Asymptotic Synchronization of a Leader-Follower Network of Uncertain Euler-Lagrange Systems

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***Abstract*—This paper investigates the synchronization of a network of Euler-Lagrange systems with leader tracking. The Euler-Lagrange systems are heterogeneous and uncertain and contain bounded, exogenous disturbances. The network leader has a time-varying trajectory which is known to only a subset of the follower agents. A robust integral sign of the error-based decentralized control law is developed to yield semiglobal asymptotic agent synchronization and leader tracking.**

***Index Terms*—Decentralized control, nonlinear dynamical systems, robust control.**

I. Introduction

CONSENSUS is a general term that has been adopted for a set of control problems where the goal is convergence of a group of agents. Typically, the consensus goal is a constant formation or constant common coordinate [1]–[4]; the latter case is sometimes called the rendezvous problem. Similar to results such as [5]–[18], this paper examines the more general problem of consensus to a dynamic desired behavior specified by a network leader or desired trajectory (often referred to as a virtual network leader). In particular, the objective in this paper is for the agents to track a network leader while also coordinating with the states of neighboring agents. This objective is important for applications where only following the leader could yield disruptive effects (e.g., graph segregation, collisions) due to the differences in the dynamic response or initial conditions of neighboring agents. In the following development, this particular consensus objective is called "synchronization".

For many consensus applications, a decentralized communication strategy yields advantages over centralized methods, wherein an omniscient agent communicates with all other agents and computes the appropriate control signal. In a decentralized architecture, communication bandwidth demands are mitigated since agents only communicate with network neighbors and make independent control decisions while still achieving a network-wide goal. Results such as [6], [8], [9], [12], [13], and [18] achieve synchronization; however, all of the agents are able to communicate with the network leader so that the developed controllers for each agent exploit explicit knowledge of the desired goal. Practical applications often dictate network topologies in which not every node receives information from the leader; for example, only vehicles in the front of a formation tracking a target may be able to directly observe the location of the target. For such scenarios, the additional design challenge is to develop a decentralized control policy so that the states of all agents converge to the leader's state using only neighbor feedback, where the leader may not be in an agent's neighborhood. Some results, such as [19]–[21], address this issue by investigating the impact of how many nodes and which nodes are connected to the leader (i.e., pinned nodes) on synchronization performance. The work in [22] further investigates network architecture solutions by determining the minimum requirement of the network structure for feasible synchronization. Unlike the pinning selection strategies, the subsequent control development, similar to results such as [5], [7], [10], [11], [16], [17], focuses on the development of decentralized controllers assuming that at least one node is connected to the leader without further investigating the structure of the network. In these results, the controllers are based on a composite error system that penalizes the state dissimilarity between neighbors and the dissimilarity between a follower agent and the leader, if that connection exists. Penalizing the tracking error with the leader and with the states of neighboring agents is motivated by applications such as automated convoys, synchronizing generators, etc. The weighted composite error system in this result (and results such as [7], [10], [11], [23]) allows a user to arbitrate between the potentially conflicting goals of leader following and maintaining coordinated behavior with the states of neighboring agents.

For centralized and decentralized communication strategies, typical agent synchronization results have focused on networks of linear dynamical systems (cf. [5], [14], [16]). Recent results such as [7], [15], and [17] investigate the more general problem where the trajectories of networked agents are described by nonlinear dynamics; specifically, the results in [7], [15], and [17] focus on Euler-Lagrange systems, which model a broad class of physical systems. The synchronization of physical systems leads to additional challenges in the sense that the trajectories of neighboring agents are less predictable due to heterogeneity, parametric uncertainty, and additive unmodeled

Manuscript received March 18, 2014; revised June 11, 2014; accepted September 7, 2014. Date of publication December 8, 2014; date of current version June 16, 2015. This work was supported in part by the National Science Foundation under award numbers 1161260 and 1217908 and in part by the AFRL Mathematical Modeling and Optimization Institute. Recommended by Associate Editor M. Bernardo.

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Digital Object Identifier 10.1109/TCNS.2014.2378875

disturbances. If these effects are not considered in the design of a synchronizing control policy, agents may suffer undesirable performance and instability; furthermore, these effects may propagate through the network and cause cascading control objective failure. In [7] and [15], decentralized controllers are developed for Euler-Lagrange systems, where exact knowledge of the agent dynamics is used within a feedback linearization approach to compensate for the nonlinear dynamics. Motivated to improve the robustness of the controller, results such as [10], [11], and [17] consider uncertainty in the nonlinear agent dynamics. In [17], a continuous controller is proposed to yield asymptotic synchronization in the presence of parametric uncertainty. In addition to parametric uncertainty, the results in [10] and [11] also consider exogenous disturbances. The result in [10] uses a neural-network-based adaptive synchronization method and the result in [11] exploits a sliding-mode-based approach. The continuous controller in [10] yields a uniformly ultimately bounded result, whereas [11] achieves exponential synchronization through the use of a discontinuous controller.

This paper (and the preliminary result in [23]) investigates the synchronization of networked systems consisting of a leader and followers, where at least one follower is connected to the leader. The networked systems are modeled by nonlinear, heterogeneous, and uncertain Euler-Lagrange dynamics which are affected by additive unmodeled disturbances. The most comparable results to the current result are [10] and [11]. In contrast to the discontinuous result in [11], the developed decentralized controller is continuous, using a robust integral sign of the error (RISE)-based strategy. In contrast to the result in [10], the RISE-based approach yields asymptotic synchronization with neighboring states and the time-varying state of the leader, despite the effects of bounded exogenous disturbances and parametric uncertainties. Lyapunov-based analysis is provided that proves asymptotic synchronization of each agent's state. A simulation of a network of second-order Euler-Lagrange systems is provided that demonstrates the practical implications of achieving an asymptotic result using a continuous controller in comparison with the results in [10] and [11, Sec. IV].

II. Problem Formulation

A. Preliminaries

Graph theory provides convenient tools to describe the information exchange between multiple agents in a network. Consider a network consisting of one leader and N followers, where the leader is indexed by 0. Let the communication of the networked followers be represented by a fixed undirected graph $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$, which has a nonempty finite set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$ and a set of edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. An undirected edge $(i,j)\in\mathcal{E}$ exists if nodes i and j mutually share information. The set of neighbors that provide information to node $i\in\mathcal{V}$ is defined as $\mathcal{N}_i\triangleq\{j\in\mathcal{V}|(i,j)\in\mathcal{E}\}$. An adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is defined such that $a_{ij}=a_{ji}>0$ if $(i,j)\in\mathcal{E}$ and $a_{ij}=a_{ji}=0$ otherwise. It is assumed that the graph is simple, that is, $(i,i)\notin\mathcal{E}$ and thus, $a_{ii}=0\ \forall i\in\mathcal{V}$. Let $\mathcal{D}\triangleq\mathrm{diag}\{D_1,D_2,\ldots,D_N\}\in\mathbb{R}^{N\times N}$ be a diagonal matrix, where $D_i\triangleq\sum_{j\in\mathcal{N}_i}a_{ij}$. The Laplacian matrix for $\mathcal{G}$ is defined as

