensing (e.g., sensor occlusion), the set of agents that can sense the target varies with time. To model random sensing of the target, let there be a Bernoulli random variable $\delta_{0i}(k)$ associated with the link $(0,i)$ such that $\delta_{0i}(k) = 1$ if the edge $(0,i)$ exists at k and $\delta_{0i}(k) = 0$ otherwise. It is assumed that, for different sensing links, each $\{\delta_{0i}(k)\}$ is statistically independent, and the random process $\{\delta_{0i}(k)\}$ evolves according to a two-state homogeneous Markov process[1] with stationary state transition probability[2] $p_{0,1}^{(0,i)} > 0$, which indicates that $\bar{\mathcal{G}}(k)$ changes randomly at the transition instant k and remains constant over each interval $[k, k+1)$.

It is assumed that each edge $(i,j) \in \bar{\mathcal{E}}$ is associated with a weight $w_{ij} \in \mathbb{R}^+$, which is assumed to be known initially and indicates how node j evaluates the information collected from node i. No self-loop is considered, i.e., $(i,i) \notin \bar{\mathcal{E}}$, and thus, $w_{ii} = 0\ \forall i \in \bar{\mathcal{V}}$. The adjacency matrix $\bar{A}(k) = [\bar{a}_{ij}(k)] \in \mathbb{R}^{(N+1)\times(N+1)}$ for the augmented graph $\bar{\mathcal{G}}(k)$ is then defined as

$$\bar{a}_{ij}(k) \triangleq \begin{cases} w_{0i}\delta_{0i}(k) & i \in \mathcal{V} \\ w_{ij} & i,j \in \mathcal{V}. \end{cases} \tag{2}$$

Since $\bar{\mathcal{G}}$ is a random graph from (2), $\bar{\mathcal{G}}$ is drawn from a finite sample space $\bar{\mathcal{G}}' = \{\bar{\mathcal{G}}_1, \ldots, \bar{\mathcal{G}}_M\}$, where the size of $\bar{\mathcal{G}}'$ is determined by the power set of $\mathcal{V}$, i.e., $M = |\bar{\mathcal{G}}'| \leq 2^{|\mathcal{V}|}$. All element graphs $\bar{\mathcal{G}}_i \in \bar{\mathcal{G}}'$, $i = \{1, \ldots, M\}$, are directed graphs, which share a common node set $\bar{\mathcal{V}}$ and differ in the edge set due to the random adjacency matrix $\bar{A}$. If a directed graph contains a directed spanning tree, every node has exactly one parent node except for one node, called the root, and the root has directed paths to every other node. Let $\bar{\mathcal{G}}_T$ denote such a directed spanning tree with node 0 (i.e., the target) as the root. Note that, as long as at least one agent can sense the target (i.e., an edge $(0,i)$ exists), the augmented graph $\bar{\mathcal{G}}(k)$ contains $\bar{\mathcal{G}}_T$ as its subgraph since $\mathcal{G}(k)$ is an undirected graph. An objective in this study is to develop a decentralized method to achieve consensus to the target position over the random graph $\bar{\mathcal{G}}(k)$. To achieve this objective, the following assumption is made.

Assumption 1: The target moves sufficiently slow and an observation period $o = mt_\Delta$ exists for $m \in \mathbb{Z}^+$ such that with arbitrarily low probability, no agent has observation of the target over the whole period o. It is further assumed that $\Pr(\cup_{k=k'}^{k'+m} \bar{\mathcal{G}}(k)$ contains $\bar{\mathcal{G}}_T) > 0$ for all $k' \in \mathbb{Z}^+$ during any observation period o.

Assumption 1 implies that with sufficiently high probability, at least one agent can sense the target over any observation period o. Assumption 1 also implies that the velocity of the target is sufficiently slower than the sampling rate of the sensor, which is generally practical, although may limit some applications. Requiring a sufficiently slow

[1]Since the edge state $\delta_{0i}(t)$, $i \in \mathcal{V}$, changes randomly between connection and disconnection with continuous-time t, the random process $\{\delta_{0i}(t)\}$ is a two-state, continuous-time, homogeneous Markov process. It is assumed that the random processes $\{\delta_{0i}(t)\}$ do not change infinitely fast and the sampling time t_Δ is selected sufficiently small such that with arbitrarily high probability $\delta_{0i}(t) = \delta_{0i}(t+t')$ if $0 \leq t' < t_\Delta$ for all $i \in \mathcal{V}$. Provided that t_Δ is sufficiently small, the continuous time $\{\delta_{0i}(t)\}$ is discretized into $\{\delta_{0i}(k)\}$, where $\delta_{0i}(t)$ remains constant during the period t_Δ and only changes randomly at each transition instant.

[2]Since target tracking is performed in an unknown environment, stationary transition probability is considered as a first attempt, which is predetermined based on available environmental information (e.g., possible measurement occlusions and sensor failures). Additional research will consider a more general sensing model with dynamic state transition probability that accounts for the time-varying relative positions between the target and agents and environmental factors (e.g., measurement noise and occasional occlusions).

target is motivated by the fact of limited sensing zone of each agent and such a constraint can be relaxed if a larger sensing zone is applicable. Assumption 1 will be true for the considered random process $\{\delta_{0i}(k)\}$. Specifically, since different edges $\{(0,i)\}$ are statistically independent, there always exists a nonzero probability that no agent is able to sense the target at a given time instant k. However, Markov properties in [33, ch. 6] indicate that the probability of staying at a state (e.g., the worst-case scenario that no agent observes the target) over a period o exponentially decreases with respect to the increase of o. Therefore, a sufficiently long observation period o can be adopted such that at least one agent can sense the target within the period o. The condition $\Pr(\cup_{k=k'}^{k'+m}\bar{\mathcal{G}}(k)$ contains $\bar{\mathcal{G}}_T) > 0$ for all $k' \in \mathbb{Z}^+$ in Assumption 1 is mild since it only requires nonzero probability of the existence of $\bar{\mathcal{G}}_T$ in the union of $\bar{\mathcal{G}}(k)$, which will be true as long as one agent senses the target during the period o.