$$\mathcal{L}\triangleq\mathcal{D}-\mathcal{A}. \tag{1}$$

Let the graph $\bar{\mathcal{G}}=\{\bar{\mathcal{V}},\bar{\mathcal{E}}\}$ represent a supergraph of $\mathcal{G}$, which is formed by inserting edges directed from the leader node to the subset of leader-aware follower nodes. The neighborhood for a node in $\bar{\mathcal{G}}$ is similarly defined as $\bar{\mathcal{N}_i}\triangleq\{j\in\bar{\mathcal{V}}|(i,j)\in\bar{\mathcal{E}}\}$. The pinning matrix B describes which follower agents are connected to the leader and is defined as $B\triangleq\mathrm{diag}\{b_1,b_2,\ldots,b_N\}$ with $b_i>0$ $(i\in\mathcal{V})$ if $0\in\bar{\mathcal{N}_i}$ and $b_i=0$ otherwise. The pinning matrix allows for a convenient description of network interactions between the leader and followers using the matrix $\mathcal{L}+B$, which is positive definite provided some assumptions given in the following section are satisfied (see Assumption 3).

B. Dynamic Models and Properties

Consider a network of $N+1$ agents, which have dynamics described by the nonidentical Euler-Lagrange equations of motion

$$M_0(q_0)\ddot{q}_0+C_0(q_0,\dot{q}_0)\dot{q}_0+F_0(\dot{q}_0)+G_0(q_0)=\tau_0 \tag{2}$$
$$M_i(q_i)\ddot{q}_i+C_i(q_i,\dot{q}_i)\dot{q}_i+F_i(\dot{q}_i)+G_i(q_i)+d_i(t)=\tau_i,\ i\in\mathcal{V} \tag{3}$$

where the zero index denotes the leader and all other agents $i\in\mathcal{V}$ are followers. The terms in (2) and (3) are defined such that $q_j\in\mathbb{R}^m$ $(j\in\bar{\mathcal{V}})$ is the generalized configuration coordinate, $M_j:\mathbb{R}^m\to\mathbb{R}^{m\times m}$ is the inertia matrix, $C_j:\mathbb{R}^m\times\mathbb{R}^m\to\mathbb{R}^{m\times m}$ is the Coriolis/centrifugal matrix, $F_j:\mathbb{R}^m\to\mathbb{R}^m$ is the friction term, $G_j:\mathbb{R}^m\to\mathbb{R}^m$ is the vector of gravitational torques, $\tau_j\in\mathbb{R}^m$ is the vector of control inputs, $d_i:\mathbb{R}_{\geq 0}\to\mathbb{R}^m$ $(i\in\mathcal{V})$ is a time-varying nonlinear exogenous disturbance, and $t\in\mathbb{R}_{\geq 0}$ is the elapsed time.

The following system properties are used in the subsequent analysis.

Property 1: The inertia matrix M_j is symmetric, positive definite, and satisfies $\underline{m}_j\|\xi\|^2\leq\xi^TM_j\xi\leq\overline{m}_j\|\xi\|^2\ \forall\xi\in\mathbb{R}^m$ $(j\in\bar{\mathcal{V}})$, where $\underline{m}_j\in\mathbb{R}$ is a positive known constant and $\overline{m}_j\in\mathbb{R}$ is a known positive bounded function [24].

Property 2: The functions M_j, C_j, F_j, and G_j $(j\in\bar{\mathcal{V}})$ are second-order differentiable such that their second derivatives are bounded if $q_j^{(k)}\in\mathcal{L}_\infty$, $k=0,1,2,3$ [25].

The following assumptions are also required for subsequent analysis.

Assumption 1: The nonlinear disturbance term d_i and its first two time derivatives are bounded by known[1] constants for all $i\in\mathcal{V}$.

Assumption 2: The leader configuration coordinate q_0 is sufficiently smooth such that $q_0\in\mathcal{C}^2$; in addition, the leader configuration coordinate and its first two time derivatives are bounded such that $q_0,\dot{q}_0,\ddot{q}_0\in\mathcal{L}_\infty$.

[1]Following the developments in [26] and [27], Assumption 1 can be relaxed such that the bounding constants can be unknown.

Assumption 3: The graph $\mathcal{G}$ is connected and at least one follower agent is connected to the leader.

The equation of motion for the follower agents may be written as

$$M\ddot{Q} + C\dot{Q} + F + G + d = \tau \tag{4}$$

where

$$\begin{aligned}
Q &\triangleq \left[q_1^T, q_2^T, \ldots, q_N^T\right]^T \in \mathbb{R}^{Nm}\\
M &\triangleq \operatorname{diag}\{M_1, M_2, \ldots, M_N\} \in \mathbb{R}^{Nm\times Nm}\\
C &\triangleq \operatorname{diag}\{C_1, C_2, \ldots, C_N\} \in \mathbb{R}^{Nm\times Nm}\\
F &\triangleq \left[F_1^T, F_2^T, \ldots, F_N^T\right]^T \in \mathbb{R}^{Nm}\\
G &\triangleq \left[G_1^T, G_2^T, \ldots, G_N^T\right]^T \in \mathbb{R}^{Nm}\\
d &\triangleq \left[d_1^T, d_2^T, \ldots, d_N^T\right]^T \in \mathbb{R}^{Nm}\\
\tau &\triangleq \left[\tau_1^T, \tau_2^T, \ldots, \tau_N^T\right]^T \in \mathbb{R}^{Nm}.
\end{aligned}$$

For convenience in the subsequent analysis, the leader dynamics are represented as

$$M_\varnothing \ddot{Q}_0 + C_\varnothing \dot{Q}_0 + F_\varnothing + G_\varnothing = \tau_\varnothing, \tag{5}$$

where $Q_0 \triangleq \underline{1}_N \otimes q_0 \in \mathbb{R}^{Nm}$, $M_\varnothing \triangleq I_N \otimes M_0 \in \mathbb{R}^{Nm\times Nm}$, $C_\varnothing \triangleq I_N \otimes C_0 \in \mathbb{R}^{Nm\times Nm}$, $F_\varnothing \triangleq \underline{1}_N \otimes F_0 \in \mathbb{R}^{Nm}$, $G_\varnothing \triangleq \underline{1}_N \otimes G_0 \in \mathbb{R}^{Nm}$, $\tau_\varnothing \triangleq \underline{1}_N \otimes \tau_0 \in \mathbb{R}^{Nm}$, $\underline{1}_N$ denotes an N-dimensional column vector of ones, and $\otimes$ denotes the Kronecker product.

Note that because the graph $\mathcal{G}$ is undirected and connected and at least one follower agent is connected to the leader by Assumption 3, the matrix $\mathcal{L} + B$ is positive definite and symmetric [28]. The customarily used Laplacian matrix is positive semidefinite for a connected undirected graph; however, the matrix $\mathcal{L}$, also known as the "Dirichlet" or "Grounded" Laplacian matrix, is designed such that $\mathcal{L} + B$ is positive definite given Assumption 3 [28].