Agents need to communicate over $\mathcal{G}(k)$ and coordinate their motion to perform cooperative target tracking. Due to the limited communication capability, agent motion can partition the position-dependent graph $\mathcal{G}(k)$, leading to permanent loss of communication and mission failure. Therefore, another objective in this paper is to design a cooperative controller to perform target tracking while ensuring the global connectivity of the graph $\mathcal{G}(k)$. Since wireless communication quality degrades with distance (e.g., path loss), the weight w_{ij} associated with each edge $(i,j)\in\mathcal{E}$ in the communication graph $\mathcal{G}(k)$ is designed as $w_{ij}(k) = f_w(x_i(k), x_j(k))$, where $f_w : \mathbb{R}^+ \to [0,1]$ is a smooth nonlinear function. The function f_w assumes a constant value of one when the distance between two agents (i.e., d_{ij}) is less than the user-defined maximum threshold $\rho_w < R$, and decreases to zero as $d_{ij} \to R$. The Laplacian matrix $L(\mathbf{x}(k)) \in \mathbb{R}^{N\times N}$ associated with $\mathcal{G}(k)$ is defined based on the pairwise interagent distance as

$$[L(\mathbf{x}(k))]_{ij} = \begin{cases} -w_{ij}(k) & i\neq j \\ \sum_{j\neq i} w_{ij}(k) & i = j \end{cases} \quad (3)$$

where $\mathbf{x}(k) \triangleq \left[x_1^T(k), \ldots, x_N^T(k)\right]^T$. Let $\lambda_1(\mathbf{x}(k)) \leq \lambda_2(\mathbf{x}(k)) \leq \cdots \leq \lambda_N(\mathbf{x}(k))$ be the ordered eigenvalues of $L(\mathbf{x}(k))$. From the definition of $L(\mathbf{x}(k))$ in (3), it is well known from [18] that $\lambda_1(\mathbf{x}(k)) = 0$ with the corresponding eigenvector $\mathbf{1}_N$ (i.e., an N-dimensional vector with all entries equal to 1), and $\lambda_2(\mathbf{x}(k)) > 0$ if and only if $\mathcal{G}(k)$ is connected. The one-hop neighbor set of node i is denoted as $\mathcal{N}_i = \{j|\,(i,j)\in\mathcal{E}\}$. Since the graph $\mathcal{G}(k)$ is undirected, $i\in\mathcal{N}_j$ indicates that $j\in\mathcal{N}_i\ \forall i,j\in\mathcal{V}, i\neq j$.

III. Target Estimation Over Random Graphs

Consider the random graph $\bar{\mathcal{G}}(k)$, which consists of all agents and the target. Let $\theta_i(k)\in\mathbb{R}^2$ be agent i's local estimate of the target position, and $\theta_0(k) = x_0(k)$ denotes the target position at time k. Note that the availability of target state $\theta_0(k)$ to the agents is time varying due to random loss of target sensing. The stacked deterministic estimates at k are denoted as $\theta_k \triangleq [\theta_0(k), \ldots, \theta_N(k)]^T \in \mathbb{R}^{2(N+1)}$. Let $\Theta_k \triangleq [\Theta_0(k), \ldots, \Theta_N(k)]^T \in \mathbb{R}^{2(N+1)}$ be the corresponding random variables of θ_k. Due to the potential random loss of target sensing, each agent $i\in\mathcal{V}$ updates its estimate of the target position according to

$$\Theta_i(k+1) = \Theta_i(k) - \sum_{j\in\bar{\mathcal{N}}_i(k)} t_\Delta \bar{a}_{ij}(k)(\Theta_i(k) - \Theta_j(k)) \quad (4)$$

where the random variable $\bar{a}_{ij}(k)$ is defined in (2), and $\bar{\mathcal{N}}_i(k) \triangleq \{j\in\bar{\mathcal{V}} \mid (j,i)\in\bar{\mathcal{E}}(k)\}$ is the time-varying neighbor set of agent i in $\bar{\mathcal{G}}(k)$. Note that (4) is a stochastic system, and $\{\Theta_k\}$ is a random sequence. Since the systems in (4) along different dimensions are decoupled, for the simplicity of presentation, $\Theta_i(k)$ will be treated as a scalar in the subsequent analysis, where $\Theta_i(k)$ can be trivially extended to two dimensional states by using the Kronecker product.

In this section, almost sure consensus to the target position is established for the agreement protocol in (4) over the random graph $\bar{\mathcal{G}}(k)$. Despite the random loss of target sensing modeled by $\{\delta_{0i}(k)\}$, a connected $\mathcal{G}(k)$ is necessary for agents to communicate to achieve consensus. Since Section IV will prove that graph connectivity is preserved for all k if $\mathcal{G}(0)$ is connected initially, the subsequent development in Section III is based on a connected $\mathcal{G}(k)$.

Since $\theta_0(k)$ denotes the target position, cooperative estimation of the target position is achieved if all local estimates of agents achieve consensus (i.e., $\theta_0(k) = \cdots = \theta_N(k)$). To capture the disagreement of local estimates, let $z(\theta_k)\in\mathbb{R}$ be defined as

$$z(\theta_k) \triangleq \max_{i\in\bar{\mathcal{V}}} \theta_i(k) - \min_{i\in\bar{\mathcal{V}}} \theta_i(k) \quad (5)$$

and let $Z_k\in\mathbb{R}$ be the corresponding random variable of $z(\theta_k)$ in (5).

To facilitate the subsequent convergence analysis, the definitions of almost sure convergence and supermartingale in a probabilistic setting are introduced.

Definition 1: [3] Let θ^* be a state in the agreement space $\mathcal{A} = \{c\mathbf{1}_{N+1} | c\in\mathbb{R}\} \in \mathbb{R}^{N+1}$. The random sequence $\{\Theta_k\}$ in $\mathbb{R}^{N+1}$ almost surely converges to θ^* , if

$$\lim_{k_0\to\infty} \Pr\left\{\sup_{k\geq k_0} \inf_{\theta^*\in\mathcal{A}} \|\Theta_k - \theta^*\| > \epsilon\right\} = 0 \quad (6)$$

for every $\epsilon > 0$, where $\Pr\{\cdot\}$ denotes probability. Almost sure convergence is also called convergence with probability one (w.p.1).

Definition 2: [33] Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a measurable space. A filtration $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \mathcal{F}_2 \ldots \subseteq \mathcal{F}_n$ is an increasing subsequence of sub-σ-algebras of $\mathcal{F}$. A sequence of random variables $Y(k)$ is adapted to a filtration $\mathcal{F}_k$ if $Y(k)$ is $\mathcal{F}_k$-measurable for all k. The pair $(Y, \mathcal{F})$ is called a supermartingale if, for all $k\geq 0$

$$\mathbb{E}[Y(k)] < \infty \text{ and } \mathbb{E}[Y(k+1)|\,\mathcal{F}_k] \leq Y(k) \quad (7)$$

where $\mathbb{E}[Y(k)]$ denotes the expected value of the random variable $Y(k)$, and $\mathbb{E}[\square|\,\lozenge]$ denotes the conditional expectation of some $\square$ under the condition of some $\lozenge$.

The supermartingale sequence $\{Y(k)\}$ in (7) indicates that $\lim_{k\to\infty} Y(k)$ exists and is finite w. p. 1. In addition, the current $Y(k)$ provides an upper bound for the conditional expectation $\mathbb{E}[Y(k+1)|\,\mathcal{F}_k]$ on the next time instant $k+1$. If the sequence $\{Y(k)\}$ is a nonnegative supermartingale with $\mathbb{E}[Y(k)] < \infty$ in (7), $Y(k)$ converges w. p. 1. to a limit [34, ch. 8].

Lemma 1: Consider the random graph $\bar{\mathcal{G}}(k)$ evolving according to the defined two-state Markov model. Suppose that a sufficiently long observation period $o = mt_\Delta,\ m\in\mathbb{Z}^+$ is selected such that Assumption 1 is satisfied. If the state $\theta(k)$ is updated according to (4), the estimate disagreement is nonincreasing in every m steps (i.e., $z(\theta_{k+m}) \leq z(\theta_k)$).

Proof: The update law in (4) can be written in a compact form as

$$\Theta_{k+1} = T\Theta_k \quad (8)$$

where $T = [T_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is the state matrix with diagonal entries

$$T_{ii} = 1 - \sum_{j\in\bar{\mathcal{N}}_i(k)} t_\Delta \bar{a}_{ij}(k) \quad (9)$$

and off-diagonal entries

$$T_{ij} = t_\Delta \bar{a}_{ij}(k) \quad (10)$$

if $j \in \bar{\mathcal{N}}_i$ and $T_{ij} = 0$ otherwise. Since node 0 (i.e., the target) only has outgoing information flow, $\bar{a}_{0j} = 0 \; \forall j \in \mathcal{V}$, which indicates that $T_{00} = 1$ and $T_{0j} = 0$. Provided the sampling time t_Δ is sufficiently small, the diagonal entries T_{ii} in (9) are positive. The off-diagonal entries T_{ij} in (10) are nonnegative, and each row in T sums to one from (9) and (10). Hence, according to the definition of convex hull,[3] given that $\Theta_i(k) = \theta_i(k)$ where $\theta_i(k)$ is a deterministic state at time k, the next time state $\theta_i(k+1)$ for each agent i is a convex linear combination of its current state $\theta_i(k)$ and its neighbors' current states $\theta_j(k)$ for $j \in \bar{\mathcal{N}}_i(k)$. However, there exists a nonzero probability that the target cannot be sensed by any agent at a time instant k. By Assumption 1, the target can be sensed during an observation period o composed of m steps, if the observation period o is sufficiently long. Combined with the fact that the target state θ_0 is available in every observation period o, the linear convex combination at each step indicates that θ_i will not move outside of the convex hull formed by its own state and its neighbors' states, which implies that nonincreasing disagreement in every m steps (i.e., $z(\theta_{k+m}) \leq z(\theta_k)$). ■

Theorem 1: Consider the random graph $\bar{\mathcal{G}}(k)$ evolving according to the defined two-state Markov model and suppose that Assumption 1 is satisfied. Almost sure consensus to the target position is established over the random graph $\bar{\mathcal{G}}(k)$ if all agents follow the update law in (4).

Proof: Consider a time sequence O_k, $k \in \mathbb{Z}^+$, with $O_{k+1} - O_k = o$, where $o = m t_\Delta$ is the observation period defined in Assumption 1. Given the disagreement $z(\theta(O_k))$, consider a function $f(O_{k+1}) \in \mathbb{R}$ as

$$\begin{aligned} f(O_{k+1}) &= \mathbb{E}\left[z(\Theta(O_{k+1})) - z(\Theta(O_k)) \middle| \Theta(O_k) = \theta(O_k)\right] \\ &= \mathbb{E}\left[z(\Theta(O_{k+1})) \middle| \Theta(O_k) = \theta(O_k)\right] - z(\theta(O_k)). \end{aligned} \tag{11}$$

In the aforementioned equation, the state $\theta(O_k)$ evolves according to (4) by following a sequence of m random graphs, where each random graph stays constant for a period of t_Δ. To capture all possible sequences of the evolution from $\theta(O_k)$ to $\theta(O_{k+1})$, let $\check{\mathcal{G}}' = \{\check{\mathcal{G}}_1, \ldots, \check{\mathcal{G}}_s\}$ denote the finite set of all possible sequences over $[O_k, O_{k+1}]$, where $s \in \mathbb{Z}^+$ denotes the size of $\check{\mathcal{G}}'$ and each entry $\check{\mathcal{G}}_i \in \check{\mathcal{G}}'$, $i \in \{1, \ldots, s\}$, denotes a possible sequence that contains m graphs (i.e., $\check{\mathcal{G}}_i \in \prod_{j=1}^m \bar{\mathcal{G}}(j)$). Let $\check{p}_i \triangleq \Pr(\check{\mathcal{G}}_i \in \check{\mathcal{G}}')$ be the probability of the occurrence of the sequence $\check{\mathcal{G}}_i$ in $\check{\mathcal{G}}'$.

Given the definition of $\check{\mathcal{G}}'$ and the associated probability for each entry in $\check{\mathcal{G}}'$, the conditional expectation in (11) is computed as

$$\mathbb{E}\left[z(\Theta(O_{k+1})) \middle| \Theta(O_k) = \theta(O_k)\right] = \sum_{j=1}^{s} z\left(\tilde{T}_j \theta(O_k)\right) \check{p}_j \tag{12}$$

where $\tilde{T}_j = \prod_{i=1}^m T_i$ corresponds to the combined state transition matrix associated with the sequence $\check{\mathcal{G}}_i$ in $\check{\mathcal{G}}'$, and each T_i is the corresponding state transition matrix in (8) for the random graph at the ith period of t_Δ within $[O_k, O_{k+1}]$. By Assumption 1, $\Pr(\cup_{k=k'}^{k'+m} \bar{\mathcal{G}}(k)$ contains $\bar{\mathcal{G}}_T) > 0$ indicates that at least one agent can sense the target within $[O_k, O_{k+1}]$, and there exists a sequence $\check{\mathcal{G}}_T \in \check{\mathcal{G}}'$ such that the probability of the graph union in $\check{\mathcal{G}}_T$ over $[O_k, O_{k+1}]$ containing a directed spanning tree $\bar{\mathcal{G}}_T$ is strictly greater than zero. Let $\check{p}_T$ be the nonzero probability of the occurrence of the sequence $\check{\mathcal{G}}_T$ in $\check{\mathcal{G}}'$. In [35], consensus is established over a switching graph provided that the union of graphs frequently has a spanning tree, which indicates that the disagreement decreases strictly if $\check{\mathcal{G}}_T$ occurs frequently. Based on the fact that $z(\theta(O_{k+1})) \leq z(\theta(O_k))$ in Lemma 1 and the nonzero probability $\check{p}_T$ from Assumption 1, $z(\theta(O_{k+1})) < z(\theta(O_k))$ during each observation period o, which indicates that $f(O_{k+1}) < 0$ in (11). Therefore, the random sequence $\{Z_k\}$ is a supermartingale. Invoking the Theorem 1 of Chapter 8 in [34] can prove that the disagreement $z(O_k) \to 0$ almost surely. That is, all local estimates almost surely converge to the agreement space $\mathcal{A}$ in Def. 1, which indicates almost sure consensus $\theta_0(O_k) = \theta_1(O_k) = \cdots = \theta_N(O_k)$ as $k \to \infty$. Note that $\theta_0(O_k) = x_0(O_k)$, which indicates that all local estimates θ_i, $i \in \mathcal{V}$, achieve consensus to the target position almost surely. ■

[3]For a set of points $y \triangleq \{y_1, \ldots, y_n\}$, the convex hull $\mathrm{Co}(y)$ is defined as the minimal set containing all points in y, satisfying $\mathrm{Co}(y) \triangleq \left\{\sum_{i=1}^n \alpha_i y_i \,\middle|\, y_i \in y, \alpha_i \geq 0, \sum_{i=1}^n \alpha_i = 1\right\}$.