III. Control Objective

The objective is to design a continuous controller which ensures that N agents asymptotically track the state of the reference node with only neighbor state feedback despite model uncertainties and bounded exogenous system disturbances. Moreover, the subsequent control design is based on the constraint that only the generalized configuration coordinate and its first derivative are measurable.

To quantify the control objective, a local neighborhood position tracking error $e_{1,i} \in \mathbb{R}^m$ is defined as [7]

$$e_{1,i} \triangleq \sum_{j\in\mathcal{N}_i} a_{ij}(q_j - q_i) + b_i(q_0 - q_i). \tag{6}$$

The error signal in (6) includes the summation $\sum_{j\in\mathcal{N}_i} a_{ij}(q_j - q_i)$ to penalize state dissimilarity between neighbors and the proportional term $b_i(q_0 - q_i)$ to penalize state dissimilarity between a follower agent and the leader, if that connection exists. The ability to emphasize either follower agent synchronization or leader tracking is rendered by assigning $a_{ij} = k_a$ if $(i, j) \in \mathcal{E}$ and $b_i = k_b$ if $0 \in \bar{\mathcal{N}}_i$, where $k_a, k_b \in \mathbb{R}$ are constant positive gains. Thus, if a control application dictates the need for close similarity in follower agents' states while approaching the leader trajectory, the gain k_a may be selected such that $k_a \gg k_b$. Alternatively, the gain k_b may be selected such that $k_b \gg k_a$ if quick convergence to the leader state is desired and the similarity in follower agents' states is not as important.

An auxiliary tracking error, denoted by $e_{2,i} \in \mathbb{R}^m$, is defined as

$$e_{2,i} \triangleq \dot{e}_{1,i} + \alpha_{1,i} e_{1,i} \tag{7}$$

where $\alpha_{1,i} \in \mathbb{R}$ denotes a constant positive gain. For brevity, let $L_B \triangleq (\mathcal{L}+B) \otimes I_m \in \mathbb{R}^{Nm\times Nm}$, where L_B is positive definite and symmetric since $\mathcal{L} + B$ is positive definite and symmetric. The error systems in (6) and (7) may be represented as [10]

$$E_1 = L_B(Q_0 - Q), \tag{8}$$
$$E_2 = \dot{E}_1 + \Lambda_1 E_1 \tag{9}$$

where $E_1 \triangleq (e_{1,1}^T, e_{1,2}^T, \ldots, e_{1,N}^T)^T \in \mathbb{R}^{Nm}$, $E_2 \triangleq (e_{2,1}^T, e_{2,2}^T, \ldots, e_{2,N}^T)^T \in \mathbb{R}^{Nm}$, and $\Lambda_1 \triangleq \operatorname{diag}(\alpha_{1,1}, \alpha_{1,2}, \ldots, \alpha_{1,N}) \otimes I_m \in \mathbb{R}^{Nm\times Nm}$. Another auxiliary error signal $R \in \mathbb{R}^{Nm}$ is defined as

$$R \triangleq L_B^{-1}(\dot{E}_2 + \Lambda_2 E_2) \tag{10}$$

where $\Lambda_2 \triangleq \operatorname{diag}(\alpha_{2,1}, \alpha_{2,2}, \ldots, \alpha_{2,N}) \otimes I_m \in \mathbb{R}^{Nm\times Nm}$ and $\alpha_{2,i} \in \mathbb{R}$ is a constant positive gain. The introduction of R facilitates the subsequent stability analysis; however, it is not measurable because it depends on the second derivative of the generalized configuration coordinate and, hence, is not used in the subsequently developed controller.

IV. Controller Development

The open-loop tracking error system is developed by multiplying (10) by M and utilizing (4), (5), and (8)–(10) to obtain

$$MR = -\tau + d + S_1 + S_2 \tag{11}$$

where the auxiliary functions $S_1 \in \mathbb{R}^{Nm}$ and $S_2 \in \mathbb{R}^{Nm}$ are defined as

$$\begin{aligned}
S_1 \triangleq{}& M(Q)M_\varnothing^{-1}\tau_\varnothing - M(Q_0)M_\varnothing^{-1}\tau_\varnothing\\
&- M(Q)f_0(Q_0, \dot{Q}_0) + M(Q_0)f_0(Q_0, \dot{Q}_0) + f(Q, \dot{Q})\\
&- f(Q_0, \dot{Q}_0) + M(Q)L_B^{-1}\left(-\Lambda_1^2 E_1 + (\Lambda_1 + \Lambda_2)E_2\right)\\
S_2 \triangleq{}& M(Q_0)M_\varnothing^{-1}\tau_\varnothing - M(Q_0)f_0(Q_0, \dot{Q}_0) + f(Q_0, \dot{Q}_0)
\end{aligned}$$

where the functional dependency of M is given for clarity, and the auxiliary functions $f_0 : \mathbb{R}^{Nm} \times \mathbb{R}^{Nm} \to \mathbb{R}^{Nm}$ and $f : \mathbb{R}^{Nm} \times \mathbb{R}^{Nm} \to \mathbb{R}^{Nm}$ are defined as

$$f_0 \triangleq M_\varnothing^{-1}(C_\varnothing \dot{Q}_0 + F_\varnothing + G_\varnothing) \tag{12}$$
$$f \triangleq C\dot{Q} + F + G. \tag{13}$$

The RISE-based (cf. [29], [30]) control input is designed for agent $i \in \mathcal{V}$ as

$$\tau_i \triangleq (k_{s,i} + 1)e_{2,i} + \nu_i \tag{14}$$

where $\nu_i \in \mathbb{R}^m$ is the generalized solution to the differential equation

$$\begin{aligned}\dot{\nu}_i &= (k_{s,i}+1)\alpha_{2,i}e_{2,i} + b_i\chi_i \text{sgn}(e_{2,i}) \\ &\quad + \sum_{j\in\mathcal{N}_i} a_{ij}\left(\chi_i \text{sgn}(e_{2,i}) - \chi_j \text{sgn}(e_{2,j})\right) \\ \nu_i(t_0) &= \nu_{iO} \end{aligned} \tag{15}$$

where $\nu_{iO} \in \mathbb{R}^m$ is an initial condition, $k_{s,i}, \chi_i \in \mathbb{R}$ are constant positive gains, and $\text{sgn}(\cdot)$ is defined $\forall\, \xi = [\xi_1\ \xi_2\ \ldots\ \xi_l]^T \in \mathbb{R}^l$ as $\text{sgn}(\xi) \triangleq [\text{sgn}(\xi_1)\ \text{sgn}(\xi_2)\ \ldots\ \text{sgn}(\xi_l)]^T$. Note that the continuous controller in (14) is decentralized: only local communication is required to compute the controller. The following development exploits the fact that the time derivative of (14) is

$$\begin{aligned}\dot{\tau}_i &= (k_{s,i}+1)(\dot{e}_{2,i} + \alpha_{2,i}e_{2,i}) + b_i\chi_i \text{sgn}(e_{2,i}) \\ &\quad + \sum_{j\in\mathcal{N}_i} a_{ij}\left(\chi_i \text{sgn}(e_{2,i}) - \chi_j \text{sgn}(e_{2,j})\right)\end{aligned} \tag{16}$$

which allows the $\text{sgn}(\cdot)$ terms to cancel disturbance terms in the Lyapunov-based stability analysis that have a linear state bound, similar to sliding-mode-based results.