IV. Preservation of Network Connectivity

The consensus algorithm developed in Section III is based on a connected communication graph $\mathcal{G}(k)$. However, due to communication constraints, agent motion may result in a disconnected $\mathcal{G}(k)$ and failure to achieve consensus on the target estimate. The objective in this section is to develop a distributed motion algorithm for each agent i to maintain the global connectivity of $\mathcal{G}(k)$ during target tracking by using only local information from its two-hop neighbors. It is assumed that the initial communication graph $\mathcal{G}(0)$ is connected. The developed control strategy is based on the algebraic connectivity, where each agent moves according to the information collected from its two-hop neighbors to maintain the positive algebraic connectivity (i.e., global network connectivity). Rather than restricting agent motion to maintain the network connectivity by preserving every existing link, the developed control algorithm allows agents to break the existing links when necessary, which relaxes motion constraints.

A. Control Design

Let $L(\mathbf{x}(k))$ and $\det(L(\mathbf{x}(k))) = \prod_{i=1}^N \lambda_i$ denote the position-dependent Laplacian of $\mathcal{G}(k)$ and its determinant, respectively. The following lemma summarizes the results developed in [20].

Lemma 2: Let $\mathcal{X}_c = \{\mathbf{x}(k) | \lambda_2(L(\mathbf{x}(k))) > 0\}$ be the set of agent positions corresponding to connected graphs. If $\det\left(L(\mathbf{x}(k)) + \mathbf{1}_N \mathbf{1}_N^T\right)$ is strictly positive, then $\mathcal{G}(k)$ is connected (i.e., $\mathbf{x} \in \mathcal{X}_c$).

Let $\mathcal{G}_i(k)$ be a local subgraph of $\mathcal{G}(k)$, which is composed of agent i's one-hop and two-hop neighbors and the associated edges and $L_i(\mathbf{x}(k))$ be the graph Laplacian of $\mathcal{G}_i(k)$. Motivated by our previous work in [14], a gradient-based approach is developed to preserve the connectivity of $\mathcal{G}_i(k)$. Given a local graph $\mathcal{G}_i(k)$ composed of $m \leq N$ agents, the potential function $\varphi_i(k) : \mathbb{R}^{2m} \to [0, 1]$ is designed as

$$\varphi_i(k) \triangleq \frac{\gamma_i(k)}{\left(\gamma_i^{\vartheta}(k) + \beta_i(k)\right)^{\frac{1}{\vartheta}}} \tag{13}$$

where $\vartheta \in \mathbb{R}^+$ is a tuning parameter, $\gamma_i(k)$ is the goal function, and $\beta_i(k)$ is the constraint function.

Based on the estimated target position $\theta_i(k)$, the target tracking objective $\gamma_i(k) : \mathbb{R}^2 \to [0, \infty)$ is encoded in (13) as

$$\gamma_i(k) \triangleq \|x_i(k) - \theta_i(k) - r_i\| \tag{14}$$

which aims to position agent i at r_i, where $r_i \in \mathbb{R}^2$ is the predetermined relative position with respect to the target that gives the best sensing.

Since $\det\left(L_i(\mathbf{x}(k)) + \mathbf{1}_m \mathbf{1}_m^T\right)$ can be used as an indicator for the network connectivity of $\mathcal{G}_i(k)$, the constraint function $\beta_i : \mathbb{R}^{2m} \to [0, \infty)$ in (13) is designed as

$$\beta_i(k) \triangleq \tanh\left(\det\left(L_i(\mathbf{x}(k)) + \mathbf{1}_m \mathbf{1}_m^T\right)\right). \tag{15}$$

The key idea of designing β_i in (15) is to treat the connectivity constraint as a virtual obstacle. Whenever $\mathcal{G}_i(k)$ is about to be disconnected (i.e., $\det\left(L_i(\mathbf{x}(k)) + \mathbf{1}_m \mathbf{1}_m^T\right) \to 0$), $\beta_i(k)$ achieves its minimum of 0. The hyperbolic tangent $\tanh(\cdot)$ is used in (15) to penalize more on the agent motion when the network is about to break (i.e., $\det\left(L_i(\mathbf{x}(k)) + \mathbf{1}_m \mathbf{1}_m^T\right)$ is close to zero), and provides more motion flexibility for agents when the network is well connected based on the saturation property of $\tanh(\cdot)$ (i.e., the gradient of β_i is close to zero for a well-connected graph indicating that the agent will be mainly driven based on the gradient of γ_i for target tracking with less motion constraint on preserving the network connectivity).

Based on the designed potential function in (13), each agent i computes a local solution $\hat{x}_{i,j}$ for its immediate neighbor j, as well as itself, as

$$\hat{x}_{i,j}(k+1) = \hat{x}_{i,j}(k) - K_i t_\Delta \nabla_{x_j} \varphi_i(k), j \in \mathcal{N}_i^* \tag{16}$$

where K_i is a positive gain, $\mathcal{N}_i^* \triangleq \mathcal{N}_i \cup \{i\}$ with $\mathcal{N}_i$ denoting the set of one-hop neighbors of agent i in $\mathcal{G}(k)$, and $\nabla_{x_j}\varphi_i(k)$ is the partial gradient of $\varphi_i(k)$ with respect to x_j at time k. The local solution $\hat{x}_{i,j}(k+1)$ computed by agent i in (16) determines a potential next movement of its one-hop neighbors $j \in \mathcal{N}_i$ and itself in $\mathcal{G}_i(k)$ such that the connectivity of $\mathcal{G}_i(k)$ is preserved. For agent $j \notin \mathcal{N}_i^*$, $\hat{x}_{i,j}(k)$ remains the same as the previous state. Note that the designed potential function $\varphi_i(k)$ has a similar form to our previous work [14]. Following similar analysis in [14], based on the gradient of $\varphi_i(k)$, agents will move to minimize the goal function γ_i in (14) to perform the target tracking while avoiding the virtual constraint β_i in (15) to ensure the network connectivity of $\mathcal{G}_i$. Note that the computation of $\hat{x}_{i,j}(k+1)$ in (16) requires information from two-hop neighbors due to the local Laplacian matrix $L_i(\mathbf{x}(k))$ in (15).

Motivated by (16) and the subsequent stability analysis, the controller for agent i is

$$u_i \triangleq \frac{1}{t_\Delta N} \sum_{j \in \mathcal{N}_i^*} \left(\hat{x}_{j,i}(k+1) - x_i(k)\right). \tag{17}$$

The control strategy behind (17) is to let agent i's one-hop neighbors $j \in \mathcal{N}_i^*$ (including itself) compute a local solution based on (16) using their local two-hop neighbors information, which are then averaged in (17) to indicate the next movement of agent i so that the global network connectivity is preserved.

B. Global Network Connectivity

Despite the local information used in (17), the following indicates that the global connectivity of $\mathcal{G}(t)$ can be preserved. Substituting the controller from (17) into the agent kinematics in (1) yields

$$x_i(k+1) = x_i(k) + \frac{1}{N} \sum_{j \in \mathcal{N}_i^*} \left(\hat{x}_{j,i}(k+1) - x_i(k)\right) \tag{18}$$

where $\hat{x}_{j,i}(k+1)$ is obtained from (16). The Laplacian $L(\mathbf{x}(k+1))$ is constructed based on the updated agent positions $x_i(k+1)$, $i \in \mathcal{V}$, in (18), which can be approximated by $\mathcal{L}(\mathbf{x}(k+1))$ based on first-order Taylor expansions as in [19]. Particularly, each entry $[L(\mathbf{x}(k+1))]_{ij}$, $i \neq j$, is approximated as

$$\begin{aligned}[\mathcal{L}(\mathbf{x}(k+1))]_{ij} &= -w_{ij}(k+1) \\ &= -w_{ij}(k) - \frac{\partial f_w}{\partial d_{ij}} \left(\frac{\partial d_{ij}}{\partial x_i} \Delta x_i(k+1) + \frac{\partial d_{ij}}{\partial x_j} \Delta x_j(k+1) \right)\end{aligned} \tag{19}$$

where $\Delta x_l(k+1) \triangleq x_l(k+1) - x_l(k)$ for $l = i, j$. Since $\frac{\partial d_{ij}}{\partial x_i} = -\frac{\partial d_{ij}}{\partial x_j}$ from the definition of $d_{ij} \triangleq \|x_i - x_j\|$, (19) can be simplified as

$$[\mathcal{L}(\mathbf{x}(k+1))]_{ij} = [\mathcal{L}(\mathbf{x}(k))]_{ij} + [\Xi(\Delta\mathbf{x}(k+1))]_{ij} \tag{20}$$

where

$$[\Xi(\Delta\mathbf{x}(k+1))]_{ij} \triangleq -c_{ij}^T \left(\Delta x_i(k+1) - \Delta x_j(k+1)\right) \tag{21}$$

where $c_{ij} \triangleq \frac{\partial f_w}{\partial d_{ij}} \frac{\partial d_{ij}}{\partial x_i}\Big|_{x_i(k), x_j(k)}$. Hence, $\mathcal{L}(\mathbf{x}(k+1))$ can be written as

$$\mathcal{L}(\mathbf{x}(k+1)) = \mathcal{L}(\mathbf{x}(k)) + \Xi(\Delta\mathbf{x}(k+1)). \tag{22}$$

Consider a graph $\tilde{\mathcal{G}}_i(\hat{\mathbf{x}}_i(k+1))$ built upon $\mathcal{G}_i(k)$ with $\hat{\mathbf{x}}_i(k+1) = [\hat{x}_{i,1}(k+1), \ldots, \hat{x}_{i,N}(k+1)]^T$. The set of one-hop neighbors $j \in \mathcal{N}_i^*$ in $\tilde{\mathcal{G}}_i$ is updated according to (16), while the set of nodes $j \notin \mathcal{N}_i^*$ remains the same as $\hat{x}_{i,j}(k)$. Similar to the derivation of (22), replacing $\mathbf{x}(k+1)$ with the local solution $\hat{\mathbf{x}}_i(k+1)$ yields

$$\mathcal{L}(\hat{\mathbf{x}}_i(k+1)) = \mathcal{L}(\mathbf{x}_i(k)) + \Xi(\Delta\hat{\mathbf{x}}_i(k+1)) \tag{23}$$

where the (i,j) entry of $\Xi(\Delta\hat{\mathbf{x}}_i(k+1))$ is

$$[\Xi(\Delta\hat{\mathbf{x}}_i(k+1))]_{ij} = -c_{ij}^T \left(\Delta\hat{x}_{i,i}(k+1) - \Delta\hat{x}_{i,j}(k+1)\right) \tag{24}$$

where

$$\Delta\hat{x}_{i,l}(k+1) = \hat{x}_{i,l}(k+1) - x_l(k), l = i, j. \tag{25}$$

Based on the development of the linearized graph Laplacian of the augmented local graph $\tilde{\mathcal{G}}_i(\hat{\mathbf{x}}_i)$, the controller in (17) can be proven to ensure the algebraic connectivity (see Theorem 2). To facilitate the analysis, two Lemmas are first established as follows.

Lemma 3: Given an initially connected local graph $\mathcal{G}_i(0)$, the local solution computed from (16) preserves the network connectivity of $\mathcal{G}_i(k)$, and implies that

$$\mathcal{L}(\hat{\mathbf{x}}_i(k+1)) + \mathbf{1}_N \mathbf{1}_N^T > 0$$

where $\mathcal{L}(\hat{\mathbf{x}}_i(k+1)) \in \mathbb{R}^{N \times N}$ is a linearized Laplacian matrix of the augmented local graph $\tilde{\mathcal{G}}_i(\hat{\mathbf{x}}_i)$ in (23).

Proof: As stated in Lemma 2, a positive $\det\left(L_i(\mathbf{x}(k)) + \mathbf{1}_m \mathbf{1}_m^T\right)$ indicates a connected local graph $\mathcal{G}_i(k)$. Since $\beta_i > 0$ in (15) implies that $\det\left(L_i + \mathbf{1}_m \mathbf{1}_m^T\right) > 0$, if $\mathcal{G}_i(k)$ is about to be disconnected, then β_i will decrease to its minimum of 0, resulting in a maximum value of the designed potential function $\varphi_i(k)$ in (13). Since $\hat{x}_{i,j}$ evolves according to the negative gradient of the potential field $\varphi_i(k)$ in (16), similar to our previous work in [14], no open set of initial conditions can be attracted to the maximum value of $\varphi_i(k)$. Hence, $\beta_i > 0$ is ensured and the network connectivity of $\mathcal{G}_i(k)$ is preserved.