After substituting (16) into (11), the closed-loop error system can be expressed as

$$\begin{aligned}M\dot{R} &= -\frac{1}{2}\dot{M}R + \tilde{N} + L_B^T N_d - L_B^T E_2 - L_B \beta \text{sgn}(E_2) \\ &\quad - (K_s + I_{Nm})(\dot{E}_2 + \Lambda_2 E_2)\end{aligned} \tag{17}$$

where (16) is expressed in block form as $\dot{\tau} = (K_s + I_{Nm}) \times (\dot{E}_2 + \Lambda_2 E_2) + L_B \beta \text{sgn}(E_2)$, with $K_s \triangleq \text{diag}(k_{s,1}, k_{s,2}, \ldots, k_{s,N}) \otimes I_m$ and $\beta \triangleq \text{diag}(\chi_1, \chi_2, \ldots, \chi_N) \otimes I_m$. In (17), the unmeasurable/uncertain auxiliary terms $\tilde{N} \in \mathbb{R}^{Nm}$ and $N_d \in \mathbb{R}^{Nm}$ are defined as

$$\tilde{N} \triangleq -\frac{1}{2}\dot{M}R + \dot{S}_1 + L_B^T E_2 \tag{18}$$

$$N_d \triangleq \left(L_B^T\right)^{-1}(\dot{d} + \dot{S}_2). \tag{19}$$

The auxiliary terms in (18) and (19) are segregated such that after utilizing (8)–(10), Properties 1-2, Assumptions 1-2, the Mean Value Theorem, and the relations $Q_0 - Q = L_B^{-1}E_1$, $\dot{E}_1 = E_2 - \Lambda_1 E_1$, and $\dot{E}_2 = L_B R - \Lambda_2 E_2$, the following upper bounds are satisfied

$$\|\tilde{N}\| \le \rho\left(\|Z\|\right)\|Z\| \tag{20}$$

$$\sup_{t\in[0,\infty)} |N_{d_l}| \le \zeta_{a_l},\ l = 1,2,\ldots,Nm \tag{21}$$

$$\sup_{t\in[0,\infty)} \left|\dot{N}_{d_l}\right| \le \zeta_{b_l},\ l = 1,2,\ldots,Nm \tag{22}$$

where $\rho : \mathbb{R}_{\ge 0} \to \mathbb{R}$ is a positive strictly increasing function (cf. [31, Lemma 3], [32, App. A]); N_{d_l} and $\dot{N}_{d_l}$ denote the lth element of N_d and $\dot{N}_d$, respectively; the elements of $\zeta_a \in \mathbb{R}^{Nm}$ and $\zeta_b \in \mathbb{R}^{Nm}$ denote some known upper bounds on the corresponding elements in N_d and $\dot{N}_d$, respectively; and $Z \in \mathbb{R}^{3Nm}$ is the composite error vector

$$Z \triangleq \begin{bmatrix} E_1^T & E_2^T & R^T \end{bmatrix}^T. \tag{23}$$

Thus, the terms which arise from the exogenous disturbance and dynamics are segregated by those which can be upper bounded by a function of the state (after use of the mean value theorem) and those which can be upper bounded by a constant. This separation clarifies how these different terms are handled robustly by the different feedback terms in the controller. Specifically, compensation for the terms in $\tilde{N}$ is achieved by using the proportional and derivative feedback terms, and compensation for the terms in N_d is achieved by using the RISE-based feedback terms. The terms N_d and $\dot{N}_d$ do not need to be known exactly to determine the corresponding sufficient upper bounds in ζ_a and ζ_b; however, obtaining numerical values for ζ_a and ζ_b involves *a priori* upper bounds related to the leader trajectory, the leader and followers' dynamics, and the exogenous disturbances. For example, a leader's future trajectory may be unknown, but practical limitations on leader behavior can guide in selecting appropriate upper bounds. In addition, developing upper bounds for the parametrically uncertain dynamics is straightforward since the uncertain coefficients (e.g., mass and friction coefficients) can easily be upper bounded. See results such as [26] and [27] for an extension to the controller for systems where the sufficient bounding constants in (21) and (22) cannot be determined.

For clarity in the definitions of the sufficient gain conditions in the following stability analysis, let the vectors $\varsigma_{ai}, \varsigma_{bi} \in \mathbb{R}^m$, $i \in \mathcal{V}$, be defined such that $\zeta_a = [\varsigma_{a1}^T\ \varsigma_{a2}^T\ \ldots\ \varsigma_{aN}^T]^T$ and $\zeta_b = [\varsigma_{b1}^T\ \varsigma_{b2}^T\ \ldots\ \varsigma_{bN}^T]^T$. Furthermore, let the auxiliary bounding constant $\psi \in \mathbb{R}$ be defined as

$$\psi \triangleq \min\left\{\lambda_{\min}(\Lambda_1) - \frac{1}{2}, \lambda_{\min}(\Lambda_2) - \frac{1}{2}, \lambda_{\min}(L_B)\right\}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue.

V. Stability Analysis

To simplify the development of the subsequent theorem statement and associated proof, various expressions and upper bounds are presented.

An auxiliary function $P \in \mathbb{R}$ is used in the following stability analysis as a means to develop sufficient gain conditions that enable the controller to compensate for the disturbance terms given in N_d; P is defined as the generalized solution to the differential equation

$$\begin{aligned}\dot{P} &= -\left(\dot{E}_2 + \Lambda_2 E_2\right)^T\left(N_d - \beta \text{sgn}(E_2)\right) \\ P(t_0) &= \sum_{l=1}^{Nm} \beta_{l,l}\left|E_{2_l}(t_0)\right| - E_2^T(t_0)N_d(t_0)\end{aligned} \tag{24}$$

where $\beta_{l,l}$ denotes the lth diagonal element of β, and E_{2_l} denotes the lth element of the vector E_2. Provided that the sufficient condition in (30) is satisfied, then $P \ge 0$ (see the Appendix), and can be included in the subsequently defined positive definite function V_L. The inclusion of P enables the development of sufficient gain conditions that ensure asymptotic tracking by the continuous controller, despite additive exogenous disturbances.