Proving that $\mathcal{L}(\hat{\mathbf{x}}_i(k+1)) + \mathbf{1}_N \mathbf{1}_N^T > 0$ is equivalent to showing connectivity of the augmented graph $\tilde{\mathcal{G}}_i(\hat{\mathbf{x}}_i(k+1))$. From the definition of $\tilde{\mathcal{G}}_i$, it is clear that the two-hop neighbors of agent i in $\mathcal{G}_i$ and the agents outside of $\mathcal{G}_i$ remain stationary at $k+1$, while only agent i's one-hop neighbors in $\mathcal{G}_i$ move according to (16). Therefore, since $\mathcal{G}_i$ remains connected under (16), then $\tilde{\mathcal{G}}_i(\hat{\mathbf{x}}_i(k+1))$ remains connected, which implies that $\mathcal{L}(\hat{\mathbf{x}}_i(k+1)) + \mathbf{1}_N \mathbf{1}_N^T > 0$. ■

Lemma 4: Given (21) and (24), the following equality holds:

$$\Xi(\Delta\mathbf{x}(k+1)) = \frac{1}{N} \sum_{m=1}^{N} \Xi(\Delta\hat{\mathbf{x}}_m(k+1)). \tag{26}$$

Proof: Using (18) and (21), each entry (i, j) of $\Xi\left(\Delta\mathbf{x}\left(k+1\right)\right)$, $i \neq j$, in (26) can be expanded as

$$\begin{aligned}
&\left[\Xi\left(\Delta\mathbf{x}\left(k+1\right)\right)\right]_{ij}\\
&= -c_{ij}^T\left(\Delta x_i\left(k+1\right) - \Delta x_j\left(k+1\right)\right)\\
&= -\frac{1}{N}\left(\sum_{j\in\mathcal{N}_i^*} c_{ij}^T\left(\hat{x}_{j,i}\left(k+1\right) - x_i\left(k\right)\right)\right.\\
&\qquad \left. - \sum_{i\in\mathcal{N}_j^*} c_{ij}^T\left(\hat{x}_{i,j}\left(k+1\right) - x_j\left(k\right)\right)\right)\\
&= -\frac{1}{N}\left(\sum_{j\in\mathcal{N}_i^*} c_{ij}^T \Delta\hat{x}_{j,i}\left(k+1\right) - \sum_{i\in\mathcal{N}_j^*} c_{ij}^T \Delta\hat{x}_{i,j}\left(k+1\right)\right).
\end{aligned}$$

Based on (24) and (25), each entry (i, j) of the right-hand side of (26) can also be expressed as

$$\begin{aligned}
&\frac{1}{N}\sum_{m=1}^{N}\left[\Xi\left(\Delta\hat{\mathbf{x}}_m\left(k+1\right)\right)\right]_{ij}\\
&= -\frac{1}{N}\sum_{m=1}^{N}\left(c_{ij}^T\left(\Delta\hat{x}_{m,i}\left(k+1\right) - \Delta\hat{x}_{m,j}\left(k+1\right)\right)\right)\\
&= -\frac{1}{N}\left(\sum_{m\in\mathcal{N}_i^*} c_{ij}^T\Delta\hat{x}_{m,i}\left(k+1\right) - \sum_{m\in\mathcal{N}_j^*} c_{ij}^T\Delta\hat{x}_{m,j}\left(k+1\right)\right)
\end{aligned}$$

where the facts that $\sum_{m\notin\mathcal{N}_i^*} c_{ij}^T\Delta\hat{x}_{k,i}\left(k+1\right) = 0$ and $\sum_{m\notin\mathcal{N}_j^*} c_{ij}^T\Delta\hat{x}_{k,j}\left(k+1\right) = 0$ from the definition of $\tilde{\mathcal{G}}_i$ are used. ■

Theorem 2: Based on the kinematics in (1) and the controller in (17), the algebraic connectivity of the global graph $\mathcal{G}\left(k\right)$, $k \in \mathbb{Z}^+$, remains positive, which implies that the communication graph is always connected.

Proof: Based on the assumption that $\mathcal{G}\left(0\right)$ is connected initially, the network connectivity of $\mathcal{G}$ is preserved if, provided a connected graph $\mathcal{G}\left(k\right)$, $\mathcal{G}\left(k+1\right)$ remains connected for every $k \in \mathbb{Z}^+$. From Lemma 2, the graph $\mathcal{G}\left(k+1\right)$ is guaranteed to be connected, if $\mathcal{L}(\mathbf{x}(k+1)) + \mathbf{1}_N\mathbf{1}_N^T$ is positive definite. Using (22) yields

$$\mathcal{L}\left(\mathbf{x}\left(k+1\right)\right) + \mathbf{1}_N\mathbf{1}_N^T = \mathcal{L}\left(\mathbf{x}\left(k\right)\right) + \Xi\left(\Delta\mathbf{x}\left(k+1\right)\right) + \mathbf{1}_N\mathbf{1}_N^T. \tag{27}$$

Note that Lemma 3 shows that $\mathcal{L}\left(\hat{\mathbf{x}}_i\left(k+1\right)\right) + \mathbf{1}_N\mathbf{1}_N^T > 0\,\forall i$, which implies that

$$\frac{1}{N}\sum_{i=1}^{N}\left(\mathcal{L}\left(\hat{\mathbf{x}}_i\left(k+1\right)\right) + \mathbf{1}_N\mathbf{1}_N^T\right) > 0.$$

Substituting $\mathcal{L}\left(\hat{\mathbf{x}}_i\left(k+1\right)\right)$ from (23) yields

$$\frac{1}{N}\sum_{i=1}^{N}\left(\mathcal{L}\left(\mathbf{x}\left(k\right)\right) + \Xi\left(\Delta\hat{\mathbf{x}}_i\left(k+1\right)\right) + \mathbf{1}_N\mathbf{1}_N^T\right) > 0$$

which can be rewritten as

$$\mathcal{L}\left(\mathbf{x}\left(k\right)\right) + \mathbf{1}_N\mathbf{1}_N^T + \frac{1}{N}\sum_{i=1}^{N}\left(\Xi\left(\Delta\hat{\mathbf{x}}_i\left(k+1\right)\right)\right) > 0. \tag{28}$$

Since Lemma 4 shows that $\Xi\left(\Delta\mathbf{x}\left(k+1\right)\right) = \frac{1}{N}\sum_{i=1}^{N}\Xi(\Delta\hat{\mathbf{x}}_i\left(k+1\right))$, then

$$\mathcal{L}\left(\mathbf{x}\left(k\right)\right) + \Xi\left(\Delta\mathbf{x}\left(k+1\right)\right) + \mathbf{1}_N\mathbf{1}_N^T > 0$$

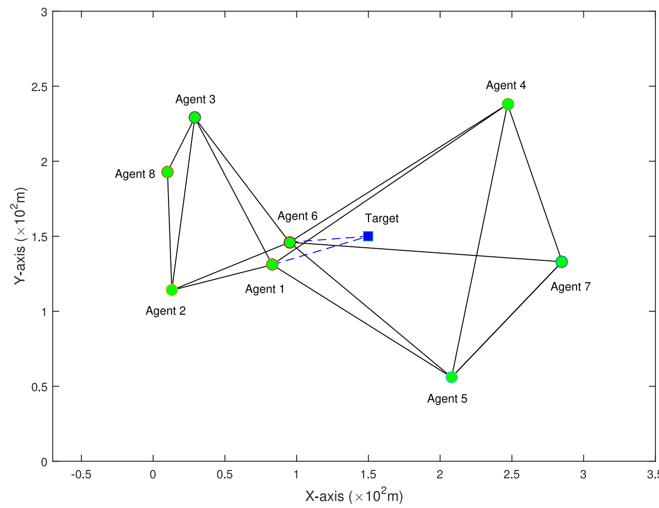

Fig. 1. Group of eight agents is initially deployed in the workspace with the goal of tracking the mobile target. The target is represented by a square, and the sensors are denoted by dots. The solid lines connecting agents indicate the interagent information exchange over the communication graph $\mathcal{G}$, while the dashed lines indicate random failures in sensing the target.

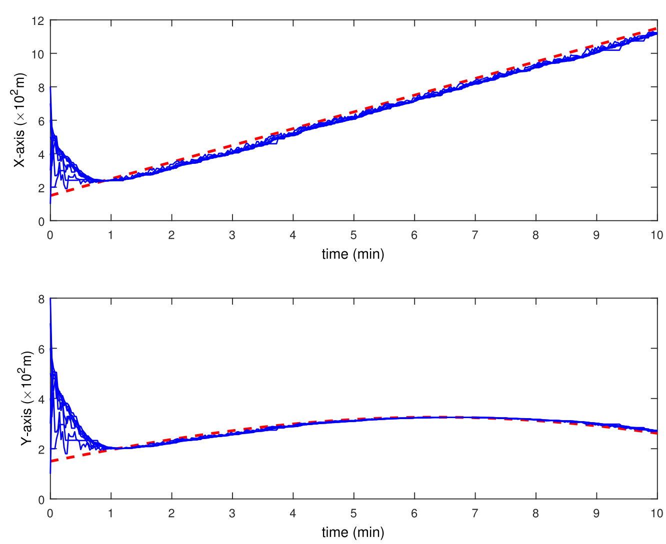

Fig. 2. Dashed line indicates the target trajectory, while the solid lines represent the local estimates of each agent.

which implies that the global network connectivity of $\mathcal{G}(k+1)$ is preserved. ■

V. Simulation

Numerical simulations are performed to illustrate the performance of the consensus algorithm in (4) over the random graph $\bar{\mathcal{G}}$, and the developed distributed algorithm for connectivity preservation of $\mathcal{G}$ in Section IV. As shown in Fig. 1, a group of eight agents with initially connected graph $\mathcal{G}$ is tasked to track a moving target. Each agent is assumed to have a limited communication zone of $R = 200$ m. The solid lines in Fig. 1 indicate available information exchange among agents over the position-dependent communication graph $\mathcal{G}$. Nodes $\{1,6\}$ are assumed to be able to sense the target initially and the set of agents that can sense the target will be time varying evolving according to

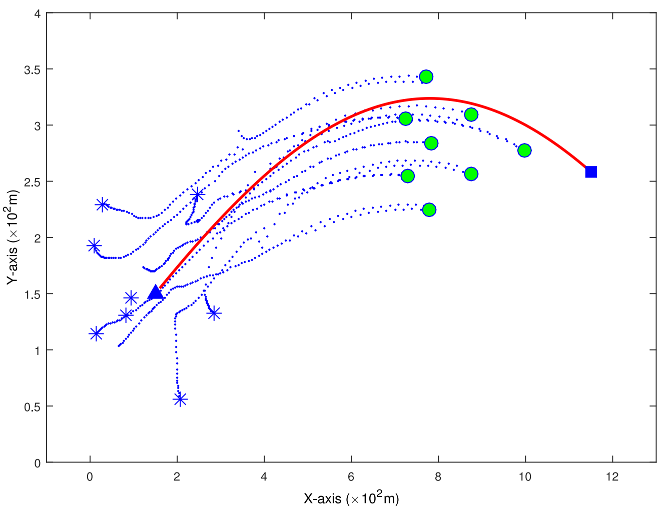

Fig. 3. Trajectories of the agents and the target, where "*" and the triangle denote the initial position of the agents and the target, respectively. The final position of the target and agents are represented by the dots and square, respectively.

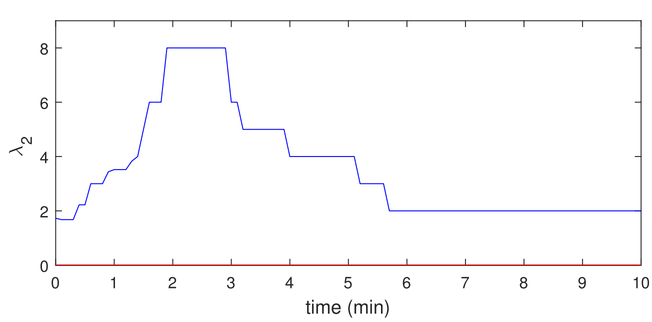

Fig. 4. Plot of algebraic connectivity (i.e., the Fiedler value) of the communication graph $\mathcal{G}$.

the Markov process defined in Section II. The dashed lines connecting the target and Agents 1 and 6 are assigned a random probability to model the random failures in sensing the target. Simulation results are provided in Figs. 2–4. Fig. 2 indicates consensus to the target position is achieved almost surely over the random graph $\bar{\mathcal{G}}$. The trajectory evolution of the agents and the target are shown in Fig. 3, where the triangle and "*" denotes the initial position of each sensor and the target, respectively, and the square and dots denote their final position. The dotted and solid lines in Fig. 3 represent the generated trajectories of the sensors and the target. To show that the network connectivity is preserved during the task, evolution of the algebraic connectivity of the proximity graph (i.e., the Fiedler value λ_2) is plotted in Fig. 4. Since the Fiedler value is always positive, network connectivity is preserved.

VI. Conclusion

Almost sure consensus is established to enable agents to estimate the target states over a random network where the agents experience random loss of target sensing. Based on the algebraic connectivity, a potential-field-based distributed motion control law is designed to preserve the network connectivity during cooperative target tracking, where only one-hop and two-hop neighboring information is required. Despite the consideration of a single target in this study, future efforts will focus on extending the current framework by allowing multiple target tracking. Additional work will also consider extending the developed Markov process by taking into account various factors, e.g., the relative positions between the target and agents, measurement noise, and measurement qualities, for improved target tracking.

References

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[2] W. Ren, R. W. Beard, and E. M. Atkins, "Information consensus in multivehicle cooperative control," *IEEE Control Syst. Mag.*, vol. 27, no. 2, pp. 71–82, Apr. 2007.

[3] Y. Hatano and M. Mesbahi, "Agreement over random networks," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1867–1872, Nov. 2005.