Remark 1: Since the derivative of the closed-loop tracking error system in (17) is discontinuous, the existence of Filippov solutions to the developed differential equations is established. Let the composite vector $w \in \mathbb{R}^{4Nm+1}$ be defined as $w \triangleq [Z^T\ \nu^T\ \sqrt{P}]^T$, where $\nu \triangleq [\nu_1^T\ \nu_2^T\ \ldots\ \nu_N^T]^T$. The existence of Filippov solutions can be established for the closed-loop dynamical system $\dot{w} = K[h_1](w,t)$, where $h_1 : \mathbb{R}^{4Nm+1} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{4Nm+1}$ is defined as the right-hand side of $\dot{w}$ and $K[h_1](\sigma,t) \triangleq \cap_{\delta>0} \cap_{\mu(S_m)=0} \overline{\text{co}} h_1(B_\delta(\sigma) \setminus S_m, t)$, where $\delta \in \mathbb{R}$, $\cap_{\mu(S_m)=0}$ denotes an intersection over sets S_m of Lebesgue measure zero, $\overline{\text{co}}$ denotes convex closure, and $B_\delta(\sigma) \triangleq \{\varrho \in \mathbb{R}^{4Nm+1} | \|\sigma - \varrho\| < \delta\}$ [33]–[35].

Let $V_L : \mathcal{D} \to \mathbb{R}$ be a continuously differentiable, positive definite function defined as

$$V_L(y,t) \triangleq \frac{1}{2}E_1^T E_1 + \frac{1}{2}E_2^T E_2 + \frac{1}{2}R^T M R + P \tag{25}$$

where $y \in \mathbb{R}^{3Nm+1}$ is defined as

$$y \triangleq [Z^T \quad \sqrt{P}]^T \tag{26}$$

and the domain $\mathcal{D}$ is the open and connected set $\mathcal{D} \triangleq \{\varrho \in \mathbb{R}^{3Nm+1} |\ \|\varrho\| < \inf(\rho^{-1}([2\sqrt{\psi\lambda_{\min}(K_s L_B)}, \infty)))\}$. The expression in (25) satisfies the inequalities

$$\lambda_1 \|y\|^2 \leq V_L(y,t) \leq \lambda_2 \|y\|^2 \tag{27}$$

where $\lambda_1 \triangleq (1/2)\min\{1, m_{min}\}$, $m_{min} \triangleq \min_{j\in\mathcal{V}}(\underline{m}_j)$, and $\lambda_2 \triangleq \max\{1, (1/2)\sum_{j\in\mathcal{V}} \sup_{q_j \in \mathbb{R}^m} \overline{m}_j(q_j)\}$. Let the set of stabilizing initial conditions $S_{\mathcal{D}} \subset \mathcal{D}$ be defined as

$$\mathcal{S}_{\mathcal{D}} \triangleq \{\varrho \in \mathcal{D} | \|\varrho\| < \sqrt{\frac{\lambda_1}{\lambda_2}} \inf\left(\rho^{-1}\left(\left[2\sqrt{\psi\lambda_{\min}(K_s L_B)}, \infty\right)\right)\right)\right\}. \tag{28}$$

Theorem 1: The controller given in (14) and (15) ensures that all system signals are bounded under closed-loop operation and that the position tracking error is semiglobally regulated in the sense that

$$\|q_0 - q_i\| \to 0 \quad as \quad t \to \infty\ \forall\, i \in \mathcal{V}$$

(and, thus, $\|q_i - q_j\| \to 0\ \forall\, i,j \in \mathcal{V},\ i \neq j$), provided that $k_{s,i}$ ($\forall\, i \in \mathcal{V}$) introduced in (14) is selected sufficiently large such that $y(t_0) \in \mathcal{S}_{\mathcal{D}}$, and the parameters $\alpha_{1,i}, \alpha_{2,i}, \chi_i$ ($\forall\, i \in \mathcal{V}$) are selected according to the sufficient conditions

$$\alpha_{1,i} > \frac{1}{2}, \quad \alpha_{2,i} > \frac{1}{2} \tag{29}$$

$$\chi_i > \|\varsigma_{ai}\|_\infty + \frac{1}{\alpha_{2,i}}\|\varsigma_{bi}\|_\infty \tag{30}$$

where χ_i was introduced in (15).

Proof: Under Filippov's framework, a Filippov solution y can be established for the closed-loop system $\dot{y} = h_2(y,t)$ if $y(t_0) \in S_{\mathcal{D}}$, where $h_2 : \mathbb{R}^{3Nm+1} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{3Nm+1}$ denotes the right-hand side of the closed-loop error signals. The time derivative of (25) exists almost everywhere (a.e.), that is, for almost all $t \in [t_0, \infty)$, and $\dot{V}_L \overset{a.e.}{\in} \dot{\tilde{V}}_L$, where

$$\dot{\tilde{V}}_L = \bigcap_{\xi \in \partial V_L(y,t)} \xi^T K \left[\dot{E}_1^T \quad \dot{E}_2^T \quad \dot{R}^T \quad \frac{1}{2}P^{-\frac{1}{2}}\dot{P} \quad 1\right]^T$$

where ∂V_L is the generalized gradient of V_L [36]. Since V_L is continuously differentiable

$$\dot{\tilde{V}}_L \subseteq \nabla V_L K \left[\dot{E}_1^T \quad \dot{E}_2^T \quad \dot{R}^T \quad \frac{1}{2}P^{-\frac{1}{2}}\dot{P} \quad 1\right]^T \tag{31}$$

where

$$\nabla V_L \triangleq \left[E_1^T \quad E_2^T \quad R^T M \quad 2P^{\frac{1}{2}} \quad \frac{1}{2}R^T \dot{M} R\right].$$

Using the calculus for $K[\cdot]$ from [34], substituting (9), (10), (17), and (24) into (31), and using the fact that L_B is symmetric, and canceling common terms yields

$$\begin{aligned}\dot{\tilde{V}}_L \subseteq\ & E_1^T(E_2 - \Lambda_1 E_1) + E_2^T(L_B R - \Lambda_2 E_2) \\ &+ R^T\left(\tilde{N} + L_B^T N_d - L_B^T E_2\right) \\ &+ R^T\left(-(K_s + I_{Nm})L_B R - L_B \beta K\left[\text{sgn}(E_2)\right]\right) \\ &- (\dot{E}_2 + \Lambda_2 E_2)^T\left(N_d - \beta K\left[\text{sgn}(E_2)\right]\right)\end{aligned} \tag{32}$$

where $K[\text{sgn}(E_2)] = \text{SGN}(E_2)$ such that $\text{SGN}(E_{2_i}) = 1$ if $E_{2_i} > 0$, $\text{SGN}(E_{2_i}) = -1$ if $E_{2_i} < 0$, and $\text{SGN}(E_{2_i}) = [-1, 1]$ if $E_{2_i} = 0$ [34]. Using the upper bound in (20) and applying the Raleigh–Ritz theorem, (32) can be upper bounded as

$$\begin{aligned}\dot{V}_L \overset{a.e.}{\leq}\ & \|E_1\|\|E_2\| - \lambda_{\min}(\Lambda_1)\|E_1\|^2 \\ &- \lambda_{\min}(\Lambda_2)\|E_2\|^2 + \|R\|\rho\left(\|Z\|\right)\|Z\| \\ &- \lambda_{\min}(L_B)\|R\|^2 - \lambda_{\min}(K_s L_B)\|R\|^2\end{aligned} \tag{33}$$