[4] C. W. Wu, "Synchronization and convergence of linear dynamics in random directed networks," *IEEE Trans. Autom. Control*, vol. 51, no. 7, pp. 1207–1210, Jul. 2006.

[5] M. Porfiri and D. J. Stilwell, "Consensus seeking over random weighted directed graphs," *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1767–1773, Sep. 2007.

[6] A. Tahbaz-Salehi and A. Jadbabaie, "A necessary and sufficient condition for consensus over random networks," *IEEE Trans. Autom. Control*, vol. 53, no. 3, pp. 791–795, Apr. 2008.

[7] Z. Kan, J. Shea, and W. E. Dixon, "Leader-follower containment control over directed random graphs," *Automatica*, vol. 66, pp. 56–62, 2016.

[8] Y. Zhang and Y.-P. Tian, "Maximum allowable loss probability for consensus of multi-agent systems over random weighted lossy networks," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2127–2132, Aug. 2012.

[9] Y. Wang, L. Cheng, W. Ren, Z.-G. Hou, and M. Tan, "Seeking consensus in networks of linear agents: Communication noises and Markovian switching topologies," *IEEE Trans. Autom. Control*, vol. 60, no. 5, pp. 1374–1379, May 2015.

[10] M. Zavlanos, M. Egerstedt, and G. Pappas, "Graph theoretic connectivity control of mobile robot networks," *Proc. IEEE*, vol. 99, no. 9, pp. 1525–1540, Sep. 2011.

[11] D. Dimarogonas and K. Johansson, "Bounded control of network connectivity in multi-agent systems," *Control Theory Appl.*, vol. 4, no. 8, pp. 1330–1338, Aug. 2010.

[12] X. Li, D. Sun, and J. Yang, "A bounded controller for multirobot navigation while maintaining network connectivity in the presence of obstacles," *Automatica*, vol. 49, no. 1, pp. 285–292, 2013.

[13] M. Ji and M. Egerstedt, "Distributed coordination control of multiagent systems while preserving connectedness," *IEEE Trans. Robot.*, vol. 23, no. 4, pp. 693–703, Aug. 2007.

[14] Z. Kan, A. Dani, J. M. Shea, and W. E. Dixon, "Network connectivity preserving formation stabilization and obstacle avoidance via a decentralized controller," *IEEE Trans. Autom. Control*, vol. 57, no. 7, pp. 1827–1832, Jul. 2012.

[15] Z. Kan, L. Navaravong, J. Shea, E. Pasiliao, and W. E. Dixon, "Graph matching based formation reconfiguration of networked agents with connectivity maintenance," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 1, pp. 24–35, Mar. 2015.

[16] Z. Feng, C. Sun, and G. Hu, "Robust connectivity preserving rendezvous of multi-robot systems under unknown dynamics and disturbances," *IEEE Trans. Control Netw. Syst.*, vol. 4, no. 4, pp. 725–735, Dec. 2017.

[17] Y. Cao, W. Ren, D. W. Casbeer, and C. Schumacher, "Finite-time connectivity-preserving consensus of networked nonlinear agents with unknown Lipschitz terms," *IEEE Trans. Autom. Control*, vol. 61, no. 6, pp. 1700–1705, 2016.

[18] C. Godsil and G. Royle, *Algebraic Graph Theory (Graduate Texts in Mathematics)*. New York, NY, USA: Springer, 2001.

[19] Y. Kim and M. Mesbahi, "On maximizing the second smallest eigenvalue of a state-dependent graph Laplacian," *IEEE Trans. Autom. Control*, vol. 51, no. 1, pp. 116–120, Jan. 2006.

[20] S. Boyd, "Convex optimization of graph Laplacian eigenvalues," in *Proc. Int. Congr. Math.*, Madrid, Spain, 2006, pp. 1311–1320.

[21] M. Zavlanos and G. Pappas, "Potential fields for maintaining connectivity of mobile networks," *IEEE Trans. Robot.*, vol. 23, no. 4, pp. 812–816, Aug. 2007.

[22] A. Simonetto, T. Keviczky, and R. Babuška, "Constrained distributed algebraic connectivity maximization in robotic networks," *Automatica*, vol. 49, pp. 1348–1357, 2013.

[23] V. M. Preciado and A. Jadbabaie, "Moment-based spectral analysis of large-scale networks using local structural information," *IEEE/ACM Trans. Netw.*, vol. 21, no. 2, pp. 373–382, Apr. 2013.

[24] V. M. Preciado, A. Jadbabaie, and G. C. Verghese, "Structural analysis of Laplacian spectral properties of large-scale networks," *IEEE Trans Autom. Control*, vol. 58, no. 9, pp. 2338–2343, Sep. 2013.

[25] M. De Gennaro and A. Jadbabaie, "Decentralized control of connectivity for multi-agent systems," in *Proc. IEEE Conf. Decis. Control*, Dec. 2006, pp. 3628–3633.

[26] P. Yang, R. A. Freeman, G. J. Gordon, K. M. Lynch, S. S. Srinivasa, and R. Sukthankar, "Decentralized estimation and control of graph connectivity for mobile sensor networks," *Automatica*, vol. 46, no. 2, pp. 390–396, 2010.

[27] T.-M.-D. K. Tran and A. Y. Kibangou, "Distributed estimation of Laplacian eigenvalues via constrained consensus optimization problems," *Syst. Control Lett.*, vol. 80, pp. 56–62, 2015.

[28] R. Aragues, G. Shi, D. V. Dimarogonas, C. Sagüés, K. H. Johansson, and Y. Mezouar, "Distributed algebraic connectivity estimation for undirected graphs with upper and lower bounds," *Automatica*, vol. 50, no. 12, pp. 3253–3259, 2014.

[29] M. Franceschelli, A. Gasparri, A. Giua, and C. Seatzu, "Decentralized estimation of Laplacian eigenvalues in multi-agent systems," *Automatica*, vol. 49, no. 4, pp. 1031–1036, 2013.

[30] H. A. Poonawala and M. W. Spong, "Decentralized estimation of the algebraic connectivity for strongly connected networks," in *Proc. Amer. Control Conf.*, 2015, pp. 4068–4073.

[31] L. Sabattini, N. Chopra, and C. Secchi, "Decentralized connectivity maintenance for cooperative control of mobile robotic systems," *Int. J. Robot. Res.*, vol. 32, no. 12, pp. 1411–1423, 2013.

[32] A. Gasparri, L. Sabattini, and G. Ulivi, "Bounded control law for global connectivity maintenance in cooperative multirobot systems," *IEEE Trans. Robot.*, vol. 33, no. 3, pp. 700–717, Jun. 2017.

[33] G. Grimmett and D. Stirzaker, *Probability and Random Processes*. London, U.K.: Oxford Univ. Press, 2001.

[34] H. J. Kushner, *Introduction to Stochastic Control*. New York, NY, USA: Holt, Reinhart and Winston, 1971.

[35] W. Ren and R. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.