where the set in (32) reduces to the scalar inequality in (33) since the right-hand side is continuous a.e.; that is, the RHS is continuous except for the Lebesgue negligible set of times when $R^T L_B \beta K[\text{sgn}(E_2)] - R^T L_B \beta K[\text{sgn}(E_2)] \neq \{0\}$.[2] Young's inequality gives $\|E_1\|\|E_2\| \leq (1/2)\|E_1\|^2 + (1/2)\|E_2\|^2$which allows for (33) to be upper bounded as

$$\begin{aligned}\dot{V}_L \overset{a.e.}{\leq}\ & \frac{1}{2}\|E_1\|^2 + \frac{1}{2}\|E_2\|^2 - \lambda_{\min}(\Lambda_1)\|E_1\|^2 \\ &- \lambda_{\min}(\Lambda_2)\|E_2\|^2 + \|R\|\rho\left(\|Z\|\right)\|Z\| \\ &- \lambda_{\min}(L_B)\|R\|^2 - \lambda_{\min}(K_s L_B)\|R\|^2.\end{aligned} \tag{34}$$

Using the gain condition in (29), (34) is upper bounded by

$$\dot{V}_L \overset{a.e.}{\leq} -\psi\|Z\|^2 - \lambda_{\min}(K_s L_B)\|R\|^2 + \rho\left(\|Z\|\right)\|R\|\|Z\|. \tag{35}$$

Completing the squares for terms in (35) yields

$$\dot{V}_L \overset{a.e.}{\leq} -\left(\psi - \frac{\rho^2\left(\|Z\|\right)}{4\lambda_{\min}(K_s L_B)}\right)\|Z\|^2. \tag{36}$$

Provided the control gains $k_{s,i}$ are selected sufficiently large such that $y(t_0) \in \mathcal{S}_{\mathcal{D}}$, the expression in (36) can be further

[2]The set of times $\Gamma \triangleq \{t \in \mathbb{R}_{\geq 0} \,|\, R^T L_B \beta K[\text{sgn}(E_2)] - R^T L_B \beta K[\text{sgn}(E_2)] \neq \{0\}\}$ is equal to the set of times $\Phi = \cup_{l=1,2,\ldots Nm} \Phi_l$, where $\Phi_l \triangleq \{t \in \mathbb{R}_{\geq 0} | E_{2_l} = 0 \wedge R_l \neq 0\}$. Due to the structure of R in (10), Φ_l may be re-expressed as $\Phi_l = \{t \in \mathbb{R}_{\geq 0} | E_{2_l} = 0 \wedge \dot{E}_{2_l} \neq 0\}$. Since $E_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}^{Nm}$ is continuously differentiable, it can be shown that Φ_l is Lebesgue measure zero [31]. Because a finite union of sets of Lebesgue measure zero is itself Lebesgue measure zero, Φ has Lebesgue measure zero. Hence, Γ is Lebesgue negligible.

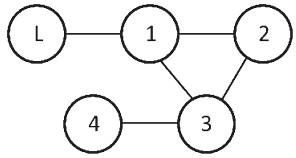

Fig. 1. Network topology.

upper bounded by

$$\dot{V}_L \overset{a.e.}{\leq} -c\|Z\|^2 \tag{37}$$

for all $y \in \mathcal{D}$, for some positive constant $c \in \mathbb{R}$.

The inequalities in (27) and (37) can be used to show that $V_L \in \mathcal{L}_\infty$. Thus, $E_1, E_2, R \in \mathcal{L}_\infty$. The closed-loop error system can be used to conclude that the remaining signals are bounded. From (37), [35, Cor. 1] can be invoked to show that $c\|Z\|^2 \to 0$ as $t \to \infty$ $\forall\, y(t_0) \in \mathcal{S}_\mathcal{D}$. Based on the definition of Z in (23), $\|E_1\| \to 0$ as $t \to \infty$ $\forall\, y(t_0) \in \mathcal{S}_\mathcal{D}$. Noting the definition of E_1 in (8) and the fact that $((\mathcal{L}+B)\otimes I_m)$ is full rank, it is clear that $\|Q_0 - Q\| \to 0$ as $t \to \infty$ if and only if $\|E_1\| \to 0$ as $t \to \infty$. Thus, $\|q_0 - q_i\| \to 0$ as $t \to \infty$ $\forall\, i \in \mathcal{V}$, $\forall\, y(t_0) \in \mathcal{S}_\mathcal{D}$. It logically follows that $\|q_i - q_j\| \to 0$ as $t \to \infty$ $\forall\, i, j \in \mathcal{V}$, $i \neq j$, $\forall\, y(t_0) \in \mathcal{S}_\mathcal{D}$. ■

Note that the region of attraction in (28) can be made arbitrarily large to include any initial conditions by adjusting the control gains $k_{s,i}$ (i.e., a semiglobal result). The distributed controller shown in (14) and (15) is decentralized in the sense that only local feedback is necessary to compute the controller. However, because the constant gain $k_{s,i}$ must be selected based on sufficient conditions involving L_B, which contains information regarding the configuration of the entire network, this gain is selected in a centralized manner before the control law is implemented.

VI. Simulation

Simulations were performed with multiple decentralized control methods for a network of robotic manipulators to compare the performance of the developed method with other related distributed control methods. The developed control policy is compared with the adaptive control policy in [10] and the sliding-mode-based control policy in [11, Sec. IV]. Simulation results are presented for the synchronization of four follower agents to a leader's state trajectory in the network shown in Fig. 1. Similar to [10] and [11, Sec. IV], each follower is modeled as a two-link robotic manipulator (a typical example of an Euler-Lagrange system) with the form

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix} -p_3s_2\dot{q}_2 & -p_3s_2(\dot{q}_1+\dot{q}_2) \\ p_3s_2\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + d_\tau$$

where q_1, q_2 denote joint angles; $c_2 \triangleq \cos(q_2)$ and $s_2 \triangleq \sin(q_2)$; τ_1 and τ_2 represent torque control inputs; and $d_\tau \in \mathbb{R}^2$ represents a vector of added disturbances. The constant unknown parameters $p_1, p_2, p_3, f_{d1}, f_{d2} \in \mathbb{R}$ differ for each manipulator. The virtual leader is defined by the trajectory $q_0 = \left[\begin{smallmatrix}2\sin(2t)\\ \cos(3t)\end{smallmatrix}\right]$, where the first and second elements are the desired trajectories for the first and second joint angles, respectively. The time-varying disturbance term has the form $d_\tau = \left[\begin{smallmatrix}a\sin(bt)\\ c\sin(dt)\end{smallmatrix}\right]$, where the constants $a, b, c, d \in \mathbb{R}$ differ for each manipulator. The model parameters for each manipulator are shown in Table I.

TABLE I
Simulation Parameters

	Robot 1	Robot 2	Robot 3	Robot 4
p_1	3.7	3.5	3.2	3.0
p_2	0.22	0.20	0.18	0.17
p_3	0.19	0.25	0.23	0.21
f_{d1}	5.3	5.1	5.2	5.4
f_{d2}	1.1	1.3	1.0	1.2
a	2.0	4.0	3.0	5.0
b	1.0	2.0	3.0	5.0
c	1.0	3.0	4.0	2.0
d	4.0	3.0	1.0	2.0

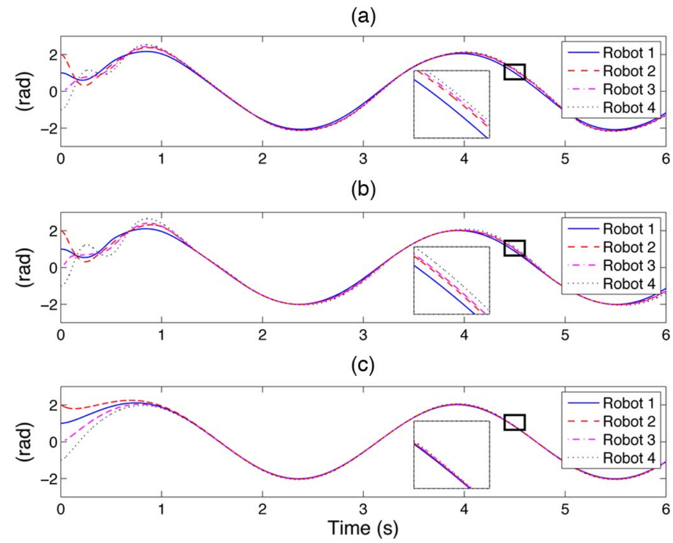

Fig. 2. Joint 1 angle trajectories using (a) [10], (b) [11, Sec. IV], and (c) the proposed controller.

The control gains for each method were selected based on convergence rate, residual error, and magnitude of control authority. The gains were obtained for each controller by qualitatively determining an appropriate range for each gain and then running 10 000 iterations with random gain sampling within those ranges in an attempt to minimize

$$J = \sum_{k=1}^{4}\int_{2}^{10} \|e_{1,k}\| d\tau \tag{38}$$

(with $a_{ij} = 1$ if $(i,j) \in \mathcal{E}$ and $b_i = 4$ if $0 \in \bar{\mathcal{N}}_i$) while satisfying bounds on the control input such that the entry-wise inequality $\tau_k(t) \leq \left[\begin{smallmatrix}500\\ 150\end{smallmatrix}\right]$, $k = 1, 2, 3, 4$, $\forall\, t \in [0.2, 10]$ is satisfied. Beginning the error integration at two seconds encourages a high convergence rate and low residual error, while monitoring the control input only after 0.2 s accommodates a possibly high initial control input.

Figs. 2 and 3 demonstrate that asymptotic synchronization of the follower agents and tracking of the leader trajectory are qualitatively achieved for the developed controller and the sliding-mode-based controller in [11, Sec. IV], despite the

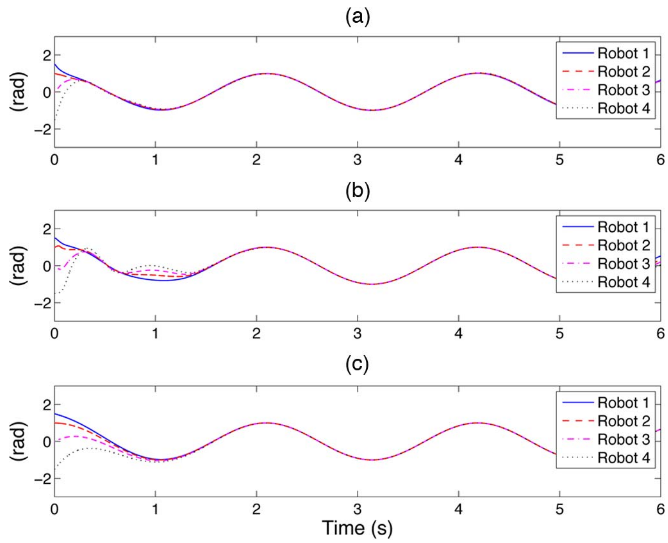

Fig. 3. Joint 2 angle trajectories using (a) [10], (b) [11, Sec. IV], and (c) the proposed controller.

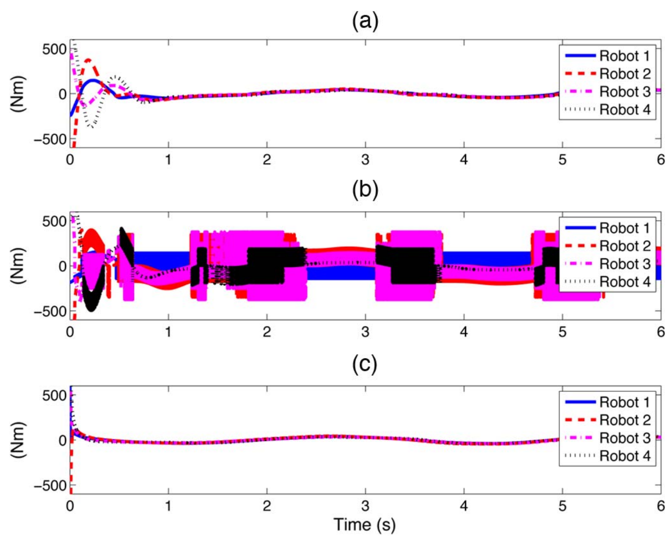

Fig. 4. Joint 1 control effort using (a) [10], (b) [11, Sec. IV], and (c) the proposed controller.

exogenous disturbances. Figs. 4 and 5 illustrate the effects of the control methods used to obtain synchronization: the controller in [11, Sec. IV] utilizes a high-frequency, discontinuous, sliding-mode-based control signal, whereas the developed controller is continuous and exhibits lower frequency content.

As shown in Figs. 2–5, the neural-network-based adaptive controller given in [10] stabilizes the system using a continuous controller which produces a control signal of moderate magnitude, but maintains a residual error. This behavior agrees with the theoretical result in [10]: the controller achieves bounded convergence.

Table II provides a quantitative comparison of the controllers, where $e_{\mathrm{rms,max}}$ is the maximum rms error taken over the time interval [2, 10] of the components of $e_{1,i}$ for each networked manipulator, $\|\tau_i\|_{\max}$ denotes the maximum instance of the norm of the simulated control efforts for each manipulator over the time interval [0.2, 10], and J is introduced in (38). The proposed controller provides a significantly improved tracking

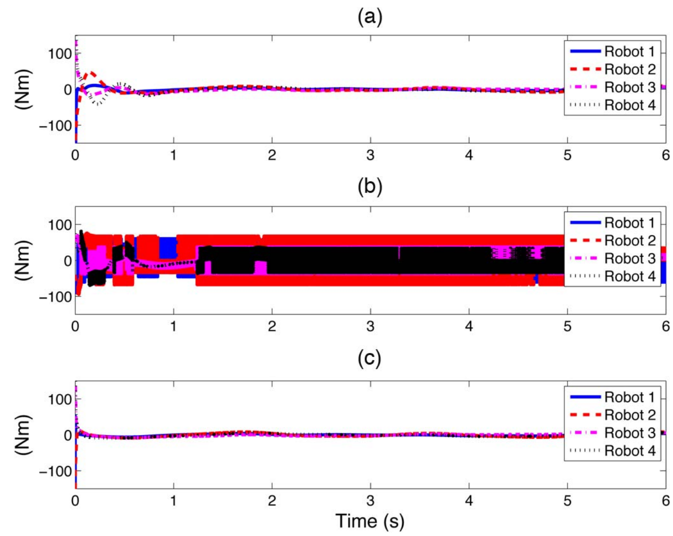

Fig. 5. Joint 2 control effort using (a) [10], (b) [11, Sec. IV], and (c) the proposed controller.

TABLE II
CONTROLLER PERFORMANCE COMPARISON

Method	$e_{\mathrm{rms,max}}$(rad)	$\|\tau_i\|_{\max}$(Nm)	J
[10]	0.1754	376	2.32
[11, Section IV]	0.0517	500	1.07
Proposed	0.0133	45.3	0.209

performance with a continuous control signal of a relatively low magnitude.

VII. CONCLUSION

A distributed RISE-based controller was developed which ensures semiglobal asymptotic tracking and synchronization of the networked followers' states toward a leader's time-varying state using a continuous control input, despite model uncertainty and exogenous disturbances, where the leader and follower agents have uncertain and heterogeneous Euler-Lagrange dynamics. The graph of the networked follower agents is assumed to be connected and at least one follower agent receives information from the leader. Simulation results are provided for the proposed decentralized controller to demonstrate its performance compared to other prominent related distributed controllers.

Although only a single network leader is considered in the current work, future research may consider the case of multiple leaders for the purpose of containment control, similar to the work in [37] and [38]. Future research may also investigate asymptotic synchronization by continuous control for a network affected by unknown disturbances wherein the network topology is directed or subject to communication delays. Results such as [9], [15], and [39] may provide insights for extending this result for digraphs and including communication delays.

APPENDIX

Lemma 1: Given the differential equation in (24), $P \geq 0$ if χ_i $(\forall i \in \mathcal{V})$ satisfies

$$\chi_i > \|\varsigma_{ai}\|_\infty + \frac{1}{\alpha_{2,i}}\|\varsigma_{bi}\|_\infty. \quad (39)$$

Proof: For notation brevity, let an auxiliary signal $\sigma \in \mathbb{R}$ be the negative of the integral of $\dot{P}$ in (24) as

$$\sigma = \int_{t_0}^{t} E_2^T(\varepsilon)\Lambda_2\left(N_d(\varepsilon) - \beta\text{sgn}\left(E_2(\varepsilon)\right)\right)d\varepsilon + \int_{t_0}^{t} \frac{\partial E_2^T(\varepsilon)}{\partial\varepsilon} N_d(\varepsilon)d\varepsilon - \int_{t_0}^{t} \frac{\partial E_2^T(\varepsilon)}{\partial\varepsilon}\beta\text{sgn}\left(E_2(\varepsilon)\right)d\varepsilon. \quad (40)$$

Integrating the last two terms in (40) yields [31]

$$\sigma = \int_{t_0}^{t} E_2^T(\varepsilon)\Lambda_2(N_d(\varepsilon) - \beta\text{sgn}(E_2(\varepsilon)))\,d\varepsilon + \sum_{l=1}^{Nm} E_{2_l}(t)N_{d_l}(t) - E_2^T(t_0)N_d(t_0) - \int_{t_0}^{t} E_2^T(\varepsilon)\frac{\partial N_d(\varepsilon)}{\partial\varepsilon}\,d\varepsilon - \sum_{l=1}^{Nm}\left|E_{2_l}(t)\right|\beta_{l,l} + \sum_{l=1}^{Nm}\left|E_{2_l}(t_0)\right|\beta_{l,l}. \quad (41)$$

The expression in (41) may then be expressed as

$$\sigma = \int_{t_0}^{t}\left(\sum_{l=1}^{Nm} E_{2_l}(\varepsilon)\Lambda_{2_{l,l}}N_{d_l}(\varepsilon)\right)d\varepsilon - \int_{t_0}^{t}\left(\sum_{l=1}^{Nm}\left|E_{2_l}(\varepsilon)\right|\Lambda_{2_{l,l}}\beta_{l,l}\right)d\varepsilon - \int_{t_0}^{t}\left(\sum_{l=1}^{Nm} E_{2_l}(\varepsilon)\dot{N}_{d_l}(\varepsilon)\right)d\varepsilon + \sum_{l=1}^{Nm} E_{2_l}(t)N_{d_l}(t) - E_2^T(t_0)N_d(t_0) - \sum_{l=1}^{Nm}\left|E_{2_l}(t)\right|\beta_{l,l} + \sum_{l=1}^{Nm}\left|E_{2_l}(t_0)\right|\beta_{l,l}. \quad (42)$$

The upper bounds in (21) and (22) are then used to upper bound (42) as

$$\sigma \leq \int_{t_0}^{t}\left(\sum_{l=1}^{Nm}\left|E_{2_l}(\varepsilon)\right|\Lambda_{2_{l,l}}\zeta_{a_l}\right)d\varepsilon - \int_{t_0}^{t}\left(\sum_{l=1}^{Nm}\left|E_{2_l}(\varepsilon)\right|\Lambda_{2_{l,l}}\beta_{l,l}\right)d\varepsilon + \int_{t_0}^{t}\left(\sum_{l=1}^{Nm}\left|E_{2_l}(\varepsilon)\right|\zeta_{b_l}\right)d\varepsilon + \sum_{l=1}^{Nm}\left|E_{2_l}(t)\right|\left(\zeta_{a_l} - \beta_{l,l}\right) - E_2^T(t_0)N_d(t_0) + \sum_{l=1}^{Nm}\beta_{l,l}\left|E_{2_l}(t_0)\right|$$

where ζ_{a_l} and ζ_{b_l} represent the lth element of ζ_a and ζ_b, respectively. Provided the gain condition for χ_i in (39) is satisfied for each $i \in \mathcal{V}$ (recall that $\beta = \text{diag}(\chi_1, \chi_2, \ldots, \chi_N) \otimes I_m$ and $\Lambda_2 = \text{diag}(\alpha_{2,1}, \alpha_{2,2}, \ldots, \alpha_{2,N}) \otimes I_m$), then $\sigma \leq \sum_{l=1}^{Nm}\beta_{l,l}|E_{2_l}(t_0)| - E_2^T(t_0)N_d(t_0)$. Thus, σ may be upper bounded as

$$\sigma \leq P(t_0). \quad (43)$$

Integrating both sides of (24) yields $P(t) = P(t_0) - \sigma$, which indicates that $P \geq 0$ from (43). ■

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